A STATISTICAL AND EMPIRICAL INVESTIGATION
OF BUSINESS RISK IN AGRICULTURAL PRODUCTION

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A Statistical and Empirical Investigation
of Business Risk in Agricultural Production

The concept of economies of size related to costs and returns has long been a significant framework for analyzing efficiency of different farm sizes. While this framework has been used extensively, little attention has been paid to risk/size relationships. These relationships may be important.

Many economists are suggesting that farms will increase in size as consolidation occurs, because of the exit of many highly leveraged farms in the current financial environment. Risk/size relationships could either hamper, be neutral, or encourage the increase in size. Thus, risk/size relationships as well as the traditional economies of size concepts are of considerable importance in understanding the forces shaping the future structure of production agriculture.

Diversification has generally been viewed as a method of reducing variability of income (Heady and Jensen). Pope and Prescott recognize this benefit, but also recognize that economies of size exist. They have suggested that there is a trade-off between the diversification benefits of reducing risk and the economies of size benefits from specialization. If there are substantial economies of size in an enterprise, then one gives up a substantial expected return to reduce the variability of return by diversifying.

Robison and Barry suggest that specialization, in some cases, may reduce variability of incomes. They argue that learning can occur or quality control may increase because of specialization. They suggest that this phenomenon may be called increasing returns to scale in risk.
These issues revolve around changing the enterprise mix for a given total resource base. The issue addressed in this paper is the possibility that increased size reduces business risk in a relative sense.

Risk has been split into two types: business risk, which refers to variation in income, and financial risk, which refers to the risk associated with increased leverage. Business risk refers to variation in net earnings because of yield, price and cost variability (Lee et al. p. 21). There is considerable emphasis currently on financial risk because of the debt crisis, but as agriculture moves out of this period, business risk will increase in relative importance.

This article focuses on business risk and the relationships between business risk and size. First, a statistical framework is developed to explore risk/size relationships. Secondly, the relationships are estimated using farm level data. Since \( \text{NET} = \text{GROSS} - \text{EXPENSES} \),

\[
V(\text{NET}) = V(\text{GROSS}) + V(\text{EXPENSES}) - 2C(\text{GROSS},\text{EXPENSES}),
\]

where \( V \) is variance and \( C \) is covariance. We will begin by analyzing the variability of gross, then expenses, then the covariance of gross and expenses. Then we will put these together to investigate the variability of net incomes.

**Variability of Gross Income**

First, let us look at the variance of gross, where gross is the sum of the revenue generated by \( n \) enterprises. In this case,

\[
\text{GROSS} = \sum_{i=1}^{n} P_i S_i T Z_i \text{ where}
\]

\( P_i = \text{price of product} \)

\( S_i = \text{share of } T \text{ devoted to enterprise } i \)
\( T \) = total size, and

\( Z_i \) = production per unit of enterprise \( i \).

In this case,

\[
V(\text{GROSS}) = V(P_1S_1TZ_1) + V(P_2S_2TZ_2) + \ldots + V(P_nS_nTZ_n)
\]

\[+ 2[\text{Covariances of } \binom{n}{2} \text{ pairs of } P_iS_iTZ_i]\]

Now, assuming \( S_i \) is constant for a farm, i.e., the enterprise combination is fixed and the size \( T \) is fixed for a farm, then

\[
V(P_iS_iTZ_i) = S_i^2T^2V(P_iZ_i),
\]

and

\[
C(P_iS_iTZ_i, P_jS_jTZ_j) = S_iS_jT^2C(P_iZ_i, P_jZ_j)
\]

So

\[
V(\text{GROSS}) = \sum_{i} S_i^2T^2V(P_iZ_i) + \sum_{i,j} S_iS_jT^2C(P_iZ_i, P_jZ_j)
\]

or, \( V(\text{GROSS}) = T^2[\sum_{i} S_i^2V(P_iZ_i) + \sum_{i,j} S_iS_j C(P_iZ_i, P_jZ_j)] \)

for \( i \neq j \).

Dividing both sides of the equation by \( T^2 \) gives

\[
\frac{V(\text{GROSS})}{T^2} = \sum_{i} S_i^2 V(P_iZ_i) + \sum_{ij} S_iS_j C(P_iZ_i, P_jZ_j)
\]

for \( i \neq j \).

If gross is used as a measure of size, then taking the square root of the left side of the equation results in the coefficient of variation of gross income.

While much of the impact of size has been eliminated from the right side of the above equation, we can still argue that the \( V(P_iZ_i) \) are functions of size. If we assume that price \( (P_i) \) and yield \( (Z_i) \) are bivariate normally distributed, then using Bohrnstedt and Goldberger, the variance of a product
is
\[ V(P_iZ_i) = E^2(P_i)V(Z_i) + E^2(Z_i)V(P_i) + 2 E(P)E(Z_i)C(P_i,Z_i) \]
\[ + V(P_i) V(Z_i) + C^2(P_i,Z_i) \]

In particular, it can be argued that \( V(Z_i) \) is a function of size, and that the variance of yield per acre will decrease as acres increase. To illustrate with an example, let

\[ Y_1 = \text{yield/acre on the first acre and} \]
\[ Y_2 = \text{yield/acre on the second acre.} \]

Then \( V(Y_1) = \text{variance of yield on acre } i \). Also, let \( V(Y_1) = V(Y_2) \), since they are similar but not identical acres. Now, let us look at the variance per acre for two acres.

\[ V\left(\frac{Y_1+Y_2}{2}\right) = \frac{1}{4} \left[ V(Y_1) + V(Y_2) + 2C(Y_1,Y_2) \right] \]

Now \( V(Y_1) = V(Y_2) \), since the acres are similar. However, it is likely that \( C(Y_1,Y_2) < V(Y_1) \) since \( C(Y_1,Y_2) = E[(Y_1 - \overline{Y_1})(Y_2 - \overline{Y_2})] \). It is not likely that both \( Y_1 \) and \( Y_2 \) will be affected exactly the same way by localized weather patterns and other phenomena, because they are similar but not identical acres. So,

\[ V \left( \frac{Y_1 + Y_2}{2} \right) = \frac{1}{4} [2V(Y_1) + 2C(Y_1,Y_2)] < \frac{1}{4} [4V(Y_1)] = V(Y_1), \]

which suggests that variance per acre decreases as the number of acres increases.

The same argument holds as a farm spreads out over more acres. The variance of yield per acre will likely decline as acres increase because of localized phenomena that affect some areas more than others. This result is a form of diversification, even though it is the same enterprise. We will call
this natural diversification. The benefits of natural diversification from differences in soil types, localized weather patterns, different planting dates and rotation schedules as well as different varieties should not be overlooked. Diminishing returns to this type of diversification could be expected. However, as farm size grows, acreage is spread over a broader area and the likelihood of localized weather affecting one area and not the other grows. In addition, as a farm grows, the difference in planting dates and other management practices may grow (given the same machinery size). Natural diversification benefits also could be expected in livestock enterprises. In this case, the additional livestock units may have substantially different characteristics, which could react differently to environmental conditions and diseases. The magnitude of benefits and the range of farm size that receives these benefits is an empirical question.

Variability of Expenses

Now, let us look at the variance of expenses. We can use the same analysis that we used for gross, if we define

\[ \text{EXPENSES} = \sum_{i=1}^{n} \sum_{j=1}^{m} P_j S_i T X_{ij} \]

where

\( P_j \) = price of input \( j \)
\( S_i \) = share of \( T \) devoted to enterprise \( i \)
\( T \) = total size
\( X_{ij} \) = quantity of input \( j \) used on enterprise \( i \).

Now the above equation can be rearranged as

\[ \text{EXPENSES} = \sum_{i=1}^{n} S_i T \sum_{j=1}^{m} P_j X_{ij} \]
Now, \[ \sum_{j=1}^{m} P_j X_{ij} \] is the cost of \( m \) inputs per unit of enterprise \( i \).

So, let \[ \sum_{j=1}^{m} P_j X_{ij} = C_i. \]

Now, using the same logic used for analyzing the variance of gross,

\[
V(\text{EXPENSES}) = V(S_1TC_1) + V(S_2TC_2) + \ldots + V(S_nTC_n) + 2\left[\text{Covariances of } \frac{n}{2} \text{ pairs of } S_iTC_i\right]
\]

Now, when \( S_i \) is constant for a farm and size \( T \) is fixed for a farm, then

\[
V(\text{EXPENSES}) = \frac{\sum S_i^2}{T^2} V(C_i) + \sum_{i \neq j} S_i S_j C(C_i,C_j)
\]

for \( i \neq j \).

Now, let us examine again whether \( V(C_i) \) and \( C(C_i,C_j) \) may be functions of size. Since \( C_i = \sum_{j=1}^{m} P_j X_{ij} \), the question of relationship to size revolves around the likelihood that \( V(X_{ij}) \) may decrease as size increases. We can again argue that it does decrease as size increases, using the same argument that we used for \( V(Z_i) \). That is, since we have similar, but not identical units, it is reasonable that the variance of input use per acre will decrease as acres increases. Thus, we could hypothesize that the ratio between variance of expenses and gross farm income squared (i.e. \( \frac{V(\text{EXPENSES})}{T^2} \)) would decrease as size increases.

**Covariance of Gross and Expenses**

The final piece of the puzzle is the covariance of gross and expenses.

Now, using previous definitions

\[
\text{GROSS} = T \sum_{i=1}^{n} S_i P_i Z_i \quad \text{and} \quad \text{EXPENSES} = T \sum_{j=1}^{n} S_j C_j.
\]
So, the covariance will be
\[ C(\text{GROSS, EXPENSES}) = T^2 \left( \sum_{i=1}^{n} S_i P_i Z_i, \sum_{j=1}^{n} S_j C_j \right). \]

This can be rewritten as
\[ C(\text{GROSS, EXPENSES}) = T^2 \sum_{i=1}^{n} \sum_{j=1}^{n} C(S_i P_i Z_i, S_j C_j) \]

Now, the covariance of one of the pairs is
\[ C(S_i P_i Z_i, S_j C_j) = S_i S_j C(P_i Z_i, C_j) \]

So,
\[ C(\text{GROSS, EXPENSES}) = T^2 \sum_{i=1}^{n} \sum_{j=1}^{n} S_i S_j C(P_i Z_i, C_j). \]

The issue now is whether \( C(P_i Z_i, C_j) \) is related to size in any way. This is the covariance per unit of production. We can argue from a logical standpoint that gross and expenses are positively correlated, since higher costs should result in higher gross. The question is, does this positive relationship increase or decrease as size of farm increases?

An argument for the hypothesis that the relationship between gross and expenses decreases as farm size increases is that the proportion of gross income used for family or personal consumption is larger for a small farm than a large farm. Therefore, when a small farm has a high gross income from high yields per unit or high product prices, the operator may purchase large or high cost inputs that are needed, such as new equipment. This type of action will increase expenses. Using this argument, we would expect the covariance per unit of production between gross and expenses to be larger for small farms than for large farms.

On the other hand, we can argue that the covariance per unit increases as size increases. From a tax standpoint, larger farms with higher incomes have been in higher marginal tax brackets. Thus, the incentive is greater for
larger farms to increase expenses when gross income is high, to reduce the tax liability. This behavior will result in a larger positive covariance for large farms than for small farms.

Variance of Net

Finally, we can put all the pieces together since

\[ V(\text{NET}) = V(\text{GROSS}) + V(\text{EXPENSES}) - 2C(\text{GROSS, EXPENSES}). \]

First, dividing each component by \( T^2 \) gives

\[ \frac{V(\text{NET})}{T^2} = \frac{V(\text{GROSS})}{T^2} + \frac{V(\text{EXPENSES})}{T^2} - 2C(\text{GROSS, EXPENSES}). \]

Now, we have argued that \( \frac{V(\text{GROSS})}{T^2} \) decreases as size increases, \( \frac{V(\text{EXPENSES})}{T^2} \) decreases as size increases, and \( \frac{2C(\text{GROSS, EXPENSES})}{T^2} \) can either increase or decrease as size increases. The relative sizes of these relationships could result in \( \frac{V(\text{NET})}{T^2} \) decreasing as size increases. If this is the case, then risk economies of size exist.

Empirical Evidence

Data from 687 farms for 13 years were used to estimate these relationships. Variances and means were calculated over the 13 years for each farm, after financial variables were deflated using the implicit gross national product deflator. Gross farm income was calculated on an accrual basis, as total sales plus government payments plus miscellaneous income. Expenses were calculated as cash operating expenses plus depreciation minus interest expenses. Interest was not included in expenses, to remove the impact of financial leverage on net incomes. Net farm income was calculated as gross farm income minus expenses as defined above.

The models developed earlier in this paper were then estimated, using the following variables. Gross farm income was used as the measure of size (T).
The shares \((S_i)\) were calculated as the share of total sales contributed by each of 15 enterprises. A diversification variable, \(\Sigma S_i S_j\) where \(i \neq j\) was used to measure the impacts of diversification.

In addition to the model developed in this paper, other variables believed to have an impact on income variability were included. These variables include government payments as a percent of gross, interest payments as a percent of gross, age of operator, machinery investment per acre, and dummy variables for location in the state.

Size, measured as gross farm income, was included as an independent variable to test the hypothesis that relative variability decreases as size increases. Evidence not reported here suggests that \(\frac{V(GROSS)}{T^2}\), \(\frac{V(EXPENSES)}{T^2}\), \(\frac{C(GROSS, EXPENSES)}{T^2}\) decrease as size increases. The relative sizes of the decreases result in \(\frac{V(NET)}{T^2}\) decreasing as size measured by gross farm income increases (see Table 1). The ratio of variance of net to gross farm income squared provides a measure of the relative variability of net. The negative (and significant) coefficient on gross farm income suggests that relative variability decreases as size increases. This empirical evidence suggests that risk economies of size exist in production agriculture. This finding supports the statistical argument given earlier in this paper.

Other variables were also significantly related to relatively variability. The diversification coefficient is an estimate of average covariance of net returns between enterprises. It is positive and significant, which indicates that net returns are often positively correlated. The government payments coefficient suggests that larger government payments as a percent of gross reduce relative variability. The interest payments coefficient suggests that larger interest payments as a percent of gross
increases relative variability. A possible explanation is that operations may have less flexibility as their financial commitment increases thus increasing their relative variability of income. This is consistent with suggestions made by Gabriel and Baker that the lender or farmer may impose restrictions altering the dispersion of net cash flows due to debt financing. The coefficient on age of operator suggests that relative variability increases with operator age. This may be due to less flexible management strategies or technologies that do not adapt as well to all circumstances. The coefficient on the machinery investment per acre variable (although not significant) suggests that a greater machinery investment is related to a lower relative net income variability.

The coefficients on the enterprise variables can be interpreted as the variance of net returns per unit of the enterprise (i.e. \( V(P_iZ_i - C_i) \)). A unit of an enterprise is one dollar of sales. This allows comparison of the variances of net between enterprises. For example, variance of net income from wheat per dollar of wheat sales is .489 compared to .400 for grain sorghum. Both coefficients are significantly (.05) different than zero, but probably not significantly different from each other.

The adjusted \( R^2 \) for the equation is .23. It can be argued that there are many additional factors related to income variability. Localized weather patterns, incidence of diseases, differing management ability, etc. could be responsible for the low \( R^2 \). These and other factors can not be measured in any reasonable fashion.

**Summary and Conclusions**

This work supports the concept of risk economies of size in agriculture. Evidence suggests that the ratios of variance of gross, variance of expenses,
and covariance of gross and expenses to size squared decrease as size increases. The result is that the ratio of variance of net to size squared decreases as size increases, i.e., risk economies of size exist.

One reason this occurs in agriculture, but does not occur in finance, is that additional units of productive resources in agriculture are similar, but they cannot be identical. Thus, localized natural phenomena cannot affect each unit in an identical fashion. Therefore, "natural diversification" results from the numerous small differences between one unit of an enterprise and the next unit.

The implications of this research are significant. First, it suggests that the concentration of resources into the hands of fewer, larger producers is encouraged not only by economies of size, but by risk economies as well. Second, major consideration should be given to the types of risk research conducted in agricultural economics and reconsideration to the types of models used. Because of the fundamental difference between agriculture and finance, we should no longer borrow techniques without question from finance. Portfolio models, in particular, contain the implicit assumption that variance of income per unit is constant as more units are used in an enterprise. While this relationship does hold in finance, it does not hold in agriculture. This suggests that our risk model results have implicitly been biased toward diversification among enterprises. Specialization in one enterprise carries along its own "natural diversification", which has not been recognized in our risk models to date.

Finally, additional research needs to be devoted to determining the magnitude of risk economies in agriculture. Are there decreasing marginal risk economies as size increases? Are risk economies eventually offset by
managerial limitations? These are questions that need to be addressed in order to ascertain the full importance of risk economies of size.
Table 1. Regression Coefficients and T Values with the Ratio of the Variance of Net Farm Income to Gross Farm Income Squared as the Dependent Variable and Other Farm Characteristics as Independent Variables.

<table>
<thead>
<tr>
<th>Independent Variables</th>
<th>Equation Coefficients</th>
<th>T Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gross Farm Income</td>
<td>-0.000000255*</td>
<td>-4.391</td>
</tr>
<tr>
<td>Measure of Diversification</td>
<td>0.626*</td>
<td>2.610</td>
</tr>
<tr>
<td>Government Payments</td>
<td>-0.467*</td>
<td>-2.381</td>
</tr>
<tr>
<td>Interest Payments</td>
<td>0.149*</td>
<td>3.165</td>
</tr>
<tr>
<td>Age of Operator</td>
<td>0.00181*</td>
<td>4.661</td>
</tr>
<tr>
<td>Machinery Investment per Acre</td>
<td>-0.000311</td>
<td>-1.360</td>
</tr>
<tr>
<td>Enterprise Shares Squared:</td>
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<td></td>
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<tr>
<td>Raised Beef</td>
<td>0.516*</td>
<td>4.109</td>
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<tr>
<td>Purchased Beef</td>
<td>0.349*</td>
<td>2.871</td>
</tr>
<tr>
<td>Raised Swine</td>
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<tr>
<td>Purchased Swine</td>
<td>0.354</td>
<td>1.756</td>
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<tr>
<td>Dairy</td>
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<td>Other Livestock</td>
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<tr>
<td>Irrigated Wheat</td>
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<tr>
<td>Irrigated Corn</td>
<td>0.308*</td>
<td>2.398</td>
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<tr>
<td>Irrigated Grain Sorghum</td>
<td>0.461*</td>
<td>2.483</td>
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<tr>
<td>Irrigated Soybeans</td>
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<td>Alfalfa Hay</td>
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<tr>
<td>Dryland Wheat</td>
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<tr>
<td>Dryland Grain Sorghum</td>
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<td>2.891</td>
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<tr>
<td>Dryland Soybeans</td>
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<td>2.851</td>
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<td>Locations:</td>
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<tr>
<td>North Central</td>
<td>0.0238</td>
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<td>South Central</td>
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<td>South West</td>
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<td>North West</td>
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</tr>
<tr>
<td>Intercept</td>
<td>-0.316*</td>
<td>-2.613</td>
</tr>
</tbody>
</table>

Adjusted $R^2$ ______________________________ ________________

0.231

* The variable is significant at the .05 level
References


