THE ESTIMATION OF A SHADOW EQUILIBRIUM EXCHANGE RATE:
A Direct Method

Terry Roe and Duty Greene
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A Direct Method

by

Terry Roe and Duty Greene*
A. Introduction

The literature on the primary determinants of the supply and demand for foreign currency and the concept of an equilibrium exchange rate is extensive. Moreover, there appears to be no generally accepted consensus on the correct procedure to estimate exchange rates that might prevail under alternative regimes (Krueger, Bacha and Taylor, Blitzer, Dasgupta, and Stiglitz) or in the presence of resource discoveries and terms of trade changes (Diaz-Alejandro).

Krueger's 1983 review of exchange rate determination points out that the literature in the 1960's focused primarily on the current account and the determinants of the balance of payments under fixed exchange rates. In the early and middle 1970's the focus was primarily on the capital account and monetary theory. She also states (Krueger, p. 103) that the more recent literature on exchange rate determination has brought the current account back into the focus of analysis because of three interrelated analytical developments, namely:

1. Recognition that changes in asset holdings can come about only through imbalances on current account so that the current account position is accompanied by asset accumulation or decumulation.

2. Rational expectation models of exchange rate determination have been developed that focus on particular links between the current and capital accounts, including (a) the recognition that the expected current account path implies an expectation of the future path of foreign asset accumulation and consequently an expectation about future prices of those assets and (b) the identification of exchange-rate changes with terms-of-trade shifts in response to real disturbances.
3. Analysis of the current account balance as an expression of savings behavior has focused attention on the intertemporal aspects of the current and capital account linkages and their role in exchange rate determination.

These recent developments have tended to support the widely-held view that financial asset market variables are the primary determinants of exchange rates in the short-run, while current account variables are the principal determinants in the longer-run. The approach posited in this note is based on an analysis of the current account and hence the estimates of the exchange rate obtained should be viewed in a long-run context.

In this note we posit a simple, static and partial equilibrium model of exchange rate determination. The approach is based on the current account. The uniqueness of the approach is that the determination of the demand and supply of foreign exchange is directly related to the trade in goods and services in a manner that facilitates the estimation of a shadow exchange rate under alternative situations.

The model is illustrated graphically in the first part. Then, algebraic forms are assumed for the excess supply and demand functions. From these equations, the foreign exchange market equations and an equation for the currency exchange rate are derived. In the final section, the model is illustrated numerically for the case of the Dominican Republic for the years 1966 to 1984. In our opinion, the approach seems so simple and straightforward that given the actual complexity and dynamics of the forces determining exchange rates it might be prone to misuse.

B. Graphic Analysis

The demand for foreign exchange is derived from the excess demand for goods and services as shown in diagram 1.a. The chart is divided into three
quadrants. The excess demand function is depicted in the first quadrant. Excess demand is expressed as a function of the domestic price of imports $P_m$. Domestic price equals the product of the exchange rate $E_1$, border price $P_w$ and implicit tariff $(1+t_m)$. Other variables that cause the excess demand function to shift, such as prices of commodity and input substitutes, income, interest rates etc. are treated as exogenous variables. In the numerical example, they are treated as unobservable variables that have some aggregate effect on the demand and supply of foreign exchange at each point in time.

Given the small country assumption, the mapping between the quantity of imports ($M$) and foreign exchange is linear. This linearity is depicted in quadrant II. The demand for foreign exchange is depicted in quadrant III. The curve in quadrant III is derived by choosing an exchange rate value, $E_1$, and moving in a clock-wise manner, the coordinate point is established at $(E_1, Q_d)$. The locus of points implied by the curve $D_f$ is obtained by holding $P_mP_w(1+t_m)$ constant and selecting other exchange rate values, such as $E_0$. Notice that the curvature of $D_f$ in $E, Q_d$ space is directly related to the curvature of $D_m$ in $P_m, M$ space since $P_w$ is linear.

The supply of foreign exchange is derived from the excess supply of goods and services and donations and remittances as shown in diagram l.b. The excess supply of goods and services, as a function of domestic price $P_x$, is expressed in quadrant I. Domestic price is a product of the exchange rate, border price $P_w$ and taxes $(1-t_x)$. Since the country's exports has no effect on world market price, the mapping between the quantity of exports ($X$) and foreign exchange is linear, quadrant II. Notice that the small country assumption only simplifies the analysis. Selecting arbitrary exchange rate values $E_1$ and $E_0$, the coordinates in quadrant III are obtained.
Figure 1.a: The excess demand for imports and the derived demand for foreign exchange.

Figure 1.b: The excess supply of exports and the derived supply of foreign exchange.
by pivoting clock-wise.

Hence, this simple graphic analysis relates the demand and supply of foreign exchange to the goods and services markets. The effects of taxes, tariffs and the terms of trade \( \frac{P_m}{P_x} \) on the demand and supply of foreign exchange can also be shown. All other variables in the excess supply and demand functions are held constant.

C. Algebraic Analysis

Let:

\[
(1) \quad M_t = B^*(Z_t)[P_{mt}]^\eta - B^*(Z_t)[E_tP_{mt}^W(1+t_{mt})]^\eta
\]

denote the excess demand for goods and services depicted in diagram 1.a where \( Z_t \) is a vector of some other exogenous variables and subscript \( t \) denotes time. The demand for foreign exchange (the curve in quadrant III) can be simply derived by multiplying both sides of (1) by the border price \( P_w \). The result is:

\[
(1') \quad Q_{dt} = B^*(Z_t)(P_{mt})^{1+\eta}[E_t(1+t_{mt})]^\eta
\]

The supply of foreign exchange can be derived likewise. Let

\[
(2) \quad X_t = A^*(W_t)[P_{xt}]^\varepsilon - A^*(W_t)[E_tP_{xt}^W(1-t_{xt})]^\varepsilon
\]

denote the excess supply of goods and services and the flows from donations and remittances depicted in diagram 1.b where \( W_t \) is a vector of some other exogenous variables, possibly including those in \( Z_t \). Multiplying both sides of (2) by the border price \( P_w \) yields the supply function for foreign exchange:

\[
(2') \quad Q_{st} = A^*(W_t)(P_{xt})^{1+\varepsilon}[E_t(1-t_{xt})]^\varepsilon.
\]

Notice that the elasticities of \( E_t \) in the foreign exchange equations are identical to the elasticities in their "parent" equations. Moreover, it is not necessary for \( Q_{dt} = Q_{st} \).

The simplicity of this framework for computing exchange rates that
might prevail if the current account and/or taxes on exports and tariffs were changed from historical levels can now be shown.

The first step is to derive values for \( B^*(Z_t) \) and \( A^*(W_t) \) for each period \( t \). From (1') we obtain the estimate:

\[
(1') \quad B^*_t = B^*(Z_t)(P_{mt}^w)^{1+\eta} - Q_{dt}/[E_t(1+t_{mt})]^{\eta}
\]

and from (2') we obtain:

\[
(2') \quad A^*_t = A^*(W_t)(P_{xt}^w)^{1+\epsilon} - Q_{st}/[E_t(1-t_{xt})]^{\epsilon}.
\]

Thus, if data exists on the value of imports \( Q_{dt} \), the nominal exchange rate \( E_t \), tariffs \( (1+t_{mt}) \), and, if the value of the elasticity \( \eta \) is known, then the "intercept" term in (1') can be computed for each period \( t \).

Likewise, knowledge of \( Q_{st}, (1-t_{xt}) \) and \( \epsilon \) permit the calculation of the "intercept" term in (2').

Substituting these estimates into equations (1') and (2'), respectively, yields:

\[
(1'') \quad Q_{dt} = B^*_t[E_t(1+t_{mt})]^{\eta}
\]

\[
(2'') \quad Q_{st} = A^*_t[E_t(1-t_{xt})]^{\epsilon}.
\]

Now, equating (1'') and (2'') and solving for \( E_t \) permits an estimate of the exchange rate that might prevail under alternative assumptions of \( t_{mt}, t_{xt} \) when the current account is balanced. To evaluate exchange rates that might prevail under alternative current account assumptions, simply substitute (1'') and (2'') into

\[
Q_{dt} = Q_{st} \cdot k,
\]

and solve for \( E_t \), where \( k \) is a parameter. Values of \( k > 1 \) imply a current account deficit, \( k < 1 \) a surplus and \( k = 1 \) implies a balanced current account.
For the case \( k = 1 \), the equation is:

\[
E_t^* = E_t = \left( \frac{B_t}{A_t} \right)^{1/(\epsilon-\eta)} \left( \frac{(1+t_{mt})^\eta}{(1-t_{xt})^\epsilon/(\epsilon-\eta)} \right)
\]

Setting \( t_{mt} = t_{xt} = 0 \), yields an estimate of \( E_t \) that might prevail for the case where the current account is in balance and tariffs and taxes have been removed. The variables \( Z_t \) and \( W_t \) are exogenous; their effects on \( E_t \) are implicit through the observed levels of the variables (\( M_t, X_t \), etc.) used to compute \( B_t \) and \( A_t \). Hence, this is a partial equilibrium model.

D. Numeric Analysis

To illustrate the model, the numerical analysis is performed based on data from the Dominican Republic for the period 1966 to 1984. For a more detailed discussion of the data and the economic characteristics of the country, see Greene and Roe (1986).

The data upon which the empirical example is based are presented in Table 1. The country's current account imbalance is contrasted with its medium and long term external loans and Dominican CPI in Figure 1. The imbalance in the country's current account corresponds to the medium and long term capital inflows and, since 1980, rising inflation. An estimate of the country's implicit tariffs, coefficients of protection and implicit export taxes appear in columns (7), (8) and (9) respectively. Overall, the level of protection has been fairly constant with tariffs rates exceeding export taxe rates throughout the period.

An official and a parallel market for foreign exchange existed in the country from 1966 until the Dominican government, in agreement with IMF recommendations, unified the official and parallel markets on January 1, 1985. These rates are reported in Columns (13) and (14) of the table. Based on
data from the country's Central Bank, the weighted average exchange was computed, column (4). These three rates are charted in Figure 2.

An estimate of the exchange rate $E_t^*$ is derived from equation (3). The estimated constants $A_t$ and $B_t$ of equations (2'') and (1'') appear in columns (10) and (11) respectively. These estimates are based on the flows of foreign exchange, columns (2) and (3) and on the excess supply ($\epsilon$) and demand ($\eta$) elasticities of 1 and -2 that were used in the study by Greene and Roe (1986). Substituting these values into equation (3) for the case of zero taxes and tariffs yields an estimate of the exchange rate $E_t^*$ that might have prevailed in the absence of protection. These estimates appear in column (12) of the table and in figure 2 (denoted by A).

In all cases, the results suggest that the currency was overvalued relative to the actual weighted exchange rate. Since the mid 1970's, the parallel market has accounted for an increasing share of foreign exchange transactions. Notice that the parallel market (denoted by + in figure 2) and the estimated equilibrium rates "track" closely during this period. This result suggests that prior analyses (e.g., Norton [1986]) of price policies based on the parallel market rate of exchange have probably been a good approximation of the marginal cost of foreign exchange to the Dominican economy.

If taxes and tariffs were eliminated and the estimated exchange rate $E_t^*$ would have prevailed, equations (1''') or (2''') can be used to estimate the value of foreign trade that might have occurred under the given assumptions. The result (denoted by 0) of this exercise appear in Figure 3. With the exception of two years (1980 and 1982) the value of trade in constant 1985 US dollars would have exceeded the observed levels of imports.
(denoted by □). Except for the three years 1975-1977, the estimated level of trade closely tracks the observed level of trade.

Given the simplicity of this model, these results are dependent on the assumption that the exogenous factors embodied in the constants $A_t$ and $B_t$ are either independent of changes in $E_t$ or that their effects are countervailing. This assumption is not likely to be valid since exchange rates are expected to have real income effects.

The approach posited here is straightforward and flexible in the sense that alternative exchange rate estimates can be generated as a function of the various levels of the variables appearing on the right hand side of (3), including levels of surplus or deficit, as measured by the constant $k$, that might be sustainable on a country's current account. However, it seems that the approach is so simple and straightforward that it might be prone to misuse since evidence in both the theoretical and empirical literature (e.g., Orden) suggests that capital flows, interest rate differentials and other dynamic forces influence a country's currency exchange rates. While these factors may be captured to some extent in the computed values of $B_t$ and $A_t$, the model is most likely far too simple to capture these effects.
Figure 2

Figure 3
Effect of Equil. Ex. Rate on Trade
Table 1: Data used to compute the equilibrium exchange rate, Dominican Republic, 1966 to 1984.

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(1) Source: International Financial Statistics: IMF
(2) Sum of goods, services and donation transfers in US$.
(3) Sum of goods and service imports in US$.
(4) Weighted average of the official and parallel exchange rate; Source: Central Bank, Monthly Bulletin, various years.
(5) Assumed supply elasticity for exports (or supply of US$).
(6) Assumed demand elasticity for imports (or demand for US$).
(7) Implicit import tariff rate. Computed from Table 2.3, p. 11, IBRD(1985).
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(8) Estimated coefficient of protection; computed from terms of trade data published by the Central Bank.
(9) Estimated implicit export tax rate (column 8*7).
(10) Based on equation (2''), (column 2/(4*9)^5).
(11) Based on equation (1''), (column 3/(4*7)^6).
(12) Estimated nominal equilibrium exchange rate; (column (11/10)^ (1.0/column 5-6)
(13) Central Bank, Monthly Bulletin, various years.
(14) Central Bank, Monthly Bulletin, various years.
REFERENCES


