The Optimal Time Path for Carbon Abatement and Carbon Sequestration under Uncertainty: The Case of Stochastic Targeted Stock

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INTRODUCTION

Although there is a consensus in the scientific world that carbon sequestration should be included in a portfolio of GHG mitigation strategies (WG II, IPCC, 2007; Richards and Stokes, 2004), the optimal timing of its implementation is still debated.

An important feature of carbon sequestration that distinguishes it from abatement technologies is its ability to actually reduce atmospheric concentrations of CO₂.

This paper explores the optimal time path of carbon sequestration and carbon abatement in stabilizing the level of carbon dioxide in the atmosphere under uncertainty in climate impacts.

MODEL

We apply backwards induction starting with the minimization problem for the second period:

\[ \min_{x_2, x_1} \left[ C_1(x_1) + C_2(x_2) + \int_{x_2}^{x_1} g(x; \theta_1) dx \right] \]

subject to:

\[ B_1 = x_1 - E - A - S \]

\[ x_1 \geq 0 \]

\[ x_2 \geq 0 \]

\[ x_1 \leq C_1 \]

\[ x_2 \leq C_2 \]

\[ A \equiv \frac{1}{2} \frac{\mu^2}{\sigma^2} \]

\[ B \equiv \frac{1}{2} \frac{\mu^2}{\sigma^2} \]

\[ S \equiv \frac{1}{2} \frac{\mu^2}{\sigma^2} \]

\[ \theta_1 \equiv \frac{1}{2} \frac{\mu^2}{\sigma^2} \]

\[ \theta_2 \equiv \frac{1}{2} \frac{\mu^2}{\sigma^2} \]

Variables list (iso 1):

\[ A \equiv \text{level of abatement in atmosphere} \]

\[ S \equiv \text{rate of sequestration} \]

\[ B \equiv \text{rate of abatement deployment} \]

\[ E \equiv \text{mean of stabilization level} \]

\[ C_1 \equiv \text{critical capacity of abatement needed} \]

\[ C_2 \equiv \text{critical capacity of seq required to meet } \theta \]

\[ \theta_1 \equiv \text{discount factor} \]

\[ \theta_2 \equiv \text{variability of the stabilization level} \]

\[ \theta_3 \equiv \text{mean of stabilization level} \]

\[ \theta_4 \equiv \text{excess sequestration capacity} \]

The above convexity properties of \( f_i(x) \) determine qualitatively the respond of the system when uncertainty in the stabilization level exists. Apply simple function forms: Let \( f_i(x) = x^{-\alpha} \), where \( \alpha > 1 \) and the rate of sequestration, \( S \), is bounded above the available sequestration capacity, \( C_2 \). Similarly, let the marginal cost of abatement be \( B(x) = \frac{1}{2} x^2 \), where \( \theta > 1 \) and the rate of abatement, \( A \), cannot exceed the rate of emissions, \( E \). In addition, assume \( C_1 > C_2 \), so that sequestration is cheaper than abatement initially.

The key innovation in our paper is that we provide an analytical treatment of the optimal timing of carbon sequestration and abatement under uncertainty. We show that uncertainty can make it optimal to use carbon sequestration either earlier or later and clarify the conditions under which different effects of uncertainty are obtained.

Uncertainty over climate damages is introduced into the model by recognizing that today we cannot be sure of the amount of warming expected at different atmospheric CO₂ concentrations. However, as time progresses, society’s understanding of the severity of climate impacts will presumably increase.

COMPARATIVE STATICS

An increase in the level of the discount factor, \( \alpha \), could either result in more or less deployment of abatement (and consequently, more or less sequestration in the first period) depending on the ratio between the elasticity of excess sequestration and the elasticity of excess abatement.

Two important factors in determining the actual outcome of the system are the magnitude of sequestration capacity and the ratio between the elasticity of excess abatement and the elasticity of excess sequestration capacity.

The existence of both the precautionary path and the ambiguous path is hinged on the availability of large volume of sequestration capacity. This assumption is consistent with results from recent simulation analysis suggesting a considerable conversion of land to forest depending on predicted carbon-price paths (Sohngen and Mendelsohn, 2003; Richards & Stokes, 2004; Sohngen and Sedjo, 2006).

DISCUSSION

1. The ambiguous path is not only combining the results for the aggressive path and the precautionary path but also relate the two paths on an abatement deployment rate scale. Here, not only the elasticity of excess sequestration with respect to excess abatement plays a role but also how cheap or expensive abatement is. The combination of both, together with large enough sequestration capacity, will dictate if we are at the precautionary path, the aggressive path or, somewhere in between where uncertainty in climate impacts does not have a real effect on current deployment of abatement and sequestration.

2. The positive dependency between the deployment of sequestration and abatement in the first period is in agreement with previous studies suggesting that sequestration and abatement are compliments in the short run rather than substitutes (Stein, 1999; Richards & Stokes, 2004). In addition, the cross-temporal dependency in sequestration under uncertainty in climate impacts reflects either complimentary or substitutable relationship depending on the ratio between the elasticity of excess abatement and the elasticity of excess sequestration capacity.