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CERTAINTY EQUIVALENT PRICES AND PRODUCER WELFARE UNDER OUTPUT PRICE UNCERTAINTY

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Abstract

Producer welfare indices under price uncertainty are derived using the concept of certainty equivalent prices. In this approach the marginal cost function is used instead of the *ex-ante* supply function; the effects of uncertainty and risk preference are captured by certainty equivalent prices. Implications for welfare evaluation are discussed.

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CERTAINTY EQUIVALENT PRICES AND PRODUCER WELFARE UNDER PRICE UNCERTAINTY

1. Introduction

Economists have long recognized the importance of uncertainty and risk aversion in the behavior of entrepreneurs. A new body of literature evolved in the 1970s around the works of Sandmo [1971] and Leland [1972] and augmented microeconomics by introducing the theory of firm decision making under uncertainty. The literature has proliferated substantially since these early works, and recent studies incorporate multi product, multi risk and multi period considerations (see e.g., Hey [1979], Just *et al.* [1982] and Newbery and Stiglitz [1981]). Less attention has been given to the welfare implications of choices made by risk averse producers. Only recently have the welfare consequences of the Sandmo-Leland framework become the focus of a growing literature (Chavas and Pope [1981], Pope *et al.* [1983], Pope and Chavas [1985], Larson [1988]). Particular attention has been given to the extension of welfare measures to situations of risk aversion and uncertainty and to the development of practical means of approximating these measures. Compensating Variation (CV), Equivalent Variation (EV) and Certainty (money) Equivalent (CE) have been the three indices considered for measuring producer welfare under uncertainty. (Section 2 provides a brief presentation of these indices.) The main tool used to approximate these measures has been the producer surplus calculated from the *ex-ante* output supply and input demand functions.

Pope and Chavas [1971], using Willig's [1976] approach, show that producer surplus can be used "without apology" instead of EV and CV. Larson [1988] offers a procedure, in the spirit of that suggested by Hausman [1981], which evaluates these indices exactly. Both approaches rely upon the *ex-ante*

output supply and input demand functions. The evaluation of these functions in actual practice is in many case cumbersome, requiring data which are hard to obtain; thus applications rarely are found.

A procedure to evaluating the producer welfare indices without the need of the *ex-ante* supply and demand functions may therefore be useful. In this paper we describe such a procedure. It relies upon the concept of the Certainty Equivalent of Price (CEP) and uses the purely technological relation of the marginal cost curve instead of the *ex-ante* supply function. By using CEP, we derive (in Section 3) the three welfare indices as the profits (abstracting from fixed costs) that would prevail under certainty at different levels of the CEP. The implications (discussed in Section 4) include a unique framework for estimating welfare changes in certain cases. Further remarks concerning possible use of the present approach for welfare evaluation in practice are given in the concluding section.

2. Welfare Measures

Consider a risk averse supplier of a single product who faces uncertainty on product price.[1] The producer maximizes expected utility, where the utility function $U(\cdot)$ is defined on wealth and satisfies $U'>0$ and $U''<0$. Wealth is composed of initial wealth, W_0, and the operating profit $PY-C(Y;\alpha)$, where P is the output price, Y is output supplied, $C(\cdot)$ is the variable cost function generated by some underlying production technology and α is a parameter vector characterizing the production technology (unless needed explicitly, α will be suppressed from the arguments of C). The uncertainty is represented by a (subjective) cumulative distribution function on the output price. This distribution function is assumed to be uniquely defined by the moments vector $\theta=(\mu,\sigma,\ldots)$, where $(\mu,\sigma,\ldots)$ represents the mean, standard deviation and

higher central moments of output price.[2]

The firm is a taker of a price distribution or, alternatively, of a vector θ. The *ex-ante* supply function, $Y(\theta,W_o)$, is the supply level that maximizes the expected utility $E\{U(PY-C(Y)+W_o)\}$. Plugging $Y(\theta,W_o)$ into the maximand yields the indirect expected utility of profit

$$V(\theta,W_o) = E\{U(W(\theta,W_o)+W_o)\}, \tag{2.1}$$

where

$$W(\theta,W_o) = PY(\theta,W_o) - C(Y(\theta,W_o)). \tag{2.2}$$

$V(\theta,W_o)$ is a non-monetary measure of the well-being of a producer endowed with initial wealth W_o which operates under output price uncertainty characterized by θ. The associated monetary measure, indicated by $\hat{W}(\theta,W_o)$, is the money income that leaves the producer indifferent to receiving it with certainty or having the random income $W(\theta,W_o)$. Thus $\hat{W}(\theta,W_o)$ is the lowest certain income the producer would be willing to receive instead of the prevailing uncertain income $W(\cdot)$; it satisfies

$$U(\hat{W}(\theta,W_o)+W_o) = V(\theta,W_o). \tag{2.3}$$

Suppose a change in the price distribution, indicated by a move $\theta^1 \longrightarrow \theta^2$, occurs. The resulting change in producer welfare is $V(\theta^2,W_o)-V(\theta^1,W_o)$. In view of the definition of $\hat{W}(\cdot)$, an obvious monetary measure of this welfare change would be the Certainty (money) Equivalent (CE) index

$$CE = \hat{W}(\theta^2,W_o) - \hat{W}(\theta^1,W_o). \tag{2.4}$$

Two other monetary indices have been considered in the literature; they are the Compensating Variation (CV) and the Equivalent Variation (EV) defined from:

$$V(\theta^1,W_o+EV) = V(\theta^2,W_o), \tag{2.5}$$

$$V(\theta^2,W_o-CV) = V(\theta^1,W_o). \tag{2.6}$$

Figure 1 provides a graphical representation of producer surplus (S), CE

and CV in the μ-Y plan, with σ and higher moments of output price held constant (since CV and EV are parallel concepts, only CV is considered). The curve indicated by Y is the *ex-ante* supply defined above. The curve labeled Y_c is the *ex-ante* supply when the producer's initial wealth is compensated to keep him or her as well off as under regime θ^1. The curve indicated by Y_h is the derivative of $\hat{W}(\cdot)$ with respect to μ , and lies to the right of the *ex-ante* supply, provided that decreasing absolute risk aversion prevails (see e.g., Pope and Chavas [1985]). The three indices CV, S and CE are given by the areas $\mu^2 ae\mu^1$, $\mu^2 be\mu^1$ and $\mu^2 cd\mu^1$, respectively. The producer that generates the curves depicted in Figure 1 exhibits decreasing absolute risk aversion with respect to wealth. Under constant absolute risk aversion, the three curves and their corresponding welfare indices coincide.[3]

Figure 1

3. Certainty Equivalent Prices and the Three Welfare Indices

The concept of Certainty Equivalent Price (CEP) is now used to derive alternative representations of the three welfare indices. The underlying idea is simple. Recalling that producer welfare under certainty (i.e., profit) is measured by the area to the left of the Marginal Cost (MC) curve and below the output price, we will show that the three welfare measures CE, EV and CV are obtained as areas to the left of MC and between appropriate CEP levels.

The CEP, denoted by $\hat{P}(\theta,W_o)$, is the least certain price that a risk averse producer, endowed with initial wealth W_o, would be willing to receive instead of the random price distributed according to θ.[4] Formally, $\hat{P}(\theta,W_o)$ satisfies:

$$\hat{P}(\theta,W_o)\cdot Q[\hat{P}(\theta,W_o)] - C(Q[\hat{P}(\theta,W_o)]) = \hat{W}(\theta,W_o), \tag{3.1}$$

where $Q[\cdot]$ is the supply under certainty determined by equating price to

marginal cost and $\hat{W}(\theta,W_o)$ is defined in equation (2.3).

It follows directly from (3.1) and (2.4) that

$$CE = \hat{P}(\theta^2,W_o)\cdot Q[\hat{P}(\theta^2,W_o)] - C(Q[\hat{P}(\theta^2,W_o)]) - \{\hat{P}(\theta^1,W_o)\cdot Q[\hat{P}(\theta^1,W_o)] - C(Q[\hat{P}(\theta^1,W_o)])\}. \tag{3.2}$$

Furthermore, EV and CV can now be represented in terms of CEP as:

$$EV = \hat{P}(\theta^2,W_o)\cdot Q[\hat{P}(\theta^2,W_o)] - C(Q[\hat{P}(\theta^2,W_o)]) - \{\hat{P}(\theta^1,W_o+EV)\cdot Q[\hat{P}(\theta^1,W_o+EV)] - C(Q[\hat{P}(\theta^1,W_o+EV)])\} \tag{3.3}$$

and

$$CV = \hat{P}(\theta^2,W_o-CV)\cdot Q[\hat{P}(\theta^2,W_o-CV)] - C(Q[\hat{P}(\theta^2,W_o-CV)]) - \{\hat{P}(\theta^1,W_o)\cdot Q[\hat{P}(\theta^1,W_o)] - C(Q[\hat{P}(\theta^1,W_o)])\}. \tag{3.4}$$

To verify (3.3), note that the first term on its right hand side equals $\hat{W}(\theta^2,W_o)$, which is also equal to $U^{-1}(V(\theta^2,W_o))-W_o$ [cf. equation (2.3)]. The second term equals $\hat{W}(\theta^1,W_o+EV) = U^{-1}(V(\theta^1,W_o+EV))-W_o-EV$. But EV satisfies $V(\theta^1,W_o+EV)=V(\theta^2,W_o)$ [cf. (2.5)], which implies (3.3). A similar argument can be used to verify (3.4).

From (3.2)-(3.4) it directly follows that the three welfare indices are obtained as areas to the left of the MC curve and between appropriate CEP levels:

$$CE = \int_{\hat{P}(\theta^1,W_o)}^{\hat{P}(\theta^2,W_o)} Q[x]dx \quad , \tag{3.5}$$

$$EV = \int_{\hat{P}(\theta^1,W_o+EV)}^{\hat{P}(\theta^2,W_o)} Q[x]dx \tag{3.6}$$

and

$$CV = \int_{\hat{P}(\theta^1,W_o)}^{\hat{P}(\theta^2,W_o-CV)} Q[x]dx \quad . \tag{3.7}$$

To verify (3.5) note, from (3.2), that CE is the difference between the quasi-rents evaluated at the CEP levels $\hat{P}(\theta^2,W_o)$ and $\hat{P}(\theta^1,W_o)$. This

difference is merely the area to the left of the MC curve between these two CEP levels. In a similar manner, (3.6) and (3.7) follow from (3.3) and (3.4). A graphical illustration is presented in Figure 2.

Figure 2

4. Implications

Representations (3.5)-(3.7) reveal that, unlike the definitions of EV and CV, the CE concept does not allow for (hypothetical) income compensations to affect decisions [CE does not enter the CEP in representation (3.2)]. The CE concept is based on a welfare comparison which views the producer as if he or she were operating in a certain world (i.e., the certainty equivalent world); under certainty wealth compensations do not affect supply decisions. In the EV and CV concepts, uncertainty is retained and income compensations are therefore allowed to affect decisions. Evidently, the situation perceived here is where producers operate under (a changing) uncertainty. It therefore appears that CV and EV are the appropriate indices to use in the present context.[5]

Nevertheless the CE index is still useful since it satisfies the welfare criterion: *regime* θ^2 *is preferred or indifferent to regime* θ^1 *if and only if* $CE \geq 0$. This property follows directly from the strict monotonicity of $U(\cdot)$ and equations (2.3) and (2.4); where the preference relation over the uncertain regimes θ is represented by the indirect utility function $V(\cdot)$. Furthermore, the CE index bounds the CV and EV indices from above and hence can serve as an upper bound on welfare changes, thereby justifying its use as a quantitative welfare measure.

The CE index is easier to evaluate than the other, variational indices. This can be seen by noting that the CE index has a closed form expression [equation (3.5)], whereas the other two indices are given as the roots of

their corresponding equations [(3.6) and (3.7)]. Thus less information is required to obtain the CE index. All three require information on the MC curve, which is a technological relationship independent of risk preferences and uncertainly. Evaluating EV and CV requires, in addition, knowledge of the behavior of the CEP function over an interval of income compensation levels. For example, from representation (3.6), the EV measure associated with a move $\theta^1 \rightarrow \theta^2$ requires the knowledge of $\hat{P}(\theta^2, W_o)$ and of $\hat{P}(\theta^1, W_o+EV)$ for various levels of EV. On the other hand, using (3.5), the evaluation of CE requires the knowledge of just two points of the CEP function, $\hat{P}(\theta^1, W_o)$ and $\hat{P}(\theta^2, W_o)$, which are evaluated at the uncompensated initial wealth W_o. Thus evaluating CE does not require wealth effect information; only effects of the price distribution (θ) are needed. Obviously, it is easier to obtain two CEP points evaluated at the actual initial wealth rather than a continuum of points defined over an interval of (hypothetically) compensated income levels.

Representations (3.5)-(3.7) also make apparent that, under decreasing absolute risk aversion, CE exceeds both CV and EV; and this relation holds true in the general case where the mean and other moments of the output price distribution vary. This is so because decreasing absolute risk aversion implies that the CEP is positively related to wealth (as the initial wealth increases producers are less bothered by the uncertainty and will demand a higher [certain] price to get rid of it). Assuming with no loss of generality that regime θ^2 is preferred to regime θ^1, it is clear from Figure 2 that CE exceeds EV and CV. Obviously, the three indices coincide when no wealth effects are present, i.e., under constant absolute risk aversion (simply note that the limits of integration in (3.5)-(3.7) are the same).

Applying the present approach in practice requires information on i) the marginal cost (MC) curve and ii) some CEP levels. The MC curve depends on the

production technology and can be evaluated from engineering data. The effects of uncertainty and risk preference are captured by the CEP. Obtaining the required CEP information is more problematic because there exists no market mechanism through which data on this variable can be observed. However, in the generic case where producers can choose the price distribution under which to operate, it is possible to use the observable discrete choices of the uncertain regime in order to obtain the CEP information. Such is the case, for instance, when the producer must decide on whether to participate in an agricultural commodity program. A participation decision entails a certain price distribution (which depends on the program's provisions) whereas the decision not to participate entails another one (which is determined by the market). Another example is where a choice must be made on a single product to produce among several possible products. The production technologies are perfectly known but the demand for each product is uncertain with uncertainty that varies across products.

As an illustration, suppose the welfare consequences of a change in the provisions of an agricultural price stabilization program are sought. Under the current program the price distribution is characterized by the moment vector θ^1 and under the new program by θ^2. Evaluating the CE index associated with this move requires knowledge of $\hat{P}(\theta^j, W_o)$, $j=1,2$. Given a form for the CEP function and given data on program participation decisions, it is described in Appendix B how one can indirectly estimate the parameters of the CEP function. Such an estimate can then be used to predict the CEP level under θ^2 and thereby to evaluate the CE index, according to equation (3.5). Furthermore, given estimates of the wealth effect in the CEP function, the CV and EV indices can easily be calculated as the roots of equations (3.6) and (3.7), respectively.

5. Concluding Remarks

Previous methods to evaluating welfare consequence of changes in price uncertainty rely on the *ex-ante* supply function. This paper presents an alternative approach which uses the marginal cost (MC) function instead, but requires knowledge of some levels of the certainty equivalent price (CEP). The MC function is a technological relation and can be estimated from engineering data. The effects of uncertainty and risk aversion are captured by the CEP, for which no data are observed. However, in cases involving discrete choices over uncertain regimes, it may be possible to indirectly extract the required CEP information using the observable discrete choices.

Even when this indirect approach is not feasible (perhaps because data on the related decisions are not available), there is still another approach, a direct one, that is worth considering; namely, eliciting the required CEP information via interviews. In view of the discussion of the previous section, such a direct approach may in particular be appropriate (in the sense of requiring the least information) when the CE index is evaluated. Experimental methods to elicit utility information have a long history in decision theories (e.g., Becker, DeGroot and Marschak [1964], Keene and Raiffa [1976]). A related literature, dealing with the valuation of public goods and other extra market benefits, appears under the heading of "contingent valuation methods" (Mithchel and Carson [1989]).

If such a direct approach is used to elicit CEP information, one may wonder why not to use the same approach in order to extract information on income compensations and thereby to obtain the EV and CV directly. True, this is also a possibility. However there is a substantial difference between the two tasks. To understand this difference we must resort to the decision

making process outlined in Section 2. Suppose an agent is asked to reveal the least certain price he or she would be willing to receive instead of the prevailing random price. Equipped with a knowledge of the level of $V(\theta, W_o)$ [cf. equation (2.1)] and of the production technology as summarized by MC, the producer must compare profits under different (certain) price levels and to choose that price level (or its associated profit) which under certainty would yield the same level of "well being" as that represented by $V(\cdot)$. Notice that in this scenario no income compensation takes place and comparisons of certainty world profits are performed.

Now suppose the same agent is asked to reveal the lowest income compensation he or she would be willing to receive in order to accept the move $\theta^1 \rightarrow \theta^2$. To derive this information the agent must make comparisons involving the *ex-ante* supply function at various income compensation levels [cf. equations (2.1), (2.2) and (2.5)]. Such uncertainty world calculations are clearly of different nature than those performed in the previous case. Thus the two tasks are different and whether either can be implemented successfully is yet an open question which must be determined empirically.

Appendix A. Uncertain Input Prices

Suppose output and input prices are uncertain. Let $q=(p,r)$ be the 1+k vector of random output and input prices whose distribution is characterized by the moment vector θ. Let $x(\theta,W_o)$ be the k by 1 vector of *ex-ante* input demand functions defined from $\underset{x}{\text{Max}}\ E\{U(py(x)-r\cdot x+W_o)\}$, where the expectation is taken with respect to the joint distribution of all prices and $y(x)$ is the production function. The indirect utility of profit is defined, analogously to equation (2.1), as $V(\theta,W_o) = E\{U(py(x(\theta,W_o))-r\cdot x(\theta,W_o)+W_o)\}$ and the certainty equivalent income $\hat{W}(\theta,W_o)$ is as defined in equation (2.3). Given $V(\cdot)$ and $\hat{W}(\cdot)$, the three welfare indices are as defined in equations (2.4)-(2.6).

Let the k functions $x[\cdot]$ represent the input demand under certainty. That is, for any given price vector, say $q_o = (p_o,r_o)$, $x[q_o]$ satisfies $Dy(x[q_o])=r_o/p_o$, where $Dy(\cdot)$ is the vector of the first derivatives of y. The certainty equivalent price vector $\hat{q} = (\hat{p}(\theta,W_o),\hat{r}(\theta,W_o))$ can now be defined as the solution to the k+1 equations:

$$\hat{p}y(x[\hat{q}]) - \hat{r}\cdot x[\hat{q}] + W_o = \hat{W}$$

$$Dy(x[\hat{q}]) = \hat{r}/\hat{p}\ .$$

Suppose a change in the uncertainty, indicated by a move $\theta^1 \rightarrow \theta^2$, occurs. Let $\hat{q}(\theta^j,Z) = (\hat{p}(\theta^j,Z),\hat{r}(\theta^j,Z))$ be the CEP under regime θ^j, j=1,2, with Z indicating the compensated initial wealth. Let $QR(\hat{q}) = \hat{p}y(x[\hat{q}]) - \hat{r}\cdot x[\hat{q}]$ represent the quasi-rent under certainty associated with the (certain) price vector $\hat{q}$. Then, using the derivation of Section 3, it is straightforward to verify that:

$$CE = QR(\hat{q}(\theta^2,W_o)) - QR(\hat{q}(\theta^1,W_o)),$$

$$EV = QR(\hat{q}(\theta^2,W_o)) - QR(\hat{q}(\theta^1,W_o+EV)),$$

$$CV = QR(\hat{q}(\theta^2,W_o-CV)) - QR(\hat{q}(\theta^1,W_o)).$$

Consider the special case where only one price is uncertain, say that of the first input, with the output price and the rest of the input prices given at a known level. Using Hotelling's lemma it can be verified that the three welfare indices are obtained as areas to the left of the first input demand function and between appropriate levels of the first input CEP:

$$CE = \int_{\hat{r}_1(\theta^1,W_o)}^{\hat{r}_1(\theta^2,W_o)} x_1[s]ds, \quad EV = \int_{\hat{r}_1(\theta^1,W_o+EV)}^{\hat{r}_1(\theta^2,W_o)} x_1[s]ds \quad \text{and} \quad CV = \int_{\hat{r}_1(\theta^1,W_o)}^{\hat{r}_1(\theta^2,W_o-CV)} x_1[s]ds ,$$

where $x_1[s]$ is the demand for the first input as a function of the first input's price, s, given that all other prices are at their fixed known level.

Appendix B. Estimating the Certainty Equivalent Price Function From Observable Data: The Case of Commodity Program Participation

It is assumed that the form of the CEP function $\hat{P}(\cdot)$ is known. This can be achieved directly by specifying the utility function and the output price distribution and proceeding along the definitions of $\hat{W}(\cdot)$ and $\hat{P}(\cdot)$ [cf. equations (2.3) and (3.1)], or indirectly by specifying a form of $\hat{P}(\cdot)$ which is consistent with some underlying utility function and output price distribution. The CEP $\hat{P}(\cdot)$ depends on its arguments, θ and W_o, via a set of unknown parameters β. We seek to estimate β. If observations on $\hat{P}(\cdot)$ were available and given data on θ and W_o, one could proceed in an obvious manner to estimate β. Unfortunately, data on $\hat{P}(\cdot)$ from observable actions are not available.

Suppose the production technology is known a priori (or can be estimated from engineering data) so that the cost function $C(\cdot)$ and the inverse marginal cost function $Q[\cdot]$ are given. Thus the certainty equivalent income function

$$\hat{W}(\theta,W_o;\beta) = \hat{P}(\theta,W_o;\beta)Q[\hat{P}(\theta,W_o;\beta)] - C(Q[\hat{P}(\theta,W_o;\beta)])$$

[cf. equation (3.1)] is known up to the parameter vector β.

A decision to participate in a commodity program entails a binary choice between two uncertain regimes θ^0 and θ^1, where θ^0 and θ^1 characterize the output price distribution for non-participants and participants, respectively. Regime 1 (0) is chosen if $V(\theta^1, W_o) > (\leq) V(\theta^0, W_o)$ or equivalently, using equation (2.3), if $\hat{W}(\theta^1, W_o) > (\leq) \hat{W}(\theta^0, W_o)$. Taking account of measurement (and possibly of specification) errors and letting v represent these errors, the discrete choice problem can be formulated in terms of a non-linear discrete choice model as:

$$d = \begin{cases} 1 & \text{if } \hat{W}(\theta^1, W_o; \beta) - \hat{W}(\theta^0, W_o; \beta) + v > 0 \\ 0 & \text{otherwise} \end{cases} .$$

Given data on participation decisions (d), on the output price distributions (θ^1 and θ^o), on wealth (W_o) and possibly on other socioeconomic attributes of growers, one can use this model to estimate the parameter vector β up to a normalization with respect to the variance of the error term v.

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Figure Captions

Figure 1. CV = area $\mu^2 ae\mu^1$; S = area $\mu^2 be\mu^1$; CE = area $\mu^2 cd\mu^1$.

Figure 2. CE = area EADH; EV = area EACG; CV = area FBDH.

Footnotes

[1]This simple case is considered for the sake of presentation clarity. The analysis extends to cases involving also input price uncertainty. In Appendix A the general case is outlined.

[2]The condition for the moments of a random variable to define a unique distribution function can be found in Rao (1965, p. 86).

[3]It is obvious that in the absence of wealth effect, i.e., under constant absolute risk aversion, $Y_c = Y(\theta, W_o)$. To see that $Y_h = Y(\theta, W_o)$ in this case, note that $Y(\theta, W_o)$ is the supply level that maximizes $E\{U(PY-C(Y)+W_o)\}$ and satisfies the first order condition: $E\{U'(W(\theta, W_o))\cdot[P-C'(Y(\theta, W_o))]\} = 0$. By differentiating equations (2.2) and (2.3) with respect to μ and using the above condition, we obtain $\partial\hat{W}(\theta, W_o)/\partial\mu = Y(\theta, W_o)\cdot H(\theta, W_o) = Y_h$, where $H(\theta, W_o) = E\{U'(W(\theta, W_o))\}/U'(\hat{W}(\theta, W_o))$. Now constant absolute risk aversion implies exponential utility. Without loss of generality, let $U(W) = 1-e^{-AW}$, A being the absolute risk coefficient, and define M as the moment generating function of W (assumed to exists). Thus $E\{U(W)\} = 1-M(-A)$. From $U(\hat{W}) = E\{U(W)\}$ it follows that $\hat{W} = -\log\{M(-A)\}/A$. Likewise $E\{U'(W)\} = A\cdot E\{e^{-AW}\} = A\cdot M(-A)$, and $U'(\hat{W}) = A\cdot e^{-A\hat{W}} = A\cdot \bar{M}(-A)$. Recalling the definition of H above, we obtain $H(\cdot)=1$.

[4]Newbery and Stiglitz (1981, p. 59) denote this price the *utility certainty equivalent price*, as opposed to the *action certainty equivalent price*. The latter is the price that under certainty would result in the supply level being equal to the *ex-ante* supply $Y(\theta, W_o)$.

[5]A similar conclusion was drawn in the related context of evaluating consumer surplus under price uncertainty (see Choi and Johnson)

FIGURE 1

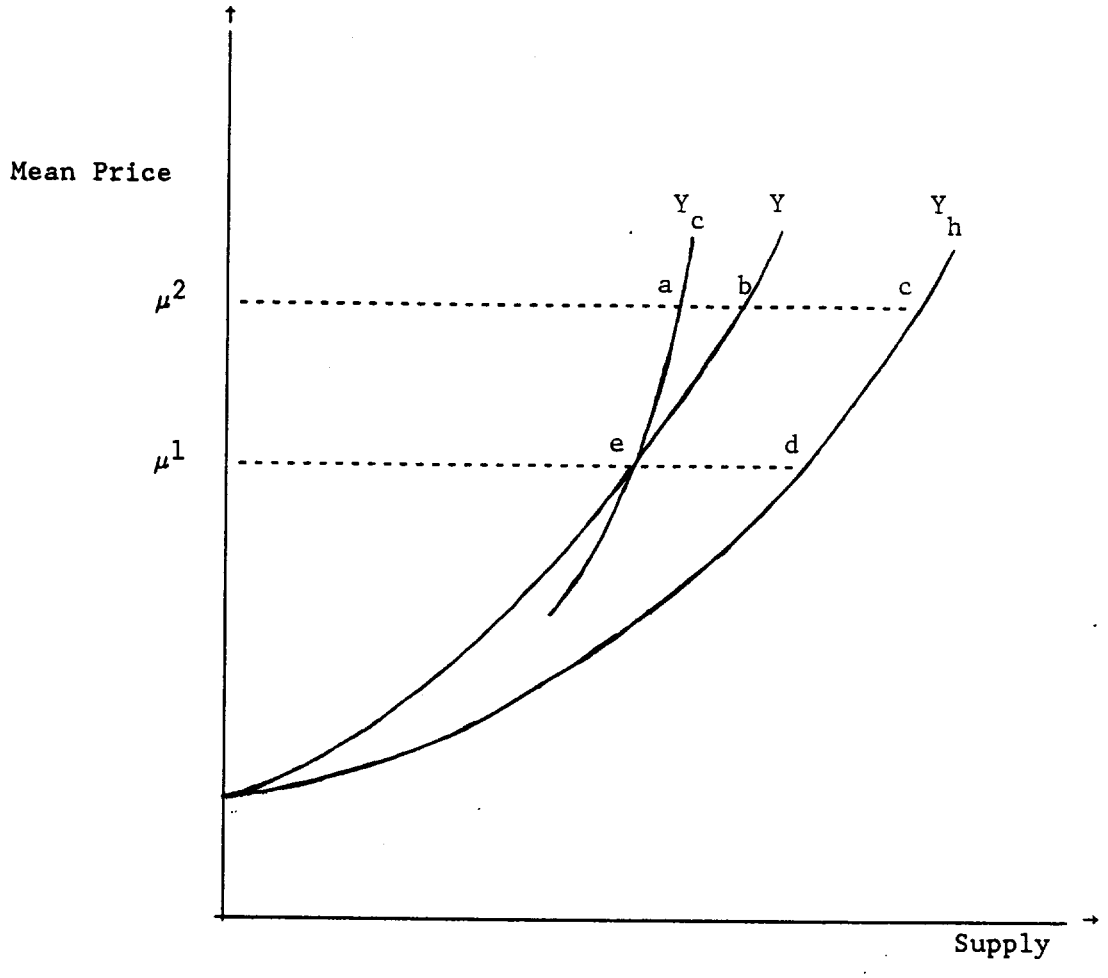
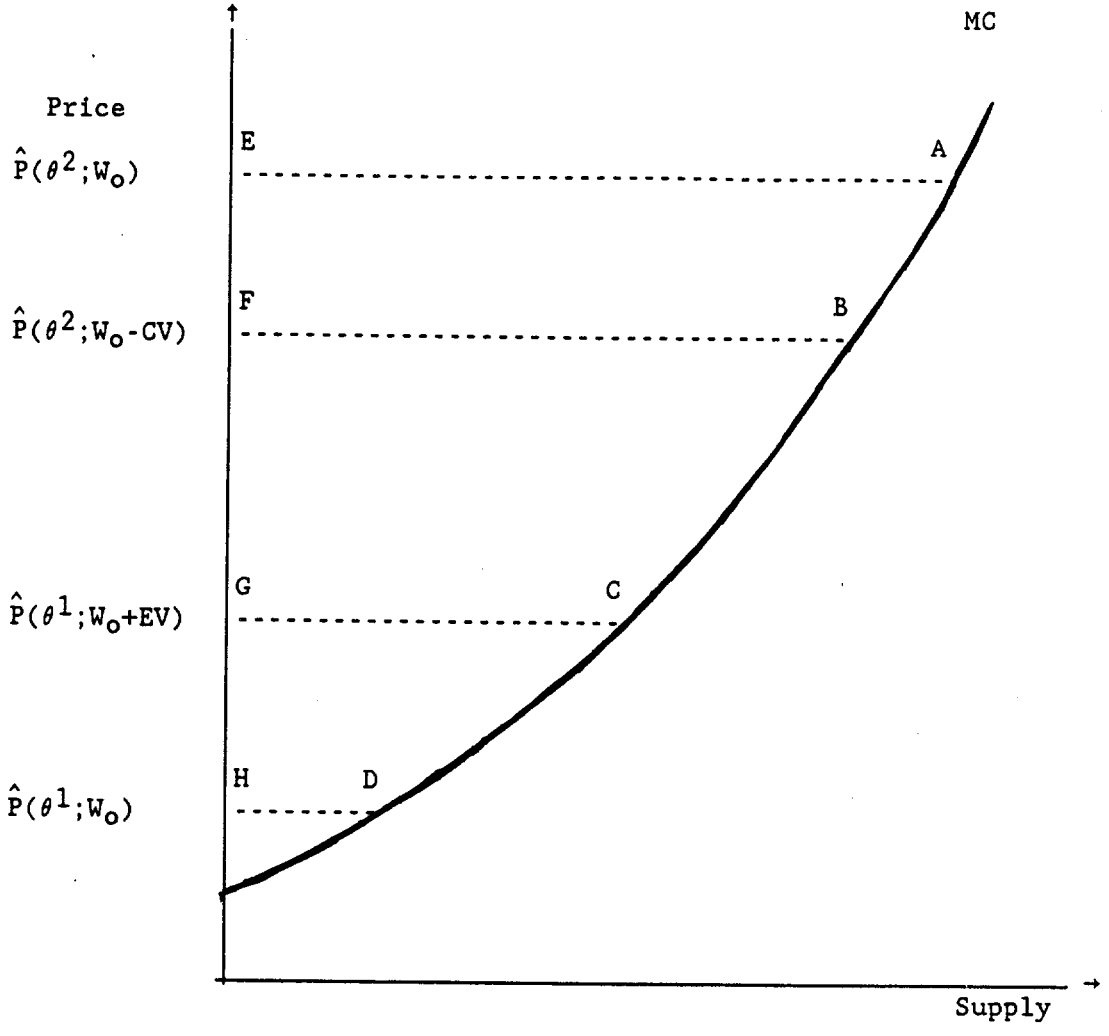


FIGURE 2


MC
Price
E
A
$\hat{P}(\theta^2;W_o)$
F
B
$\hat{P}(\theta^2;W_o\text{-}CV)$
G
C
$\hat{P}(\theta^1;W_o+EV)$
H
D
$\hat{P}(\theta^1;W_o)$
Supply