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CAPITAL MARKETS AND THE DYNAMICS OF GROWTH

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CAPITAL MARKETS AND THE DYNAMICS OF GROWTH

Mathew Shane*

Development of a dynamic monetary framework has been a prime objective of monetary theorists since the publication of Tobin's article "Money and Economic Growth" in 1965. These efforts[1/] have generally been limited to investigating the impact of introducing money into a neoclassical aggregate growth model. In this paper, however, I investigate the effects of introducing an equity security into money and growth analysis. This approach has several distinct advantages. In a model which considers only money and capital, saving is merely a decision to invest directly in real capital or money balances, whereas the introduction of a financial market allows us to analyze directly the mechanism by which saving and investment are separated. In terms of developing a policy tool, this approach also provides a basis for directly relating policy actions which are financial in nature to the effects on real variables such as income and capital intensity.

This paper is further distinguished by an explicit consideration of the stock-flow nature of the "monetary" adjustment process and an analysis of global asymptotic stability.[2/] There are three potential formulations of the stock-flow relationship. In Tobin's model, both the supply and demand for money, and saving and investment are assumed equal. This may be viewed as an equilibrium-equilibrium stock-flow model. In some analyses of financial markets and investment [see articles by H. Grossman (1971), D. Jaffee (1971) and D. Purvis (1973)], a stock disequilibrium adjustment hypothesis has been postured. In our context these are disequilibrium-disequilibrium theories, since neither the supply and demand for money nor saving and investment are equal off the growth path. A third stock-flow adjustment hypothesis, however, implies an equilibrium-disequilibrium model, following an approach developed by George Horwich (1955, 1964). It is this hypothesis which will be investigated in this paper.

The formulation of an equilibrium-disequilibrium model offers two clear advantages over other models: (1) Instantaneous stock equilibrium provides a mechanism for establishing prices and interest rates in the short run, while still allowing for a long-run adjustment process generated by excess flow demands. Since the emphasis of this paper is on long-run adjustment, short-run stock equilibrium is a justifiable simplification.[3/] (2) Global stability can be investigated without the assumption of instantaneous adjustment. Thus the addition of a security market serves three roles: it acts as the link between real and monetary behavior; it moderates financial disturbances by permitting changes in real variables to occur over an extensive period; and it provides the medium for separating saving and investment decisions.

I. Capital Market Growth Model

The model consists of two markets: (1) a neoclassical production and factor market, and (2) an asset market for real capital, securities and real balances. The flow of funds generated from saving and investment behavior by acting cumulatively on the asset markets changes its composition and thus provides the dynamic elements of the model. Associated with the two markets are two basic sectors, one banking intermediary and one monetary control sector. Households hold securities issued by firms and money in the form of currency and deposits. Their saving is a flow demand for these assets. Firms hold the entire capital stock and issue equity securities to finance the accumulation of the capital stock and real balances. The banking intermediary purchase securities as its only asset and issued demand deposits to both the household and the firm sectors. Lastly, the monetary control sector increases the stock of outside money through open market operations. An explicit presentation of the model follows.

A. Production-Factor Market

It is assumed that a single good $(Y(t))$ is produced by a linear homogeneous neoclassical production function of two factors, labor $(N(t))$ and capital $(K(t))$, and that technological change $(A(t))$ is of the labor embodied type. In reduced form, this relationship is:

$$y = f(k) \tag{1.1}$$

where $y = Y(t)/K(t)$ and $k = A(t)N(t)/K(t)$.

Using the production relationship and assuming competition in the factor markets, it follows directly that the real rental of capital (r) and the real wage rate (w) are:

$$r = f(k) - kf'(k) = r(k) \tag{1.2}$$

$$w = Af'(k) = w(k,t), \tag{1.3}$$

where both r_k and $w_k < 0$. Further, the labor force and technological change are assumed to be growing at exogenous rates n and α respectively:

$$\dot{N}/N = n \tag{1.4}$$

$$\dot{A}/A = \alpha. \tag{1.5}$$

Both the capital stock and labor force are assumed given at some initial point (t_o).

B. Asset Markets

Household money wealth (PW_h) consists of money balances (M_h), deposits and currency, plus the value of security holdings $(P_s S_{qh})$,

$$PW_h = M_h + P_s S_{qh}. \tag{1.6}$$

Firm's assets (PW_f) consist of capital (PK) and money balances (M_f):[4/]

$$PW_f = M_f + PK. \tag{1.7}$$

Combining (1.6) and (1.7), another wealth term is derived:

$$PW = M_h + M_f + PK + P_s S_{qh}. \tag{1.8}$$

The money supply (M) is divided between an outside (M^1) and inside (M^2) component:

(1.9) $$M = M^1 + M^2,$$

which is distributed between household (h), business (f) and banking (b) sectors as follows:

(1.9.1) $$M^1 + M^2 = M^1_h + M^1_f + M^1_b + M^2_h + M^2_b.$$

The banking system is assumed to hold securities (P_sS_{qb}) and currency (M^1_b) and to issue deposits (M^2),

(1.10) $$M^2 = M^1_b + P_sS_{qb},$$

while the central bank issues outside money (M^1) in exchange for securities (P_sS_{qc})[5/],

(1.11) $$M^1 = P_sS_{qc}.$$

Equations (1.9.1), (1.10) and (1.11) can be used to derive a simplified form of (1.8). Thus the money holdings of the household and business sectors can be divided between their inside and outside component:

(1.8.1) $$PW = M^1_h + M^2_h + M^1_f + M^2_f + PK + P_sS_{qh}.$$

Using (1.9,1) this can be rewritten as:

(1.8.2) $$PW = M^1 - M^1_b + M^2 + PK + P_sS_{qh}.$$

Lastly, by substituting (1.10) for M^2 and (1.11) for M^1, the following relationship results:

(1.8.3) $$PW = PK + P_sS_q$$

where by definition

(1.8.4) $$P_sS_q = P_sS_{qh} + P_sS_{qb} + P_sS_{qc}.$$

We will now consider the demand for securities in value terms.[6/] The household demand for securities (P_sD_{qh}) is a function of money income, the security interest rate and wealth. Thus:

(1.12) $$P_sD_{qh} = d_{qh}(PY,\ i^*_s,\ PW).$$

The demand for securities by the banking system (P_sD_{qb}), their earning asset, is a simple function of outside money:

(1.13) $$P_s D_{qb} = \theta(i_s^*)\ M^1.$$

Substitutiong (1.11) into (1.13) yields:

(1.13.1) $$P_s D_{qb} = \theta(i_s^*)\ P_s S_{qc}.$$

It is further assumed that the supply and demand for securities by the control sector is equal:

(1.14) $$P_s D_{qc} = P_s S_{qc}.$$

Summing equations (1.12), (1.13.1) and (1.14) give an aggregate demand for securities $(P_s D_q)$ 7/

(1.15) $$P_s D_q = D_{qh}(PY,\ i_s^*,\ PW) + [1 + \theta(i_s^*)]\ P_s S_{qc} = D_q(PY, i^*, PW).$$

The rate of open market operations is assumed to be exogenous:

(1.16) $$\frac{\dot{M}^1}{M^1} = \frac{P_s \dot{S}_{qc}}{P_s S_{qc}} = u.$$

The supply of securities in value terms is positively related to the profits of the business sector $[(r-\delta)PK]$, negatively related to the real interest rate on securities (i_s),

(1.17) $$P_s S_q = S_q[(r-\delta)PK,\ i_s^*),$$

and is linear homogeneous in profits. Since Walras' Law holds in the asset markets, the remaining stock equations--the demand for money of the household and business sectors--are determined and the asset equations are complete.

The interest rate on securities (i_s^*) is determined by instantaneous asset market equilibrium. Thus setting (1.15) equal to (1.17),

(1.18) $$P_s S_q = P_s D_q,$$

and solving for i_s^* yields:

(1.19) $$i_s^* = D_q^{-1}\ (PY,\ P_s S_q,\ PW).$$

Lastly, it is assumed that the factor markets are always in equilibrium. This is consistent with the full employment neoclassical framework. Thus, the short run behavior of the asset markets is fully determined.

Real saving is a function of real income, the real return on securities and real wealth,

(1.20) $$S = S(Y, i_s, W),$$

and is linear homogeneous in income and wealth. Real investment is a function of real profits $[(r-\delta)K]$, the rental of capital (r) and the capital stock (K),

(1.21) $$I = I[(r-\delta)K, r, K],$$

and is linear homogeneous in profits and capital. The rate of actual capital formation $(\dot{K}/K)$ is assumed to be a composit function of the desired rate of investment (I/K) and the difference between the rate of savings and investment. Thus:

(1.22) $$\frac{\dot{K}}{K} = \frac{I}{K} + \gamma\left(\frac{S}{K} - \frac{I}{K}\right), \quad 0 < \gamma < 1.$$

Although real saving and investment are the central flow equations, additional flow relationships must be specified. Total household saving in nominal terms (S^h) is equal to the household flow demand for securities (S^h_{sec}), deposits (S^h_{dep}) and currency (S^h_{cur}):

(1.23) $$S^h = S^h_{sec} + S^h_{dep} + S^h_{cur}.$$

At the same time, the business sectors new issues of securities (I_{sec}) must equal its demand for deposits (S^f_{dep}), currency (S^f_{cur}) and investment (I^n):

(1.24) $$I_{sec} = S^f_{dep} + S^f_{cur} + I^n.$$

By adding (1.23) and (1.24) and making the following substitutions,

(1.25) $$\dot{M}^1 = S^c_{sec}$$

(1.26) $$I_{dep} = S^b_{sec} + S^b_{cur}$$

where S^c_{sec}, S^b_{sec} are the new demand for securities by the control agent and banking system and I_{dep}, S^b_{cur} are new deposits and reserves in the banking system, we get:

(1.27) $$S^h + I_{sec} = S^h_{sec} + S^b_{sec} + S^c_{sec} + I^n.$$

Rewriting this, where S^d_{sec} is the sum of S^h_{sec}, S^b_{sec} and S^c_{sec} yields:

(1.27.1) $$S^h - I^n = S^d_{sec} - I_{sec},$$

an excess of nominal saving over nominal investment likewise implies an excess flow demand for securities. This is an important point and demonstrates the interrelationship between the financial and real behavior. Using (1.27.1) we get that the actual rate of security issues ($\varepsilon = P_s\dot{S}_q/PK$) is equal to the desired rate (I_{sec}/PK) plus an adjustment coefficient which depends on the excess of nominal saving over nominal investment:

(1.27.2) $$\varepsilon = \frac{I_{sec}}{PK} + \emptyset\ \left(\frac{S^h - I^n}{PK}\right)^{8/} \qquad 0 < \emptyset < 1.$$

Letting $s = S^h/PK = S/K$ and $v = I^n/PK = I/K$, we can rewrite (1.27.2) as:

(1.27.3) $$\varepsilon = \frac{I_{sec}}{PK} + \emptyset\ (s-v).$$

The desired rate of security issues (I_{sec}/PK) must be equal to the desired additions to real balances plus desired investment. This is a flow portfolio decision which is a function of rates of return (r, i_s):

(1.27.4) $$\frac{I_{sec}}{PK} = \Phi(r, i_s) + v,$$

where Φ_1, $\Phi_2 < 0$. Substituting (1.27.4) back into (1.27.3) yields:

(1.27.5) $$\varepsilon = \Phi(r, i_s) + v + \emptyset(s-v).$$

With the flow relationships specified, we can now preceed to hypothesize adjustment functions.

The rate of inflation (p) is assumed to be a function of the excess of relative real investment over real saving:

(1.28) $$p = \lambda p\ \left(\frac{I}{K} - \frac{S}{K}\right),$$

where $\lambda p(0) = p^e$ and $\lambda_p > 0$. The change in the expected rate of inflation ($\dot{p}^e$) is a function of the difference between the actual and expected rate of inflation,

an adaptive expectation hypothesis,

(1.29) $$\dot{p}^e = \beta(p-p^e), \qquad 0 < \beta < 1.$$

The rate of change in the rental of capital is derived by taking the time differential of (1.2)

(1.30) $$\dot{r} = kf''(k)\dot{k}.$$

Since $f''(k)$ is negative, this is a positive function of the difference between the effective labor and real capital growth rates.

II. Reduced Form Model

The extensive model presented above is not in a convenient form for analysis. Since we are interested in the long run properties of the model, a reduced form of the model will be developed in this section. This will involve deriving reduced equations in the fundamental variables capital intensity (k), wealth ratio ($w=P_oS_q/PK$) and price expectations (p^e).

Dividing (1.20) by K gives:

(II.1) $$s = s\ [f(k),\ i_s,\ 1 + sq[(r-\delta),\ i_s+p^e]]$$

where $s_1 > 0$, $s_2 < 0$, and $s_3 < 0$.[9] Similarly, from (1.21), we get:

(II.2) $$v = v(r-\delta,\ r,\ 1) = v(r(k);\ \delta)$$

where $v = I/K$ and $v_r > 0$. The reduced wealth equation is derived by dividing (1.18) by PK and substituting $i_s^* = i_s + p^e$,

(II.3) $$w = w^d\ (f(k),\ i_s + p^e,\ r,\ 1 + w(r-\delta,\ i_s + p_e))$$

where $w_1^d > 0$, $w_2^d > 0$, $w_3^d < 0$, $w_4^d > 0$.

Only the variable i_s prevents the (II.3) from being in reduced form. By deriving the partial inverse function of (II.3) for i_s in terms of k, w, p^e, we will have the basis for presenting the reduced model. From (II.3), we get:

(II.4) $$i_s = i_s\ (k,\ w,\ p^e)$$

where

$$i_{s1} = -J_{is}\ (\overset{+}{w_1^d}\ \overset{+}{f'}(k) + \overset{-}{w_3^d}\ \overset{+}{r_k} + \overset{+}{w_4^d}\ \overset{+}{w_1}\ \overset{+}{r_k})$$

$$i_{s2} = J_{is}\ (1 + \overset{+}{w_4^d}\ \overset{-}{w_2})$$

$$i_{s3} = -J_{is}\ (\overset{+}{w_2^d} + \overset{+}{w_4^d}\ \overset{-}{w_2})$$

and

$$J_{is} = 1/(\overset{+}{w_2^d} + \overset{+}{w_4^d}\ \overset{-}{w_2})$$

The assumption that w_4^d is a fraction while $w_2^d \geq -w_2$ implies that J_{is} is positive and i_{s3} negative. In addition, if $w_4^d w_2$ is less than one, i_{s2} is positive. It will be further assumed that i_{s1} is negative, i.e., that $w_1^d f'(k)$ plus $w_4^d w_1 r_k$ dominates $w_3^d r_k$.

We have thus completed the prerequisites and can now present the reduced differential dynamic model. Assuming equilibrium in the labor market by (1.4), (1.5) and (1.22), we get:

(11.5) $$\frac{\dot{k}}{k} = n + \alpha - v - \gamma(s-v)$$

and

(11.6) $$\dot{w} = \varepsilon - [p + v + \gamma(s-v)]w$$

From (11.5), (11.2) and (11.1), we find:

(11.5.1) $$\dot{k} = [n + \alpha - v(r(k);\ \delta) - \gamma(s(f)(k)),\ i_s(k,w,p^e),$$
$$1 + sq(r(k)-\delta,\ i_s(k,w,p^e) + p^e)) - v(r(k);\ \delta)]k$$

Similarly, from (11.6) and (1.27.5) we get:

(11.6.1) $$\dot{w} = \Phi(r,i_s) + v + \emptyset(s-v) - [\lambda_p(v-s) + v - \gamma(s-v)]w.$$

Rearranging terms and expressing explicit functional forms gives:

(11.6.2) $$\dot{w} = \Phi(r(k),\ i_s(k,w,p^e) + v(r(k);\delta)\ [1-\emptyset-(1+\gamma)w]$$
$$+ (\emptyset+\gamma w)\ [s(f(k),\ i_s(k,w,p^e),\ 1 + sq(r(k)-\delta,\ i_s(k,w,p^e) + p^e))]$$
$$- \lambda_p(v-s)w.$$

Finally from (1.28) and (1.29), we get:

$$\dot{p}^e = \beta(\lambda p(v(r(k);\delta) - s(f(k),\ i_s(k,w,p^e),\ 1 + sq(r-\delta, i_s(k,w,p^e) + p^e)) - p^e). \tag{II.7}$$

Equations (II.5.1), (II.6.1) and (II.7) are three autonomous differential equations in three variables $(k,\ w,\ p^e)$. To determine the properties of the stationary solution we set the differentials equal to zero and solve. This leads to the following results:

$$n + \alpha = v(v(k);\ \delta) \tag{II.7}$$

[10]

$$\frac{\varepsilon}{w} = \lambda p\ (v(r(k);\ \delta) - s(f(k),\ i_s(k,m,p^e),\ 1 + sq(r-\delta),(i_s+p^e)),\ - v(r(k);\ \delta). \tag{II.8}$$

$$p^e = \lambda p(v(r(k);\ \delta) - s(f(k),\ i_s(k,m,p^e),\ 1 + sq((r-\delta),(i_s+p^e)) \tag{II.9}$$

Thus we find that:

$$p = p^e = \frac{\varepsilon}{w} - n - \alpha = u - n - \alpha \tag{II.10}$$

since $\frac{\varepsilon}{w} = u$ on the growth path.[11] Further, given the equilibrium values of (k,m,p^e) we can find both the real rental and return on securities. Finally, we can make the following statement about the long run growth path. Real income, capital, securities and wealth grow at the rate $n + \alpha$, that of the effective labor rate. The rate of inflation and expected inflation is equal to the rate of money creation less that of labor growth. And finally, the real and money wage rates grow at the rate α and $u-n$.

III. Stability[12]

In this section we will investigate the global asymptotic stability properties of the model.[13] A system of differential equations is said to be globally asymptotically stable if it has the property that when a solution for small $t \geqq 0$ exists, then the solution exists for all $t \geqq 0$ and the solution converges to the

equilibrium solution as time (t) grows indefinitely large. The criteria to be employed for investigating this property reduce to the following inequality:

(III.1) $$f(y) \quad f(y) \leqq 0$$

where

(III.1.1) $$f(y) = (\dot{k}, \dot{w}, \dot{p}^e),$$

the system (II.5.1), (II.6.1) and (II.7) while

(III.1.2) $$J(y) = \begin{bmatrix} \dot{k}_1 & \dot{k}_2 & \dot{k}_3 \\ \dot{w}_1 & \dot{w}_2 & \dot{w}_3 \\ \dot{p}^e_1 & \dot{p}^e_2 & \dot{p}^e_3 \end{bmatrix}$$

is its system of partial derivatives. This condition, in other words, is that the quadratic form derived by taking the system of differential equations and its partial derivatives is negative definite or semi-definite.[14] A necessary and sufficient condition for (III.1) to be negative definite is that the naturally ordered principal minors of J(y) alternate in sign,[15] i.e.,

(III.2) $$\dot{k} < 0, \quad \begin{vmatrix} \dot{k}_1 & \dot{k}_2 \\ \dot{w}_1 & \dot{w}_2 \end{vmatrix} > 0, \quad \begin{vmatrix} \dot{k}_1 & \dot{k}_2 & \dot{k}_3 \\ \dot{w}_1 & \dot{w}_2 & \dot{w}_3 \\ \dot{p}^e_1 & \dot{p}^e_2 & \dot{p}^e_3 \end{vmatrix} < 0$$

To determine whether condition (III.2) is satisfied, we must first interpret the partial derivatives of system. These partial derivatives follow:

(III.2.1) $$\begin{aligned} \dot{k}_1 = k[&-v_r\, r_k - \gamma(s, f'(k) + s_2 i_{s1} \\ &+ s_3(sq_1\, r_k + sq_2 i_{s1}) - v_r\, r_k) \\ &+ n + \alpha - v - \gamma(s-v)], \end{aligned}$$

(III.2.2) $$\dot{k}_2 = k[-\gamma(s_2 i_{s2} + s_3 sq_2 i_{s2})],$$

(III.2.3) $$\dot{k}_3 = k[-\gamma(s_2 i_{s3} + s_3 sq_2\, (i_{s3} + 1)],$$

(III.2.4) $$\begin{aligned} \dot{w}_1 = {} & \Phi_1 r_k + \Phi_2 i_{s1} + v_r r_k\, (1 - \emptyset - (1+\gamma)\, w - \lambda'_p) \\ & + (\emptyset + \gamma w - \lambda'_p)[s_1 f'(k) + s_2 i_{s1} + s_3(sq_1 r_k + sq_2 i_{s1})], \end{aligned}$$

(III.2.5) $$\dot{w}_2 = \Phi_2 i_{s2} - v + \gamma(s-v) + (\phi+\gamma w+\lambda_p')(s_2 i_{s2} + s_3 sq_2 i_{s2}) - \lambda_p(v-s),$$

(III.2.6) $$\dot{w}_3 = \Phi_2 i_{s3} + (\phi + \gamma w + w\lambda_p')(s_2 i_{s3} + s_3 sq_2\ (i_{s3} + 1)),$$

(III.2.7) $$\dot{p}_1^e = \beta\lambda_p'(v_r r_k - s_1 f'(k) - s_2 i_{s1} - s_3(sq_1 r_k + sq_2 i_{s1})),$$

(III.2.8) $$\dot{p}_2^e = \beta\lambda_p'(-s_2 i_{s2} - s_3 sq_2 i_{s2}),$$

(III.2.9) $$\dot{p}_3^e = \beta(\lambda_p'(-s_2 i_{s3} - s_3 sq_2(i_{s3} + 1)) - 1).$$

Given the conditions which have so far been imposed on the system none of the partial derivatives are of determinate sign. However, with the addition of two plausible conditions all but (III.2.1) become determinate. These two conditions are:

(III.3.1) $$s_2 > - s_3 sq_2$$

(III.3.2) $$0 < - i_{s3} < 1.$$

To determine the sign of $\dot{k}_1$ additional conditions need to be imposed. Thus,

(III.3.3) $$\dot{k}_1 < 0 \quad \text{if}$$

$$\gamma[-v_r r_k + s_1 f'(k) + s_3 sq_1 r_k + s_2 i_{s1}] + v - \gamma(s-v) > -v_r r_k - \gamma s_3 sq_2 i_{s1} + n + \alpha.$$

This is plausible since for all k greater than k equilibrium, v is greater than $n + \alpha$. It has already been assumed that s_2 is greater than $-s_3 sq_2$ which implies $\gamma s_2 i_{s1}$ is greater than $-\gamma s_3 sq_2 i_{s1}$. Thus the crucial problem is whether the three remaining negative terms are greater than $-v_r r_k$. Although this seems logical, there is no way to argue this on a priori grounds. With the addition of (II.3.1) - (II.3.3), we find that:

(III.1.2.1) $$J(y) = \begin{bmatrix} - & + & - \\ + & - & + \\ - & + & - \end{bmatrix}.$$

Notice that the symmetry condition is satisfied and that the diagonal elements are negative. Thus the first of the three conditions of (III.2) is met, i.e., $\dot{k}_1 < 0$.

The remaining two conditions of (III.2) are guaranteed if the following hold:

(III.3.4) $$1 > \gamma(1+kX_1)$$

(III.3.5) $$-v_r r_k - \Phi_2 i_{s2} > \Phi_1 r_k$$

(III.3.6) $$-v_r r_k + \Phi_1 r_k + 2\lambda_p' X_3 > -v_r r_k w + \Phi_2 i_{s2}$$

(III.3.7) $$\gamma > \lambda_p'$$

(III.3.8) $$\emptyset + 2\lambda_p' > 1,$$ 16/

where

$$X_1 = s_1 f'(k) + s_2 i_{s1} + s_3(sq_1 r_k + sq_2 i_{s1})$$

$$X_3 = s_2 i_{s3} + s_3 sq_2(1 + i_{s3}).$$

(III.3.4) implies that the cumulative effect of the forced saving rate times the effect of a change in capital intensity on savings must be less than one. (III.3.5) and (III.3.6) together imply that the effect of changing capital intensity on investment must be greater than the indirect effects through the flow portfolio decisions. Finally (III.3.7) and (III.3.8) relate to relative adjustment coefficients. Thus the rate of forced saving must be greater than the rate of adjustment in inflation while the total effect of the security market flow adjustment must be greater than one.

Conditions (III.3.1) - (III.3.8) are sufficient to guarantee the stability of the model. These additional conditions are of two kinds. The first set relate to the slopes of saving investment terms and to the relative size of direct as compared with indirect effects. It has been assumed throughout that the direct effects are greater than the indirect effects. The second conditions relate to the relative efficiency of the different markets in the adjustment process. It is a consistent attribute of money and growth models that the adjustment in the rate of inflation tend to be destabilizing. As (III.3.7) implies it is crucial that the rate of forced

saving and by implication the rate of security market adjustment be greater than that of the rate of inflation. Thus a policy which would tend to inhibit changes in the rate of interest would tend to lead to prolonged periods of inflation. Although this is only posed here as a hypothesis, it is one that is well worth further investigation.

IV. Conclusions

This paper investigated the effects of introducing an equity security into a money and growth framework. The model was developed explicitly in terms of securities and capital instead of the more traditional treatment in terms of money and capital. The model was reduced to three differential equations in capital intensity, the wealth ratio and price expectations. Based on this reduced system equilibrium and global stability was analyzed. The stability of the model was found to depend crucially on whether the direct interest rate effect was greater than the offsetting wealth effect and whether the rate of forced saving was greater than the adjustment in the rate of inflation. It is proposed that the crucial role of financial markets in general, and security markets in particular is to take the short-run impact of economic disturbances and allow the rate of inflation to adjust slowly to changes in the economic environment.

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14/ See Hartman (1964) pp. 537-540 and particularly note exercise (B) on p. 538 and corollary 11.3 on p. 540. Although this negative definite condition is not sufficient by itself, it will be assumed that the remaining conditions such as uniqueness of solution path given initial conditions are met.

15/ See Hadley (1961, p. 262).

16/ In addition, there are two more conditions which guarantee that $n + \alpha - v - \gamma(s-v)$ will not dominate the determinant sign. Since these are rather complex terms whose economic meaning is relatively difficult to interpret, they will be omitted from discussion here.

FOOTNOTES

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1/ See M. Hadjimichalakis (1971), D. Levhari and D. Patinkin (1968), K. Nagatani (1970), M. Shane (1969), M. Sidrauski and D. Foley (1970), J. Stein (1969), F. Takahaski (1971).

2/ However, it was George Horwich (1955, 1964) who first explicitly recognized the stock-flow nature of the macro-monetary mechanism.

3/ Evidence presented by Fama (1970) and others indicates that the security markets adjust very rapidly to new information, and thus the short run equilibrium assumption about the equity markets is a reasonably good approximation of reality. On the other hand, recent evidence on banking presented by D. Jaffee (1971) indicates that stock disequilibrium might be the usual state of that market.

4/ A unique feature of equity securities as distinct from other methods of financing such as bonds is that equities are not a liability of the firm in the traditional sense. This fact is recognized by Steve Taylor (1969, p. 1, 23) in the following statement: "Corporate equities have an ambiguous position in the financial scheme in that they are only residual claims and, in a legal sense at least, not a burden on the issuer."

5/ It is implicitly assumed that neither the banking system nor the central bank has a net worth. The earnings of each of these sectors is redistributed to the non-banking public sector so as to not effect relative portfolio allocation.

6/ This section could be developed in terms of the demand for money of the household and business sector. However, since the emphasis of the paper is on the role of security markets, in this case the only financial instrument, this alternative development is preferred.

7/ Equation (1.13.1) implies a simplified money supply function (see Takahaski 1971) and thus the aggregate demand for securities become a composite function of the three non-business sectors.

8/ Dividing nominal saving and investment by PK gives the real rate of saving and investment. It is clear from the above presentation that households are the only sector which generates net saving.

9/ The notation s_j for the partial derivative of s with respect to the j-th variable will be used throughout the remaining part of the paper.

10/ Since s = v in long run equilibrium the later part of the actual capital formation function drops out.

11/ $\frac{\varepsilon}{W}$ is the rate of security issues. If that rate was different from u in equilibrium, the portfolio composition would be changing over time, i.e., the share of the control agent would increase or decrease as $\frac{\varepsilon}{W} \lessgtr u$.

12/ Comparative statis properties of the model were not presented. The results of this analysis could be summarized as follows. The effect of issuing the rate of money issuance is to lower capital intensity, a result consistent with Tobin's model. Thus monetary policy in this type of model is non-neutral.

13/ All previous authors in the money and growth area have restricted their analysis to local stability, i.e., investigating the stability of the linear approximations of Taylor series expansions.