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RISK, UNCERTAINTY, AND FUTURES TRADING, IMPLICATIONS FOR HEDGING DECISIONS OF BEEF CATTLE FEEDERS

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RISK, UNCERTAINTY, AND FUTURES TRADING, IMPLICATIONS FOR HEDGING DECISIONS OF BEEF CATTLE FEEDERS

William A. Elder[1/]

Conventional short-run static theory of the firm assumes the existence of an economic world in which the operators possess perfect knowledge with respect to alternative possible behavioral strategies, actions necessary to effect the strategies, and outcomes which will flow from specific actions. With perfect knowledge, production-marketing decisions of primary producers (e.g. farmers) can be made with certainty. The nature of reality, however, precludes economic operators from attaining the "practical omniscience" required for perfect decisions.

Risk and uncertainty arise in the absence of perfect knowledge. Imperfect knowledge of strategies, actions, and consequences may lead to decisions which result in an economically non-optimal allocation of resources on both an *ex ante* and an *ex post* basis for both micro and macro economic units. Risk and uncertainty are thus manifest in actual outcomes which differ from expected outcomes.

Literature on the theory of futures trading and the hedging use of futures markets has long recognized the role of hedging as a means of shifting commodity price level risks of merchants, or holders of commodity stocks, to specialized risk takers, or speculators. Futures

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theory also recognizes the function of futures trading (combined with trading in the cash commodity) as a guide to optimal allocation of storable commodity stocks and resources among uses over time. While traditional hedging concepts deal with commodity price risk and uncertainty problems facing inventory holders, they do not fully apply to the more general risk and uncertainty problems of primary producers. Further, the recent emergence of livestock futures trading, primarily live cattle futures, has created the need to modify traditional hedging theory to deal with hedging decisions of primary producers of non-storable commodities.

A recent study has attempted to extend hedging theory to problems of primary producers.[2/] Separate studies have attempted to modify traditional futures theory for application to futures markets for livestock commodities, primarily live beef cattle.[3/] This paper draws from each of these studies, and with certain additional extensions and modifications, attempts to develop a hedging decision model for primary producers of non-storable commodities. Throughout the paper, specific attention is given to the problems faced by cattle feeders in the economic short-run. Although aggregate implications are unavoidable, the basic analysis is micro-economic and limited to the individual farm firm.

2/Ronald McKinnon outlined a theoretical hedging decision model for grain producers in connection with his study of the price and income policy uses of futures markets /13/.

3/Paul and Wesson pioneered the "price of feedlot services" concept in cattle futures /14/ and R. L. Ehrich extended and applied the concept in his empirical analysis of the spread between cash prices of feeder cattle and futures prices for fat cattle /2/.

Section I of the paper reviews the role of knowledge in microeconomic decisions and the sources of risk and uncertainty for primary producers. Section II examines basic concepts of hedging, and Section III develops a hedging decision model.

I. Knowledge, Risk and Uncertainty

Among the most important assumptions in the traditional short-run static theory of the firm are the assumptions regarding knowledge. This section reviews (1) the perfect knowledge assumption, (2) optimum decisions under perfect knowledge, (3) definitions of risk and uncertainty, (4) sources of risk and uncertainty, and (5) alternative methods of dealing with risk and uncertainty.

Perfect Knowledge Assumption

Traditional theory assumes the existence of perfect knowledge or, in Knight's terminology, "practical omniscience" [12, p. 197]. Such an assumption requires the decision-maker to possess sufficient information and foresight to be able to (1) identify all the relevant production alternatives open to him and (2) identify the single outcome (from among the set of possible outcomes) which will result from each contemplated action. Once the alternatives and outcomes are identified, the decision-maker must specify a rank or order among strategies in accordance with his preferences for the outcomes associated with the specific actions which are required to effect the various alternative strategies. The actions eventually taken will demonstrate a rational choice or decision

as long as (1) the preference order of outcomes is consistent with the individual's value structure, and (2) the strategy chosen is consistent with the individual's preference order.

In the ideal world of perfect knowledge and complete rationality, micro-economic decisions are infallible. Indeed, in such a world, it might be argued that decisions, per se, are non-existent, in that actions become automatic and unplanned responses to objective facts and subjective values. Errors of fact can not exist and actions based on such facts, consistent with subjective values, can not be wrong. An element of choice or decision remains in this ideal world, however, to the extent that differences or changes in individual objectives may lead to the selection of different strategies.[4/]

Optimal Decisions Under Perfect Knowledge

Basic theory of the firm identifies three broad types of production decisions. Management must decide (1) what products to produce, (2) how much of each to produce, and (3) how to produce each product. In the ideal world of perfect knowledge and complete rationality, the answers

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[4/] If infallible micro-economic decisions result under perfect knowledge, then the perfect knowledge assumption implies that operators possess perfect predicted future knowledge or perfect foresight as well as perfect knowledge of the past and present. Allowing changes in an individual's values and objectives, or differences among individuals in an economy regarding values and objectives, is not inconsistent with the perfect foresight implication of the perfect knowledge assumption. However, in a macro-economic context the two assumptions (perfect knowledge and changes in objectives) are intuitively incompatible.

to these questions are easily obtained from a simple textbook model which assumes a net returns or profit maximizing objective for a firm operating under perfect competition. For example, a general model for multiple inputs and outputs may be written to maximize net revenue subject to given production functions or physical transformation relations between inputs and outputs [4, pp. 72-75].[5]

$$\text{Max NR} = \sum_{i=1}^{n} p_i q_i + \lambda F(q_i, \ldots, q_n) \qquad (1)^{6}$$

where:

NR = revenue minus costs

p_i = price of input or output i

q_i = quantity of input or output i

λ = unspecified scalar

The solution may be found by taking the partial of NR with respect to q_i and λ, setting the partials equal to zero, and solving the resulting equations for the values which satisfy the familiar first- and second-order optimizing conditions.

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[5] Other models may of course be written to maximize net revenue subject to budget or resource constraints. The calculus may be used to solve problems involving continuous functions with continuous first- and second-order partial derivatives in which the constraint is to be exhausted. A linear programming approach may be used to solve problems involving linear functions subject to a system of linear inequality constraints [4, p. 75].

[6] Notation is simplified by letting $[q_i, \ldots, q_n]$ include both inputs and outputs where outputs are numbered $[i = (1, \ldots, s)]$ and inputs (written as negative outputs) are numbered $[i = (s + 1, \ldots, n)]$.

Definitions of Risk and Uncertainty

In popular usage, the term risk refers to the possibility of unfavorable events (e.g. financial losses) arising as undesirable, and to some extent, unexpected contingency outcomes of chosen strategies and actions. In classical economic usage, risk is defined in terms of objective probability. Knight defines risk as measurable uncertainty of either an *a priori* or statistical variety [12, pp. 215-239]. To qualify as a risk situation in the classical sense, the distribution of outcomes from a group of instances is known either by calculation from basic mathematical principles or by calculation from statistics of past experience.

For both *a priori* and statistical probability, a distribution of outcomes is possible and measurable only because a valid basis exists for classifying instances. Under uncertainty, however, no valid basis exists for classifying instances and thus uncertainty is characterized by highly subjective probabilities which Knight calls "estimates" [12, pp. 215-239]. This third type of probability or estimate may be considered an estimate of an estimate. The decision-maker must not only form a subjective estimate of the probable outcomes of decisions but also must estimate the probability that his estimate is correct.

Although Johnson and others have provided different classifications of the objective and subjective imperfect knowledge states, it is sufficient for this paper to distinguish between risk and uncertainty on the broad basis of the measurability of outcome distributions [9]. However, for the ease of discussion, the term uncertainty is used

throughout the paper as a general term to include all decision situations under imperfect knowledge. Likewise, the term risk is used in its more popular sense denoting the attitude of entrepreneurs toward unfavorable contingencies. Circumstances requiring a more rigorous distinction of the meanings of risk and uncertainty are identified as such and the meanings of these terms are specified in each case.

Sources of Uncertainty

According to Knight, errors occur in the decision process since "we do not perceive the present as it is and in its totality, nor do we infer the future from the present with any high degree of dependability, nor yet do we accurately know the consequences of our own actions." [12, p. 202]. Further errors occur since "we do not execute actions in the precise form in which they are imaged and willed" [12, p. 202]. Thus, as a consequence of imperfect knowledge, decision-makers are faced with risk and uncertainty traceable to subjective failures of perception, inference, foresight, and execution.

The sources of uncertainty are not confined to the subjective and internal factors listed above. Human limitations of the individual entrepreneur certainly cause him to err in his decisions, and in regard to failures of perception and execution, the responsibility for errors in decisions is largely that of the individual alone. On the other hand, errors due particularly to limitations in inference and foresight are compounded by forces external to the firm and by internal factors not controlled by the decision-maker. A suitably designed classification

of the sources of uncertainty must therefore account not only for factors internal to the firm and its management, but also for (1) external and uncontrolled internal factors, (2) conditions resulting from the interaction of internal and external factors, and (3) conditions facing the entrepreneur resulting from the aggregate of decisions made by all individuals in the economy.

If we assume that the short-run production process of farmers starts with the acquisition of variable inputs (given a set of fixed resources) and extends through the sale of products to off-farm buyers, and if we assume that the short-run planning horizon is practically concomitant with the actual production period, then we can identify two major sources of entrepreneurial uncertainty. Technical uncertainty arises from imperfect knowledge of the production function and the quantitative or physical relationships among inputs and outputs associated with and derived from the production function. Market uncertainty arises from imperfect knowledge of present and future prices of inputs and outputs. Market uncertainty also may be related quantitatively to imperfect knowledge regarding the future availability of inputs or the existence of market outlets.

Technical uncertainty leads to variations in the quantity of output from a given input package or to variations in the quantity of inputs required to produce a given level of output. Physical production efficiency is thus reduced since vagaries in inputs or outputs become increased costs of production over time. Market uncertainty leads to variations in the value of goods produced or purchased due to changes

in prices over time. The short-run effect of both technical and market uncertainty is that production-marketing decisions made on the basis of expectations, even if expectations prove to be correct, will lead to a misallocation of the firm's resources.[7/]

In many respects, the forces leading to market uncertainty are more complex than those leading to technical uncertainty. For this reason, it is useful to distinguish further categories of uncertainty applicable primarily to market factors.

Houthakker has divided uncertainty into social and individual uncertainty [7, pp. 141-144]. Social uncertainty refers to a situation in which individuals are certain or definite about their own decisions and actions under different sets of economic conditions but are uncertain about the decisions of other individuals. Social uncertainty faces farmers as a group, is "due exclusively to the fact that many individuals take part in production and consumption," and creates uncertainty with respect to economic conditions resulting from the aggregate of individual decisions [7, p. 141].

Individual uncertainty refers to a situation in which individuals are uncertain or indifferent regarding their own appropriate decisions as well as those of other people. Individual uncertainty results in part from the existence of a state of social uncertainty since a primary reason for individuals not knowing, *ex ante*, the correct *ex post*

7/D. Gale Johnson outlines an argument which supports the point of this statement [8, pp. 44-45].

decision is the fact that they do not know the future economic conditions resulting from the aggregate of individual decisions.

In a similar fashion, Hicks has identified three possible sources of uncertainty disequilibrium in an economy in which expectations of individuals are definite, namely, (1) inconsistent price expectations among individuals, (2) inconsistent buying and selling plans among individuals, and (3) incorrectly foreseen wants by individuals [5, pp. 133-135]. A fourth type of uncertainty disequilibrium may result if individual decisions are based on discounted expected futures prices accounting for risk rather than on discounted expected prices not accounting for risk [14, pp. 133-135].

Venkataramanan states that social uncertainty is comparable to the first two of Hick's categories which Hicks calls "inconsistency disequilibrium," and that individual uncertainty is related to the third and fourth of Hicks' categories [18, p. 8]. Both the Hicks and Houthakker classifications refer to knowledge and rationality states within individuals as they are related to the aggregate uncertainty existing in an economic environment composed not only of concrete things but also of imperfect human beings possessing multiple abstract ideas and diverse values.

Methods of Dealing with Uncertainty

Diversification, flexibility, insurance, buffer stocks, and forward contracting are frequently advocated as means of coping with risks arising from uncertainty. Each is inadequate,

Diversification refers to the practice of producing several commodities during a production period rather than only one. It may also refer to multiple marketings or the use of different combinations of inputs to produce quantities of the same product. The effect of diversification is to reduce the variations in total farm income between production periods usually at the sacrifice of expected mean income. The justification for dispersing a firm's resources among several production processes rests on the assumption that unfavorable and unforeseen events will affect the production-marketing outcomes selectively. To the extent that forces affecting outcomes of one process are independent of those affecting other processes, total farm returns can be protected from wide fluctuations. Johnson has pointed out, however, that technical and market factor variations for a firm's alternative production processes tend to be highly correlated, indicating that the possible reduction in income variations from diversification is probably no greater than 25 percent [8, p. 51]. He further indicates that diversification does little to reduce the range of income, concluding that diversification "is not an effective measure to protect a firm from abnormal price or production conditions"[8, p. 53].

Flexibility in production refers to the degree, ease, and frequency with which resource uses may be shifted and production processes altered within the firm in response to changes in economic conditions or expectations. Over several production periods, increased flexibility should result in higher average profit expectations at the expense of increased average variable costs of production. However, it is

Johnson's view that for the individual firm, "the types of flexibility required for short-run shifts of the relative outputs...are largely unnecessary and are an outgrowth of mistaken price expectations" [8, p. 56]. That is, short-run flexibility has a high individual and social cost.

Insurance is one means of protecting the individual firm from certain kinds of production losses. For example, a farmer can insure farm buildings against destruction by fire. He may also purchase wind or hail crop damage insurance. These events are insurable because they are independent in occurrence, the probabilities of occurrence are calculated for large numbers, and the probabilities of occurrence decline as underwriters' commitments increase. However, risks of loss (to holders of inventory or primary producers) due to a decline in prices are uninsurable risks in the normal sense [1, p. 1]. For instance, a price decline affects all stocks of the commodity equally (thus unfavorable events are not independent in occurrence). Further, the risks of loss increase rather than decrease with the size of commitments. "This explains why neither risks due to technical uncertainties affecting total or a very large portion of total supplies (e.g. vagaries of the weather affecting the entire crop) nore risks of fluctuations in market values due to other causes are convertible into 'cost' by means of ordinary insurance" [1, p. 1].

Maintaining buffer stocks or buffer funds is another action a farmer might take in response to technical and market uncertainties. Buffer stockholding requires the farmer to invest in and store

quantities of the commodities he produces. In years of reduced output, he withdraws and sells a portion of his stocks, while in years of higher than average output he increases the inventory out of current production. The effect of buffer stockholding is to even out fluctuations in physical output and thus stabilize period-to-period variations in income due to technical uncertainty. Buffer funds operate similarly except that instead of holding actual commodity stocks, the farmer keeps a reserve of highly liquid but stable valued assets to draw down and increase in periods of below and above average income, respectively.

McKinnon has shown that buffer stocks and funds can help farmers reduce some of the adverse effects of uncertainty under selected conditions [13, pp. 852-855]. To be effective, however, buffer stocks must be used in conjunction with a system of forward contracting since alone, buffer stocks are a relatively poor means of dealing with price fluctuations in combination with output fluctuations. Also, buffer stocks would be inappropriate for non-storable commodities such as beef cattle. The principal disadvantage of a buffer fund is that it requires a substantially greater capital commitment to maintain income stability than is desirable for the firm, particularly the firm facing tight external capital rationing.

The deficiencies of cash forward trading are not so clear as the inadequacies of the previously mentioned strategies for dealing with uncertainty. Cash forward trading, or forward contracting, consists of buyers and sellers entering into a formal and enforceable agreement to transfer ownership and possession of a commodity at a later date.

For example, a farmer with 10,000 bushels of #2 corn in on-farm bin storage could in October make an agreement with the local grain merchant to deliver 10,000 bushels of #2 corn to the merchant's elevator on a specified day (or days) in the coming January. Assuming no quantity or quality changes in the stored corn, the price agreement made in October removes all of the farmer's uncertainty with respect to the returns from his stored corn. That is, the farmer is protected from risks of loss due to a decline in corn prices from October to January.

So long as the forward price agreed upon is satisfactory to the participants, changes in the price of corn between October and January should be irrelevant. In the case of the producer, two conditions may disturb this bliss, however. For example, suppose the market price of corn rises substantially between October and January, so that in January the farmer could have sold his corn for much more than he agreed to in October. Since his contract is binding, the producer must deliver and simply regret his earlier decision and lack of foresight. Furthermore, suppose that rodents infested the corn and destroyed or reduced the quality of some of the 10,000 bushels the farmer intended to deliver. The farmer would be forced to acquire additional supplies of corn from reserve stocks or the open market in order to meet his contract. If corn prices have gone up between October and January or if the quality of corn needed is in short supply, then acquiring replacement stocks may be very costly. A similar example could be drawn for beef production. In short, forward trading may increase risks of loss due to technical uncertainties and may increase or at least not reduce risks of regret.

Houthakker argues that "social uncertainty can be eliminated by forward trading..." [7, pp. 141-144]. If "everyone accepts the forward price as a perfect prediction of the (future) spot price" and if "every individual participates in forward trading to the full extent of his foreseeable position," then the forward price resulting from transactions between forward buyers and sellers will "be equal to the spot price that would prevail in the absence of social uncertainty and unintentional stocks...will not emerge" [7, p. 142].

Forward trading cannot, however, eliminate individual uncertainty. The possibility of regret associated with a forward transaction decision plus the technical uncertainties present influence the willingness of persons to participate in forward trading. (The willingness to participate in forward trading is at least one indication of the degree of individual uncertainty present.)

Complete coordination of economic activity can only be achieved if all individuals participate in forward trading. Yet "under conditions of individual uncertainty, even complete participation...would be no guarantee that the forward price would agree with the ultimate spot price" [7, p. 143]. In such circumstances, plans made on the basis of a forward price will result in an allocation of stocks which is partly uneconomic under the ultimate spot price. For example, Houthakker cites the case of a crop turning out to be smaller than anticipated. The ultimate spot price is above the forward price on which many decisions and commitments were made. "The result of forward trading therefore has been not that risks of a small crop have

been eliminated, but merely that they have been shifted from forward buyers to forward sellers. Conversely, the risks of a large crop have been shifted from forward sellers to forward buyers" [7, p. 143].

Since the risks of individual uncertainty can not be eliminated but merely transferred from one group to another, the question arises as to what group would agree to accept risks. Usually the participants in forward trading are persons whose business income is derived from the transformation of inputs into outputs or the providing of time, form, and place utility with respect to a particular commodity. They may reap entrepreneurial rewards for the risk bearing associated with their activities, but their prime motive is usually to earn stable returns to other resources and reduce as much income variation due to uncertainty as possible. In order for both conventional parties to a forward trade to shift the risks of unfavorable events, a third, outside group of persons must be induced to assume these risks. That is, speculators willing to put up "risk capital" to forward buy or sell commodities in anticipation of gains due to favorable price changes must be induced to participate in forward trading. Cash forward trading, however, greatly limits the amount of "risk capital" entering to assume such burdens since cash forward trading requires the outright purchase or sale of the physical goods and thus requires large amounts of money capital. Further, cash forward trading also ultimately involves the actual delivery and possession of the commodity even though in the case of transferrable forward contracts, the

rights to possession or disposition of the commodity may have changed hands several times.

The five methods discussed above do not exhaust the strategies available to primary producers for dealing with risk and uncertainty. For example, a farmer's participation in government price support programs would tend to reduce his risks of loss due to major commodity price declines. Vertical coordination arrangements (in addition to forward contracting which has already been discussed) might also be expected to moderate the effects of market and technical risk and uncertainty. Some firms might be able to exercise a degree of market or bargaining power to obtain favorable trade agreements which shift some risks to others, such as in the selection of an FOB point for pricing commodities. It is beyond the scope of this paper to examine each of these additional methods. However, such an examination would reveal limitations in these methods equally as serious as those of the five methods discussed in detail.

Deficiencies of the frequently advocated means of dealing with risk and uncertainty are prime reasons for hoping that futures markets might improve primary producers' decisions under imperfect knowledge. We now turn to the consideration of concepts basic to hedging through futures markets.

II. Hedging Concepts

This section examines basic concepts of hedging through futures markets and elements of traditional hedging theory.

Conventional theory defines hedging as taking a position in futures markets which is equal and opposite to a similar position already held or anticipated in physical units of the cash commodity. In actual practice, hedging activities usually do not strictly conform to this definition. The closest correspondence between the definition and actual practice occurs in the case of a grain merchant or processor who "short hedges" his total inventory during the storage period. That is, a pure hedger hedges all stocks carried forward by selling futures contracts equivalent to the quantity of the commodity which he has in storage. A pure hedger might also be a "long hedger" if he buys futures contracts equivalent to the quantities of the commodity which he expects to acquire through the cash market at a later date.

Hieronymus identifies four principal ways futures can be used in connection with the farm business /6/:

1. to fix the price of a crop before harvest
2. to fix the price of grain in storage for later delivery
3. to fix the cost of feed without taking immediate delivery
4. to speculate in the price of a crop that has been produced but for which storage is not available.

Each of the above uses of futures can be interpreted to include livestock as well as crop enterprises.[8/] For either livestock or crop production, the first two uses involve short futures positions while the latter two involve long futures positions. Only the first three uses qualify as hedging, however.

Since hedging involves the interrelation of both cash and futures markets, it is helpful to recognize categories of market participants and the kinds of prices which the markets generate.

Peston and Yamey divide market participants into categories based on the type of business activity and futures trading they undertake /15, pp. 355-357/.

1.	pure hedgers:	hedge all stocks carried forward
2.	pure merchants:	carry forward only unhedged stocks
3.	pure speculators:	deal in futures but not in cash commodities
4.	mixed traders:	carry forward both hedged and unhedged stocks
5.	mixed speculators:	carry unhedged stocks and buy futures as well.

The above categories refer to both long and short positions.

8/For livestock enterprises the wording of these uses can be changed to the following:

1. to fix the price of livestock prior to marketing
2. to fix the price of livestock in production for later delivery (uses 1 and 2 become the same thing for livestock production)
3. to fix the cost of feed without taking immediate delivery
4. to speculate in the price of livestock that has been produced but which cannot be stored.

Blau defines four basic prices generated by cash and futures markets [1, p. 7].

p = cash spot price = current price ruling in cash market for spot delivery

p^* = cash forward price or expected cash price = current price ruling in cash market for future delivery

q = near futures price = current price ruling in the futures market for near term delivery

q^* = distant futures price = current price ruling in the futures market for deferred delivery

All the above prices are defined and exist at the same point in time, say t_0. Thus, at t_0 relationships exist between any combination of these prices. Through time the relationships existing at each point in time establish price relation paths.

The above prices may be used to indicate the returns from hedging. Kaldor, Blau, and others have considered that the net carrying costs per unit of stocks held in inventory consist of the physical costs of storage, a risk premium, and a convenience yield.[9]

$$m'(s) = w'(s) - c'(s) + r'(s) \qquad (2)$$

where $w'(s)$ = marginal physical cost of storage

$c'(s)$ = marginal convenience yield

$r'(s)$ = marginal risk premium

(s) = level of stock holding

9/Convenience yield refers to the return realized by inventory holders (applicable mainly to processors) due to having stocks on hand which can be used to meet immediate needs, for example, in the event the flow of processing supplies is interrupted [1].

The expected return from holding a unit of any commodity from time t_0 to time t_1 ($u^*_{t_0t_1}$) is equal to the expected cash price at t_0 for t_1 ($P^*_{t_0t_1}$) minus the current spot price ($P_{t_0t_0}$) and the marginal net carrying costs to carry stocks from t_0 to t_1 ($m'_{t_0t_1}$)

$$u^*_{t_0t_1} = P^*_{t_0t_1} - P_{t_0t_0} - m'_{t_0t_1} \tag{3}$$

The expected return from holding a unit of hedged stocks ($h^*_{t_0t_1}$) is similarly defined as the expected return from holding a unit of unhedged stocks plus the per unit gains or losses expected from the futures operation:

$$h^*_{t_0t_1} = (u^*_{t_0t_1}) + (q^*_{t_0t_1} - q^{**}_{t_0t_1} - n) \tag{4}$$

where

$q^*_{t_0t_1}$ = current futures price at t_0 for delivery at t_1

$q^{**}_{t_0t_1}$ = futures price expected at t_0 to prevail at t_1 for delivery at t_1 (a double expectation)

n = futures transaction costs for hedging one unit of the commodity. Includes brokerage fees and interest charged on margin money.

Rewriting equation (4):

$$h^*_{t_0t_1} = (q^*_{t_0t_1} - P_{t_0t_1}) - (q^{**}_{t_0t_1} - P^*_{t_0t_1}) - (m' + n') \tag{5}$$

Thus, expected returns from hedging can be defined as the change in basis from t_0 to t_1 minus the costs of storage and futures transactions. (Basis is the term used to refer to the spread or margin between cash and futures prices.)

Certain aspects of the theory of hedging through futures trading have been the subject of considerable controversy.[10/] Keynes and Hicks were among the early writers to argue that hedging by holders of inventory is motivated by the desire to reduce the risks of loss due to adverse changes in prices during the storage period [5, 11]. Thus, the Keynes-Hicks theory of "normal backwardation" rests on the basic premise that hedgers pay a "risk premium" to speculators for the privilege of shifting their price level risks to speculators. Since hedgers are net short in futures and speculators net long, the theory of normal backwardation says that the current price prevailing in the futures market for deferred delivery falls below the expected spot price by the amount of the risk premium.

It has been argued that if the theory of normal backwardation holds, and a risk premium is paid to long speculators, then (1) futures prices should trend upward during the life of any futures contract (i.e. futures prices are downward biased estimates of future spot prices) and (2) long speculators should make money on the balance of their futures transactions and short hedgers should lose on futures transactions. Empirical studies testing for the existence of these implications have provided a basis for questioning the validity of the Keynes-Hicks theory of normal backwardation [18, pp. 28-50].

[10/] Venkataramanan provides a good discussion of the conflicting views regarding hedging theory [18, pp. 21-50].

In contrast to the Keynes-Hicks views, Working has argued that hedgers are not motivated primarily by the desire to reduce or shift risks [19]. Rather, hedging is undertaken primarily to gain from anticipated favorable changes in the basis, or the cash-futures price spread, over time. That is, hedging is a form of arbitrage involving the cash and futures price relation, or more radically, hedging is speculation in the basis. In support of his views, Working cites evidence of selective hedging and the lack of a stable basis. [19,20].

More recently, the traditional theory of hedging has been reformulated by Johnson and Stein. Johnson has concluded that "hedging activities appear to be motivated by the desire to reduce risk, as described in traditional theory, but levels of inventory held appear to be not independent of expected hedging profits, as emphasized by Working" [10, p. 142]. Johnson thus developed a model to determine the level of hedging to minimize the variance of expected returns given subjective estimates of expected returns and variance as well as the hedger's utility function. In a similar fashion, Stein developed a model to determine the optimum combination of hedged and unhedged stockholding to minimize risks and at the same time to infer the nature of forces producing changes in spot and futures prices [16].

The Johnson-Stein reformulation of the theory of hedging is used to develop the general hedging decision model in Section III.

III. Hedging Decision Model

Our development of a hedging decision model for cattle feeders begins by tracing out the important and relevant aspects of the Johnson-Stein formulation of the theory of hedging. The model is then modified to include considerations of the more general technical and market uncertainties facing primary producers. The third step in the development involves extending the model to cover specific decisions of cattle feeders or other producers of non-storable commodities.

Johnson-Stein Formulation

Johnson defines a hedge as a position in market j of x^*_j units such that, given x_i units of the actual commodity held in market i, the price risk is minimized for holding x_i and x_j from t_0 to t_1 [10, pp. 142-143]. Price risk is measured by the variance of a subjective probability distribution, or the standard deviation of a subjective probability density function. Price risk is thus wholly a subjective estimate (possessed by the operator at t_0) of the change in prices to occur from t_0 to t_1. The actual price change from t_0 to t_1 is considered a random variable. The total variance of return V(R) due to price changes from t_0 to t_1, is equal to the variance of return or price risk in holding x_i units ($x_i^2\sigma_i^2$) plus the variance of return or price risk in holding x_j units ($x_j^2\sigma_j^2$) plus the covariance of returns due to price change between i and j markets ($2x_ix_j\ cov_{ij}$), or:

$$V(R) = x_i^2\sigma_i^2 + x_j^2\sigma_j^2 + 2x_ix_j\ cov_{ij} \qquad (6)$$

From the combination of positions x_i and x_j both actual returns R and expected returns E(R) may be written as:

$$R = x_i B_i + x_j B_j \tag{7}$$

and

$$E(R) = x_i u_i + x_j u_j \tag{8}$$

where

B_i = actual price changes from t_0 to t_1 in i

B_j = actual price changes from t_0 to t_1 in j

u_i = price changes from t_0 to t_1 in i expected at t_0

u_j = price changes from t_0 to t_1 in j expected at t_0

To find the value of x^*_j which minimizes the variance of return for $x_i x^*_j$, differentiate equation (6) with respect to x_j and set the derivative equal to zero:

$$\frac{\partial V(R)}{\partial x_j} = 2x_j \sigma_j^2 + 2x_i \, cov_{ij} = 0 \tag{9}$$

$$x^*_j = - \frac{x_i . cov_{ij}}{\sigma_j^2}$$

Substituting (9) back into (6) and defining V(R)* as the total variance of return of (x_i, x^*_j), the minimum variance with hedging is defined:

$$V(R)^* = x_i^2 \sigma_i^2 + \frac{x_i^2 \, cov_{ij}^2}{\sigma_j^2} - \frac{2x_i^2 \, cov_{ij}^2}{\sigma_j^2} \tag{10a}$$

or

$$V(R)^* = x_i^2 (\sigma_i^2 - \frac{cov_{ij}^2}{\sigma_j^2} \tag{10b}$$

but if the coefficient of correlation between price changes in i and j is:

$$\rho = \frac{cov_{ij}}{\sigma_i \sigma_j}$$

then

$$V(R)^* = x_i^2 \sigma_i^2 (1 - \rho^2) \tag{11}$$

Since the absolute values for the coefficient of correlation (ρ) are less than or equal to 1, the higher these values, the greater the reductions in price risk which result from hedging. That is, if the trader at t_0 believes that any price changes in x_i from t_0 to t_1 will be perfectly correlated with price changes in x_j, then $\rho = 1$ and $V(R)^* = 0$.

In Johnson's formulation, the effectiveness of hedging (e) is measured by computing a ratio of the variance from the combined ($x_i x^*_j$) position ($V(R)^*$).

$$e = (1 - \frac{V(R)^*}{x_i^2 \sigma_i^2}) = \rho^2 \tag{12}$$

As in the case of price risk itself, the effectiveness of hedging is a subjective concept measuring the reduction in variance resulting from hedging as viewed <u>ex ante</u> by the trader. That is, hedging is perfectly effective, in the trader's view, when the total <u>ex ante</u> variance of a combined position in cash and futures markets is zero

in the presence of positive ex ante variance in returns from the cash market alone. Unlike a perfectly effective hedge in traditional futures theory, neither actual nor expected returns need equal zero since in the Johnson formulation, hedging effectiveness is not measured by comparing the actual returns in hedged relative to unhedged positions.

Both Johnson and Stein illustrate geometrically the selection of the optimum combination of hedged and unhedged stocks. The two techniques are somewhat different but largely equivalent. For expositional convenience the Stein version is presented [16, pp. 1013-1015].

Stein assumes that the possessor of stocks can either forward contract to sell stocks at a fixed price or hold stocks for later sale at an uncertain price. If stocks are held for later sale, they may be held in either hedged or unhedged form. Stein's thesis is that the stock holder will allocate his inventory between hedged and unhedged positions in such a way as to maximize his expected utility.

Similar to equations (3) and (5) in Section II, expected per unit returns from holding unhedged and hedged stocks, respectively, can be written:

$$u = p^* - p - m \tag{13}$$

and

$$h = (p^* - p) - (q^* - q) - m \tag{14}$$

where

p^* = spot price expected to prevail at a later date

p = current spot price

m = marginal net carrying costs consisting of marginal costs of storage and marginal convenience yield

q = current futures price

q* = price of futures expected at a later date

For hedging at t_0, stocks are bought at price (p) and futures sold at (q). At t_0 the hedger expects to sell his stocks at t_1 for price (p*) and repurchase the previously sold futures for price (q*). So long as stocks may be used for delivery on the futures (if it turns out that it would be less costly at t_1 to deliver futures rather than buy futures and sell spot) the losses from stock holding are limited to the value represented by (q - p - m) [16, p. 101].

Like Johnson, Stein assumes a symmetrical probability density function of returns and measures risk with the variance of expected returns. Thus, risk on a unit of unhedged stocks is equal to the variance of (u), which is equal, given (p) and (m) to the variance of (p*). Risk on a unit of hedged stocks equals the variance of (h) which is equal to (var p* + var q* - 2 cov p*q*) if given (p), (m), and (q). As the proportion of hedged stocks varies from zero to 100 percent of total inventories, expected returns range from (u) to (h) and the risks vary from (var. u) to (var. h), respectively.

It is assumed that traders possess a declining marginal utility of income and an indifference curve between risk and expected returns which is convex from below when expected returns are measured on the Y-axis (Figure 1).[11/] To remain on the same indifference curve,

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11/For livestock producers the marginal utility of wealth may be as relevant a determinant of behavior as the marginal utility of income.

increases in variance of returns must be accompanied by proportionately greater increases in expected returns. Points on the same indifference curve represent combinations of risk and expected returns which yield equal subjective utility. Successively higher indifference curves represent successively higher levels of utility (I_0, I_1, I_2, etc.).

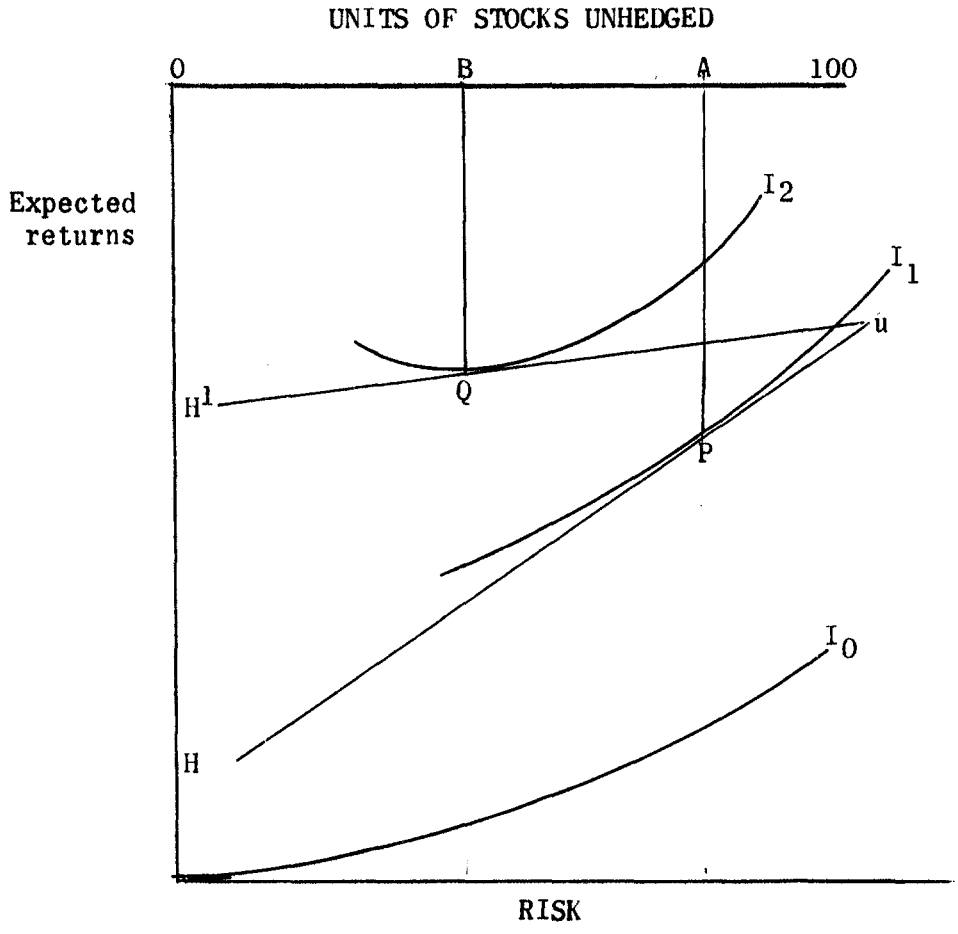

Figure 1.

Line HU in Figure 1 represents an opportunity locus of risk and expected returns for 100 units of inventory as the proportion of inventory hedged varies from zero to 100 percent. If no stocks are hedged, (Point U), expected returns = U and risk = var U while if all stocks are hedged (point H), expected returns = h and risk = var h. Line HU assumes that unhedged stocks have both higher risks and expected returns than hedged stocks. Given HU and I_1, the optimum quantity of unhedged stocks equals OA and hedged stocks = 100 - OA defined by point P which maximizes expected utility from holding 100 units of stocks.

A new opportunity line H'U may be defined if the expected returns from holding hedged stocks increases due possibly to a ceteris paribus increase in the futures price (q). In such a case, point Q defines a new optimum combination of hedged and unhedged stocks. Such a change produces both income and substitution effects.

Extension to Primary Producers of Crops

In Section II it was suggested that farmers could use futures markets to fix the price of a crop prior to planting. The hedging models discussed so far have provided a means of determining the optimal hedge for an inventory holder whose income variations or risk depends only on market uncertainty or variations in prices. A fundamental difference exists, however, between merchants holding stocks and farmers or primary producers. The primary producer faces technical uncertainty, or variations in output as well as market uncertainty, or variations in prices. It is clear from the previous

analysis that the optimal hedging decisions of primary producers will thus depend on the variance of his "output relative to the variance of prices and on the way in which the two are related" [13, p. 845].

Following McKinnon, assume that a particular planting decision has been made in accordance with an assumed fixed production opportunity [13, p. 846]. To determine the optimum futures sale at planting time, assume that harvest time (t_1) output (X) is a random variable at planting time (t_0) with fixed and known mean (μ_x) and variance (σ_x^2). Further assume that the harvest time spot price (p) is a random variable with known mean (μ_p) and variance (σ_p^2) but that in the absence of transactions costs and backwardation, the futures price at planting time (P_f) for harvest time delivery is the expected harvest time spot price. Income or returns (Y) then is a function of variables (X) and (P), the parameter (P_f) and the level of hedging (X_f).

$$Y = PX + (P_f - P)\, X_f \tag{15}$$

Expected income (E(Y)) is determined independently of the level of hedging (X_f):

$$E(Y) = E\,(P \cdot X) + X_f E(P_f - P) = E(P \cdot X) \tag{16}$$

The variance of income (σ_y^2) does, however, depend on the level of hedging (X_f). To specify the value of (X_f) which minimizes (σ_y^2) the correlation between (X) and (P) must be considered. McKinnon assumes that (X) and (P) have a bivariate normal distribution described by parameters (μ_x), (σ_x), (P_f), (σ_p) and (ρ) where:

$$\rho = \frac{\text{cov}\ (X,P)}{\sigma_x \sigma_p} \tag{17}$$

From the manipulation of equations (15) and (17), the variance of income can be expressed:

$$\begin{aligned} \sigma_y^2 &= E(Y^2) - [E(P \cdot X)]^2 \\ &= P_f^2 \sigma_x^2 + \mu_x^2 \sigma_p^2 + 2P_f \mu_x \rho \sigma_x \sigma_p \\ &\quad + (1 + \rho^2)\ \sigma_x^2 \sigma_p^2 - 2X_f P_f \rho \sigma_x \sigma_p \\ &\quad - 2X_f \mu_x \sigma_p^2 + X_f^2 \sigma_p^2 \end{aligned} \tag{18}$$

The optimal level of the hedge (X_f^*) is obtained by differentiating equation (18) with respect to (X_f) and setting the expression equal to zero:

$$X_f^* = \rho P_f \frac{\sigma_x}{\sigma_p} + \mu_x \tag{19}$$

or, as a proportion of expected output:

$$\frac{X_f^*}{\mu_x} = \rho \frac{\sigma_x / \mu_x}{\sigma_p / P_f} + 1 \tag{20}$$

where σ_x/μ_x is the coefficient of output variation and σ_p/P_f is the coefficient of price variation. The coefficient of output variation is a measure of the relative variation in output, compared to expected output while the coefficient of price variation is a measure of relative variation in spot price compared to the expected price (i.e. the futures

price for t_1 at t_0). McKinnon concludes from expression (20) that: "(a) the greater output variability is relative to price variability, the smaller will be the optimal forward sale, and (b) the more highly negatively correlated price and output, the smaller will be the optimal forward sale" [13, pp. 848-849]. It will be recalled that this conclusion is similar to Johnson's conclusion regarding the effect of correlation of spot and futures price changes.

By considering the optimum hedge in equation (19) and replacing (X_f) in equation (18) by the value of (X^*_f), the minimum variance of returns after hedging may be defined:

$$\sigma_y^{*2} = (1 - \rho^2) \; P_f^2 \sigma_x^2 + (1 + \rho^2) \; \sigma_x^2 \sigma_p^2 \qquad (21)$$

"In general, the optimum hedge reduced income variance by $(\rho \sigma_x P_f + \mu_x \sigma_p)^2$ as compared to no hedging" [13, p. 849].

Hedging Decision Model for Cattle Feeders

Cattle feeding differs in several basic ways from holding inventories or producing crops. Cattle feeding is a dynamic production process in which product form is continually in transformation during the production period. Cattle feeding is thus yielding time, form, and location utility rather than only time utility which is created by holding stocks. Furthermore, live "fat" cattle are non-storable commodities in the normal sense. Cattle can not physically be held in unchanging form nor can they economically be held in changing form for long periods after optimal market weights have been reached.

Unlike crop production, cattle feeding is not technically forced into a seasonal production pattern although seasonality of aggregate beef production and marketing may result from economic factors.

The technical and market risk and uncertainty facing beef cattle feeders is similar to that facing other primary commodity producers. At the time production or resource allocation decisions must be made, the cattle feeder does not fully know the consequences of alternative strategies. However, technical uncertainty takes a slightly different form in cattle feeding than in crop production. In crop production, technical uncertainty is manifest mainly in the variations of output around some level of expected mean output for a given level of controlled inputs. The technical uncertainty in beef production is manifest in a form of output variability based on the variation in input quantities (and qualities) and the variation in the length of feeding period necessary to produce the desired quantity. That is, variation in output, per se, is not so important a source of technical uncertainty as the variation in technical coefficients of the production function (including the length of feeding period) for producing a given level of output. Of course, holding inputs (including time) constant, allows the expression of technical uncertainty in terms of output variation as of a given point in time regardless of the specific sources of that variation.

Returns from cattle feeding can be stated ex post in terms of the familiar accounting identity [17, pp. 640-641]:

$$R_1 = W_s P_s - W_b P_b - (W_s - W_b)\ C \tag{22}$$

where

R_1 = returns per unit produced (e.g. per head or per drove), above the cost of the feeder animal and the cost of feed

W_b = beginning weight of feeders (pounds)

W_s = selling weight of finished cattle (pounds)

P_b = purchase price of feeders (dollars per pound)

P_s = selling price of finished cattle (dollars per pound)

C = feed cost (dollars per pound of gain)

Expression (22) for cattle feeders is the ex post equivalent to the expected returns from holding unhedged inventories if market or transactions costs are considered to be embedded in the prices paid and received. That is, we assume that the selling price of finished cattle (P_s), for example, includes a price received per pound (P_r) minus the average cost per pound of marketing the finished cattle (m).

If the cattle feeder enters into beef futures market transactions to hedge quantities of cattle fed, then his ex post returns are merely the sum of the net returns from cattle feeding and the net gain or loss on his futures transactions:

$$R = R_1 + [(Q_s - Q_b - N)\ X_h] = R_1 + R_2 \tag{23}$$

where

Q_s = selling price of futures (dollars per pound)

Q_b = repurchase price of futures (dollars per pound)

N = hedging transaction costs (dollars per pound)

X_h = quantity of beef hedged in futures (pounds)

It is obvious that, given the decision to feed cattle, hedging should be undertaken so long as $Q_s > Q_b + N$ since total combined returns from cattle feeding and hedging would be greater than returns from cattle feeding alone. Moreover, the quantity of cattle hedged under such conditions would be endogenously unlimited and exogenously limited only by the total quantity of fed cattle produced.

Optimal ex ante cattle feeding and hedging decisions are not as simple as ex post decisions. In the economic short run of say one production period, given a set of fixed resources or a fixed production plant (e.g. buildings and equipment), the cattle feeder is able to choose among several alternative production processes. Generally the class, grade, weight, sex, and age of feeders; the grade, weight, and age of finished cattle; the type of ration; the source of variable inputs; and the timing of purchases and sales are choice variables largely controlled by the cattle feeder in the short run. Given the existence of hedging opportunities, the cattle feeder is also able to decide whether or not to hedge and what quantities to hedge. Fundamentally, the cattle feeder must decide whether to use his resources to feed cattle, shift their use to other enterprises, or, if they have no alternative uses, let them remain idle. All of these decisions involve some degree of risk and uncertainty.

The existence and use of a live cattle futures market may materially improve cattle feeders' returns in principally two ways.

First, given the decision to feed a particular quantity of cattle in a particular production process, hedging may protect the cattle feeder from losses (or gains) due to changing conditions during the production period. That is, hedging may "fix" the outcomes from cattle feeding within a narrow range of possibilities so that optimal decisions of the firm at t_0 will remain in some sense optimal at t_1, even though different decisions might have been made if conditions had been perfectly foreseen. Second, beef futures and the prices they generate may be used directly in the decision process as a decision tool to aid farmers in choosing optimal product and input combinations. In other words, a hedging decision model may be used (1) to protect an optimal decision or (2) to make an optimal decision. In this paper we consider only the protection of an optimal decision.

Assume that the cattle feeder has already decided to feed cattle and has decided the type and magnitude of production to undertake in accordance with conventional optimizing procedures. Given his expected net returns from cattle feeding, he wishes to establish a short futures position of optimum size at t_0 which will minimize the variance of net returns or risk from the combined operation of feeding cattle and hedging. Expression (23) may be simplified to consider actual returns as:

$$R = x_i V_i + x_j V_j \tag{24}$$

where

x_i = number of cattle on feed

x_j = number of cattle units hedged

V_i = actual change in per head net value of cattle on feed from t_0 and t_1

V_j = actual change in unit value of futures contracts

Expected returns may be written:

$$E(R) = x_i v_i + x_j v_j \tag{25}$$

where

v_i = change in per head value of cattle on feed from t_0 to t_1 expected at t_0

v_j = change in per unit value of futures position from t_0 to t_1 expected at t_0

The variables (v_i) and (v_j) are mean values of the probability distributions of returns from cattle feeding and hedging, respectively. The total variance of returns from changes in values in feeding and hedging can be written as:

$$V(R) = x_i^2 \sigma_i^2 + x_j \sigma_j^2 + 2x_i x_j \, cov_{ij} \tag{26}$$

Equation (26) is differentiated with respect to x_j and set equal to zero, defining the optimum number of units to hedge to minimize the total variance of returns.

$$X_j^* = - \frac{x_i \, cov_{ij}}{\sigma_j 2} \tag{27}$$

Substituting (x_j*) for (x_j) in expression (26) and simplifying, the total variance of return with optimum hedging becomes:

$$V(R)^* = x_i^2 \sigma_i^2 (1 - \rho^2) \tag{28}$$

where the correlation coefficient between changes in the value of cattle on feed and the value of futures contracts, is

$$\rho = \frac{cov_{ij}}{\sigma_i \sigma_j} \tag{29}$$

The above model is identical to the one provided by Johnson [10]. We can conclude, as does Johnson, that the higher the absolute value of ρ the greater the reduction in the variance of returns due to hedging. We can also express the effectiveness of hedging as the variance of return from the combined cattle feeding and hedging operations relative to the variance of returns from cattle feeding alone, or as

$$e = \rho^2 \tag{30}$$

Unfortunately, the above model of optimal hedging decisions suffers from many limitations in interpretation and application. The change in the value of futures contracts from t_0 to t_1 is a simple function of the change in futures prices over the period. The change in the net value of cattle on feed, however, is a function of changes in the weight of cattle, the cost of feeding and the prices of finished cattle. That is, V_i and v_i, defined on a per head basis, result from changes in quantities and factor costs as well as product prices. Thus, V_i and v_i obscure relationships between the technical and market factors which give rise to net returns variability and thus also make difficult the interpretation of σ_i^2 and ρ.

To overcome these difficulties, a more specific model is needed which breaks out the variability in net returns due to technical and market factors. To develop such a model assume that hedging involves

no transactions costs. Actual net returns from cattle feeding and hedging may then be rewritten from expression (23) as:

$$R = [W_s P_s] - [(W_b P_b) + C\ (W_s - W_b)] + [X\ (Q_s - Q_b)] \tag{31}$$

In expression (31), the per head weight of cattle sold at t_1 (W_s), the price received per head sold at t_1 (P_s), the per head costs per pound of gain (C) and the futures price at t_1 for t_1 delivery (Q_b) are viewed at t_0 as random variables whose means (expected values) and variances are fixed and known from statistical calculations of past experience and other available economic data. The cost per pound of gain (C), is considered a random variable due to imperfect knowledge regarding (1) the actual sales weight of cattle at t_1 (i.e. uncertainty regarding the gain when the feeding period is fixed), (2) the actual efficiency of feed conversion, and (3) the factor prices for any inputs purchased during the production period. At t_0, the weight of feeders purchased (W_b), the price paid for feeders (P_b), and the current (t_0) futures price for t_1 delivery (Q_s) are known with certainty. The single choice variable at t_0 is the level of hedging (X) in terms of the number of fixed weight cattle units to be sold forward by establishing a short futures position.

For simplicity, we assume that the cost of feeder cattle per head sold plus the cost of gain per head sold can be written as a single random variable (V) with known mean and variance:

$$V = [(W_b P_b) + C\ (W_s - W_b)] \tag{32}$$

We also assume that at t_1 the actual spot price received (P_s) is equal to the actual futures price for lifting the hedge (Q_b)

$$\text{at } t_1, \quad P_s = Q_b \tag{33}$$

Expression (33) will hold in the absence of transactions costs, location or quality difference in the cattle priced in the cash and futures market, or other disturbances due to imperfect competition and arbitrage limitations in the two markets.

Substituting (32) and (33) into equation (31), net returns from cattle feeding and hedging may be rewritten as:

$$R = [(W_s P_s) - (V) - X\,(P_s - Q_s)] \tag{34a}$$

and dropping the subscripts:

$$R = (WP) - (V) - X\,(P - Q) \tag{34b}$$

Again assuming no transactions costs and no Keynes-Hicks normal backwardation the expected value of the t_1 spot price (P) is the current (t_0) futures price (Q):[12/]

$$E(P) = Q \tag{35}$$

12/This assumption relates to a significant difference between livestock and grain cash-futures price relations. Ehrich has argued that since grain is a storable commodity it can be reasonsed that expectations regarding future supply and demand conditions affect current cash prices and current future prices about equally [3]. Finished cattle or other livestock can not be "stored" in unchanging or low cost forms. Thus, current spot prices for cattle will reflect current supply-demand conditions while current futures prices for distant delivery will reflect expected supply-demand conditions. Near the delivery period both cash and futures price for cattle will be affected by current S-D conditions.

Given that the expected value of the t_1 spot price is the current futures price, the expected value of net returns is independent of the level of hedging and dependent only on the expected returns from feeding cattle:

$$E(R) = E\,(W \cdot P) - E(V) \tag{36}$$

The total variance of returns from cattle feeding and hedging is not independent of the level of hedging, however, and may be written in generalized form as:

$$\sigma R^2 = E\,[R - E(R)]^2 \tag{37}$$

Expression (37) may be expanded in general form assuming no particular underlying distribution relating the variables (W), (V), and (P). Expanding (37) and recombining terms:

$$\begin{aligned}\sigma R^2 = \Big\{&[E\,(W^2 \cdot P^2) + [E(V^2)] - 2\,[E(W \cdot P \cdot V)] - [E(W \cdot P)]^2 \\ &+ 2\,[E(W \cdot P)\,E(V)] - [E(V)]^2\Big\} + \Big\{X^2\,[E(P - Q)^2] \\ &- 2X[E(W \cdot P)\,(P - Q)] + 2X[E(V)\,(P - Q)]\Big\}\end{aligned} \tag{38}$$

The first term on the right hand side of (38) represents the variance of net returns from feeding cattle while the second term represents the variance of net returns from hedging. A more exact expression for the variance of returns from cattle feeding requires specific assumptions regarding the distributions of $(W \cdot P)$ or assumptions regarding the higher order moments of W, P, and perhaps V. According to McKinnon [13], such

assumptions are not needed to derive an expression of the optimal hedge (X*) which minimizes total variance of net returns (σ_R^2). To derive the optimal hedge, expand the second term of (38):

$$-2XE[(P - Q + Q)(W - \mu_W + \mu_W)(P - Q)] + 2XE[(V - \mu_V + \mu_V)(P - Q)] + X^2\sigma_p^2$$

$$= -2X\left\{E[(P - Q)^2(W - \mu_W)] + [Q\rho_1\sigma_W\sigma_p + \mu_W\sigma_p^2]\right\} + 2X(\rho_2\sigma_V\sigma_p) + X^2\sigma_p^2 \qquad (39)$$

To minimize (39), differentiate with respect to X, set equal to zero, then divide the resulting expression by (-2) and multiply by $(1/\sigma_p^2)$ to get the following:

$$X^* = Q\rho_1\frac{\sigma_W}{\sigma_p} + \mu_W - \rho_2\frac{\sigma_V}{\sigma_p} + \frac{1}{\sigma_p^2}E[(P - Q)^2(W - \mu_W)] \qquad (40)$$

Expression (40) is somewhat difficult to interpret since the economic meaning of the fourth term on the right-hand side is unclear. If, however, we assume that the third moment lininking W and P is zero (i.e. that the joint distribution linking W and P is symmetrical), then this fourth term is zero and the optimal level of hedging (X*) is given by the first three terms on the right-hand side of (40):

$$X^* = Q\rho_1\frac{\sigma_W}{\sigma_p} + \mu_W - \rho_2\frac{\sigma_V}{\sigma_p} \qquad (41)$$

The optimal level of hedging can also be written as a proportion of the expected level of output (μ_w) by dividing expression (41) by (μ_w):

$$\frac{X^*}{\mu_w} = 1 + \rho_1 \frac{\sigma_w/\mu_w}{\sigma_{p/Q}} - \rho_2 \frac{\sigma_v}{\sigma_p} \cdot \frac{1}{\mu_w} \tag{42}$$

The optimal proportion of expected output hedged thus depends on the degree of correlation between the finished weight of cattle and the price received for finished cattle (ρ_1) as well as the degree of correlation between the variable costs of production and the price received for finished cattle (ρ_2). Specifically, the less variation in finished weight compared to the expected weight relative to variation in the price received compared to the expected price, the greater the optimal proportion of output hedged since $-1 \leq \rho_1 \leq 0$. In comparison to ρ_1, ρ_2 is expected to be small and positive. It is also expected that the variation in costs of production is less than the variation in the price of finished cattle. Thus, the smaller the correlation between production costs and output prices, and the smaller the variation in costs relative to prices, the larger will be the optimal proportion of expected output hedged.

IV. Concluding Note

The major effort of this paper has been to develop a theoretical hedging decision model for cattle feeders. The paper attempts to provide a background for such a model by (1) reviewing the nature, sources and strategies for meeting the problems of risk and uncertainty

faced by primary producers and (2) outlining basic concepts of hedging through futures markets.

It was noted in Section III that hedging decision models may be developed for two types of production - marketing decision situations. Models may be developed to determine the optimal hedge given prior production-marketing decisions, or they may be used to determine optimal production-marketing strategies while also determining an optimal hedge. This paper has attempted to develop only a model of the first type i.e. to prescribe an optimal hedge for cattle feeders given basic production marketing decisions. For this reason the model closely resembles earlier models provided by Johnson [10], Stein [16], and McKinnon [13].

In its present form, the model developed does not readily lend itself to direct application by cattle feeders. Work is presently underway to obtain empirical estimates of the parameters and distributions required to "apply" the model. Further work is underway to extend the hedging decision model for application to basic production - marketing decisions as well as to the optimal hedging decision. The results of these efforts will be reported in subsequent publications.

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