A YIELD CURVE BASES RULE FOR CHOOSING AMONG FARM MORTGAGES

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A YIELD CURVE BASED RULE FOR CHOOSING AMONG FARM MORTGAGES

Xiaolong Luo¹, Timothy G. Baker, Kenneth A. Foster and Paul V. Preckel

INTRODUCTION

The agricultural mortgage market is a major source of funds for farm firms in the United States. Among the 115 billion-dollar total US farm debt as of December 1994, 76 billion dollars was in real estate debt, which was raised largely through the mortgage market. A popular mortgage instrument since the 1930s has been the conventional fixed rate mortgage, FRM, which is a level payment instrument often with an option to prepay the outstanding balance with little or no penalty. While market interest rates vary over time, the fixed rate protects borrowers from upward movements of interest rates, and the option to prepay gives them the advantage to refinance at a lower cost when interest rates go down. High and increasingly volatile interest rates during the 1970s motivated considerable experimentation with new mortgage contracts. The principal result was the adjustable rate mortgage (ARM). The Federal Land Banks began using ARMs in the early 1970s with interest rate adjustments based on their average cost of funds. ARMs were introduced in residential lending in the early 1980s. A typical residential ARM has a long term but the interest rate is repriced yearly based on an index such as the one-year Treasury bill rate. An ARM provides a long-term loan but places the interest rate risk with borrowers, and thus makes it possible for the ARM rates to be generally lower than FRM rates.

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Market acceptance of ARMs depends on various elements. A study by Joseph Hu in 1992 showed that ARMs were particularly popular when FRM rates or FRM-ARM interest rate differentials were high. During the period from 1984-1990, ARMs accounted for more than 50 percent of total mortgage origination in the years 1984, 1985, 1987, and 1988 when the differentials were high. In 1986 and late 1989 when fixed mortgage rates were at single-digit levels, ARMs accounted for less than 30 percent of the total originations.

The interest rates on mortgages are determined by the current market situation. Borrowers face a set of mortgage alternatives whose repricing procedures vary. Besides the fully adjustable rate, there is a spectrum of interest repricing periods. The longest is the conventional fixed rate loan. A major component of the decision, also the focus of this article, is to choose the length of time that the rate is fixed.

The problem of valuing mortgage products from the perspective of a farm borrower is not straightforward. It is complicated by the need to include in the analysis: the stochastic nature of interest rates (yield curve), the option to refinance in the future, the farmer's portfolio and diversification issues, and risk aversion. This study determines the likely effect on farm mortgage choice of varying market yield curves. Specially, the choice among mortgages with different interest repricing periods is examined under the condition that any existing loan can be refinanced over the planning horizon. The sensitivity of the choice to initial yield curve conditions is examined.

The decision making model used in this study is driven by economic conditions that follow a stochastic process. The random variables include market interest rates, the value of farmland, and crop returns per acre. The stochastic process of market interest rates, (a yield curve that was made up of six Treasury interest rates with different time to maturity), is
estimated using the state space modeling method, developed by Aoki and Havenner (1991). The land value market is based on a model proposed by Featherstone and Baker (1988) that combines fundamental economic factors with market expectation adjustment.

Based on these stochastic processes, a simulation model was built to determine the preferable mortgage choice for a farmer borrower under different starting economic conditions. The simulation period was set to 20 years.

THE SIMULATION MODEL AND DATA

The Utility Function and Consumption Decision Rule

Mortgage loans are long term debt instruments, and the decision for initial loan choice must be considered under a multiperiod framework. It can be expressed as

\begin{equation}
\text{Max } E \left[ U(C, W_T) \right],
\end{equation}

where \( C \) is the random vector of consumption flow, \( W_T \) is terminal wealth, \( U(\cdot) \) is the multiple period utility function, and \( E \) is expectation operator. The specific utility function used in this study is a functional form suggested by Ingersoll (1987). Under the assumption that utility function form for consumption in each time period is the same, and that time preference is the form \( \delta^t \) (constant rate of time preference), the utility function can be expressed as

\begin{equation}
U(C) = \gamma^{-1}\left\{ \exp\left[ \gamma \sum_{t=0}^{T-1} \delta^t u(c_t) + \delta^T u(W_T) \right] \right\},
\end{equation}

where \( c_t \) denotes consumption expenditures in time \( t \). In this additive form the utility of current consumption is independent of past or future consumption and intertemporal risk aversion is zero.
The consumption decision-rule used in the simulation model is based on the dynamic programming solution to maximizing the above objective function subject to minimal restrictions. The final solution (see Ingersoll 1987) for time $t$ is:

$$C_t^* = \frac{1-\delta}{1-\delta^{-T-t+1}} W_t.$$ 

The objective function (utility) is arbitrary with respect to units. To improve understanding, we convert the expected utility into an annuity certainty equivalent. This annualized certainty equivalent is defined as follows:

$$\gamma^{-1} \exp[\gamma \sum_{i=0}^{T-1} \delta^i \ln(CE_A)] = EU,$$

where $CE_A$ denotes the annual certainty equivalent and $EU$ expected utility. This annual certainty equivalent is the constant consumption stream yielding indifference to the expected utility of the uncertain consumption stream.

**Interest Rate Movement and Time Series Modeling**

Since the 1970s, various econometric models have been used to estimate the movement of (the term structure of) interest rates. Many of these models were time series (reduced form) models. The major difficulty in time series modeling is the problem of model specification. This is especially true when it comes to yield curve modeling.

Inefficiency occurs when a large number of parameters need to be estimated. Aoki (see also Aoki and Havenner (1991)) proposed a new time series modeling approach based on the Linear Systems State Space Model (SSM). Essentially the State Space represents a first order augmentation of the conventional ARMA model, and thus it is different in form but equivalent mathematically. State space modeling is especially useful in multivariate
settings because it mitigates the need to explicitly specify the cross lag structure among the series being modeled. The cross lag structure in the SSM is implicitly determined by the choice of states that represent the dynamics of the system. This is accomplished by imposing restrictions on the parameters of equivalent ARMA representation. The implied restrictions are determined using information contained in the data.

In the state space approach to time series modeling, time series are described with the help of auxiliary variables (the state variables). They are linearly related to the observed process. SSMs contain state variables with at most one lag. This representation of dynamics by the single lag leads to a uniform treatment of time series and facilitates construction of algorithms for processing data. Among the several versions of the state space representation, the commonly used form is the one called innovation form:

\[(5a) \quad Z_{t+1} = AZ_t + B\varepsilon_t,\]
\[(5b) \quad Y_t = CZ_t + \varepsilon_t,\]

where \(Z_t\) is the vector of unobservable states; \(Y_t\) is the time \(t\) vector of variables to be forecast; \(A, B,\) and \(C\) are coefficient matrices to be estimated; and the noise vector \(\varepsilon\) has zero mean and a constant covariance matrix.

The first matrix equation is the state equation or dynamic equation. It describes the dynamic process of the system. All of the important characteristics of the process, both deterministic and stochastic, are specified by this equation. The second equation is called the observation equation, which relates the state variable, \(Z_t\), to the observed data.

Aoki's innovation was to suggest using equivalence of two alternative decompositions of the Hankel matrix as a means to derive parameter estimates. The Hankel matrix is a matrix of own and cross autocorrelations (or autocovariances) of the series. The
Kronecker theorem ensures that the rank of the Hankel matrix is equal to the number of states needed to specify the model. Substituting sample values for the autocorrelations in the Hankel matrix and applying singular value decomposition provides a means to approximate the rank and thus the number of states.

One of the unique properties of the state space model is that it always contains the most important variables no matter what the resulting model. Estimates of the matrices A, B, and C are closely related to the singular vector associated with the included singular values. As a result, the leading principal coefficient submatrices will always be those related to the largest singular values due to the fact that the states are brought into the model in the order of decreasing singular values, which implies the most important effects are modeled first. More important, adding more states into the model will not change the existing coefficients. Therefore, even in the face of model misspecification, the model is still consistent. And if, for example, the model is too small, the most important effects are included and only the less important are omitted.

The interest rate data used in the estimation are from “Analytical Record of Yields and Yield Spreads” (Solomon Brothers Inc.) which contains the monthly yields of US Government securities. Six maturities (3-month, 1, 3, 5, 10, and 20 years) are used in the model. The time period over which conditional forecast is made is on the quarterly basis. The overall time horizon of the data used is from March 1954 to April 1995. Therefore, the actual number of the observations used in the estimation is 165. The means, variance, and the skewness of the raw data are calculated and presented in Table 1.

The historical interest rate distributions are not symmetric, but rather positively skewed, suggesting that the commonly used normality assumption is not applicable.
Considering that the skewness is positive and increases with the variance, the lognormal
distribution assumption is a good candidate. The lognormal assumption also fits the fact that
the interest rates are non-negative.

Before estimation, the unit root hypothesis was tested for the six time series using the
Dickey-Fuller test statistic of the following form: \( \Delta Y_t = a + (b - 1)Y_{t-1} + \varepsilon_t \). The \( t \) statistics
for 3-month, 1, 3, 5, 10, and 20-year series are -3.29, -3.35, -3.16, -2.95, -2.66 and -2.63,
respectively. Based on the critical value for the Dickey-Fuller test at 90% confidence level (-
2.57) we rejected the hypothesis of unit root for all the 6 series.

The singular values obtained from Hankel matrix are presented in Table 2. It is apparent
that there is a big decrease between the first and the second singular values, indicating that
the first state is the dominant state. Three models were estimated: one with only one state,
one with two states, and the other with 3 states. The first and third models are not
significantly different from the 2-state model. As the first state is associated with a long run
stochastic trend component that is nearly nonstationary, it is prudent to include at least one
more state to allow for short-run dynamic components. However, the trade-off between
model generality and sampling error is the crux of time series modeling. Consequently, since
the 3-state model does not add much, we concluded that a two-state model is appropriate.
The estimated parameters are presented in Table 3.

Using the model structure, we have derived the theoretical forecast means and variances.

For the first period, the forecast variance is

\[
Var(Y_t) = C \text{var}(Z_t)C^T + \text{var}(\varepsilon_t) = \Psi
\]

Substituting continuously as is done above, we get the general form for period \( t \) forecast
variance

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\[ \text{Var}(Y_t) = \Psi + \sum_{i=0}^{t-2} CA^i B \Psi (CA^i B)^T \quad t=2,3,4,... \]

Note that the variances are independent of the initial values. They depend only on the model parameters A, B, C, and \( \Psi \), and on the length of the forecast. Variance increases with length of time but at a decreasing rate (as long as the model is stable).

The theoretical means of the forecast are calculated by eliminating the randomness brought about by the exogenous error (i.e., by assuming all the errors are zero). By substituting recursively we have

\[ E(Y_t) = CA^t C^{-1} Y_0. \]

The means are dependent on the initial conditions. The initial conditions have an effect that diminishes with increases in the time horizon, and the long run expectations converge to the sample means, which is one of the very important properties of the stationary ARMA models.

The Farm Land Market

The method used to model land value process is the following two equation:

\[ R_t = a_1 + b_{i1} I_{t-1} + \sum_{i=1}^{m} b_{ij} R_{t-i} \]

\[ L_t = a_2 + b_{20} R_t + \sum_{j=1}^{n} b_{2j} L_{t-j}, \]

where \( R_t \) is the cash rent for time \( t \), \( I_{t-1} \) is the operating income from farming in the previous period, and \( L_t \) is the market value of land at time \( t \). The lags, \( m \) and \( n \), are determined by a test for autocorrelation of the residuals of the regression.

The cash rent and land value data from 1972 to 1995 used in the estimation are from the Indiana Crop and livestock reporting service. The data on the net return to farmland is calculated on the assumption that the land is planted in a corn-soybean rotation. A 50/50
% weight is given to the returns from both corn and soybean. The residual return data, from
which the net return to farmland is calculated, are the estimates of the actual residual returns
to land for corn and soybean in Tippecanoe County, Indiana. All of the values are on a
nominal basis.

The regression is first run to determine the appropriate lag for both the land value and
cash rent equations. Box-Pierce and Ljung-Box test procedures are applied because: 1) it is
correct for models with random explanatory variables, and 2) it tests not only for first order
but also higher autocorrelations. The estimates for the final model (with m=1 and n=2) are
presented in Table 4.

The Refinancing Rule

The refinancing rule is first based on the assumption that the decision-maker considers
the expected cost of the loan as the primary factor. In the simulation model expected costs
for the current and alternative loans are calculated for n-periods ahead. These expected costs
depend on the terms of the existing loan, the current yield curve, and expectations of future
interest rates. The first parameter for the refinancing policy is the number of periods over
which the expected cost is calculated. The refinancing rule is secondly based on the
assumption that different loans have different risk implications. In terms of expected cost for
a risk averse producer, one dollar of cost with certainty is preferred to one dollar of
(uncertain) expected cost. Thus, loans with different interest rate repricing periods are not
entirely comparable using expected cost. We chose to decrease the expected cost of
alternatives using an "adjustment factor" AF, which is defined as $AF_y = (1 + \alpha)^{-y}$. Y is the
number of periods the interest rate is fixed (1, 3, 5, 10, or 20). Thus for the refinancing
policy, the parameters to be calibrated are: 1) n, the number of periods over which the
expected present value cost is calculated, and 2) \( \alpha \), the weight that determines the degree of
discount for loans with longer fixed interest rates among loans. The calculation period \( n \) is
varied over the range of \{1,2,3,4,5\}, and the parameter \( \alpha \) starts from zero, with an increment
of .0025 for 21 times. The combination of \( n \) and \( \alpha \) values with the highest objective
function values are chosen.

Two Assumptions about the Time Path of the Mortgage Loan

Mortgages are amortized according to their terms, the longer the time to maturity, the
less payment on the principal each time period and inevitably the higher interest cost each
time period because of the higher level of debt. The assumption made about the terms of the
refinancing loan is that the future principal balances of the new loan are the same as the
current loan.

Liquidity and Bankruptcy

Continued high levels of interest cost, declining land values, and/or extremely low net
returns to the farm operation, might put a farmer borrower into financial difficulties. In the
simulation model, short-term debt is the primary source of liquidity. In addition, surplus
cash is also accumulated in a liquid interest earning account.

However, there are times when cash needs are so large that these sources of funds are not
sufficient. Under these situations the farmer is in financial stress. A general rule of the
thumb with most of the financial institutions in the United States has been that a farm with
debt-to-equity ratio higher than two will be considered as vulnerable. When the farmer
borrower's debt-to-equity ratio reaches this point, it becomes very difficult to increase
borrowing. USDA and others have similarly used three debt-to-asset positions: 20\%, 40\%,
and 70\% to represent stable, moderately stressed, and severely stressed situations (Batte,
Far and Lee, 1989). Based on this view, the assumption made in the simulation model is that the farmer can borrow for liquidity purpose up to a debt-to-asset ratio of 65%. Once this point is reached, no more short-term debt can be obtained. The liquidity need is then met by selling farmland at the current market price.

Selling of the farmland asset might alleviate the financial stress, and as the conditions improve, the farm firm will recover from the adverse situation and remain in the farm business. The other possibility is that the conditions keep worsening and eventually bankruptcy will occur. Following Collins and Gbur (1991), we assume bankruptcy will occur before the net worth of the farmer borrower goes to zero, and that bankruptcy occurs when the debt-to-asset ratio reaches 95%.

EXPERIMENTAL DESIGNED

The experimental design employed here takes into account two major characteristics of yield curves: the position and the slope. The mean of the historical yield curves for quarterly data from the years 1954 to 1995 is chosen as the base yield curve. The effects of the changes in the position as well as in the slope of the yield curve are investigated by transforming the base yield curve into 12 more yield curves that are different from the base curve. The resulting set is made up of 13 yield curves (including the base) with different combinations of positions and slopes.

We obtain these curves by one or both of two transformations, which are called, parallel shifts and slope changes. A parallel shift moves each point on the base curve up or down by the same amount (number of interest percent points). There are two upward shifts from the
base of 0.5% and 1.0%, respectively. There are two downward shifts of 0.5% and 1.0%, respectively.

The slope changes are obtained by either “flattening” or “steepening” curves obtained by the parallel shifts. The 13 yield curves on which our experiments are based are displayed in Figure 1. The curves are indexed from the highest to the lowest. The maximum 20-yr rate in this experiment is 7.89, while the minimum 1-yr rate is 5.17. The base curve is curve number 7, with a spread between 1-yr rate and 20-yr rate of 0.72. The spread goes down as the curves flatten, and the minimum is 0.22 percent. The spread increases as the curves steepen, and the maximum is 1.22 percent. All the curves are “normal” yield curves with rates that increase with the length of the loan.

Figure 1. The Yield Curves of Selected Interest Rate Scenarios
Other model parameters of the simulation are the initial conditions, the interest rate margins, caps, and the transaction cost. These are presented in Table 5.

THE SIMULATION RESULTS

The Benchmark Curves and Simple Decision Rule

The preferred mortgage choice for each of the 13 interest rate scenarios is determined by comparing the annual certainty equivalent value \( CE_A \) of all loan choices. The mortgage that results in the highest \( CE_A \) is chosen as the best mortgage. For each simulation 10,000 iterations were run. Each of the 13 yield curve scenarios was simulated five times, once with each of the five mortgages available as the initial choice. Every simulation had all 5 loans as alternatives for refinancing. The resulting annual certainty equivalents for the 65 simulation experiments are presented in Table 6. In addition, the difference between the 20-yr loan and 1-yr loan is presented in the last column.

When the yield curve is in a higher position, the short-term loan becomes more attractive. This position effect can be observed by looking at any set of the yield curves that are parallel to each other. First, for curves that are parallel to the base (curve 3, 6, 7, 10, and 13), the differences in certainty equivalent of the 20-yr loan and 1-yr loan increase as the position of the yield curve moves downward. Second, the difference between curves 1 and 4, and between curves 2 and 5 increase from 195 to 348, and from -79 to 101, as the curves move downward. Finally, the differences between curves 8 and 11, curves 9 and 12, increase from -43 to 318, and from -269 to 89, as the curves move downward.

From the above observations two conclusions can be drawn: 1) the higher the position of the yield curve, the more attractive the short term loan becomes, and 2) the wider
the spread between the long and the short rates, the more attractive the short term loan
become.

To get quantitative information on how to choose among different loans, the t statistic
for significance is used for testing difference between the expected utilities of the one and
twenty-year loans.

\[ t = \frac{DE}{SD/(n)^{1/2}} \]

Where DE is the difference between the expected utilities of two loans, SD is the estimated
standard deviation of the utility differences between the two loans, and n is the number of
iterations (sample size). The statistics are presented in Table 7.

For one loan to be significantly different from the other, the difference between the
expected utility of the two loans should be greater than .02576. This difference converted
into certainty equivalent terms is approximately $145. We are comfortable with the number
of iteration at 10,000 on the basis that below $145, differences in certainty equivalents are
not economically significant. For the DE to be statistically non-zero, the absolute value of t
must exceed a certain critical value. Based on a 99 percent confidence criterion, the critical
value from the t table is 2.576. Therefore with 99 percent confidence, we can claim that
among the 13 yield curves we are able to determine the single best mortgage for the farmer
borrower in 9 cases. They are curves 1, 3, 4, 6, 7, 9, 10, 11, and 13. Among the 9 cases,
there are 3 cases where the 1-yr loan is better than the 20-yr loan (curves 3, 6, 9), and 6 cases
where the 20-yr loan is better than the 1-yr loan (curve 1, 4, 7, 10, 11, 13). Excluding those
curves that are parallel, we are left with 5 curves that can be used as the benchmark curves (6,
9 for 1-yr loan and 1, 7, 11 for 20-yr loan) for developing a rule of thumb.
There are some periods in the 90s when both the rates are relatively low and the spreads are high. In these cases, the above benchmark curves are insufficient for decision making. To make the set of benchmarks more practicable another simulation was performed on the yield curve of 4th quarter of 1993 (3.49, 4.32, 4.93, 5.55, 5.83), and is used as an additional benchmark curve for which the 1-yr loan is the preferable. The t-statistics for this experiment is -4.38. Based on all the information provided by the benchmark curves available, a simple rule of thumb is developed as follows:

A. The 1-yr loan is preferred if the treasury yield curve has any of the following 3 features:
   1) a 20-yr rate greater than or equal to 7.39, and a spread greater than or equal to 0.72;
   2) a 20-yr rate greater than or equal to 6.89, and a spread greater than or equal to 1.22;
   3) a 20-yr rate greater than or equal to 5.83, and a spread greater than or equal to 2.34.

B. The 20-yr loan is preferred if the yield curve has any of the following 3 features:
   1) a 20-yr rate less than or equal to 7.89, and a spread smaller than or equal to 0.22;
   2) a 20-yr rate less than or equal to 6.89, and a spread smaller than or equal to 0.72;
   3) a 20-yr rate less than or equal to 6.39, and a spread smaller than or equal to 0.97.

C. The final rule is that for concave inverted yield curves (20-yr rate less than 1-yr rate) the 20-yr loan is preferred.

**Application of the Simple Decision Rule**

The decision rule-of-thumb developed so far gives us a tool for selecting the most preferable mortgage under most interest rate scenarios. In this section, this rule is applied to the historical data of past 30 years. The interest yield curves examined are the quarterly average yield curves for the 30 years from the 3rd quarter of 1965 to the 2nd quarter of 1995, totaling 120 yield curves. A summary of these results is reported in Table 8.
During the period from 1965 to 1975, the 1-yr loan was not a good choice. It is preferred in only 2.5% cases to the other longer-term loans. On the other hand, the 20-yr loan is preferred in 65% of the cases. Including the 15% of the curves where the 10-yr loan is preferred, the chances are that in 80% of the cases, long term loan is preferred. This pattern has changed during the period from 1976 to 1985. The rate of preference for the 1-yr loan over other terms increased from 2.5% to 57.5%, i.e., by a factor of more than 20 times, accounting more than half of the total. The rate of preference for the 20-yr loan dropped to 28.2% of the total cases, or a bit more than half of the rate in the previous period. In the 1965-75 period long term loans were usually preferred, but during the 1976-1985 period, short term loans were the most preferred. The tide of change from preference for the long-term loan to short-term loan continues into the period 1986-1995. During this period, the rate of preference for the 20-yr term loan dropped to 5%, about the same as that of 1-yr loan in the years from 1965 to 1975. Meanwhile the rate of preference for the 1-yr loan rises to 87.5%, becoming dominant over the 20-yr loan. If we focus only on the 90s, this is even more the case, with 95.5% of the situations preferring for the 1-yr loan.

While the overall trend has been the change from the long-term loan to the short-term loan, the factors that are at work are not all the same. Though the general interest rate levels of late 60s and early 90s are basically in the same range, the slopes of the yield curves are quite different. Most of the curves in the 60s are flat and many are even inverted, while the curves in 90s are upward sloped. This explains a lot of the difference between the earlier and the more recent periods. In the transitional second period (1976-1985), the shift from the 20-yr loan to the 1-yr loan appears to have been caused in part by the high level of interest
rates, especially during the period of early 80s when most of the yield curves are at 2 digit level. In this period, both the position effect and the slope effect are at work.

Finally, it is very important to note that those cases that are indicated as statistically insignificant to draw conclusion (4th row in Table 6.4) are in fact cases where there is little economic difference between the two loans. This should come as no surprise. Under an efficient financial market, one would not expect the market to price loans so that one is always better than the other.

SUMMARY

The objective function adopted is the utility function suggested by Ingersoll (1989). According to this utility function, the welfare of the decision-maker is determined by the consumption flow generated from the farm land investment. Consumption in each time period is based on a function derived from recursively solving the dynamic programming problem. The refinancing strategy used in this study is a function of both the length of the time periods over which the expected cost of the mortgage is calculated and the difference in risk assumed by the farmer in taking loans with different repricing periods.

Results of the study indicate that the market interest rates, represented by the yield curves, affect the mortgage decision through changes in both the position and slope of the yield curve. The change in maximum annual certainty equivalents obtainable under different yield curve scenarios suggested that other things being equal, a higher (lower) yield curve makes the loan with shorter (longer) repricing period more attractive, while a flatter (steeper) yield curve makes the loan with shorter (longer) repricing period more attractive.

A simple rule-of-thumb was developed based on the above information and then applied to the historical data of past 30 years. The results show that the transition in market
demand for mortgages with different repricing period during the past 3 decades can be largely explained by the changes in the position and slope of the yield curves. The dominant demand for long term loans in late the 1960s and early 1970s were mostly due to the general low position of the yield curves. The transition period of the late 1970s to early 1980s during which the 1-yr loan became a better choice more frequently than the 20-yr loan can be explained mostly by the high position of the yield curves, many of which were at 2 the digit level. The general interest rate level came to the range of 5 to 7% since late 1980s, but the trend from demanding for long term loans to short term loans continued and the 1-yr loan became dominant during this period. It can be shown that the most likely force that contributed to the high demand for 1-yr loan was the changes in the slope of the yield curves from relatively flat to quite up ward sloped curves.
Table 1. Descriptive Statistics of the Raw Data (%)

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<th></th>
<th>3month</th>
<th>1 year</th>
<th>3 year</th>
<th>5 year</th>
<th>10 year</th>
<th>20 year</th>
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<tr>
<td>Mean</td>
<td>5.72</td>
<td>6.17</td>
<td>6.55</td>
<td>6.75</td>
<td>6.89</td>
<td>6.97</td>
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<td>Variance</td>
<td>9.04</td>
<td>9.43</td>
<td>8.66</td>
<td>8.45</td>
<td>8.26</td>
<td>8.37</td>
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<tr>
<td>Skewness</td>
<td>27.79</td>
<td>26.38</td>
<td>20.30</td>
<td>17.10</td>
<td>16.78</td>
<td>15.42</td>
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Table 2. Singular Values of the States

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<th>Singular value</th>
<th>Ratio of singular value</th>
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<td>1</td>
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<td>2</td>
<td>0.1474</td>
<td>0.0265</td>
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<td>3</td>
<td>0.0073</td>
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<td>B</td>
<td>-0.2951 -2.7696 -1.0108 -0.2338 -2.1178 3.5360</td>
<td></td>
</tr>
<tr>
<td></td>
<td>2.4257 2.5320 -3.8559 2.1249 7.2492 -10.984</td>
<td></td>
</tr>
<tr>
<td>C</td>
<td>-0.5094 -0.4234 -0.4421 -0.5286 -0.4613 -0.4411</td>
<td></td>
</tr>
<tr>
<td></td>
<td>0.0947 -0.0687 -0.0851 0.1253 0.0157 -0.0258</td>
<td></td>
</tr>
<tr>
<td>Ψ</td>
<td>0.0320 0.0153 0.0136 0.0323 0.0245 0.0204</td>
<td></td>
</tr>
<tr>
<td></td>
<td>0.0153 0.0089 0.0079 0.0156 0.0126 0.0110</td>
<td></td>
</tr>
<tr>
<td></td>
<td>0.0136 0.0079 0.0079 0.0141 0.0111 0.0096</td>
<td></td>
</tr>
<tr>
<td></td>
<td>0.0323 0.0156 0.0141 0.0360 0.0240 0.0202</td>
<td></td>
</tr>
<tr>
<td></td>
<td>0.0245 0.0126 0.0111 0.0240 0.0199 0.0167</td>
<td></td>
</tr>
<tr>
<td></td>
<td>0.0204 0.0110 0.0096 0.0202 0.0167 0.0143</td>
<td></td>
</tr>
</tbody>
</table>
Table 4. Estimated Parameters for the Land Value and Cash Rent Equations

<table>
<thead>
<tr>
<th></th>
<th>Parameter</th>
<th>T-ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Land value equation</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Intercept</td>
<td>-344.543</td>
<td>-1.433</td>
</tr>
<tr>
<td>Cash rent</td>
<td>13.344</td>
<td>2.706</td>
</tr>
<tr>
<td>First lag</td>
<td>0.993</td>
<td>4.600</td>
</tr>
<tr>
<td>Second lag</td>
<td>-0.590</td>
<td>-4.088</td>
</tr>
<tr>
<td><strong>Cash rent equation</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Intercept</td>
<td>14.261</td>
<td>1.868</td>
</tr>
<tr>
<td>Economic rent</td>
<td>0.026</td>
<td>2.705</td>
</tr>
<tr>
<td>First lag</td>
<td>0.821</td>
<td>12.351</td>
</tr>
</tbody>
</table>

$R^2$ for land value and cash rent equations are .92 and .89, respectively. The standard deviations of the regression errors of the two equations, the land value and the cash rent, are 119.55 and 6.03, respectively.

Table 5. The Initial Conditions and the Economic Parameters

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mortgage rate margin</td>
<td>2.00 (%)</td>
</tr>
<tr>
<td>Land bought</td>
<td>1,000 (acre)</td>
</tr>
<tr>
<td>Initial net worth</td>
<td>750,000 ($)</td>
</tr>
<tr>
<td>Initial debt</td>
<td>600,000 ($)</td>
</tr>
<tr>
<td>Caps</td>
<td></td>
</tr>
<tr>
<td>Period cap</td>
<td>2.00 (%)</td>
</tr>
<tr>
<td>Lifetime cap</td>
<td>6.00 (%)</td>
</tr>
<tr>
<td>Farmland value</td>
<td></td>
</tr>
<tr>
<td>First period</td>
<td>1,350 ($)</td>
</tr>
<tr>
<td>Second period</td>
<td>1,450 ($)</td>
</tr>
<tr>
<td>Net return to farm land</td>
<td></td>
</tr>
<tr>
<td>Mean</td>
<td>111.11 ($)</td>
</tr>
<tr>
<td>Standard deviation</td>
<td>36.19 ($)</td>
</tr>
<tr>
<td>Transaction cost</td>
<td>0.03*loan +350</td>
</tr>
</tbody>
</table>
Table 6. The Highest Obtainable Annual Certainty Equivalents for the 13 Yield Curves and the Difference Between the Certainty Equivalents for the 1-yr Loan and the 20-yr Loan

<table>
<thead>
<tr>
<th>Curve No.</th>
<th>1-yr</th>
<th>3-yr</th>
<th>5-yr</th>
<th>10-yr</th>
<th>20-yr</th>
<th>diff</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>83683</td>
<td>83006</td>
<td>83265</td>
<td>83495</td>
<td>83563</td>
<td>195</td>
</tr>
<tr>
<td>2</td>
<td>83683</td>
<td>83211</td>
<td>83387</td>
<td>83598</td>
<td>83604</td>
<td>-79</td>
</tr>
<tr>
<td>3</td>
<td>83953</td>
<td>83348</td>
<td>83509</td>
<td>83647</td>
<td>83654</td>
<td>-299</td>
</tr>
<tr>
<td>4</td>
<td>84074</td>
<td>83770</td>
<td>83998</td>
<td>84351</td>
<td>84422</td>
<td>348</td>
</tr>
<tr>
<td>5</td>
<td>84358</td>
<td>83901</td>
<td>84115</td>
<td>84401</td>
<td>84459</td>
<td>101</td>
</tr>
<tr>
<td>6</td>
<td>84653</td>
<td>84051</td>
<td>84228</td>
<td>84503</td>
<td>84507</td>
<td>-146</td>
</tr>
<tr>
<td>7</td>
<td>85327</td>
<td>84764</td>
<td>85011</td>
<td>85392</td>
<td>85497</td>
<td>169</td>
</tr>
<tr>
<td>8</td>
<td>85572</td>
<td>85014</td>
<td>85226</td>
<td>85520</td>
<td>85529</td>
<td>-43</td>
</tr>
<tr>
<td>9</td>
<td>85821</td>
<td>85250</td>
<td>85436</td>
<td>85534</td>
<td>85552</td>
<td>-269</td>
</tr>
<tr>
<td>10</td>
<td>86000</td>
<td>85507</td>
<td>85808</td>
<td>86335</td>
<td>86534</td>
<td>534</td>
</tr>
<tr>
<td>11</td>
<td>86231</td>
<td>85772</td>
<td>86032</td>
<td>86461</td>
<td>86549</td>
<td>318</td>
</tr>
<tr>
<td>12</td>
<td>86478</td>
<td>86017</td>
<td>86253</td>
<td>86478</td>
<td>86567</td>
<td>89</td>
</tr>
<tr>
<td>13</td>
<td>86653</td>
<td>86269</td>
<td>86625</td>
<td>87302</td>
<td>87595</td>
<td>942</td>
</tr>
</tbody>
</table>

Table 7. Statistics Regarding the Difference in Expected Utility of the 20-yr Loan and the 1-yr Loan

<table>
<thead>
<tr>
<th>Curve No.</th>
<th>DE</th>
<th>SD</th>
<th>t value</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.036</td>
<td>1.009</td>
<td>3.57 *</td>
</tr>
<tr>
<td>2</td>
<td>-0.015</td>
<td>0.971</td>
<td>-1.54</td>
</tr>
<tr>
<td>3</td>
<td>-0.055</td>
<td>0.924</td>
<td>-5.95 *</td>
</tr>
<tr>
<td>4</td>
<td>0.064</td>
<td>1.141</td>
<td>5.61 *</td>
</tr>
<tr>
<td>5</td>
<td>0.018</td>
<td>0.988</td>
<td>1.82</td>
</tr>
<tr>
<td>6</td>
<td>-0.027</td>
<td>0.940</td>
<td>-2.87 *</td>
</tr>
<tr>
<td>7</td>
<td>0.031</td>
<td>1.019</td>
<td>3.04 *</td>
</tr>
<tr>
<td>8</td>
<td>-0.008</td>
<td>0.995</td>
<td>-0.80</td>
</tr>
<tr>
<td>9</td>
<td>-0.048</td>
<td>0.991</td>
<td>-4.84 *</td>
</tr>
<tr>
<td>10</td>
<td>0.095</td>
<td>0.945</td>
<td>10.05 *</td>
</tr>
<tr>
<td>11</td>
<td>0.057</td>
<td>0.921</td>
<td>6.19 *</td>
</tr>
<tr>
<td>12</td>
<td>0.016</td>
<td>8.881</td>
<td>1.82</td>
</tr>
<tr>
<td>13</td>
<td>0.166</td>
<td>0.911</td>
<td>18.22 *</td>
</tr>
</tbody>
</table>

a: Average difference in expected utilities of 1-yr rate and 20-yr rate loans
b: Standard deviation of the differences
c: The critical value is 2.576 at the 1% significance level
Table 8. Percentage of Loan Terms Preferred During the Past 30 Years (%)

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1-yr</td>
<td>2.5</td>
<td>57.5</td>
<td>87.5</td>
<td>49.2</td>
<td>95.5</td>
</tr>
<tr>
<td>20-yr</td>
<td>65.0</td>
<td>28.2</td>
<td>5</td>
<td>32.5</td>
<td>-</td>
</tr>
<tr>
<td>10-yr</td>
<td>15.0</td>
<td>-</td>
<td>-</td>
<td>5</td>
<td>-</td>
</tr>
<tr>
<td>Indeterminate *</td>
<td>17.5</td>
<td>14.3</td>
<td>7.5</td>
<td>13.3</td>
<td>4.5</td>
</tr>
<tr>
<td>Total</td>
<td>100.0</td>
<td>100.0</td>
<td>100.0</td>
<td>100.0</td>
<td>100.0</td>
</tr>
</tbody>
</table>

a: indeterminate because the difference between the loans is not sufficient to draw a conclusion
References


Solomon Brothers Inc. *Analytical Record of Yields and Yield Spreads* 1995