INTEGRATION OF PRODUCTION AND FINANCIAL THEORY IN ANALYZING FIRM BEHAVIOR

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Section I

Typically economists have tried to simplify the firm level decision problem by separating production and finance decisions. Production decisions are often analyzed as if adequate capital is available at some constant cost. Finance decisions are frequently examined assuming that the production decision has already been made. The use of the weighted average cost of capital in analyzing long term production decisions is a classical example of the attempt to separate production and finance. The constant weights in the cost of capital imply that the finance decision can be made separately from the investment or production decision. The separation of production and finance decisions has been a productive avenue for economic thought. It has facilitated relatively simply analysis of complex problems.

Production and finance decisions can, however, not always be separated. This is especially true for the small, closely held firms that characterize the farm sector. Financially strapped operators may not have adequate capital to use the level of inputs that is optimal from a purely production efficiency standpoint. Additional capital may be available only at a substantially higher cost than the capital currently used. For example, the marginal cost of debt may rise with additional borrowing because the lender perceives the loan as more risky and charges a risk premium, because additional borrowing requires changing to higher cost lenders or because of the higher transaction cost created by additional documentation and supervision. Special financing arrangements offered by input suppliers may affect the type and amount of inputs used. The decisions to adopt new technology may depend on the cost of capital and the terms under which financing occurs.

In empirical work the production finance interaction has been recognized (see for instance Boehlje and White, Boehlje and Griffin, Held and Helmers, or Richardson and Condra), but the theoretical arguments behind these empirical constructions have not been well developed. Baker develops some heuristic arguments on including the cost of financing in the factor cost. In the general economics literature, Vickers has developed a static model of production with financing considered as a constraint. The theoretical framework has not included the effects of capital gains and losses on production assets, cashflow problems, uncertainty, or limits on assets or capital availability. Some researchers have successfully applied the arguments of the static Vickers model in empirical analysis of dynamic problems (Boehlje and Griffin), but a more rigorous theoretical foundation would facilitate work in this area.

The objective of this paper is to outline a general framework within which production and finance choices can be analyzed under perfect knowledge and to discuss the basic decision rules generated within this framework. Choices under uncertainty are considered by Lowenberg-DeBoer and Boehlje,

and Lowenberg-DeBoer. The focus of the paper is on the framework and its implications for understanding firm level decisionmaking; the optimization techniques and other mathematical details are covered by Lowenberg-DeBoer. The second section of this paper outlines a general model of the production-finance interaction based on Vickers model. The third section examines the basic decision rules for the growing firm. The effect of cashflow problems and leasing are examined in the fourth and fifth sections. The decisions of a disinvesting firm are considered in the sixth section.

Section II

The conceptual framework of this research is based on a modified Vickers model of firm level decision making. The model assumes that the firm's owners seek to maximize wealth subject to the constraint that equity plus debt must equal the capital absorbed in acquiring inputs. Wealth is calculated at the present value of current net cashflow from production, plus a portion of unrealized capital gain and the liquidation value of the firm at the end of the horizon. The maximization of present value allows consideration of the timing of cash flows. Through the discount rate information on returns to alternative assets enter the problem. The wealth maximization approximates utility maximization for cases in which utility is primarily a function of money income and the owners may borrow against or save cash returns to achieve the desired consumption pattern.

The objective function can be written as the discounted sum of current cashflow and capital gain that can be substituted for current cashflow in determining wealth, minus equity investment, plus a terminal value that captures the discounted value of the firm at the end of the horizon. The continuous time form is:

$$(1) \quad \max Z = \int_0^T e^{-\rho_t(1-\tau)t} (\pi_t + \phi\dot{\alpha} - u_{1t})dt + S - K_o$$

where: T = terminal time,

π_t = the after tax cashflow in period t,

ρ_t = the discount rate,

τ = the average tax rate,

$\dot{\alpha}$ = the change over time in capital absorbed by inputs,

ϕ = the vector of income substitution coefficients,

u_{1t} = equity investment,

S = the salvage value of the firm's assets,

K_t = equity capital invested in the firm, and

t = variable of integration, time.

The discount rate is assumed to reflect returns to alternative equity investments. The average tax rate is used instead of

the marginal rate because the analysis is being conducted on a whole firm basis. The use of a constant average tax rate helps make the model mathematically tractable, but a more detailed model would allow for a progressive tax system. The model outlined here assumes that alternative investments produce ordinary income for tax purposes. If the alternative investments which are used to define the discount rate are tax sheltered, for instance because they generate income which can be treated as capital gain, a separate tax rate can be specified for use in the after tax discount rate term ($\rho(1-\tau)$). The discount rate like all other prices and returns in the model is assumed to be nominal.

The vector α contains coefficients which specify the amount of capital per unit required to finance each input. In detailed models which allow for a variety of debt repayment arrangements, the capital absorption coefficients are equivalent to the input's price and the change in the capital absorption values over time have the interpretation of capital gains or losses. In simpler models, such as the one developed by Vickers, specialized financing terms can be subsumed in the capital absorption coefficient and the change of that coefficient over time is directly related to the capital gain or loss, but is not equivalent to it. For example, in the U.S., land is regularly seller financed at interest rates lower than those applied to other inputs. This could be included in the simple model with one credit source by reducing the capital absorption coefficient for land below the price; it is as if the full interest rate is paid on a smaller amount of capital. In a more detailed model a separate seller financed real estate debt source could be defined. To simplify interpretation of the capital absorption change ($\dot{\alpha}_t$), it will be assumed throughout this paper that the credit market is modelled in sufficient detail to accomodate special financing arrangements.

Unlike the Vickers model, the objective function includes a proportion of unrealized capital gains or losses as a substitute for current income ($\dot{\alpha}_1, \Phi_1, \dot{\alpha}_2, \Phi_2$). The argument for the substitutability of unrealized capital gains for current income is based on the idea that with perfect capital markets unrealized capital gains or losses would be a perfect substitute for current income because one could reduce savings in other forms or borrow against the appreciated value without restriction and without transactions cost (Bhatia, pp. 866, 869). In an imperfect capital market with restrictions on borrowing against certain types of security and for some purposes, and with transactions costs, unrealized capital gains are no longer a perfect substitute for cash, but it is reasonable to assume that for at least some agents the unrealized capital gains are an imperfect substitute for cash in determining wealth. The proportions in Φ reflect the degree to which the farm decision maker is willing to substitute accrued

capital gains or losses for cash gains or losses. Plaxico and Kletke (1980, p. 263) argue that because unrealized capital gains can not be used to meet cash obligations, the proportion of capital gain recognized as a substitute for income will be smaller if cashflow is a problem.

The financial constraint is:

$$(2) \quad K_t + J'D_t - \alpha_t' X_t + J'G_t = 0$$

where: D_t = a vector of debt of various types,

X_t = a vector of inputs,

G_t = a vector of unrealized capital gains on production inputs, and

J = a vector of ones used to sum elements of other vectors.

Throughout the paper the dimensions of the J vector are assumed to be conformable for the multiplication indicated. In the financial constraint the capital invested in production assets is assumed to be the sum of the purchase prices or equivalently the market value (αXt) minus accumulated unrealized capital gain or loss (ωG_t).

The debt vector includes all the types of credit that the decisionmaker may consider. For example, it may include real estate debt from several sources, merchant and dealer credit, and short term debt from commercial banks and other lenders. Constraints may be imposed to specify relationships between debt and asset types. For instance, based on legal requirements the amount of real estate debt held with the Federal Land Bank might be constrained to be less than or equal to 85 percent of the market value of the real estate.

The cashflow term can be more fully specified as the after tax net current cashflow from production plus cashflow from asset sales and minus principal payments and adjustment cost:

$$(3) \quad \pi_t' = \{P_t f(X_t, t) - \gamma_t' X_t - r\left(\frac{J'D_t}{K_t + \omega G_t}, t\right)' D_t\} (1-\tau)$$

$$- u_2' g(G_t) - J'h(D_t) - a(u_{2t}, u_{3t})$$

where: P_t = the output price vector,

γ_t = the current costs of using inputs,

$f(.)$ = a vector valued production function,

$r(.)$ = a vector valued debt cost function,

ω = the vector of proportions of unrealized capital gain or loss that can be substituted for equity in the financial negotiation,

$g(.)$ = a function which defines the after tax capital gain or loss realized by selling assets,

$h(.)$ = a function which defines principal payments for each type of debt,

$a(.)$ = a function defining adjustment costs in changing input use,

u_{2t} = a vector of asset sales, $u_{2t} \geqq 0$, and

u_{3t} = a vector of asset purchases, $u_{3t} \geqq 0$.

The production function specification recognizes that processes are often interrelated and that a single process may yield more than one product. The products may be both "goods" and "bads". For example, use of a given chemical may increase production, but also pollute water sources. A more complete specification of the production function depends on the technological facts of the situation. In some cases, when various commodities are produced independently, all factors are allocatable to specific commodities, and by-products are of negligible importance, the vector valued production function may become a vector of individual commodity production functions. In other cases, multiproduct production functions are needed. It is also possible to separate production from marketing and supply inventory behavior by defining functions for these activities. Explicit treatment of marketing and inventory choices may be especially important when cashflow is a problem and the timing of sales and purchases is an important part of the decision environment. In the production function the time variable is assumed to capture technological change.

As in a conventional Vickers model, the current cost terms (γ) are the costs of inputs actually consumed in the production process. For nondurable inputs, like most pesticides, the current cost is the full purchase price. For durable nonland inputs, the current cost includes maintenance and depreciation. Property taxes are a primary component of the current cost of real estate ownership. For nondurables the current cost coefficient can be equal to the capital absorption coefficient in (α), but for durable inputs the current cost will usually be smaller than the purchase price of the

input because the purchase price is allocated over the useful life of the asset.

The cost of debt is assumed to rise with increasing leverage. The argument is that when debt is larger relative to equity, the lender incurs more risk and this risk cost is passed through in the form of higher debt costs (Vickers, pp. 67-68). The time variable in the debt cost function is assumed to reflect changing financial conditions which may shift interest rates for all borrowers and may affect the premium charged for risk. The substitutability of unrealized capital gains or losses for equity in the debt cost function is based on the common practice in agricultural lending of valuing collateral assets at a "conservative market value". This is a market value adjusted for selling costs, taxes, and the uncertainty about whether the gain or loss will ever be realized. For conciseness, the ratio of debt to the sum of invested equity and the proportions of unrealized capital gain or loss that are recognized as changes in net worth will be referred to as the "market value leverage ratio".

A linear conservative market value adjustment is used in income expression (3), but in the most general case the adjustment may be a function of the sign and size of the unrealized capital gain or loss, as well as other factors. For example, the accounting rule use by many nonfarm business requires that assets be valued at the lower of cost or market value. In that case the equity substitution coefficients (ω_i) would be one for capital losses and zero for capital gains. Lenders play an important role in determining the amount of a capital gain or loss that is recognized as a permanent change in net worth. If they are confident about the future of the industry and that the asset will maintain its value, a large part of any capital gain may be included in net worth ($\omega_i \to 1$). If they are concerned about the financial position of the industry and that asset values may decline if large numbers of borrowers are forced to liquidate assets in a short period, they may be unwilling to recognize capital gain as permanent change in net worth ($\omega_1 \to 0$). In a more general model additional arguments could be included in the debt cost function. For example, the ratio of cashflow to debt service requirements may be included to reflect the lenders concern for repayment ability. The liquidity values of individual assets could also be explicitly included as separate arguments, instead of including that information in the conservative market value estimation procedure.

Ideally, a firm level model should include the possibility of disinvestment, as well as firm growth. Hence, the income expression includes a term for capital gains or losses on asset sales. In the general case, the function (g) will be very complicated. A precise accounting of the capital gain or loss would demand a record of when assets were acquired. The

function may be discontinuous. For example, in the U.S. the tax treatment of capital gains and losses is not symmetric and a g function which includes both gains and losses would be discontinuous when accumulated capital gains for some input were zero. In the most general case, the order of assets sales may be an important part of the decision problem; for instance, tax reasons may make it advantageous to sell the assets with the lowest capital gain first. For empirical work a precise accounting of capital gains and losses may be possible and useful. For example, a simulation model could acquire inputs in discrete units and record the accumulated capital gain or loss for each unit separately. For analytic purposes the essential features of the problem may be visible in a more simplified environment. A growing firm may not consider asset sales a viable option; in that case the disinvestment term would not appear in the cashflow expression. A financially troubled firm may recognize that the amount and timing of disinvestment is the real problem. If it can be further assumed that the assets have an equal initial cost or purchase price, within each input type, the g function simplifies to current capital value minus the initial value multiplied by an adjustment term for taxes and capital gains and losses that were previously recognized as substitutes for current income.

$$(4) \quad g(G_t) = (\alpha_t - \alpha^*)(\text{vector of adjustment factors})$$

where α^* = the vector of initial costs per unit of input.

Liquidity losses may be an important factor in disinvestment. These losses can be incorporated into the simplified disinvestment model by subtracting a liquidity cost from the right hand side of equality (4). The liquidity cost can be modeled as a fixed sum, a function of time or proportional to the input price.

The function defining principal payments (h) is vector valued; it specifies net principal payments as a continuous outflow of funds for each type of debt. In simpler models (Lowenberg-DeBoer) if it can be assumed that cashflow is adequate to meet principal payments, the principal payments can be subsumed in the equity investment term (u_{1t}).

Adjustment costs may include transactions costs incurred in purchasing or selling assets, and additional management charges due to problems encountered in changing firm size. Liquidity losses on asset sales may also be included. The adjustment cost may be a function of the size of the asset sale (u_{2t}) or purchase (u_{3t}). Large changes may be more costly than small changes. Separate asset sale and purchase variables are required because unrealized capital gains and losses must be accounted for in asset sales and because of the

tax treatment of those capital gains and losses is not symmetric. Capital purchases are not taxed nor are they directly tax deductible, but sales of capital items may generate taxable income or a tax deduction.

The salvage value can be expressed as the present value of liquidating the firm at the end of the horizon. Because the firm liquidation at the end of the horizon is planned, there should be ample time to search for and find the appropriate buyers. Hence, liquidity losses should be small. Liquidity losses can be incorporated by recognizing only a proportion of the assets terminal market value. Because the tax treatment of capital gains and losses is not symmetric in the U.S., the salvage value term will differ depending on the input price path and the input acquisition strategy. If capital gains are earned, and liquidity losses are negligible, the salvage value term can be written:

$$(5) \quad S = e^{-\rho(1-\tau)T}[K_T + (J-\phi-\Psi\tau_T)'G_T]$$

where: Ψ = the vector of proportions of capital gains that are taxable, and

τ_T = equals the average tax rate after the business is terminated.

This specification assumes that all the assets are sold for their full market value at the end of the horizon and all debt is paid off. Hence, the terminal cash flow is the value of invested equity plus the after tax capital gain income $[(1-\Psi\tau_T)G_T]$. To prevent double counting of capital gain, the proportion of the unrealized capital gain that has already been recognized (ϕG_T) is subtracted.

In the capital loss case, the salvage value must account for the partial deductibility of capital losses from taxable income. The value of capital loss deductions is limited because only a proportion of the loss can be deducted (currently 50 percent of long-term capital losses) and because the annual capital loss deduction is constrained (to the lower of the taxable income over the zero bracket amount or $3,000 under current law). The salvage value term is then the sum of the invested equity, the capital loss which has not yet been recognized, and the value of the capital loss deduction. If the decision maker lives long enough to use the entire deduction, if income is at least high enough to allow the maximum deduction each period, and the discount rate and average tax rate are constant, then the salvage value term may be written (Lowenberg-DeBoer , p. 28):

$$(6)\quad S = e^{-\rho(1-\tau)T}[K_T + (J-\phi)'G_T + \frac{\tau_T \varepsilon}{\rho(1-\tau)}[1-e^{-\rho(1-\tau)\xi' G_T/\varepsilon}]]$$

where ξ = the vector of proportions of the capital loss that are tax deductible, and

ε = the annual limit on capital loss deductions.

The last term in the capital loss salvage value expression (5) is the present value of the stream of tax benefits from the capital loss deduction. The period during which these deductions occur is calculated as the total deductible loss (ξG_T) divided by the annual deduction (ε). If the tax treatment of capital gains and losses is sysmetric with ($\delta=\psi$) and ($\varepsilon\to\infty$), then equation (5) simplifies to equation (4).

A cashflow or equity constraint may also be imposed in the form:

$$(7)\quad \pi_t - u_{1t} \geqq 0.$$

As stated the constraint (7) requires all debt service obligations to be met out of current cashflows, which include all the components of the income term and equity disinvestment ($u_{1t} < 0$). Inequality (7) implies that equity investment is limited to retained earnings. The availability of outside earnings or wealth to meet firm commitments can be explicitly modeled by including an outside income term in the cashflow.

Section III

This section characterizes the general decision approach for a growing firm implied by carrying out the maximization of wealth in a framework which combines production and finance choices. The model outlined in Section I can be approached as an optimal control problem with state variables: inputs, X_t; debt, D_t; equity, K_t; and unrealized capital gains and losses, G_t. The control variables are: equity investment, u_{1t}; assets sales, u_{2t}; and asset purchases, u_{3t}. If the growing firm assumption that input levels are steady or increasing is used and if there are no adjustment costs, a tractable singular control problem is defined. The problem is called singular because the control variables enter the problem linearly and the second derivative of the Hamilitonian with respect to the control variables is a singular, null matrix. General techniques for solving singular problems are outlined by Bryson and Ho (Chapter 8). Static formulations are also possible if it is assumed that input and financial variable levels are set initially, and maintained throughout the planning period (Lowenberg-DeBoer and Boehlje).

The decision approach implied by the integrated production finance framework will be characterized by examining the interior (singular) solutions for input variables. Control paths which include limit level segments for the inputs are discussed by Lowenberg-DeBoer. For expository convenience a simple scalar model with two inputs, one output, and a single debt source will be used. The cashflow constraint is not imposed in this section and the capital gains tax is assumed to be symmetric. The production function is assumed to be strictly concave and the debt cost function convex. For simplicity, the income and equity substitution parameters $(\Phi_1, \Phi_2, \omega_1, \omega_2)$ are taken to be constant over time and the required principal payment is specified as a fixed proportion of outstanding debt.

The simplified problem can be written as:

$$(8.1)\quad \max \int_0^T e^{-\beta(1-\tau)t} \{[Pf(X_{1t}, X_{2t}) - \gamma_{1t}X_t - \gamma_{2t}X_t - r[\frac{D_t}{K_t + \omega_1 G_{1t} + \omega_2 G_{2t}} D_t]^{(1-\tau)} + \Phi_1 \dot{\alpha}_{1t} + \Phi_2 \dot{\alpha}_{2t} - u_{1t} - hD_t\}dt - K_o + e^{-\beta(1-\tau)T}[K_T + G_{1T}(1 - \Phi_1 - \Psi_1 \tau_T) + G_{2t}(1 - \Phi_2 - \Psi_2 \tau_T)]$$

subject to the dynamic constraints:

$$(8.2)\quad \dot{K}_t = u_{1t} + hD_t,$$

$$(8.3)\quad \dot{X}_{1t} = u_{31t},$$

$$(8.4)\quad \dot{X}_{2t} = u_{32t},$$

$$(8.5)\quad \dot{D}_t = \alpha_{1t}u_{31t} + \alpha_{2t}u_{32} - u_{1t} - hD_t,$$

$$(8.6)\quad \dot{G}_{1t} = \dot{\alpha}_{1t}X_{1t},$$

$$(8.7)\quad \dot{G}_{2t} = \dot{\alpha}_{2t}X_{2t},$$

the control constraints:

$$(8.8)\quad u_{31t} \geqq 0,$$

$$(8.9)\quad u_{32t} \geqq 0,$$

initial conditions:

$$K_0 = K(0),\ D_0 = D(0),\ X_{10} = X_1(0),$$

$$X_{20} = X_2(0),\ G_{10} = G_1(0),\ G_{20} = G_2(0),$$

and non-negativity constraints on the inputs and financial variables. The Hamiltonian is:

$$(9)\quad H = e^{-\rho(1-\tau)t}\ \{[Pf(X_{1t},X_{2t}) - \gamma_{1t}X_t - \gamma_{2t}X_t$$

$$- r\left(\frac{D_t}{K + \omega_1 G_1 + \omega_2 G_{2t}}\right)D_t(1-\tau) + \Phi_1\dot{\alpha}_{1t} + \Phi_2\dot{\alpha}_{2t} - u_{1t}$$

$$- hD_t\} + \lambda_{1t}(u_{1t} + hD_t) + \lambda_{2t}u_{31t} + \lambda_{3t}u_{32t}$$

$$+ \lambda_{4t}(\alpha_{1t}u_{31t} + \alpha_{2t}u_{32t} - u_{1t} - hD_t) + \lambda_{5t}\dot{\alpha}_{1t}X_{1t}$$

$$+ \lambda_{6t}\alpha_{2t}X_{2t}$$

where: λ_i = adjoint variables, i = 1, ..., 6.

In this simple model without a cashflow or equity constraint the financial structure is independent of the input choice decision. The optimal leverage is characterized the equality of the marginal cost of equity and the marginal cost of debt at every point on the wealth maximizing path.

$$(10)\quad \rho - r'\left(\frac{D_t}{K + \omega_1 G_{1t} + \omega_2 G_{2t}}\right)^2 = r + r'\left(\frac{D_t}{K_t + \omega_1 G_{1t} + \omega_2 G_{2t}}\right)$$

On the left hand side of equality (9) the marginal cost of equity is the return of alternative investments minus the debt cost reduction due to additional equity investment. The righthand side is the current cost of borrowing plus the higher cost imposed on additional debt.

Repayment terms do not affect the capital structure in this case because cashflow is assumed to be adequate for the repayment plan and because the possibility of locking in fixed interest rates has not been included in the model. If the functional form of the debt cost relationship were specified, equality (10) would in theory be solved for the optimal leverage ratio as a function of the discount rate. If the debt cost function is independent of time, the optimal leverage ratio is constant throughout the planning horizon. The leverage ratio is sensitive to credit market conditions. For instance, if debt cost is increased by a change in monetary policy or by the removal of subsidies so that the righthand side of the financial structure equation (10) is larger at every leverage ratio, a lower leverage ratio would be needed to satisfy the equation. If the debt cost function becomes less sensitive to the leverage ratio, because of a change in lender risk attitudes or because of government guarantees, the optimal leverage ratio will be increased.

It should be noted that the assumptions of a perfect capital market have not been invoked. The model assumes that agents may have different information and that information is costly. In addition, transaction costs and taxes are acknowledged to exist. Under these conditions the existence of an optimal leverage ratio is plausible and not inconsistent with conventional financial theory.

Taxes do not affect the optimal leverage ratio defined by equation (10) because alternative investments were assumed to generate ordinary income for tax purposes. If the alternative investments were taxed at a different rate from the firm's production income, taxes would not cancel out; the discount rate would be multiplied by the tax rate which applies to those alternative investments and the other terms in equality (10) would be multiplied by the tax rate for current production income.

If additional sources of credit are included in the model without other constraints on their use, the optimality conditions require that the marginal cost of all sources of capital be equated. If sources of credit are tied to certain input or if other constraints are imposed the independence of the financial structure from production decisions does not generally hold.

The optimal control path implied by equality (9) requires new debt to be incurred and new equity investment to be made in amounts that will maintain the equality while providing the capital required to finance newly acquired inputs. If unrealized capital gains or losses are recognized in the financial negotiation ($\omega_1 > 0$ or $\omega_2 > 0$), debt and equity levels may change even when no new inputs are acquired. If capital gains occur and borrowing against unrealized capital gain is permitted, the optimal leverage ratio can be maintained with a smaller amount of equity capital. Therefore, equity disinvestment may be generated by capital gains. In a period of capital losses, equity infusions may be needed to offset the capital losses which would otherwise tend to increase the leverage ratio. The model suggests that all other things equal, inflationary periods will tend to lead to equity disinvestment if borrowing against unrealized capital gains is allowed.

The market value leverage ratio will not be affected by inflation unless the debt cost function changes, but the amount of invested equity relative to debt may be decreased. In the context of the U.S. agricultural sector the model suggests that the increase in farm debt in the 1970s can be linked to the farmland capital gains of the period. Because agricultural lenders generally recognized at least some unrealized capital gains as additions to net worth, a given amount of invested equity could support a larger volume of debt when capital gains are occuring than it could under stable asset prices.

Along the optimal singular path the marginal rate of substitution between the two inputs will satisfy:

$$(11.1) \quad \frac{f_1}{f_2} = \frac{[\gamma_{1t}+\alpha_{1t}R_{1t}](1-\tau)\,\dot{\alpha}_{1t}[R_{2t}+\omega_1 R_{3t}+\phi_1+R_{4t}(1-\phi_1-\Psi_1\tau_T)]}{[\gamma_2+\alpha_2+R_{1t}](1-\tau)\,\dot{\alpha}_{2t}[R_{2t}+\omega_2 R_{3t}+\phi_2+R_{4t}(1-\phi_2-\Psi_2\tau_t)]}$$

where: f_i = the first derivative of the production function with respect to input X_i, $i = 1, 2$,

$$R_{1t} = r + r\left(\frac{D_t}{K_t + \omega_1 T_{1t} + \omega_2 G_{2t}}\right)$$

$$R_{2t} = \int_t^T e^{-\rho(1-\tau)(s-t)} R_{1s}(1-\tau)ds$$

$$R_{3t} = \int_t^T e^{-\rho(1-\tau)(s-t)} r\left(\frac{D_s}{K_s + \omega_1 G_{1s} + \omega_2 G_{2s}}\right)(1-\tau)ds$$

$$R_{4t} = e^{-\rho(1-\tau)(T-t)}$$

When cashflow is not a problem, the factor cost ratio on the righthand side of equality (11.1) is known and the optimal input mix can be defined for each point in time. This is possible because the leverage ratio is defined by equality (10).

The initial terms in each factor cost, the current cost (γ_{1t} or γ_{2t}) and the marginal financial cost ($\alpha_{1t}R_{1t}$ or $\alpha_{2t}R_{2t}$), are identical to those found in the static Vickers model. Unlike the Vickers model the factor cost also includes terms reflecting input price changes. The term R_{2t} captures the benefit of buying early in a rising market or the cost of buying too soon when asset prices are falling; the benefit (cost) is the reduced (increased) financial cost over the remaining horizon. The benefit or cost of recognizing input price changes as changes in net worth for financial purposes is indicated by R_{3t}. A benefit of borrowing against unrealized capital gain is the debt cost reduction over the remaining horizon. A cost of capital losses is increased debt cost because net worth is reduced in the financial negotiation. The benefit (cost) of recognizing unrealized capital gain (loss) as a substitute for income is specified by the term in the income substitutability proportion (Φ_1 or Φ_2). The benefit (cost) of realizing the capital gain (loss) at the end of the planning period is found in the R_{4t} term.

Under the assumptions of the model, the input price change multiplier term is positive so that the if an input price is are rising, the capital gains offset the cost of using that input. If input costs are falling, the capital losses increase the costs of input use. It is important here to separate the impact of input price level and asset price change. If the input price (α_{1t} or α_{2t}) is high, the financial cost of ownership will be high. If the financial costs of the two inputs differ, all other things equal, the input mix will use relatively more of the lower financial cost

input. The input price change effects can offset or reinforce the financial cost effects. Ownership of a capital intensive input can be advantageous if its price is also rising rapidly, generating a large capital gain. A relatively low capital investment input can be undesirable if its price is dropping.

The impact of a change in the debt cost function on the input mix depends on both the input price and its associated capital gain or loss. If the input price is constant, an increase in the marginal cost of debt will shift the input mix toward the relatively low capital investment input; the factor cost in equation (11.1) will be increased more for the input with the largest capital value. A decrease in the marginal cost of debt has the opposite effect. Hence, in a constant price environment a change in monetary policy which allowed higher interest rates would provide incentives to shift production practices toward less capital intensive practices. However, in the capital gains case the multiplier of the capital gain term is also increased with an increase in the marginal cost of debt. This increases the benefit of buying now and of using accumulated capital gain as a substitute for equity and tends to offset the effect of the higher marginal debt cost in the financial cost term ($\alpha_{1t}R_{1t}$ or $\alpha_{2t}R_{1t}$).

In an environment of stable input prices, taxes would cancel out of the factor cost ratio in equality (10.1). Taxes are important when input prices are changing because of the tax preference for capital gain. Because capital gains and losses enter the tax calculation only when realized, the income substitution term ($\dot{\alpha}_{1t}\Phi_1$ or $\dot{\alpha}_{2t}\Phi_2$) is not taxed. Only a proportion (Ψ_1 or Ψ_2) of the realization at the terminal date is taxed.

In the U.S. farm sector the primary source of capital gains and losses has been farm land. Nominal capital gains and losses have occurred on equipment, livestock and other assets, but these are dwarfed by the land price changes. If in equation (11.1) X_{1t} is taken to be nonland assets and X_{2t} is land, then, all other things equal, the model suggests that during period of farmland capital gains, such as the period in the early part of the century through World War I or the 1970s, there were incentives to expand farm acreage beyond that indicated by production relationships. Similarly, during periods of falling farmland prices there was incentive to farm more intensively on a smaller acreage. This occurs because the capital gains (loss) effects offset (increase) the costs of ownership.

The capital gains effects have implications for government policy and lending practices. Government price and income support programs which create the expectation of rising farmland prices indirectly provide incentive for farm acreage

expansion. In the U.S. the stated goal of government farm policy has often been to preserve relatively small farms. The model suggests that the longer run effects of these programs may be directly opposite of their stated aims if they generate farmland capital gains.

Lenders can encourage farm acreage expansion in a period of rising farmland prices by allowing decisionmakekrs to borrow against appreciated land values. This makes land a more valuable asset; it earns not only through use in production, but also by generating more credit capacity. Because farmland is an illiquid asset, the long run effect of permitting borrowing against unrealized capital gains may be to make farmers more financially vulnerable. The model suggests that the financial problems of farmers in the mid-1980s may be at least partially due to the willingness of lenders to provide credit based on appreciated land values of the 1970s.

Section IV

If the cash flow constraint is imposed on the problem defined by (8.1-8.9), the importance of noncash and future benefits and costs in determining the optimal input mix is reduced. When the cashflow constraint is binding, equality (10) does not hold; in this case the marginal cost of debt is greater than the marginal cost of equity. If the marginal cost of debt were lower than the marginal cost of equity, the solution would be to increase debt use and restore the equality. Equity investment or disinvestment is determined by the need to maintain adequate cash flow to meet financial obligations. The debt level must be determined simultaneously with input levels. The MRS equation takes the form:

$$(11.2)\quad \frac{f_1}{f_2} = \frac{[\gamma_t+\alpha_{1t}R_{1t}](1-\tau)-\dot{\alpha}_{1t}[R^*_{2t}+\omega_1 R^*_{3t}+R_{5t}\Phi_1+R^*_{4t}(1-\Phi_1-\Psi_1\tau_T)]}{[\gamma_{2t}+\alpha_{1t}R_{1t}](1-\tau)-\dot{\alpha}_{2t}[R^*_{2t}+\omega_2 R^*_{3t}+R_{5t}\Phi_2+R^*_{4t}(1-\Phi_1-\Psi_2\tau_T)]}$$

where: $$\rho^*_s = \int_t^s \{[R_{1s} + r'(\frac{D_s}{K_s + \omega_1 G_s + \omega_2 G_{2S}})^2](1-\tau)\}ds$$

$$R^*_{2t} = \int_t^T e^{-\rho^*_s} R_{1s}(1-\tau)ds$$

$$R^*_{3t} = \int_t^T e^{-\rho^*_s} r'(\frac{D_s}{K_s + \omega_1 G_s + \omega_2 G_{2S}})(1-\gamma)ds$$

$$R^*_{4t} = e^{-\rho^*_T}$$

$$R^*_{5t} = e^{-[\rho^*_T-\rho(1-\tau)(T-t)]}$$

The MRS equation in the cashflow constraint problem does not unambiguously define the input use path because the debt level must be determined simultaneously with input use. The MRS equation does, however, suggest the decision mechanism involved in adjusting input use to meet the cashflow constraint. In the factor cost ratio all costs and benefits that are not immediately cash flows are discounted at an alternative discount rate ($\rho*$) that is higher than the rate of return on alternative investments ($\rho(1-\tau)$). The alternative discount rate is a function of the marginal cost of debt and the marginal return to equity. As the cashflow problems become more severe, more debt must be used to meet financial obligations, the marginal cost of debt and the marginal return to equity investment rise, and noncash and future benefits are

discounted more heavily in determining the optimal input mix. Hence, when cash flow is a problem the benefits of capital gains from future financial costs savings, substitution for income in wealth determination and realization at the end of the period offset a smaller part of the costs of ownership. Similarly, capital losses add less to the costs of ownership. In the extreme, as the marginal cost of debt and marginal return to equity become very high, the input mix approaches the myopic profit maximizing combination, which yields the maximum possible immediate cashflow. If constraints were imposed on equity disinvestment to meet principal payments or new debt acquisition, the repayment terms would affect the solution. If marketing and inventory behavior were explicitly defined in the model, a wider range of cashflow responses could be considered.

Section V

Finance leases can generally be treated as another form of debt in the model, especially when a purchase option is included in the lease agreement. Operating leases demand a different treatment because input price change effects are absent. Short term leases do not generate capital gains or losses. If owned and rented inputs are equally productive, the total input use in the firm can be defined as the sum of owned and rented input levels:

$$X^*_{1t} = X_{1t} + L_{1t}$$

$$X^*_{2t} = X_{2t} + L_{2t}$$

where: X^*_{1t} = total owned and rented input 1,

X^*_{2t} = total owned and rented input 2,

L_{1t} = leased input 1, and

L_{2t} = leased input 2.

The objective function can be written as:

$$(8.1) \quad \max \int_0^T e^{-\rho(1-\tau)t}\{[Pf(X_{1t}+L_{1t}, X_{2t}+L_{2t}) - \gamma_{1t}X_t - \gamma_t X_{2t}$$

$$- \gamma_{3t}L_{1t} - \gamma_{4t}L_{2t} - r\left(\frac{D_t}{K_t + \omega_1 G_{1t} + \omega_2 G_{2t}}\right)D_t](1-\tau)$$

$$+ \Phi_1\dot{\alpha}_{1t} + \Phi_2\dot{\alpha}_{2t} - u_{1t} - hD_t\}dt - K_o$$

$$+ e^{-\rho(1-\tau)T}[K_T + G_{1T}(1-\Phi-\Psi_1\tau_T) + G_{2t}(1-\Phi_2-\Psi_2\tau_T)]$$

where: α_{it} = capital absorbed by leased inputs, i = 3, 4, subject to constraints (7.2-7.4, 7.6-7.9), the initial conditions, the non-negativity constraints, the additional dynamic constraints:

$$\dot{L}_{1t} = u_{5t}$$

$$\dot{L}_{2t} = u_{6t}$$

and the modified debt acquisition constraint

$$D_t = \alpha_{1t}u_{3t} + \alpha_{2t}u_{4t} + \alpha_{3t}u_{5t} + \alpha_{4t}u_{6t} - u_{1t}$$

The current cost of leased inputs (γ_{3t}, γ_{4t}) is primarily the rental payment. Because rental payments are often due in advanced, rented outputs can absorb some capital.

In a deterministic model with adequate equity capital a corner solution for rented versus owned inputs results. The input with the lowest marginal cost will be used. If capital gains are large, there will be an incentive to own the input to capture the benefit of those rising asset values. If capital losses are occuring, the costs of ownership are increased and renting becomes relatively more advantageous. When the availability of equity capital is a binding constraint, part ownership solutions can result. The simultaneous determination of debt levels and input use can equate the factor cost of owned and leased inputs:

$$[\gamma_{3t} + \alpha_{3t}R_{1t}](1-\tau) = [\gamma_{1t} + \alpha_{1t}R_{1t}](1-\tau)$$

$$+ \dot{\alpha}_{1t}[R^*_{2t} + \omega_1 R^*_{3t} + R_{5t}\phi_1 + R^*_{4t}(1 - \phi_1 - \psi\tau_t)]$$

$$[\gamma_{4t} + \alpha_{4t}R_{1t}](1-\tau) = [\gamma_{2t} + \alpha_{2t}R_{1t}](1-\tau)$$

$$+ \dot{\alpha}_{2t}[R^*_{2t} + \omega_2 R^*_{3t} + R_{5t}\phi_2 + R^*_{4t}(1 - \phi_2 - \psi\tau_t)]$$

For example, if the factor cost of an owned input is below that of the rented input, because ownership costs are offset by capital gains effects, but not enough equity is available to finance a full owner operation at the optimal leverage ratio, there will be an incentive to use rented inputs which usually require a smaller capital investment. If the capital gain is increased, all other things equal, the incentives for choosing a higher proportion of the owned input is increased. Capital losses would have the opposite effect. By increasing ownership costs they would push the solution toward more rented inputs.

The model suggests that one of the incentives behind the increase in part-owner farming operations in the U.S. since the 1940s is the almost continuous farmland capital gains of the period. Farmers who would have chosen to be tenants under stable land price conditions, chose to be part owners to capture some farmland capital gains. Similarly, in the mid-1980s the model suggests that, all other things equal, the farmland capital losses should provide incentives for farmers to choose to rent land.

Section VI

A general investment-disinvestment model would be difficult to solve and interpret, but some insight into the disinvestment problem can be achieved by considering a simple situation in which no input increases are considered, only reductions, and all inputs of a given type have the same original cost. The growing firm model can be modified for the disinvestment problem by adding the income from immediate realization:

$$u_{21t}(\alpha_{1t}-\alpha_1^*)(1-\phi-\Psi_1\tau_t) + u_{22t}(\alpha_{2t}-\alpha_2^*)(1-\phi_2-\Psi_2\tau_t)$$

where: α_i^* = the original capital value of input i, i = 1, 2 to the income term, substituting the input sale controls $(-u_{21t}, -u_{22t})$ for the input purchase control in constraints (7.3, 7.5) subtracting the capital gain or loss realized the gain or loss accumulation equation:

$$\dot{G}_{1t} = X_{1t}\,\dot{\alpha}_{1t} - u_{21t}(\alpha_{1t}-\alpha_1^*)$$

$$\dot{G}_{2t} = X_{2t}\,\alpha_{2t} - u_{2t}(\alpha_{2t}-\alpha_1^*)$$

and replacing the constraints (7.6-7.7) with:

(7.6') $u_{21t} \geqq 0$

(7.7') $u_{22t} \geqq 0$

If adequate equity capital is available the optimal leverage ratio is the same as in the growing firm case. The MRS equation becomes:

$$(11.3)\quad \frac{f_1}{f_2} = \frac{[\gamma_{1t}+\alpha_{1t}R_{1t}](1-\tau)-\dot{\alpha}_{1t}[R_{2t}+(1-\Psi_1\tau)] + (\alpha_{1t}-\alpha_1^*)R_{6t}}{[\gamma_{2t}+\alpha_{2t}R_{1t}](1-\tau)-\alpha_{2t}[R_{2t}+(1-\Psi_2\tau)] + (\alpha_{2t}-\alpha_2^*)R_{7t})}$$

where: $$R_{6t} = [P(1-\phi_1-\Psi_1\tau_T) - \omega_1 r'(\frac{D_t}{K_t+\omega_1G_1+\omega_2G_{2t}})^2](1-\tau)$$

$$R_{7t} = [P(1-\phi_1-\Psi_1\tau_T) - \omega_2 r'(\frac{D_t}{K_t+\omega_1G_1+\omega_2G_{2t}})^2](1-\tau)$$

after collecting terms and simplifying. The multiplier of the asset price change term $(R_{2t}+(1-\Psi_i\tau_t)$ can be interpreted as the benefit or cost of continuing to hold the asset; capital gains reduce ownership costs and capital losses increase cost just as in the growing firm model. In this context the asset price change term is composed of the financial cost reduction (R_{2t}) and the after tax realization value $(1-\Psi_i\tau)$. Under the assumptions of the model the input price change multiplier is positive and capital gains reduce the cost of ownership and capital losses add to the cost. The capital gain and loss term is simpler than in the growing firm case because some effects cancel out with the immediate realization costs and benefits.

The third term in the factor cost is the opportunity cost (benefit) of immediate realization. It is composed of a term reflecting the return on the disinvested funds:

$$(\alpha_{1t}-\alpha_1^*)\ P\ (1-\Phi_1-\Psi_{1T}\tau)(1-\tau)$$

$$(\alpha_{2t}-\alpha_2^*)\ P\ (1-\Phi_2-\Psi_{2T}\tau)(1-\tau)$$

minus the increase (decrease) in debt cost because some accumulated capital gain (loss) is realized and is no longer augments (offsets) invested equity in the financial negotiation:

$$(\alpha_{1t}-\alpha_1^*)\omega_1 r'\left(\frac{D_t}{K_t+\omega_1 G_{1t}+\omega_2 G_{2t}}\right)^2(1-\tau)$$

or

$$(\alpha_{2t}-\alpha_2^*)\omega_2 r'\left(\frac{D_t}{K_t+\omega_1 G_{1t}+\omega_2 G_{2t}}\right)^2(1-\tau)$$

It is important to note that the immediate realization term can be of either sign depending on the substitutability of unrealized capital gain or loss for income $(\Phi_1$ or $\Phi_2)$, the tax cost or benefit $(\Psi_1\tau_T$ or $\Psi_2\tau_T)$ and the substitutability of unrealized capital gain or loss for equity (ω_1,ω_2). For example, if the income substitution proportion approaches one, the immediate realization term will be primarily the capital gains tax cost (capital loss tax deduction) and the debt cost effect. In that case immediate realization term would increase ownership costs; it would be the opportunity cost of delaying capital gain tax deduction and putting off reducing

the capital loss which cuts into net worth in the financial negotiation. But in the capital gains case, the realization term would be negative, reducing ownership cost. Delaying the tax liability and the continued use of the capital gain to augment equity, represent incentives to continue holding the asset. Clearly, if most of the capital gain can be recognized as income without realization ($\Phi_1 \rightarrow 1$ and $\Phi_2 \rightarrow 1$), there is little reason to incur the capital gains tax liability or to reduce the effective equity in the financial negotiation.

In contrast, if little unrealized capital gain or loss is substitutable for income, the tax impact is small and the asset price changes are not recognized as a change in net worth for financial purposes, so that:

$$(12) \quad (1 - \Phi_i - \Psi_i \tau_T) \geqq \omega_i, \quad i = 1, 2$$

then the multiplier of the immediate realization term (R_{6t} or R_{7t}) is positive. This can be seen by noting that if inequality (12) holds, the immediate realization multiplier is greater than or equal to the righthand side of the financial structure equation (10), which is equal to the marginal cost of debt and is always positive. This means that in this situation holding accumulated capital gains represents an opportunity cost. When the substitutability of capital gains for income or equity is small, the benefit of that gain must be achieved primarily by realization. Delaying that realization then represents a major cost of holding the asset. Holding capital losses in this case avoids or delays most of the cost of those losses; and thus capital losses in the immediate realization term offsets the cost of ownership and provides incentives to continue holding the property.

The growing firm model suggests that in the stagnant or declining farmland price environment of the mid 1980s, the optimal farmland ownership level for new farm firms may be smaller than it was in the 1970s. The capital losses on farmland provide incentives to farm more intensively and to rent farmland, instead of purchasing it. The disinvestment model indicates that existing firms may not find it optimal to adjust land ownership to the new conditions. For some firms the wealth maximizing strategy will be to delay recognition of the capital losses.

Summary and Conclusions

This paper outlines a general model of the farm firm under certainty that incorporates finance and production decisions. A wealth maximizing decisionmaker is assumed to choose input levels and financial structure subject to the constraint that the capital absorbed of inputs can not exceed the sum of debt and equity. A cashflow constraint is also considered. Capital gains and losses are assumed to affect the decisionmakers wealth in three ways: through selling the asset, through the ability to recognize unrealized capital gains and losses as a substitute for income in determining wealth, and through the opportunity to recognize capital gains and losses as a permanent change to net worth in negotiating financing.

Models developed by Baker and Vickers suggested that the financial cost or benefit of using an input should be considered part of the factor cost in determining optimal input use. The framework outlined here is a step toward modeling those financial costs and benefits more explicitly. The financial costs included in the decision rules derived using the model include not only the interest cost on the capital invested, but also the impact of holding or selling inputs on the financing terms, the effect of timing changes in input acquisition on the capital requirement and the cashflow effects of production decisions.

Under simplifying assumptions the model can be solved analytically. The solutions suggest that inputs which earn capital gains will be a larger part of the input mix if that capital gain can be substituted for income in determining wealth and if the accumulated capital gain can be substituted for equity in the financial negotiation. Capital losses have the opposite effect. When the availability of equity capital is a binding constraint, when cashflow is a problem or when financing terms are tied to certain inputs, the wealth maximizing financial structure is not independent of production choices. In this relatively simple model, cashflow problems push the solution toward myopic profit maximization. In the model ownership versus rental choices are affected by the relative capital requirements of each input procurement strategy and the marginal cost of that capital. Higher capital costs and capital losses push the solution toward renting inputs. The solutions suggest that the disinvestment choice depends not only on the selling price of inputs, but also on how selling the input affects future financial arrangements and on tax costs or benefits. Capital losses do not necessarily trigger disinvestment because it may be a wealth maximizing strategy to delay recognition of the loss.

The model leads to testable hypotheses. For example, the analytic solutions suggest the following hypothesis about recent changes in the U.S. farming sector:

1) the farmland capital gains in the 1970s are positively linked to the increased investment in real estate by farmers during the period;

2) the farmland capital gains in the 1970s are positively related to the increased debt use by U.S. farmers relative to cost basis net worth;

3) the reduced liquidity of farm firms in the 1970s can be at least partially traced to farmland capital gains and the incentive they provide to hold less liquid of farm assets;

4) the willingness of lenders to recognize unrealized capital gain as a permanent addition to net worth played an important part in the ability of the farm sector to shift its investment mix toward real estate; and

5) credit market conditions, including the availability of subsidized credit, and the use of market values in measuring the financial position of the firm, have encouraged farm business to expand beyond the size indicated by production relationships and to use more capital intensive methods than would otherwise be the case.

Testing of these hypotheses could use multiperiod mathematical programming models or econometrics. Exploratory empirical work using a deterministic dynamic programming model of an Iowa farm firm supports hypotheses 1-4 (Lowenberg-DeBoer); hypothesis 5 has not been tested within this framework.

The farm financial stress of the mid 1980s indicates a pressing need for a better understanding of farm firm management behavior. The framework outlined in this paper is a contribution toward a more rigorous analysis which combines the effects of financial and production conditions. Production and finance interaction are certainly not the only explanation for recent economic events, but they do appear to offer substantial explanatory power in examining the closely held firms of the farm sector.

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