The Role of Mining in an Australian Business Cycle Model

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Declaration

Except where appropriately acknowledged this thesis is my own work, has been expressed in my own words and has not previously been submitted for assessment.

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Abstract

The purpose of this paper is to evaluate a business cycle model that includes a mining sector, with the cyclical variations of the Australian Economy. Large quantities of mineral deposits are found in Australia and there exists high demand for these minerals from developing nations. This results in the mining sector contributing to a high proportion of GDP. Surprisingly, the inclusion of a mining sector has not previously been studied in a business cycle model. Australia is a small open economy however, due to a lack of prior literature then, as a first attempt, we assume an economy without a foreign sector. The model built upon a neoclassical growth model, and results were simulated from solving this model via the Blanchard-Kahn method. The statistics generated show that some variables are capable to closely model some of the elements of the Australian economy. However, other variables display standard deviations and contemporaneous correlations, which are substantially different to the actual data. This is implying that the inclusion of the basic mining mechanism alone does not provide the perfect representation of the Australian economy. As the importance of mining is growing in Australia, research should be undertaken to evaluate the impact of the mining sector in economic models.

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1 Introduction

In recent years Macroeconomists have found that when comparing the stochastic one-sector neoclassical growth model (known as the baseline Real Business Cycle model) with the fluctuating behaviour of the actual economy, the model does not successfully imitate the economy mainly due to its lack of volatility in output. In Australia, a significant proportion of GDP is contributed by the domestic mining sector. There are many people employed in mining and a significant amount of money is being invested to improve the efficiency of the extraction and exploration processes. Australia is a small open economy however, since including a mining sector into a business cycle model has not been previously studied then, as a first attempt, we assume an economy without a foreign sector. The purpose of this paper is to investigate whether the inclusion of a mining sector into an Australian business cycle model replicates the cyclical variability observed in the actual data of the economy.

The key ingredient of this model, which builds upon Kydland and Prescott's (1982) baseline RBC[1] model, is that search and extraction of the non-renewable resource[2] must first be undertaken before the good can be used in production to form the consumption good. There are separate firms which search, extract and produce the final good, with productivity shocks to the final goods firm and the search firm. Productivity shocks to the search sector attempt to replicate the notion that search of minerals is variable. The timing of the extraction firm plays an important role in this model as new deposits, which are bought from the search firm, cannot be extracted until the following period and the non-renewable resource which is extracted today cannot be sold until the following period. Therefore the extraction firm must build expectations about next period's demand (this will fluctuate due to perturbations of productivity) for the non-renewable resource to ensure the correct amount will be above-ground next period. The extraction firm has incentives to behave according to this expectation as the extraction production function exhibits diminishing returns to each factor input, thus making it costly to extract more than required. These features provide the framework of the mining mechanism analysed in this paper.

[1] The terms Real Business Cycle Model and business cycle model are interchangeable throughout this paper

[2] Section 2 explains the definition of a non-renewable resource

When comparing the mining business cycle model with the Australian economy, it produces a variety of results. Consumption, the real wage, search labour and non-mining labour display similar results to their empirical counterparts, however lack similarities in persistence. For the most part, contemporaneous correlations between variables are of different strengths and often display the opposite cyclical relationship than what is observed in the actual data. Nevertheless, these results indicate that the inclusion of a mining sector has the ability to replicate some of the cyclical volatilities observed in the Australian economy.

There have been many previous studies of business cycle models. This paper is differentiated from the current RBC literature, as the inclusion of an entire mining sector into an RBC model has not been previously attempted. Studies have been undertaken to include oil shocks (Kilian 2008) and energy shocks (Kim and Loungani 1992), although they did not include the production process to obtain these goods. Benhabib, Perli and Sakellaris (2005) studied the inclusion of multisector business cycle models, with consumption and investment sectors, however as will be explained in the following section, there are key characteristics of mining which make this vastly different to other inputs of production.

This paper proceeds as follows. Section 2 provides a brief literature review on the theory of business cycles and the reasoning behind undertaking this research. It explains from where business cycles theory originated, what this paper adds to the literature and the key characteristics of a mining sector. Section 3 describes how the empirical counterparts of the variables used in the model were constructed and the specific sources of the data. Section 4 provides an explanation of the measurement of business cycles using the Hodrick-Prescott filter. This section includes facts and statistics about Australian business cycles which the results created, by the model, can be compared with. Section 5 explains the components of the model in detail. Section 6 provides an example of the model in functional form, the subsequent steps taken to obtain a solution and how the model's parameters were calibrated. Section 7 reports the model's results, section 8 gives an overall discussion about the model and section 9 provides concluding remarks.

2 Background

Business cycle theory studies the causes of regular expansions and contractions in aggregate economic activity that occur in many countries around the world (King and Rebelo 1999). By the 1960s Keynesian models dominated macroeconomics and Keynesian business cycle models were recognised as a good representation of the behaviour of the economy in the short run. The 1970s brought the revolutionary Rational Expectations theory instigated by Lucas (1976). The key principles were: (1) Macroeconomic models should be based upon the microeconomic principles of preferences, endowments, technology and the optimising behaviour of households and firms; and (2) equilibrium models were the best method for macroeconomic modeling. In response to these principles, Kydland and Prescott (1982) were the first to create a model which used standard theory of economic growth which was subject to stochastic perturbations to productivity. They found that growth models could predict fluctuations in the economy if the stochastic perturbations to productivity were persistent and, of the right magnitude. Prescott (1986) found that these shocks could be well represented by the Solow residual. A major criticism of the model is that estimates of Solow residuals imply a probability of technical regress of 40 per cent (King and Rebelo 1999); which seems improbable to most economists as it is unsure of what could produce shocks of such variation. Studies have been conducted to recalculate the Solow residual to mainly correct for the mismeasurement of the unobserved effort and capacity utilisation. This has resulted in more realistic perturbations being produced as shocks are smaller and less cyclically volatile.

When comparing some of the unconditional moments generated by the standard (baseline) RBC model with those from the U.S. economy (see King and Rebelo 1999 for an in-depth explanation on the baseline RBC model using U.S. data) it can be observed that output and consumption are not as volatile as the data. Labour, wages and interest rates are too volatile compared to output in the model. Moreover, wage and labour are highly correlated in the model but almost uncorrelated in the data and the real rate of return is highly procyclical in the model whereas, in the data, it is almost acyclical. This could have been the end of the RBC

model, however many extensions have been applied to the baseline RBC model to attempt to better replicate U.S. business cycles. For example, King and Rebelo (1999) created a model to include capital utilisation, so that households could work their capital harder in productive periods, which successfully replicated data from the U.S. despite smaller and less volatile productivity shocks. Other extensions include investment specific shocks (Papanikolaou, 2011), oil shocks (Barsky and Killian 2004), the inclusion of a government sector with additional shocks to government spending and monetary shocks (Christiano and Eichenbaum 1992)[3].

The majority of all work in this field uses data from the United States. This paper creates an extension of the RBC model using Australian data. Very few studies have been undertaken using data from Australia to evaluate business cycles models. The most recognised studies are from Backus and Kehoe (1992), Fisher, Otto and Voss (1994) and Crosby and Otto (1995). In general they found that the results from an Australian business cycle model were consistent with results using U.S. data, apart from wages, which is explained in a subsequent section.

Mining in Australia is a major industry. In 2009-10, according to the Australian Bureau of Statistics' measure of national accounts[4], mining contributed to approximately 10 percent of total GDP. The mining industry was the second highest contributor behind financial and insurance services which contributed 11 percent to GDP. Australia has some of the largest mineral deposits in the world. For example Australia is home to the world's largest known deposit of Uranium, is second largest producer of Iron Ore, Zinc and Nickel and has the world's fourth largest deposit of Coal, to name a few. With the continuation of strong growth in emerging economies such as China and India, the demand for Australian minerals is increasing and as a result, more resources are being placed into the mining sector. Studies have previously been undertaken to include energy price shocks and oil price shocks into a business cycle model, where these goods were either imported or simply available for use in the production process. This paper could simply test a commodity price shock in an Australian business cycle model, however this does not seem appropriate as Australia is actually producing these miner-

[3] see Rebelo (2005) for several examples of various extensions of the baseline RBC model
[4] www.abs.gov.au

als. Due to the importance of mining in Australia, the addition of this sector has the potential to produce a convincing business cycle model.

Boadway and Keen (2009) defined the key characteristics of a mining sector and explained how the goods produced from mining were so different to other inputs of production. These ideas were built upon to create the basis of the model presented in this paper which focuses specifically on, non-renewable resources that are deep in the ground. Therefore the good can only be used once and search must be undertaken to find the good. The key processes of the mining sector are: (1) Search - to find the deposit, (2) Extraction - remove the non-renewable resource from the ground, and (3) Final goods firm - use the resource in production to form the consumption good. Many of the characteristics at these stages are very similar to an economy which is engaging in research and development. For example there are large uncertainties involved in relation to discoveries and the size of the deposits. The main difference between a mining sector and research and development is the feature of exhaustibility. In research and development once a discovery has been made it can simply be replicated over and over. With a non-renewable resource there is a finite potential of production; if more is extracted today, there is less available to extract tomorrow. Thus, search must be continually engaged in to replenish stocks of the non-renewable resource. The complex nature of a non-renewable resource differentiates this from other inputs of production.

The lack of studies of the mining process in business cycle models, let alone in Australian business cycle models, places this paper as an interesting addition to the current literature.

3 Data

In order to analyse the results of the multi-sector RBC model presented in a later section, quarterly data is required to represent the equivalent of variables in the model: output (Y), consumption (C), investment (I), capital (K), labour from the extraction sector (N_E), labour from the search sector (N_S) and labour from the rest of the economy (N_F)[5], the real rate of return (R) and the real hourly compensation (W). Other aspects of the model did not have a suitable

[5] The various components of the model will be explained in detail further into the paper

empirical counterpart and therefore are not included in the analysis of the model's performance. This section explains how the data equivalents of the variables were constructed and what data was used in this process.

The data series chosen is between the first quarter in 1985 to the first quarter in 2010 due to availability. All data, apart from the real rate of return, is sourced from the Australian Bureau of Statistics[6](ABS). Specific catalogue numbers and tables numbers are listed in the appendix.

Gross Domestic Product (GDP) is the market value of all final goods and services produced within an economy over a certain period of time and hence can be used to describe output in this model. GDP is normally calculated to equal the sum of consumption, investment, government expenditure and net exports. As the model, to be described in due course, represents a closed economy with no government expenditure then consumption and investment must be calculated in such a way that their sum must be equal to GDP.

In this model there is only one good available in the economy that expenditure on consumption can be made upon. Therefore final consumption expenditure from all sectors in the economy is used as the empirical counterpart. This is following the method of Farmer and Guo (1994).

Cooley (1997) stated that for a one-sector economy, investment should be constructed in such a way that it was equal to the gross fixed capital formation from all sectors, consumption of durable goods, changes in inventories and net exports. Consumption of durable goods were included in this construction of investment as they were seen as additions to the household's stock of capital. Net exports were included as the model represents a closed economy. Although the RBC model presented in this paper is not a one-sector economy, due to the fact that the household owns all capital resources in this economy and capital is not firm specific, investment can still be calculated for the aggregate using the definition from Cooley (1997). The ABS does not report on a suitable statistic for consumer durables, hence this is not included in the measure for investment. Since there is not separate data for consumption on durables this has been included in the empirical counterpart for con-

[6] ABS - www.abs.gov.au

sumption. Figure 1[7] shows the sum of consumption and investment against output. It can be seen that the two measures are very close to equality. This is consistent with the household's market clearing budget constraint, which will be described in a later section.

The ABS has an annual series to report Australian capital stock. Private non-farm inventory levels are also included in the measure of capital to be consistent with the measure of investment. This is not available in a quarterly series from either the ABS or any other statistical agency and thus, a quarterly series is constructed to closely match the annual data available. Due to the fact that the household owns all the capital stock in the economy and that capital is not firm specific, using a measure of aggregate capital stock is appropriate.

Employment was used to represent the different sectors in the model. The ABS reports on total employment in the mining sector, as well as those employed specifically for mining exploration. Subtracting exploration employment from total mining employment gives a measure for those employed in the extraction sector of the economy. In this sector there are workers who are digging for the non-renewable resource and workers who are involved in selling the non-renewable resource to the final goods firm. It is not possible to find the two specific employment figures, however finding data to represent the sector gives a better indication than simply total employment in the entire economy. Total mining employment was subtracted from total employment to gain a value for non-mining employment.

To calculate the real hourly compensation, the average weekly wage was taken and divided by the average hours worked per week to obtain the average hourly wage. This is expressed in current prices, therefore a consumer price index needed to be constructed with the base year in 1985 in order to obtain the real average hourly compensation. This was calculated using the following formula

$$CPI = \frac{Nominal}{Real} * 100$$

Data from the Reserve Bank of Australia[8] on the cash rate was used to represent the rate of return. This is reported monthly there-

[7] Figures are shown in appendix A at the end of the paper
[8] www.rba.gov.au

fore quarterly averages were taken to transform the series. The quarterly data set then had to be converted to obtain the real rate of return. This was achieved using the Fisher equation (Fisher 1977),

$$\frac{1 + Interest\,Rate}{1 + Inflation\,Rate} - 1 = Real\,Interest\,Rate$$

where the Reserve Bank of Australia's measure of inflation was used to adjust the interest rate.

4 Stylised Facts of Australian Aggregate Activity

Lucas(1977) found that "business cycles are all alike." This suggests that there are common features to business cycles, therefore stating that country-specific peculiarities and factors from institutions such as central banks and governments do not trigger business cycles. To evaluate the outcome of the model created in this paper, the cyclical component needs to be extracted from the data. This section explains the method used to obtain these fluctuations.

4.1 The Hodrick-Prescott Filter

The theory of business cycles is focused on the cyclical fluctuations of the economy around trending growth. Most economic data grows over time, thus creating the need to separate data into the trending and the cyclical components, in order to evaluate the latter of the two. A popular method to decompose data in this manner is to use a Hodrick Prescott filter, commonly known as the HP filter, created by Hodrick and Prescott (1980). The HP filter takes a data set, y_t and separates this into the trend component, y_t^g, and the cyclical component, y_t^c

$$y_t = y_t^g + y_t^c$$

The trend component is produced by solving the following minimisation problem

$$min_{\{y_t^g\}_{t=0}^{\infty}} \sum_{t=1}^{\infty} \{(y_t - y_t^g)^2 + \lambda[(y_{t+1}^g - y_t^g) - (y_t^g - y_{t-1}^g)]^2\}$$

The first term in the equation measures the degree of fit between y_t and y_t^g and the second term measures the smoothness of y_t^g across

adjacent periods. The trade-off between fit and smoothness depends on the weight placed on the parameter λ, where different values are placed depending on the frequency of the data. For monthly data $\lambda = 10000$ and for quarterly data $\lambda = 1600$. Prescott (1986) has expressed that the HP filter eliminates stochastic components with periodicities greater than 32 quarters (8 years). This means that by using the HP filter, this defines the fluctuations of the business cycle in the time-series with periodicities of less than or equal to 32 quarters (8 years). Therefore low-frequency movements of the data, which may impact the business cycle fluctuations, will be omitted by this definition.

The natural log of the data is taken and then passed through the HP filter to obtain the trending component. The trending component is then subtracted away from the logarithmic data to acquire the cyclical values. Figure 2 shows Australia's quarterly GDP in log form and the HP trend of this data. All other data was passed through the same process with the exception of the real rate of return.

4.2 Australian Business Cycles

Using data from 1985(1) to 2010(1)[9], Figure 3 compares the cyclical component of the relevant data against the cyclical component of GDP. From observing the relationship between output (GDP) and other data sets the following characteristics of Australian business cycles can be made:

- Consumption is less volatile than output and is procyclical
- Investment is more volatile than output and is procyclical
- Capital is less volatile than output and is procyclical
- Non-mining Labour is slightly more volatile than output and is procyclical
- Extraction Labour is more volatile than output and is procylical
- Search Labour is significantly more volatile than output and is procyclical

[9] The number in the brackets refers to the quarter of the reference year

- Real Hourly Compensation is slightly more volatile than output and is acyclical

Table 1 provides the magnitude of fluctuations (measured by the standard deviation), the magnitude of fluctuations relative to output (measured by relative standard deviations), the quarterly autocorrelation and the correlation coefficients between each of the variables.

Table 1: Australian Economy Statistics

	Y	C	I	N_F
Standard Deviation	0.0104	0.0083	0.0280	0.0122
Relative Standard Deviation	1.0000	0.7932	2.6804	1.1701
Quarterly Autocorrelation	0.8135	0.7249	0.5341	0.7551
Correlation Matrix				
Y	1	0.51849	0.6815	0.5448
C	-	1	-0.1383	0.4855
I	-	-	1	0.1784
N_F	-	-	-	1
N_E	-	-	-	-
N_S	-	-	-	-
W	-	-	-	-
R	-	-	-	-

	N_E	N_S	W	R
Standard Deviation	0.0535	0.1384	0.0122	0.0223
Relative Standard Deviation	5.1322	13.2710	1.1664	2.1267
Quarterly Autocorrelation	0.3314	0.3684	0.3319	0.9411
Correlation Matrix				
Y	0.0302	0.2992	-0.20337	0.2880
C	-0.1108	0.1108	-0.0561	0.3832
I	0.0633	0.1594	-0.1817	-0.0021
N_F	0.2644	0.2051	-0.2431	0.4172
N_E	1	0.0615	-0.0716	0.1136
N_S	-	1	-0.0398	0.0956
W	-	-	1	-0.0090
R	-	-	-	1

From the table it can be seen that all data, except from real wages, experience a positive contemporaneous correlation with output. The

result, that real wage is negatively correlated with output, is different to that observed when using US data, where wages are procyclical (see King and Rebelo 1999). A procyclical result is also observed when using data for the United Kingdom (see Blackburn and Ravn 1992). Fisher et al (1994) found evidence that suggested the real wage in Australia was countercyclical in the 1970s and 1980s. In the RBC literature it is a long established fact that there is a lack of relationship between real wages and employment (see Tarshis 1939). This feature is observed with Australian data as the contemporaneous correlations between the real wage and non-mining labour, the real wage and extraction labour and the real wage and search labour are -0.2431, -0.0716 and -0.0398 respectively.

5 Model

The model to be described in this section builds upon the standard decentralised Kydland-Prescott (1982) baseline RBC model; a one-sector neoclassical growth model with variable labour supply. In this model there are a continuum of identical households, final goods firms, extraction firms and search firms, all with unit mass. Households own labour and capital and receive profits from the firms. Households rent their labour and capital to the firms and make consumption and investment decisions. Households take the prices of labour, capital and output as given when determining how much to supply/consume of each.

Output is made using labour, capital and the non-renewable resource, all of which are essential to production. The non-renewable resource must first be searched for and then extracted from the ground before it can be used in production to form the consumption good. There are different firms which handle each stage of this process. The final goods firms and the search firms experience fluctuations in productivity to their production functions. At the beginning of the period productivity is revealed, which is perfectly observable to all firms and the households. All firms choose their labour and capital demands. The extraction firms choose their extraction level, deposit demand and the supply of the non-renewable resource. The final goods firms choose their non-renewable resource demand and households choose their labour and capital supply as well as make their consumption and investment decisions.

In this model exogenous fluctuations to productivity in the final goods firm and the search firm create business cycle variations. Variations in productivity create incentives to change labour and capital supplies and demands, as well as, the demand and supply of the non-renewable resource, hence creating fluctuations in the business cycle.

5.1 The Household

The infinitely lived representative household has preferences over consumption and leisure that are defined by

$$E_0 \sum_{t=0}^{\infty} \beta^t U(C_t, L_t)$$

where C is consumption of the final goods firm's output (the price is normalised to one), L is leisure and $\beta \in (0,1)$ is the subjective discount factor. E_0 denotes the expectation of future values of C and L based on the information available at time zero. The period utility function of the household, $U(C_t, L_t)$ has the property that $\lim_{c \to 0} U_C(C, L) = \infty$, to ensure households will never choose zero consumption. The utility function also has the properties of $U_C(C, L) > 0$ to ensure non-satiation in consumption, $U_{CC}(C, L) < 0$ to ensure the marginal utility of consumption diminishes with an increase in consumption and $U_L(C, L) > 0$ to ensure marginal utility of leisure is positive.

Each household is endowed with one unit of time to divide between total time working (N_t) and leisure (L_t). For each period the household's time constraint is:

$$N_t + L_t = 1.$$

In this economy, the household can provide labour services into four different areas; the final goods firm; the extraction firm for digging labour; the extraction firm for sales labour and the search firm. Therefore total labour supplied can be written as

$$N_t = N_{F,t} + N_{Ed,t} + N_{Es,t} + N_{S,t}$$

where $N_{F,t}$ is labour supplied to the final goods firm, $N_{Ed,t}$ is digging labour supplied to the extraction firm, $N_{Es,t}$ is sales labour supplied

to the extraction firm and $N_{S,t}$ is the labour supplied to the search firm. Leisure can therefore be written as $L_t = 1 - (N_{F,t} + N_{Ed,t} + N_{Es,t} + N_{S,t})$. The household's period budget constraint relates consumption and investment decisions

$$C_t + I_t \leq W_{F,t}N_{F,t} + W_{Ed,t}N_{Ed,t} + W_{Es,t}N_{Es,t} + W_{S,t}N_{S,t} + R_{F,t}K_{F,t} + R_{E,t}K_{E,t} + R_{S,t}K_{S,t} + \Pi_{F,t} + \Pi_{E,t} + \Pi_{S,t}$$

where $W(\cdot)_t$ is the period wages paid from the respective firm for labour service, $R(\cdot)_t$ is the rental rate paid to the household for their capital services given to the respective firm, $K(\cdot)_t$ is the household's period supply of capital to the respective firm, the household owns the capital, and $\Pi(\cdot)_t$ is the profits the household receives from the respective firm. Investment, I_t, is defined as

$$I_t = K_{t+1} - (1-\delta)K_t.$$

where $\delta \in (0,1)$ is the depreciation rate of capital. K_t is the total capital the household owns and with this, every period the household decides how much capital to allocate to each sector. Substituting the investment equation into the household's budget constraint gives

$$K_{t+1} \leq W_{F,t}N_{F,t} + W_{Ed,t}N_{Ed,t} + W_{Es,t}N_{Es,t} + W_{S,t}N_{S,t} + R_{F,t}K_{F,t} + R_{E,t}K_{E,t} + R_{S,t}K_{S,t} + (1-\delta)K_t - C_t + \Pi_{F,t} + \Pi_{E,t} + \Pi_{S,t}$$

The household therefore solves the maximisation problem of

$$Max_{\{C_t, N(\cdot)_t, K(\cdot)_{t+1}\}_{t=0}^{\infty}} E_0 \sum_{t=0}^{\infty} \beta^t U(C_t, 1-N_t)$$

s.t.

$$\begin{aligned} K_{t+1} &\leq W_{F,t}N_{F,t} + W_{Ed,t}N_{Ed,t} + W_{Es,t}N_{Es,t} + \\ &\quad W_{S,t}N_{S,t} + R_{F,t}K_{F,t} + R_{E,t}K_{E,t} + R_{S,t}K_{S,t} + (1-\delta)K_t \\ &\quad -C_t + \Pi_{F,t} + \Pi_{E,t} + \Pi_{S,t}. \\ K_t &= K_{F,t} + K_{E,t} + K_{S,t}. \end{aligned}$$

The household's budget constraint will hold with equality due to the assumption of positive marginal utility of consumption, as the household will never want to waste any resources. Solving by method of

lagrange the first order conditions are

$$
\begin{aligned}
C_t &: U_C(C_t, 1-N_t) = \lambda_t && (1)\\
N_{F,t} &: U_L(C_t, 1-N_t) = \lambda_t W_{F,t} && (2)\\
N_{Ed,t} &: U_L(C_t, 1-N_t) = \lambda_t W_{Ed,t} && (3)\\
N_{Es,t} &: U_L(C_t, 1-N_t) = \lambda_t W_{Es,t} && (4)\\
N_{S,t} &: U_L(C_t, 1-N_t) = \lambda_t W_{S,t} && (5)\\
K_{F,t+1} &: \lambda_t = \beta E_t[(1+R_{F,t+1}-\delta)\lambda_{t+1}] && (6)\\
K_{E,t+1} &: \lambda_t = \beta E_t[(1+R_{E,t+1}-\delta)\lambda_{t+1}] && (7)\\
K_{S,t+1} &: \lambda_t = \beta E_t[(1+R_{S,t+1}-\delta)\lambda_{t+1}] && (8)\\
\lambda_t &: K_{F,t+1}+K_{E,t+1}+K_{S,t+1} = W_{F,t}N_{F,t}+W_{Ed,t}N_{Ed,t}+\\
&\quad W_{Es,t}N_{Es,t}+W_{S,t}N_{S,t}+R_{F,t}K_{F,t}+R_{E,t}K_{E,t}+R_{S,t}K_{S,t}-C_t\\
&\quad +(1-\delta)(K_{F,t}+K_{E,t}+K_{S,t})+\Pi_{F,t}+\Pi_{E,t}+\Pi_{S,t} && (9)
\end{aligned}
$$

where λ is the lagrangian multiplier on the budget constraint. $K_{F,t}+K_{E,t}+K_{S,t}$ was substituted in for K_t . From substituting in (1) for λ into (6), (7) and (8) derives the intertemporal consumption trade-offs, which are given by

$$
\begin{aligned}
U_C(C_t, 1-N_t) &= \beta E_t[(1+R_{F,t+1}-\delta)U_C(C_{t+1}, 1-N_{t+1})] && (10)\\
U_C(C_t, 1-N_t) &= \beta E_t[(1+R_{E,t+1}-\delta)U_C(C_{t+1}, 1-N_{t+1})] && (11)\\
U_C(C_t, 1-N_t) &= \beta E_t[(1+R_{S,t+1}-\delta)U_C(C_{t+1}, 1-N_{t+1})] && (12)
\end{aligned}
$$

which states that the household cannot be made better off by re-allocating consumption, via investment in capital for the final good, extraction or search firms, across time. Substitution of (1) for λ into (2), (3), (4)and (5) gives the household's intratemporal consumption - leisure trade-offs, which are

$$
\begin{aligned}
\frac{U_L(C_t, 1-N_t)}{U_C(C_t, 1-N_t)} &= W_{F,t} && (13)\\
\frac{U_L(C_t, 1-N_t)}{U_C(C_t, 1-N_t)} &= W_{Ed,t} && (14)\\
\frac{U_L(C_t, 1-N_t)}{U_C(C_t, 1-N_t)} &= W_{Es,t} && (15)\\
\frac{U_L(C_t, 1-N_t)}{U_C(C_t, 1-N_t)} &= W_{S,t}. && (16)
\end{aligned}
$$

These equations imply that the household cannot be made better off by trading a unit of leisure, for an extra unit of labour in any of the three firms, which will lead to extra consumption.

5.2 The Final Goods Firm

This model is very similar to the firm's problem in the standard baseline RBC model. Given the market price, the representative final goods firm chooses labour, capital and the mined non-renewable resource, S_t, to produce output through the firm's production function, $A_{F,t}F(K_{F,t}, z_t N_{F,t}, S_t)$. S_t is the non-renewable resource which is bought from the extraction firm to use in production to form the consumption good. z_t is trend growth to labour productivity, which follows a deterministic linear growth process so $z_{t+1} = \gamma_z z_t, \gamma_z > 1$. The production function is exposed to temporary productivity shocks, $A_{F,t}$. These shocks are exogenous and follow the AR(1) process

$$\ln A_{F,t+1} = (1 - \rho_F) \ln A_F + \rho_F \ln A_{F,t} + \epsilon_{t+1}, \rho_F \in (0, 1)$$

such that $\epsilon \sim N(0, \sigma_\epsilon^2)$.The firm's production function exhibits the usual concave assumptions, so the marginal products of labour, capital and the non-renewable resource are positive but experience decreasing returns to scale. Each factor of production is essential, therefore $F(0, z_t N_{F,t}, S_t) = F(K_{F,t}, 0, S_t) = F(K_{F,t}, z_t N_{F,t}, 0) = 0$. The Inada conditions are $\lim_{K_F} \to F_{K_F}(\cdot) = \lim_{N_F} \to F_{N_F}(\cdot) = \lim_S \to F_S(\cdot) = \infty$, which ensure that the firm will always want to produce output.

Each period the representative final goods firm solves

$$Max_{\{K_{F,t}, N_{F,t}, S_t\}} \Pi_{F,t} = A_{F,t}F(K_{F,t}, z_t N_{F,t}, S_t) - W_{F,t}N_{F,t} - R_{F,t}K_{F,t} - P_{E,t}S_t$$

where $P_{E,t}$ is the market clearing price of the non-renewable resource bought from the extraction firm. The price of output, which is sold to the household, is normalised to one. Solving the profit maximisation function gives the final goods firm's capital demand, labour demand and non-renewable resource demand

$$R_{F,t} = A_{F,t}F_{K_F}(K_{F,t}, z_t N_{F,t}, S_t) \quad (17)$$

$$W_{F,t} = A_{F,t}F_{N_F}(K_{F,t}, z_t N_{F,t}, S_t) z_t \quad (18)$$

$$P_{E,t} = A_{F,t}F_S(K_{F,t}, z_t N_{F,t}, S_t) \quad (19)$$

5.3 The Extraction Firm

The extraction firm's problem is more complicated and timing is important in this model. The representative extraction firm is initially endowed with Ig_0 units of in-ground inventory and each period faces decisions of how much to extract, how many deposits to buy, how much to sell of the non-renewable resource from their above ground inventory and how much labour and capital to use. The extraction firm purchases deposits, X_t, of the non-renewable resource from the search firm at a price, $P_{S,t}$, which is added to the extractions firm's in-ground inventory, Ig_t, next period. The firm decides how much of this period's in-ground inventory to extract, T_t. Therefore the extraction firm's flow constraint for their in-ground inventory is

$$Ig_{t+1} = Ig_t + X_t - T_t.$$

To ensure the timing that new deposits, bought this period, cannot be extracted until next period, the inequality constraint $Ig_t \geq T_t$ is imposed.

When the deposit has been extracted it is added to the firm's above ground inventory, H_t, next period. Each period the extraction firm decides how much of their current above ground inventory they want to sell to the final goods firm, S_t, for a price $P_{E,t}$. The extraction firm can only sell the non-renewable resource once it has been extracted from their in-ground inventory and added to their above ground inventory. The flow constraint for the extraction firm's above ground inventory is

$$H_{t+1} = H_t - S_t + T_t.$$

To ensure the extraction firm can only sell from this period's above ground inventory, the inequality constraint $H_t \geq S_t$ is imposed. The firm requires labour and above-ground inventory to sell the good to the final goods firm. This can be thought of as marketing of the good and is determined by the production function

$$S_t = A_H F(H_t, z_t N_{Es,t}).$$

where z_t is the same growth rate as in the final goods firm and A_H is an exogenous component to productivity. In the extraction firm there are no shocks to this productivity. The sales production function exhibits the usual concave assumptions and both inputs are

essential to production. Extraction, T_t produced via the production function

$$T_t = A_E F(K_{E,t}, z_t N_{Ed,t}, Ig_t).$$

where z_t is the same growth rate as in the final goods firm and A_E is an exogenous component to productivity. In the extraction firm there are no shocks to this productivity. The firm's extraction production function exhibits the usual concave assumptions and all inputs are essential to production.

The representative extraction firm solves the following problem

$$Max_{\{S_t, N_{Ed,t}, N_{Es,t}, K_{E,t}, X_t, Ig_{t+1}, H_{t+1}\}_{t=0}^{\infty}} \Pi_{E,t} = E_0 \sum_{t=0}^{\infty} \beta^t \{P_{E,t} S_t$$
$$-R_{E,t} K_{E,t} - W_{Ed,t} N_{Ed,t} - W_{Es,t} N_{Es,t} - P_{S,t} X_t\}$$

s.t.

$$\begin{aligned} H_{t+1} &= H_t - S_t + A_E F(K_{E,t}, z_t N_{Ed,t}, Ig_t) \\ Ig_{t+1} &= Ig_t + X_t - A_E F(K_{E,t}, z_t N_{Ed,t}, Ig_t) \\ S_t &= A_H F(H_t, z_t N_{Es,t}). \end{aligned}$$

where $\beta \in (0,1)$ is the subjective discount factor and E_0 denotes the expectation of future values of H and Ig based on the information available at time zero. The discount factor and the expectation have been included, as this not a static one period problem, as the firm must form expectations about the future in order to have the optimal amount of the non-renewable resource extracted and ready to sell in subsequent periods. Solving using a lagrangian, where λ_t is the multiplier on the above ground inventory flow constraint and ϕ_t is the multiplier on the in-ground inventory flow constraint (S_t is substituted out for the selling production function), gives the

following first order conditions

$$
\begin{aligned}
K_{E,t} &: R_{E,t} = (\lambda_t - \phi_t) A_E F_{K_E}(K_{E,t}, z_t N_{Ed,t}, Ig_t) && (20)\\
N_{Ed,t} &: W_{Ed,t} = (\lambda_t - \phi_t) A_E F_{N_{Ed}}(K_{E,t}, z_t N_{Ed,t}, Ig_t) z_t && (21)\\
N_{Es,t} &: W_{Es,t} = (P_{E,t} - \lambda_t) A_H F_{N_{Es}}(H_t, z_t N_{Es,t}) z_t && (22)\\
H_{t+1} &: \lambda_t = \beta E_t[\lambda_{t+1}(1 - A_H F_H(H_{t+1}, z_{t+1} N_{Es,t+1})) + \\
& \quad P_{E,t+1} A_H F_H(H_{t+1}, z_{t+1} N_{E,st+1})] && (23)\\
Ig_{t+1} &: \phi_t = \beta E_t[\phi_{t+1} + (\lambda_{t+1} - \phi_{t+1}) \\
& \quad A_E F_{Ig}(K_{E,t+1}, z_{t+1} N_{Ed,t+1}, Ig_{t+1})] && (24)\\
X_t &: P_{S,t} = \phi_t && (25)\\
\lambda_t &: H_{t+1} = H_t - A_H F(H_t, z_t N_{Es,t}) + A_E F(K_{E,t}, z_t N_{Ed,t}, Ig_t) && (26)\\
\phi_t &: Ig_{t+1} = Ig_t + x_t - A_E F(K_{E,t}, z_t N_{Ed,t}, Ig_t) && (27)
\end{aligned}
$$

Substitution of (22) for λ_t and (25) into (20) gives the extraction firm's capital demand equation

$$
R_{E,t} = \left(P_{E,t} - \frac{W_{Es,t}}{A_H F_{N_{Es}}(H_t, z_t N_{Es,t}) z_t} - P_{S,t} \right) A_E F_{K_E}(K_{E,t}, z_t N_{Ed,t}, Ig_t). \tag{28}
$$

Substitution of (22) and (25) into (21) gives the extraction firm's digging labour demand equation

$$
W_{Ed,t} = \left(P_{E,t} - \frac{W_{Es,t}}{A_H F_{N_{Es}}(H_t, z_t N_{Es_t}) z_t} - P_{S,t} \right) A_E F_{N_{Ed}}(K_{E,t}, z_t N_{Ed,t}, Ig_t) z_t. \tag{29}
$$

Substitution of (25) and (22) into (24) gives the intertemporal in-ground inventory trade-off

$$
\begin{aligned}
P_{S,t} &= \beta E_t[P_{S,t+1} + (P_{E,t+1} - \frac{W_{Es,t+1}}{A_H F_{N_{Es}}(H_{t+1}, z_{t+1} N_{Es,t+1}) z_{t+1}} \\
& \quad - P_{S,t+1}) A_E F_{Ig}(K_{E,t+1}, z_{t+1} N_{Ed,t+1}, Ig_{t+1})] && (30)
\end{aligned}
$$

which states that the firm cannot be made better off by reallocating purchases of in-ground deposits across time. Substitution of (22) into (23) gives the firms intertemporal above ground inventory trade-off

$$
P_{E,t} - \frac{W_{Es,t}}{A_H F_{N_{Es}}(H_t, z_t N_{Es,t}) z_t} = \beta E_t[P_{E,t+1} - \frac{W_{Es,t+1}}{A_H F_{N_{Es}}(H_{t+1}, z_{t+1} N_{Es,t+1}) z_{t+1}} (1 - A_{H,} F_H(H_{t+1}, z_{t+1} N_{Es,t+1}))] \tag{31}
$$

which states that the firm cannot be made better off by re-allocating between selling above inventory today and selling above ground inventory tomorrow.

5.4 The Search Firm

There exists a measure of unit mass search firms in the economy. The representative search firm in each period uses labour, $N_{S,t}$, and capital, $K_{S,t}$ to search for in-ground deposits of the non-renewable resource subject to a probability function. The probability of finding a unit of the non-renewable resource deposit is represented by $P(e_f)$, where e_f represents effort exerted. Effort is measured by the production function $e_f = A_{S,t}F(K_{S,t}, z_t N_{S,t})$, with the marginal products of labour and capital both being positive but experience decreasing returns to scale. Therefore the probability of finding the non-renewable resource deposit, μ, will increase, at a decreasing rate, with the more labour and capital the search firm uses. Each input is essential to production, therefore $P(e_f) = P(0) = 0$. z_t is the trend growth rate of labour productivity, which is the same as the growth process in the final goods and extraction firms. The production function is also exposed to temporary productivity shocks, $A_{S,t}$. These shocks are exogenous and follow the AR(1) process

$$\ln A_{S,t+1} = (1 - \rho_S) \ln A_S + \rho_s \ln A_{S,t} + \epsilon_{t+1}, \rho_s \in (0, 1)$$

such that $\epsilon \sim N(0, \sigma_\epsilon^2)$. The productivity shock is realised at the beginning of the period and the firm then decides how much labour and capital to use in an attempt to find the deposit. The search firm, in each period, must pay wages and rent to the household for their labour and capital, respectively and receives a price, $P_{S,t}$, for selling the discovered, non-renewable resource deposit to the extraction firm. The search firm solves a static problem and thus every period sells all of that period's discovered deposits to the extraction firm. The search firm solves

$$Max_{\{K_{S,t}, N_{S,t}\}_{t=0}^{\infty}} \Pi_{s,t} = P_{S,t} P(e_f)\mu - W_{S,t} N_{S,t} - R_{S,t} K_{S,t}$$

Solving the maximisation problem gives the first order conditions of

$$R_{S,t} = P_{S,t} P_{K_S}(A_{S,t} F(K_{S,t}, z_t N_{S,t}))\mu A_{S,t} F_{K_S}(K_{S,t}, z_t N_{S,t}) \quad (32)$$

$$W_{S,t} = P_{S,t} P_{N_S}(A_{S,t} F(K_{S,t}, z_t N_{S,t}))\mu A_{S,t} F_{N_S}(K_{S,t}, z_t N_{S,t}) z_t \quad (33)$$

which are the capital and labour demands for the search firm.

5.5 The Competitive Equilibrium

Definition 1. *The competitive equilibrium is a sequence of allocations* $\{C_t, K_{t+1}, N_{F,t}, N_{Ed,t}, N_{Es,t}, N_{S,t}, S_t, X_t\}_{t=0}^{\infty}$ *and prices* $\{R_{F,t}, R_{E,t}, R_{S,t}, W_{F,t}, W_{Ed,t}, W_{Es,t}, W_{S,t}, P_{E,t}, P_{S,t}\}_{t=0}^{\infty}$ *such that given a sequence of productivity* $\{A_{F,t}, A_{S,t}\}_{t=0}^{\infty}$

1. *Households satisfy their optimal policies*
2. *All firms satisfy their optimal policies*
3. *All markets clear*

for all dates t = 0,1,...

In equilibrium we see that wages in all firms are equal, $W_F = W_{Ed} = W_{Es} = W_S$. If this was not the case then all labour would be allocated to the firm which is offering the highest wage which would cause other firms to adjust their wages, hence not an equilibrium. Also, in equilibrium the rental rates on capital in all firms are equal, $R_F = R_E = R_S$, for a similar reason to above.

Equating supplies and demands by substituting out prices, characterises the market clearing conditions. Aggregate labour, capital stock and consumption is determined by summing across all households. By assuming there is a continuum, this is achieved by integrating across all households. As there is a unit interval and all households are identical, then the aggregate values are the same as individual values. By substituting (17) into (10) gives the capital market clearing conditions for the final good firm.

$$\begin{aligned} U_C(C_t, 1-N_t) &= \beta E_t[(1 + A_{F,t+1}F_{K_F}(K_{F,t+1}, z_{t+1}N_{F,t+1}, S_{t+1}) - \delta) \\ &\quad U_C(C_{t+1}, 1-N_{t+1})] \end{aligned}$$

Since $R_F = R_E = R_S$, then in equilibrium this clears the other capital markets in the economy.

By equating (18) and (13) gives the labour market clearing conditions for the final good firm.

$$\frac{U_L(C_t, 1-N_t)}{U_C(C_t, 1-N_t)} = A_{F,t}F_{N_F}(K_{F_t}, z_tN_{F,t}, S_t)$$

As $W_F = W_{Ed} = W_{Es} = W_S$, then in equilibrium this clears the remaining labour markets in the economy.

As there are a continuum of extraction firms and search firms in this large competitive economy, to find the market clearing condition for the deposits of the non-renewable resource we need to determine aggregate supply from the search firm and aggregate demand from the extraction firm. This can be determined by integrating across all firms. Giving a name to each firm by indexing firms with the letter i, $i \in [0, 1]$ then,

$$X_t = \int_0^1 X_t(i)di.$$

The individual search firm on average is discovering $P(e_f)\mu$ deposits of the non-renewable resource therefore, aggregating across all firms (normalising the number of firms to one) gives the total supply. The probability function will cause some firms will discover more than the average amount and some firms will discover less. Due to the assumption of a large number of firms, then by the law of large number when aggregating across all the firms, the average outcome will result. Therefore the market clearing condition for the deposits of the non-renewable resource is

$$X_t = P(e_f)\mu.$$

Clearing the goods market involves taking the household's budget constraint and substituting out wages and the rent on capital for the respective demand equations. Period profits from the firms must also be substituted into the budget equation then wages and rents must be substituted out of those to remove prices. This gives the goods market clearing condition as

$$K_{t+1} = A_{F,t}F(K_{F,t}, z_t N_{F,t}, S_t) - C_t + (1 - \delta)K_t.$$

The final market that needs to be cleared is the market for the non-renewable resource between the final goods firm and the search firm. Since all other markets have been cleared then by Walras' law[10] the market for the non-renewable resource between the final goods firm and the extraction firm will clear too.

[10] This law states that if the economy is in equilibrium and every market in the economy but one have been cleared, the last market will clear too

6 Quantitative Examination

This section provides a functional form example of the RBC-Mining model presented in the previous section. A linear approximation of the model is taken in order to achieve a steady state such that the model can then be solved using the method of Blanchard and Kahn (1980) to simulate outcomes of the economy. Finally, this section provides an explanation of the procedure used to calibrate the model's parameters.

6.1 The Transformed Economy

Assume the household's consumption/leisure preferences are given by

$$u(C_t, L_t) = log(C_t) + \frac{\theta}{1-\eta}\left[(1-N_t)^{(1-\eta)} - 1\right],$$

the final goods firm's production technology is

$$Y_t = A_{F,t} K_{F,t}^{\alpha_1} (z_t N_{F,t})^{\alpha_2} S_t^{1-\alpha_1-\alpha_2},$$

the extraction firm's extraction technology and selling technology respectively are

$$\begin{aligned} T_t &= A_E K_{E,t}^{\zeta_1} (z_t N_{Ed,t})^{\zeta_2} Ig_t^{1-\zeta_1-\zeta_2} \\ S_t &= A_H H_t^{\nu} (z_t N_{Es,t})^{1-\nu} \end{aligned}$$

and the search firm's probability function is given by

$$P(A_{S,t} F(K_{S,t}, z_t N_{S,t})) = 1 - e^{-(A_{S,t} K_{S,t}^{\kappa} (z_t N_{S,t})^{1-\kappa})}.$$

Due to the inclusion of the non-stationary variable z_t the economy needs to be transformed in order to solve for the steady state. Dividing the household and the firms' problems by z_t will achieve a detrended, transformed economy. Therefore the representative household's problem is[11].

$$Max\, E_0 \sum_{t=0}^{\infty} \beta^t log(c_t) + log(z_t) + \frac{\theta}{1-\eta}\left[(1-N_t)^{(1-\eta)} - 1\right]$$

[11] where lower case variables represent the original variable divided by the labour-augmenting technology (e.g. $x_t = \frac{X_t}{z_t}$).

s.t. the detrended[12] budget constraint

$$\gamma_z k_{t+1} = w_{f,t}N_{F,t} + w_{ed,t}N_{Ed,t} + w_{es,t}N_{Es,t} + w_{s,t}N_{S,t}$$
$$+R_{F,t}k_{f,t} + R_{E,t}k_{e,t} + R_{S,t}k_{s,t} - c_t + (1-\delta)k_t + \Pi_{F,t} + \Pi_{E,t} + \Pi_{S,t}.$$

Therefore the household's intertemporal consumption trade-offs can be written as

$$\begin{aligned}
\frac{1}{c_t} &= \frac{\beta}{\gamma_z}E_t\left[(1+R_{F,t+1}-\delta)\frac{1}{c_{t+1}}\right] \\
\frac{1}{c_t} &= \frac{\beta}{\gamma_z}E_t\left[(1+R_{E,t+1}-\delta)\frac{1}{c_{t+1}}\right] \\
\frac{1}{c_t} &= \frac{\beta}{\gamma_z}E_t\left[(1+R_{S,t+1}-\delta)\frac{1}{c_{t+1}}\right]
\end{aligned}$$

and the labour supply can be written as

$$\begin{aligned}
\frac{\theta c_t}{(1-(N_{F,t}+N_{Ed,t}+N_{Es,t}+N_{S,t}))^\eta} &= w_{f,t} \\
\frac{\theta c_t}{(1-(N_{F,t}+N_{Ed,t}+N_{Es,t}+N_{S,t}))^\eta} &= w_{ed,t} \\
\frac{\theta c_t}{(1-(N_{F,t}+N_{Ed,t}+N_{Es,t}+N_{S,t}))^\eta} &= w_{es,t} \\
\frac{\theta c_t}{(1-(N_{F,t}+N_{Ed,t}+N_{Es,t}+N_{S,t}))^\eta} &= w_{s,t}.
\end{aligned}$$

In the transformed economy the final goods firm solves

$$max\,\Pi_{F,t} = A_{F,t}(k_{f,t}^{\alpha_1}N_{F,t}^{\alpha_2}s_t^{1-\alpha_1-\alpha_1}) - w_{f,t}N_{F,t} - R_{F,t}k_{f,t} - P_{E,t}s_t$$

which gives the capital, labour and non-renewable resource demands for the transformed economy as

$$\begin{aligned}
\alpha_1 A_{F,t}(k_{f,t}^{\alpha_1-1}N_{F,t}^{\alpha_2}s_t^{1-\alpha_1-\alpha_2}) &= R_{F,t} \\
\alpha_2 A_{F,t}(k_{f,t}^{\alpha_1}N_{F,t}^{\alpha_2-1}s_t^{1-\alpha_1-\alpha_2}) &= w_{f,t} \\
(1-\alpha_1-\alpha_2)A_{F,t}(k_{f,t}^{\alpha_1}N_{F,t}^{\alpha_2}s_t^{-\alpha_1-\alpha_2}) &= P_{E,t}.
\end{aligned}$$

In the transformed economy the extraction firm solves

$$max\,\Pi_{E,t} = E_0\sum_{t=0}^{\infty}\beta^t P_{E,t}s_t - R_{E,t}k_{e,t} - w_{ed,t}N_{Ed,t} - w_{es,t}N_{Es,t} - P_{S,t}x_t$$

[12] the detrended budget constraint was obtained by dividing both sides by z_t and multiplying the left hand side by $\frac{z_{t+1}}{z_{t+1}}$

s.t.

$$\begin{aligned}
\gamma_z h_{t+1} &= h_t - s_t + A_E k_{e,t}^{\zeta_1} N_{Ed,t}^{\zeta_2} ig_t^{1-\zeta_1-\zeta_2} \\
\gamma_z ig_{t+1} &= ig_t + x_t - A_E k_{e,t}^{\zeta_1} N_{Ed,t}^{\zeta_2} ig_t^{1-\zeta_1-\zeta_2} \\
s_t &= A_H h_t^{\nu} N_{Es,t}^{1-\nu}
\end{aligned}$$

which results in the following optimality equations

$$\begin{aligned}
R_{E,t} &= \left(P_{E,t} - \frac{w_{es,t}}{(1-\nu) A_H h_t^{\nu} N_{Es,t}^{-\nu}} - P_{S,t} \right) \zeta_1 A_E k_{e,t}^{\zeta_1 - 1} N_{Ed,t}^{\zeta_2} ig_t^{1-\zeta_1-\zeta_2} \\
w_{ed,t} &= \left(P_{E,t} - \frac{w_{es,t}}{(1-\nu) A_H h_t^{\nu} N_{Es,t}^{-\nu}} - P_{S,t} \right) \zeta_2 A_E k_{e,t}^{\zeta_1} N_{Ed,t}^{\zeta_2 - 1} ig_t^{1-\zeta_1-\zeta_2}
\end{aligned}$$

$$\begin{aligned}
\gamma_z P_{S,t} &= \beta E_t \{ \left(P_{E,t+1} - \frac{w_{es,t+1}}{(1-\nu) A_H h_{t+1}^{\nu} N_{Es,t+1}^{-\nu}} - P_{S,t+1} \right) \\
&\quad (1-\zeta_1-\zeta_2) A_E k_{e,t+1}^{\zeta_1} N_{Ed,t+1}^{\zeta_2} ig_{t+1}^{-\zeta_1-\zeta_2} + P_{S,t+1} \}
\end{aligned}$$

$$\begin{aligned}
\gamma_z \left(P_{E,t} - \frac{w_{es,t}}{(1-\nu) A_H h_t^{\nu} N_{Es,t}^{-\nu}} \right) &= \beta E_t \{ P_{E,t+1} - \frac{w_{es,t+1}}{(1-\nu) A_H h_{t+1}^{\nu} N_{Es,t+1}^{-\nu}} \\
&\quad + \frac{\nu w_{es,t+1} N_{Es,t+1}}{(1-\nu) h_{t+1}} \}.
\end{aligned}$$

Lastly in the transformed economy the representative search firm solves

$$Max\, \Pi_{s,t} = P_{S,t}(1 - e^{-(A_{S,t} k_{s,t}^{\kappa} N_{S,t}^{1-\kappa})})\mu - w_{s,t} N_{S,t} - R_{S,t} k_{s,t}$$

which gives the search capital and labour demand equations of

$$R_{S,t} = P_{S,t} e^{-(A_{S,t} k_{s,t}^{\kappa} N_{S,t}^{1-\kappa})} \mu \kappa A_{S,t} k_{s,t}^{\kappa-1} N_{S,t}^{1-\kappa} \tag{34}$$

$$w_{s,t} = P_{S,t} e^{-(A_{S,t} k_{s,t}^{\kappa} N_{S,t}^{1-\kappa})} \mu (1-\kappa) A_{S,t} k_{s,t}^{\kappa} N_{S,t}^{-\kappa}. \tag{35}$$

Removing prices, by equating supplies and demands of the optimality conditions, and removing the time subscripts, will give the steady state allocations. Due to the complex nature of the model the steady state variables cannot be solved analytically however, the steady state equations have been simplified and solved via a system of equations. The following equations are used to solve for the steady state[13]. Firstly equating the demand of capital with the

[13] Variables without time subscripts are representative of the steady state

household's intertemporal trade-off's gives[14]

$$\begin{aligned}
\frac{\gamma_z}{\beta} - 1 + \delta &= \alpha_1 A_F (k_f^{\alpha_1 - 1} N_F^{\alpha_2} s^{1-\alpha_1-\alpha_2}) \\
\frac{\gamma_z}{\beta} - 1 + \delta &= P_S (e^{-(A_S k_s^{\kappa} N_S^{1-\kappa})} \mu \kappa A_S k_s^{\kappa-1} N_S^{1-\kappa} \\
\frac{\gamma_z}{\beta} - 1 + \delta &= \{(1-\alpha_1-\alpha_2) A_F (k_f^{\alpha_1} N_F^{\alpha_2} s^{-\alpha_1-\alpha_2}) - \frac{w_{es}}{(1-\nu) A_H (h^{\nu} N_{Es}^{-\nu})} \\
&\quad - P_S\} \zeta_1 A_E (k_e^{\zeta_1 - 1} N_{Ed}^{\zeta_2} i g^{1-\zeta_1-\zeta_2}).
\end{aligned}$$

Equating supplies and demands of final goods labour, extraction digging labour and search labour in the steady state gives

$$\begin{aligned}
\frac{\theta C}{(1-(N_F+N_{Ed}+N_{Es}+N_S))^{\eta}} &= \alpha_2 A_F (k_f^{\alpha_1} N_F^{\alpha_2 - 1} s^{1-\alpha_1-\alpha_2}) \\
\frac{\theta C}{(1-(N_F+N_{Ed}+N_{Es}+N_S))^{\eta}} &= P_S e^{-(A_S k_s^{\kappa} N_S^{1-\kappa})} \mu (1-\kappa) A_S k_s^{\kappa} N S^{-\kappa} \\
\frac{\theta C}{(1-(N_F+N_{Ed}+N_{Es}+N_S))^{\eta}} &= \{(1-\alpha_1-\alpha_2) A_F (k_f^{\alpha_1} N_F^{\alpha_2} s^{-\alpha_1-\alpha_2}) \\
&\quad - \frac{w_{es}}{(1-\nu) A_H (h^{\nu} N_{Es}^{-\nu})} - P_S\} \\
&\quad \zeta_2 A_E (k_e^{\zeta_1} N_{Ed}^{\zeta_2 - 1} i g^{1-\zeta_1-\zeta_2})
\end{aligned}$$

Solving for the steady state of the intertemporal trade offs for below and above ground inventories, respectively gives

$$\begin{aligned}
\left(\frac{\gamma_z}{\beta} - 1\right) P_S &= \{(1-\alpha_1-\alpha_2) A_F (k_f^{\alpha_1} N_F^{\alpha_2} s^{-\alpha_1-\alpha_2}) - \frac{w_{es}}{(1-\nu) A_H (h^{\nu} N_{Es}^{-\nu})} \\
&\quad - P_S\} (1-\zeta_1-\zeta_2) A_E (k_e^{\zeta_1} N_{Ed}^{\zeta_2} i g^{-\zeta_1-\zeta_2})
\end{aligned}$$

$$\begin{aligned}
\left(1 - \frac{\beta}{\gamma_z}\right) \frac{w_{es}}{(1-\nu) A_H (h^{\nu} N_{Es}^{-\nu})} + \frac{\beta}{\gamma_z} \frac{\nu w_{es} N_{Es}}{(1-\nu) h} &= \left(1 - \frac{\beta}{\gamma_z}\right) (1-\alpha_1-\alpha_2) \\
&\quad A_F (k_f^{\alpha_1} N_F^{\alpha_2} s^{-\alpha_1-\alpha_2}).
\end{aligned}$$

The budget constraint and the flow constraints are also obtained for

[14] P_E has been substituted out of the extraction capital steady state equation using the demand equation for PE from the final goods firm, P_S and w_{es} were not substituted out in an attempt to maintain some simplicity

the steady state, giving

$$\begin{aligned} c &= (1-\delta-\gamma_z)(k_f+k_e+k_s)+A_F(k_f^{\alpha_1}N_F^{\alpha_2}s^{1-\alpha_1-\alpha_2}) \\ (\gamma_z-1)h &= -s+A_E(k_e^{\zeta_1}N_{Ed}^{\zeta_2}ig^{1-\zeta_1-\zeta_2}) \\ (\gamma_z-1)ig &= x-A_E(k_e^{\zeta_1}N_{Ed}^{\zeta_2}ig^{1-\zeta_1-\zeta_2}). \end{aligned}$$

The steady state of the market clearing condition for the deposits of the non-renewable resource, final goods firm's labour demand, and the constraint $s = A_H(h^{\nu}N_{Es}^{1-\nu})$ were also used in the system to solve for the steady state variables to equate the number of equations and the number of unknowns.

All first order conditions, from all agents, are log-linearised[15] as well as the market clearing condition for the deposits of the non-renewable resource (as the market clearing condition for this market is not derived from marginal demand and marginal supply). The equations are then stacked into the appropriate matrices to solve for the observation and state equations of the Blachard-Kahn method (1980)[16] to simulate the business cycle.

6.2 Calibration

The term calibration takes on two distinctive definitions. One definition, known as strict calibration, refers to choosing the model's parameters such that they are consistent with economic theory and gives the model a 'sensible' solution. The other definition, known as classic calibration, refers to choosing parameters such that they are consistent with empirical studies on micro-level data about the economy. In Benhabib et al (2005) they used both classic and strict methods to calibrate their model. They used classic calibration to determine parameters with available empirical counterparts. They calibrated other parameters using values that are standard to the RBC literature and selected remaining parameters in order to generate a significant perturbation mechanism. This paper will follow their method, due to the lack of appropriate data, to form the necessary estimates. Two sets of parameter values will be evaluated, as there is no reason why one set should be more correct than the other. Both results will be compared against the statistics generated

[15] see appendix B for log-linearised equations and how they were derived

[16] see appendix C for the observation and state equation in the Blachard-Kahn method and the values which were inserted into the matrices

by the cyclical behaviour of the Australian economy to assess their performance.

The household's discount factor, β is set to 0.99, and the depreciation rate on capital, δ is set to 0.025, which equates to 10 per cent per annum. Over 1985(1) and 2010(1) the average weekly hours worked were 35.3 hours. Burnside and Eichenbaum(1996) used 15 hours as the daily time endowment, as sleep was not included. Multiplying 15 by 7 gives the weekly time endowment of 105 hours per week. By using this method, dividing the weekly time endowment by 35.3 hours gives 1/3. θ is chosen such that the steady state value of total labour is equal to 1/3. The AR(1) coefficients and the standard deviation for ϵ are $\rho_f = \rho_s = 0.979$ and $\sigma_f = \sigma_s = 0.0072$ (see King and Rebelo 1999). The growth rate of technical progress, γ_z, is determined by the average growth rate of Australian GDP per capita over the reference period. This equates to an average of 17 per cent per annum which implies the quarterly rate of approximately $\gamma_z = 1.004$. A_F, A_E, A_H, A_S and μ are parameters which only affect the scale of the model, therefore can be normalised to one. Elasticity of marginal utility of leisure is set to $\eta = 1.001$. The input shares in all the production functions in the model are the variables that are being changed between the two sets of results. A list of parameters, used in both sets, are described in Table 2 and Table 3 below. The importance of the non-renewable resource in the model has been reduced in the second parameter set. This provides an interesting point of differentiation when comparing the two models.

Table 2: Parameter Values - Set 1

Parameter	Value	Description
β	0.99	Household's discount factor
α_1	0.28	Capital Share in Final Goods Firm's Production Function
α_2	0.52	Labour Share in Final Goods Firm's Production Function
ζ_1	0.38	Capital Share in Extraction Firm's Extracting Function
ζ_2	0.42	Labour Share in Extraction Firm's Extracting Function
ν	0.42	Inventory Share in Extraction's Selling Function
κ	0.72	Capital Share in Search Firm's Production Function
η	1.001	Elasticity of Marginal Utility of Leisure
γ_z	1.004	Growth Rate in Labour Productivity
δ	0.025	Depreciation of Capital
ρ_f	0.979	AR(1) Coefficient on Final Goods Firm's TFP Process
ρ_s	0.979	AR(1) Coefficient on Search Firm's TFP Process
σ_f	0.0072	Standard Deviation of Shock in Final Goods Firm
σ_s	0.0072	Standard Deviation of Shock in Search Firm

Table 3: Parameter Values - Set 2

Parameter	Value
β	0.99
α_1	0.37
α_1	0.53
ζ_1	0.43
ζ_2	0.47
ν	0.18
κ	0.32
η	1.001
γ_z	1.004
δ	0.025
ρ_f	0.979
ρ_s	0.979
σ_f	0.0072
σ_s	0.0072

6.3 Impulse Responses for Parameter Set 1

6.3.1 TFP Shock to the Final Goods Firm

Given the parameter values from Table 2, Figure 4 illustrates the effects of a positive one percent shock to total factor productivity (TFP) in the final goods firm. This lasts for one period and impacts on all variables in the model. As can be seen from the graphs, a one time TFP shock to the final goods firm creates a range of different responses.

The TFP shock increases the household's lifetime wealth and thus, increases their lifetime consumption ability. The concave utility function of the household, with respect to consumption, creates the desire to spread this gain in lifetime consumption across all periods. The gain in lifetime wealth, increases not only the household's consumption ability but also leisure. The concave utility function, with respect to leisure, gives the incentive for the household to spread this gain in leisure across all periods. This is the income effect, created as a result of the TFP shock.

The shock also creates a substitution effect as the marginal benefit from an extra unit of labour has temporarily increased. Therefore, the household is willing to substitute current leisure for future leisure. The household has the incentive to increase their current labour supply and work less when productivity has decreased. This is the reason for the intratemporal substitution of labour observed in the graphs.

As the level of output has temporarily increased, the household has the incentive to smooth consumption through savings. This intertemporal trade off occurs as the marginal product of capital and the real rate of return are temporarily high. The household has the incentive to work harder when productivity is high, build up their capital stock and, in the future, substitute labour for capital when the positive effects from the TFP shock have diminished. As a result, investment increases.

The consumption smoothing effect as a result of increased lifetime wealth and the intratemporal substitution of leisure for labour, in order to build up capital stock, are two major mechanisms at work as a result of the TFP shock. Although total labour and capital increases, as this is a multisector economy, the household must decide where this should be allocated in order to maximise the benefit from

the shock.

The TFP shock to the final goods firm increases the productivity of the factors of production in that particular firm. As the non-renewable resource is used in the production of output, the final goods firm must first obtain more of this resource in order to make the best use of the productivity increase. However, more of the non-renewable resource must first be searched for and extracted and therefore capital and labour are initially allocated to the search firm. As a result, final goods firm's labour and capital immediately falls, search labour and capital increases as does the number of deposits sold to the extraction firm. Although the TFP shock does not affect the search firm, the household will still allocate more labour and capital as discovering more non-renewable resource deposits allows the extraction firm to extracted higher levels and build up their inventory. Therefore, labour can be substituted for capital and the non-renewable resource.

The extraction firm has a stock of the non-renewable resource which is held in their in-ground inventory. Therefore, higher levels of capital and digging labour are also initially allocated to this sector in order to bring this inventory above ground and hence extraction increases. As new deposits from the search firm are added to the firm's in-ground inventory next period, the stock of in-ground inventory initially falls before the continuation of new deposits flow through. Once the non-renewable resource has been extracted, it can only be sold to the final goods firm in the following period. This explains the initial fall in extraction selling labour and then followed by the dramatic increase. Above ground inventory is used in the selling function of the non-renewable resource. Therefore the extraction firm has the incentive to build up as labour can be substituted for the above-ground inventory.

Once more deposits have been searched for, added to in-ground inventory, extracted, added to above-ground inventory and sold to the final goods firm, the level of capital and labour allocated to the final goods firm will at last rise in order to benefit from the increased marginal returns to factor input. When more labour and capital has finally been allocated to the final goods firm, the amount of the non-renewable resource sold to the final goods firm also increases and therefore at last, output increases.

GDP is constructed such that it is equal to output from each of

the three sectors, weighted by their prices. When there is a TFP shock to the final goods firm output from each firm increases, in order to meet the larger demand for the extracted non-renewable resource, and hence GDP increases.

The initial shock creates a rigid movement in some of the impulse responses as the economy is quickly switching capital and labour allocations to extract the non-renewable resource. The economy displays smooth responses once there is a continual flow of the non-renewable resource being supplied to the in-ground inventory and then to the above ground inventory.

6.3.2 TFP Shock to the Search Firm

Figure 5 illustrates the effects of a positive one percent shock to total factor productivity (TFP) in the search firm. The TFP shock increases the household's lifetime consumption. The concave utility function of the household, with respect to consumption, creates the desire to spread this gain in lifetime consumption across all periods. The gain in lifetime consumption creates the incentive for the household to enjoy more leisure. The concave utility function, with respect to leisure, gives the incentive to spread this gain in leisure across all periods. This is the income effect, created as a result of the TFP shock.

The TFP shock increases the probability of finding a deposit of the non-renewable resource. Placing more capital and labour into the search firm will further cause it to increase. As it is a probability, it can only increase to a certain extent and will quickly reach high diminishing returns. Therefore the marginal benefit of an extra unit of labour is not as high as the case with the TFP shock to the final goods firm and the household is not willing to substitute current leisure for future leisure. This creates the intratemporal substitution of leisure and hence, total labour decreases.

The level of output in the economy only increases in the initial period before falling below the pre-shock value. When output is high the household has the incentive to smooth consumption through savings. This intertemporal trade off occurs as the marginal product of capital and the real rate of return are temporarily higher. As a result, investment initially increases. When output falls below zero, this means there is less for the household to divide between consumption and investment. As the marginal benefit of capital has

not increased by a significant amount, the household would prefer to maintain a smooth consumption profile rather than invest in capital. Therefore investment falls below zero.

An explanation of what occurs in the rest of the economy is as follow. The household initially allocates more labour and capital to the search firm as this, together with the TFP shock, will increase the probability of finding deposits of the non-renewable resource. As a result deposits of the non-renewable resource rise. Higher levels of the deposit mean the extraction firm is willing to sell off their above ground inventory as they are now certain this will be replenished in the future. Therefore, a higher level of selling labour is initially allocated to sell this to the final goods firm. As the amount of the non-renewable resource sold to the final goods firm has momentarily increased, the household will initially allocate more capital and labour to this firm. This is the reasoning behind the one period increase in output. Output falls in subsequent periods as labour and capital are reallocated to other sectors.

The extraction firm has the incentive to build up their above-ground inventory of the non-renewable resource, as this can be used as a substitute for labour in the selling production function. They also have the incentive to build up their in-ground inventory as this can be used as a substitute for labour and capital in the extraction production function.

Search labour and capital fall below zero in subsequent periods as there is no increased benefit to the household from finding more deposits. This is due to the extraction firm hoarding the non-renewable resource in its above and below inventory, to use in place of labour and capital, rather than selling higher amounts of the resource to the final goods firm.

When there is a TFP shock to the search firm, GDP falls below the initial level. Although the search firm is producing more output this is offset by the falls in output from the extraction and final goods firms.

Due to the construction of the extraction firm, a TFP shock to the search does not produce high levels of output for the following periods. Since there is no increase to productivity in the final goods firm, there is no increased demand of their factors of production which are used to produce output. The extraction firm therefore uses the non-renewable resource as a substitute for other factors of

production. Whereas, a TFP shock to the final goods firm creates benefits which flow through the entire economy.

6.4 Impulse Responses for Parameter Set 2

6.4.1 TFP Shock to the Final Goods Firm

Given the parameter values from Table 3, Figure 6 illustrates the impact of a one percent shock to total factor productivity (TFP) in the final goods firm. These responses follow similar patterns to parameter set 1, in terms of total labour, total capital, output, consumption, investment, the real rate of return and the real wage. As the importance of variables in the production functions have changed, this has resulted in differences in how specific labour and capital are allocated in the economy, as well as decisions relating to the non-renewable resource.

The importance of the non-renewable resource in the final goods production function has been reduced. This is also the case for the production functions with the above ground or below ground inventories. Therefore, when there is a productivity shock to the final goods firm, the household would rather allocate more labour and capital to the final goods firm from the beginning, rather than use these resources to initially search and extract for more of the non-renewable resource. This is because labour and capital are much more important to the production process than the non-renewable resource. This results in labour and capital for the extraction and search firms to decrease and thus so does the stocks of the non-renewable resource. Once productivity starts to falls, the household will allocate more labour and capital to the search firm and then to the extraction firm, in order for more of the non-renewable resource to be available for use in production.

6.4.2 TFP Shock to the Search Firm

Given the parameter values from Table 3, Figure 7 illustrates the impact of a one percent shock to total factor productivity (TFP) in the search firm. As labour has more of an importance in the search process, and in other production functions, total labour now follows a similar response to a productivity shock in the final goods firm. This is also the case for total capital, consumption, the real rate of

return and the real wage

As search labour and capital increase so does the amount of deposits of the non-renewable resource found. Since these deposits are sold this period to the extraction firm, more labour and capital are initially allocated to this task, which increases the level of extraction. Higher levels of deposits mean the extraction firm is willing to sell off their above ground inventory as they are now certain this will be replenished in the future. Therefore, a higher level of selling labour is initially allocated to sell the non-renewable resource to the final goods firm. The increase in the non-renewable helps to offset the loss in output, as labour and capital are allocated away from the final goods firm until the boom of the non-renewable resource is available to use in production. Since the non-renewable resource is not as important in the production process as labour and capital, output initially falls before increasing above the steady state level.

Changing the importance of the factors of production has resulted in a very different outcome. GDP in the economy now increases from a productivity shock in the search firm and the benefits of the shock flow through the entire economy. As the above ground inventory is less important in the selling production function, the extraction firm is willing to sell more of this to the final goods firm instead of using it as a substitute for labour and thus output from the final goods firm increases.

7 Results

To evaluate the performance of the business cycle model presented in section 5, the statistics, generated by the model, are compared against those observed in the Australian economy presented in section 4.

7.1 Assessing the Model Generated by Parameter Set 1

In figure 8, the series generated by the model is plotted against the actual series generated by the Australian data. Table 4 provides the relevant statistics to compare the model against the actual data. GDP (Y) of the model, was constructed to equal output from each of the firms in the economy, to be consistent with the actual data.

Table 4: Statistics From Parameter Set 1

	Y	C	I	N_F
Standard Deviation	0.0486	0.0035	0.0873	0.0212
Relative Standard Deviation	1	0.0714	1.7940	0.4371
Quarterly Autocorrelation	0.1303	0.9256	0.2406	0.0965
Correlation Matrix				
Y	1	-0.4367	-0.6805	-0.8415
C	-	1	0.4106	0.4351
I	-	-	1	0.9643
N_F	-	-	-	1
N_E	-	-	-	-
N_S	-	-	-	-
W	-	-	-	-
R	-	-	-	-

	N_E	N_S	W	R	TFP
Standard Deviation	0.2236	0.0541	0.0060	0.0549	0.0091
Relative Standard Deviation	4.5869	1.1107	0.1240	1.1107	0.1865
Quarterly Autocorrelation	0.0949	0.7810	0.8561	0.8739	0.6854
Correlation Matrix					
Y	0.9804	0.5592	0.2919	0.4899	0.6867
C	-0.4595	-0.0930	0.3935	-0.2227	-0.1380
I	-0.7375	0.2066	0.4081	-0.0006	0.0523
N_F	-0.8735	-0.0376	0.1946	-0.1647	-0.1949
N_E	1	0.4630	0.1885	0.4310	0.5936
N_S	-	1	0.8630	0.6535	0.9776
W	-	-	1	0.4922	0.8377
R	-	-	-	1	0.6650
TFP	-	-	-	-	1

The model is able to produce a good replication of real wages, consumption, non-mining labour and search labour. The standard deviation of wages is 50 percent smaller compared to the actual data and the standard deviation of consumption is 58 percent smaller. When evaluating the labour market, search labour is 60 percent smaller and non-mining labour is 73 percent larger than their respective empirical counterparts. The remaining variables are not replicated well by the model, with vastly different standard deviations. Output is over 450 percent more volatile than the cyclical

fluctuations in Australian GDP.

Another weakness of the model is the contemporaneous correlations between the variables. Almost all of these are extremely diverse to those observed in the data, with the exception of the correlations between consumption and non-mining labour; consumption and extraction labour; investment and search labour; and investment and the real rate of return. Interestingly, in the actual data, wages have an acyclic relationship with all other variables whilst in the model this relationship is procyclical. Wages displaying a acyclical correlation with other variables is consistent with data from the United States.

When evaluating the persistence of the model, some variables exhibit stronger inertia than the data, whilst for some variables this is smaller. The persistence displayed by output is almost 85 percent smaller than what is observed in the actual data.

7.2 Assessing the Model Generated by Parameter Set 2

Figure 9 displays the series of the model, plotted against the actual series of the Australian economy. Table 5 provides the statistics of the model, using the parameters from Table 3. These are compared against the statistics of the actual data, which are displayed in Table 1, to test the validity of the model.

Table 5: Statistics From Parameter Set 2

	Y	C	I	N_F
Standard Deviation	0.0020	0.0052	0.0291	0.0051
Relative Standard Deviation	1	2.6477	14.9936	2.6194
Quarterly Autocorrelation	0.6814	0.7082	0.6765	0.6713
Correlation Matrix				
Y	1	0.9069	0.9171	0.8869
C	-	1	0.9646	0.9480
I	-	-	1	0.9951
N_F	-	-	-	1
N_E	-	-	-	-
N_S	-	-	-	-
W	-	-	-	-
R	-	-	-	-

	N_E	N_S	W	R	TFP
Standard Deviation	0.0033	0.0264	0.0070	0.0223	0.0091
Relative Standard Deviation	1.7364	13.7585	3.5744	11.5726	4.6625
Quarterly Autocorrelation	0.9115	0.5734	0.6956	0.9011	0.6837
Correlation Matrix					
Y	0.1574	-0.3218	0.9344	0.6149	0.9333
C	-0.1766	-0.5795	0.9949	0.6193	0.9819
I	-0.0718	-0.6266	0.9815	0.6527	0.9956
N_F	-0.0793	-0.6805	0.9669	0.6501	0.9861
N_E	1	0.2157	-0.1214	-0.0248	-0.0937
N_S	-	1	-0.5642	-0.3691	-0.5893
W	-	-	1	0.6376	0.9945
R	-	-	-	1	0.6499
TFP	-	-	-	-	1

This version of the model provides standard deviations that are closer to the results observed in the actual data. The version is capable of producing a real rate of return that perfect matches the standard deviation observed in the real data. Investment is modeled at 104 percent of its empirical counterpart, consumption at 63 percent, the real wage at 57 percent and non-mining labour at 42 percent. For the majority of values, the persistence generated is also closer to that displayed by the actual economy than parameter set 1. When assessing the contemporaneous correlations between the variables, parameter set 2 improves the results of some relationships, whilst is worse for others. As a result one parameter set cannot be stated as being strictly superior to another.

8 Discussion

This paper aimed to produce a business cycle model with the inclusion of a mining sector to successfully replicate the cyclical fluctuations of the Australian economy. A neoclassical growth model with stochastic perturbations was built upon to create a multisector, mining dominant, economy. Mining involves searching and extracting the non-renewable resource, before it can be used in the production of the consumption good. Unconditional moments of this model were then generated to provide a basis for comparison with the em-

pirical counterparts.

Comparing the results for the two parameter sets did not present a clearly superior model. However as parameter set 2 modeled output with slightly more accuracy, then this could be seen as the preference. Parameter values could be further tested to evaluate whether the model could be improved upon. A weakness of the model is its inability to capture the observed contemporaneous correlations between variables in the model and the actual data series. The model also failed to accurately replicate the persistence and the volatility observed in some of the variables. Adjusting the model's parameters between the two sets did not have a significant impact on these results. As there exists no similar studies to this model, there are no results outside of this paper to compare the outcomes of this model with, apart from the actual data.

Due to the multisector economy, a positive shock in one firm created movement of labour and capital between all of the firms. Although in reality all labour and capital cannot be freely moved between sectors, to some extent movement does occur, as captured by the model.

A limitation of the model is that a foreign sector had not been included. This is a possible extension of the model as the addition of a foreign sector would further replicate the small open economy of Australia and hence could potentially produce more accurate results. Many features of the model do not have a comparable empirical counterpart and therefore have been excluded from the evaluation of the model. The household can only allocate their labour and capital to either final goods production or the mining sector. This fictitious notion of the model suggests that the data used for these empirical counterparts may be skewed, as in reality the household has many more options. Remeasurement of the data, to be consistent with the theory of the model, could possibly eliminate some of the variability between the statistics observed. This is not a feasible task.

Australia has recently entered, what is referred to as, 'Mining Boom Mark II.' Mining is expected to contribute a higher proportion to GDP and generate more employment. Top mining companies are investing more in Australian mining, to further expand mines and extraction capabilities. This second mining boom is being generated from the persistently high growth of developing nations. As the importance of mining is predicted to increase, this suggests that

future data, which will incorporate this boom, may improve the performance of this model against the Australian economy. The Australian Government is planning to impose a 'super profit' tax in 2012. This tax is mainly aimed at the mining companies who are earning supernormal profits as a result of the mining boom. Policy makers should develop economic models to include a mining sector to evaluate the effects of the proposed tax.

9 Concluding Remarks

This paper has assessed the effects of the inclusion of a mining sector into a business cycle model by comparing the results against the cyclical volatility of the Australian economy. The analysis was structured around a multisector neoclassical growth model, representing a closed economy and results were simulated by solving this model via the Blanchard-Kahn method. This framework provided analysis which could be used to determine the model's success in replicating the cyclical variations of the Australian economy.

The unconditional moments generated by the model showed that it was capable to closely replicate some of the factors of the Australian economy. This indicates that a mining sector may be important in determining the cyclical fluctuations. Due to other variables displaying standard deviations and contemporaneous correlations, which were substantially different to the actual data, this suggests that the inclusion of the basic mining mechanism alone, does not provide the perfect representation.

The importance of mining is expected to increase as a second mining boom in Australia has only recently commenced. This presents future opportunities for further research into the role of mining when modeling the Australian economy.

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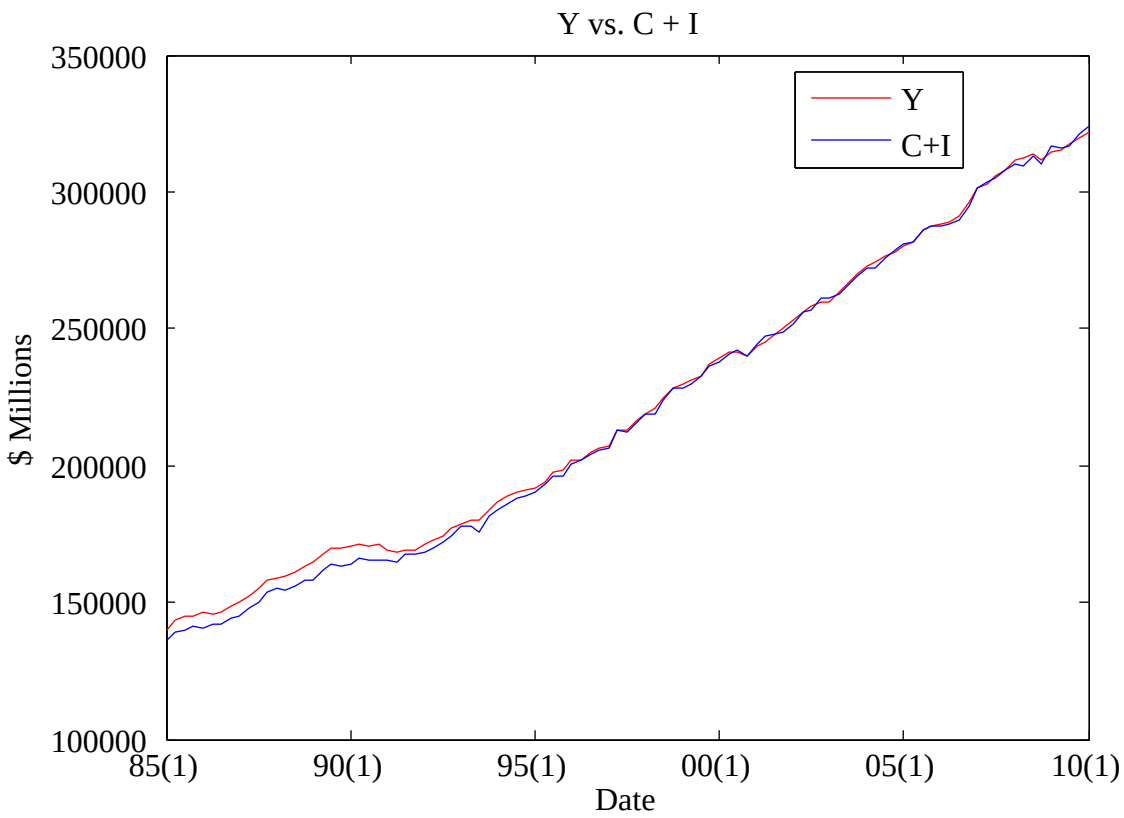
A Appendix A: Figures

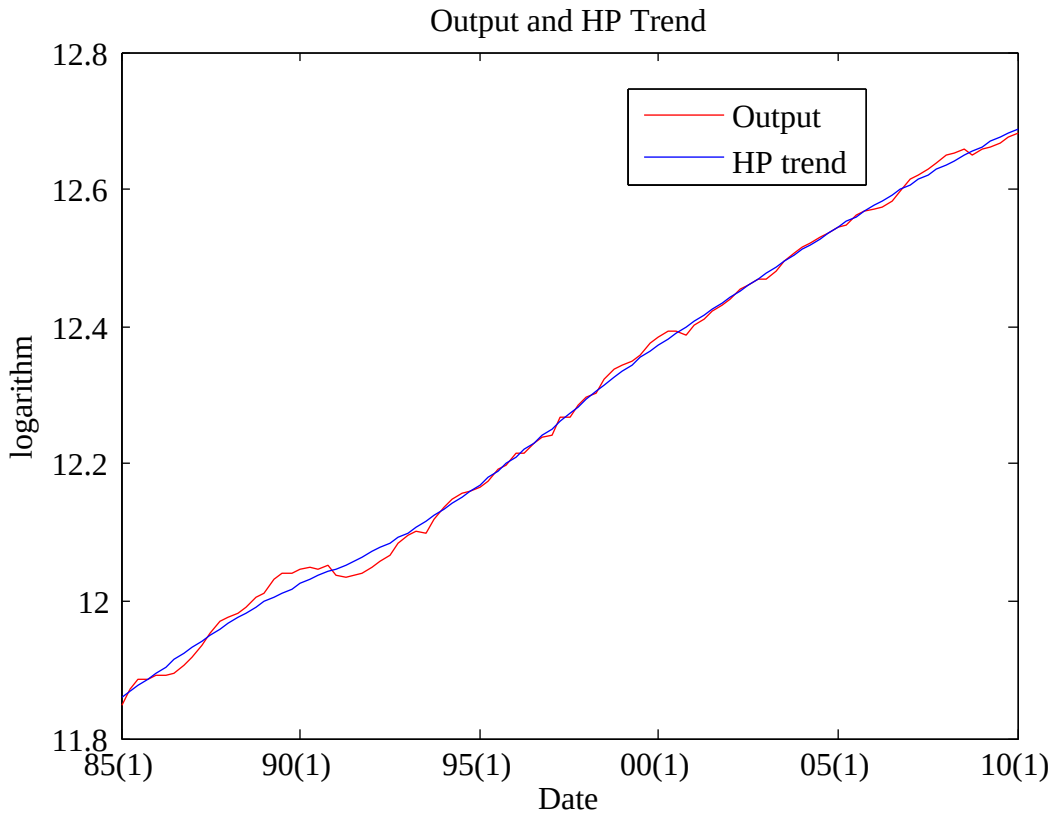
Appendix A displays the figures which are discussed in the paper.

Figure 1: Output Construction



Sample Period is 1985(1) to 2010(1)

Figure 2: HP Trend



Sample Period is 1985(1) to 2010(1)

Figure 3: Cyclical Comparisons

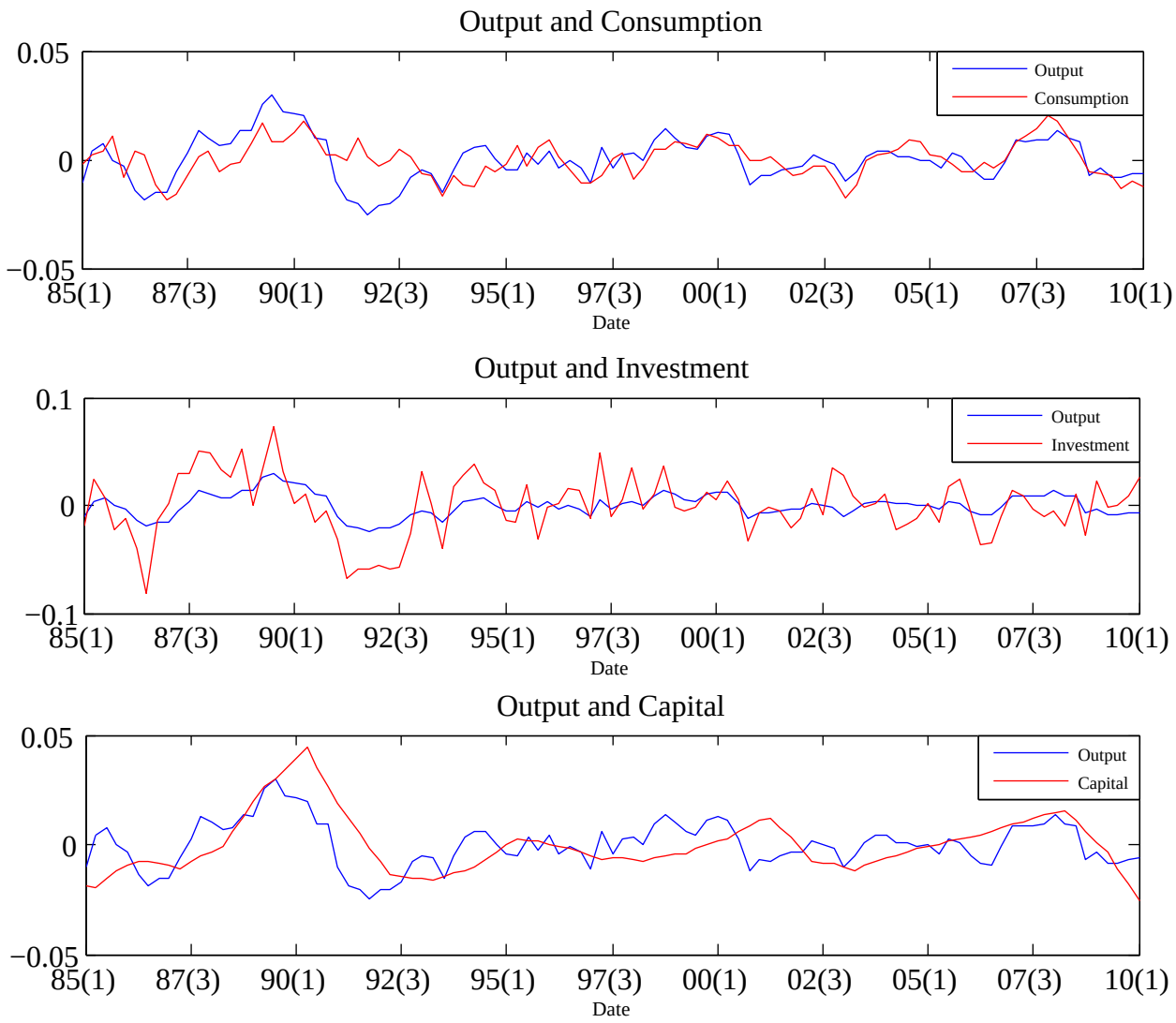
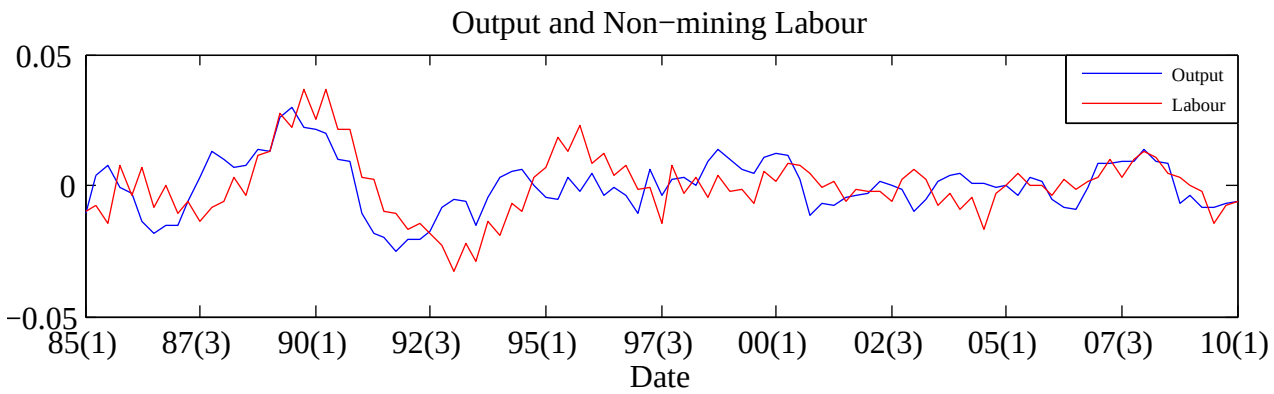


Figure 3 cont.



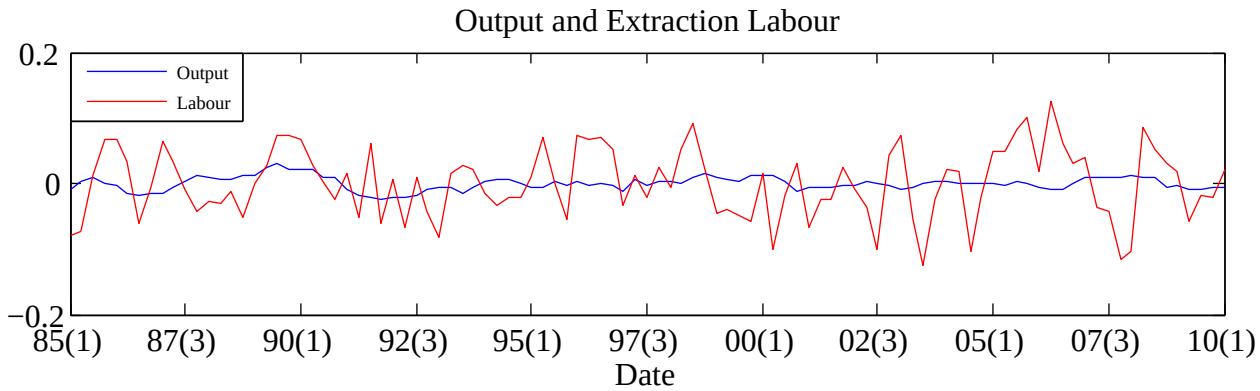


Figure 3 cont.

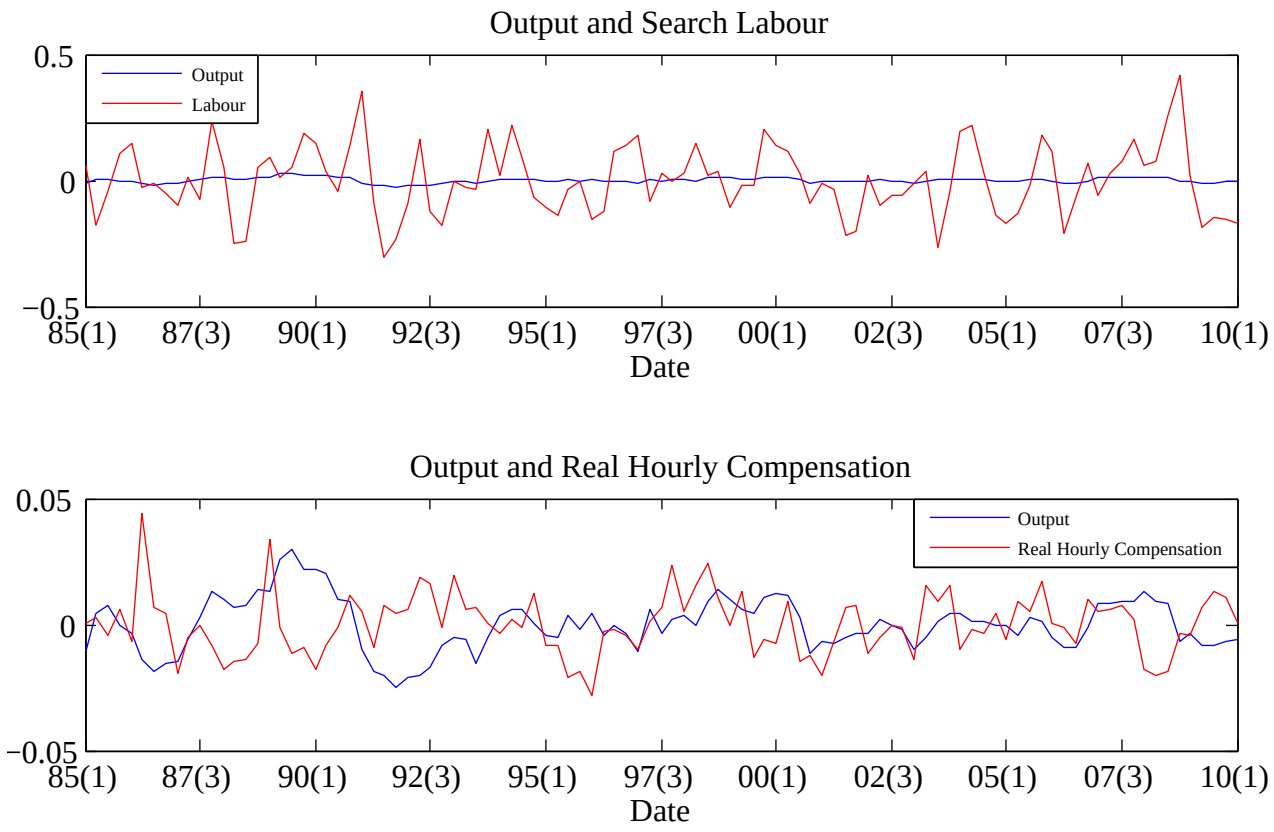


Figure 4

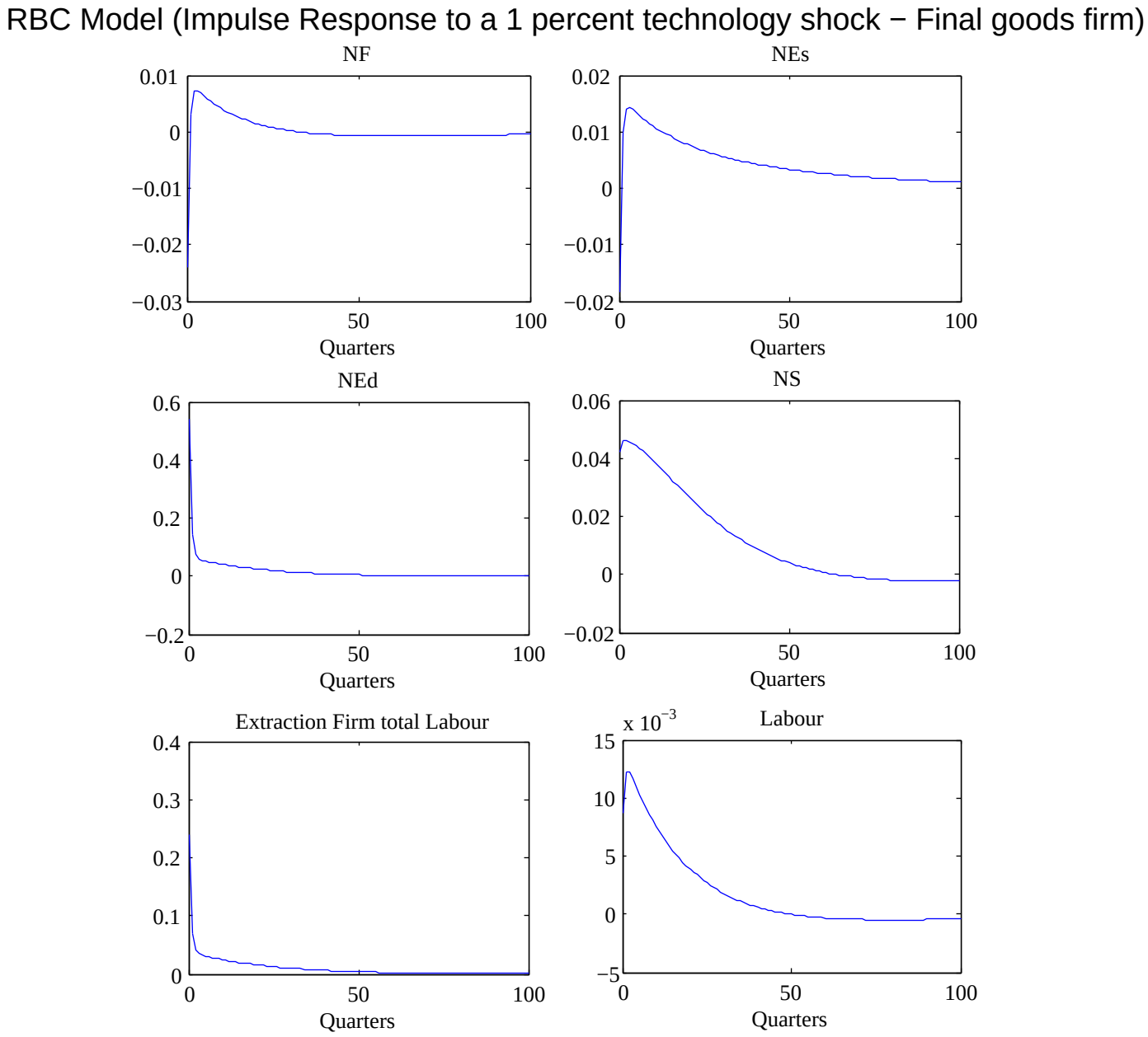

RBC Model (Impulse Response to a 1 percent technology shock − Final goods firm)
NF
NEs
NEd
NS
Extraction Firm total Labour
Labour
x 10^−3
Quarters

Figure 4 cont.

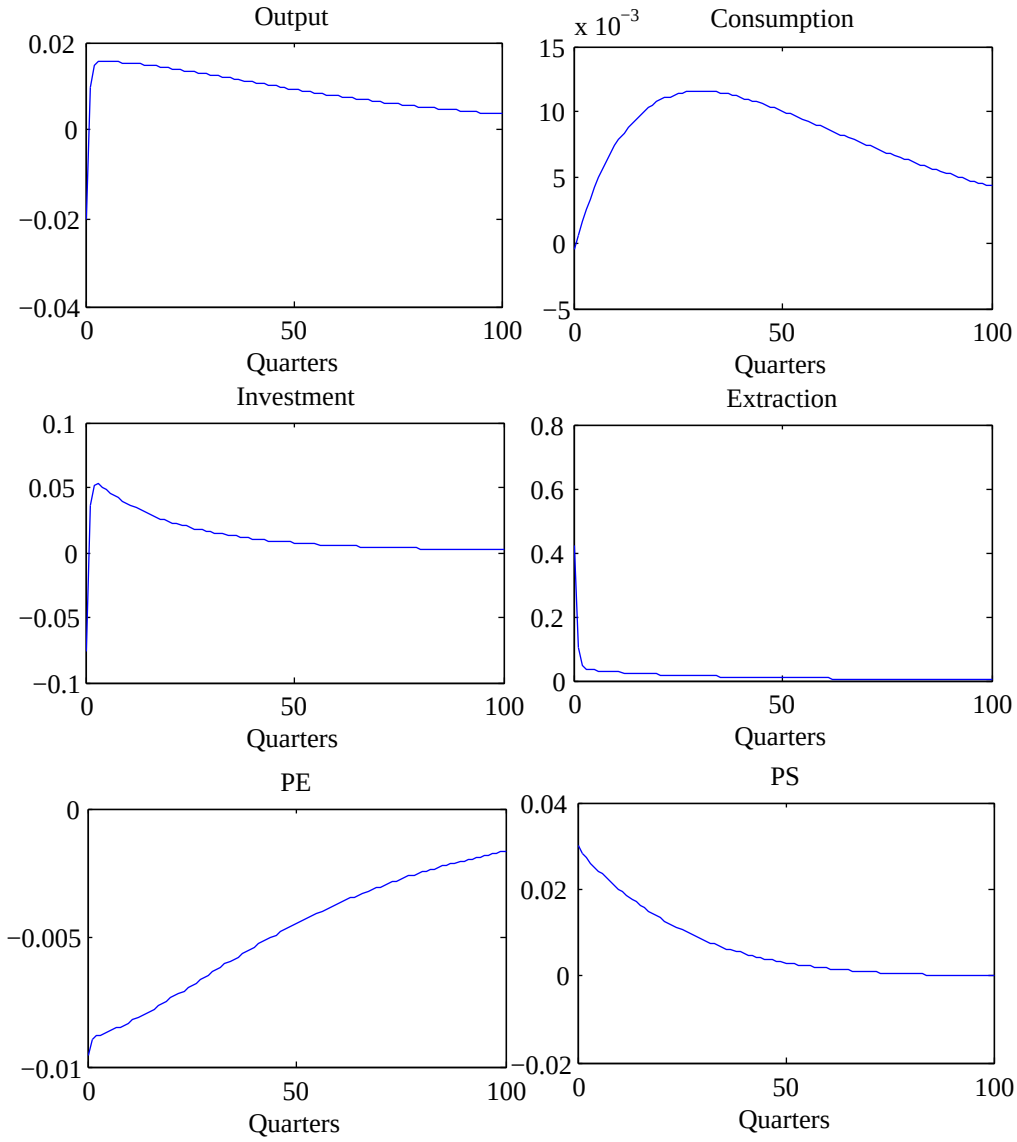


Figure 4 cont.

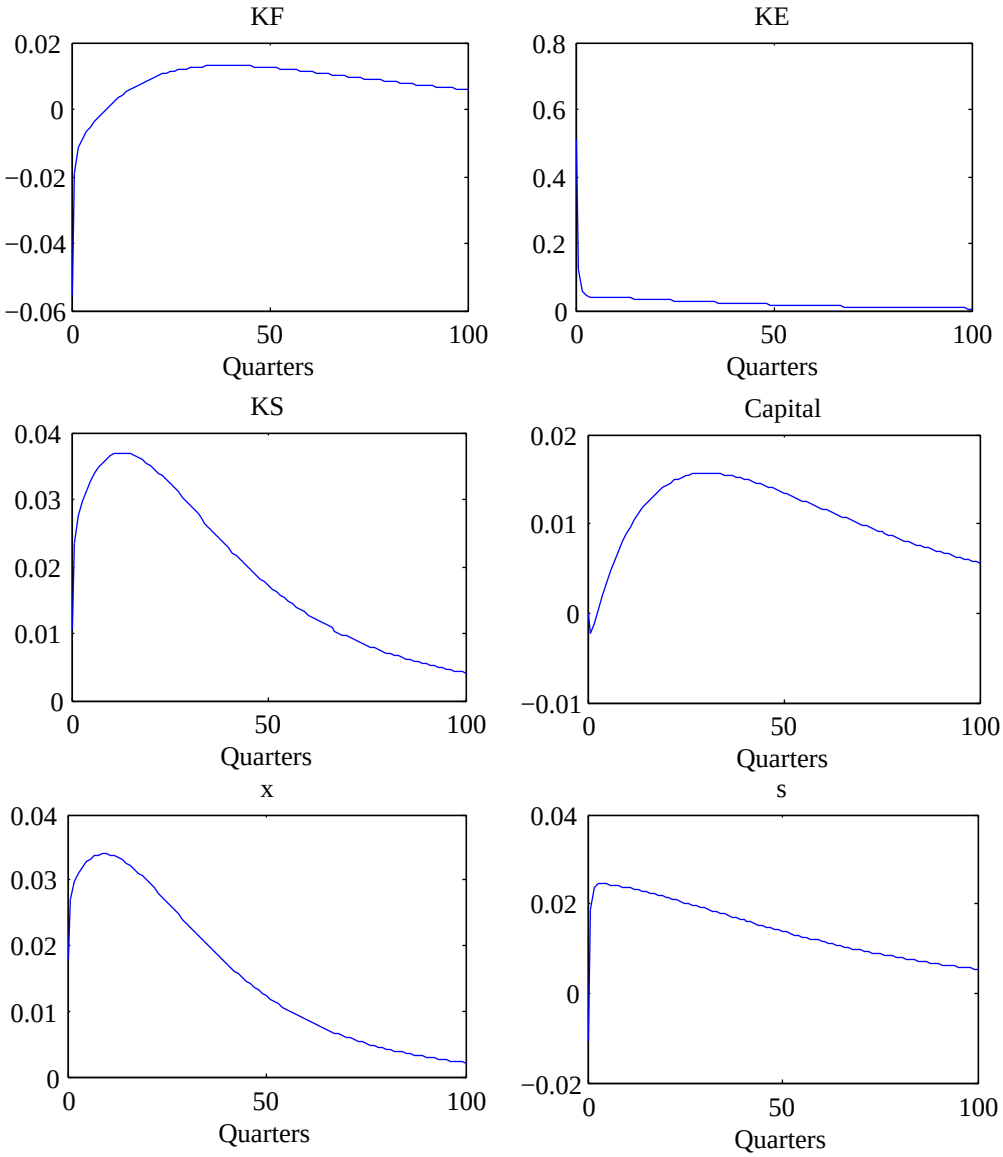


Figure 4 cont.

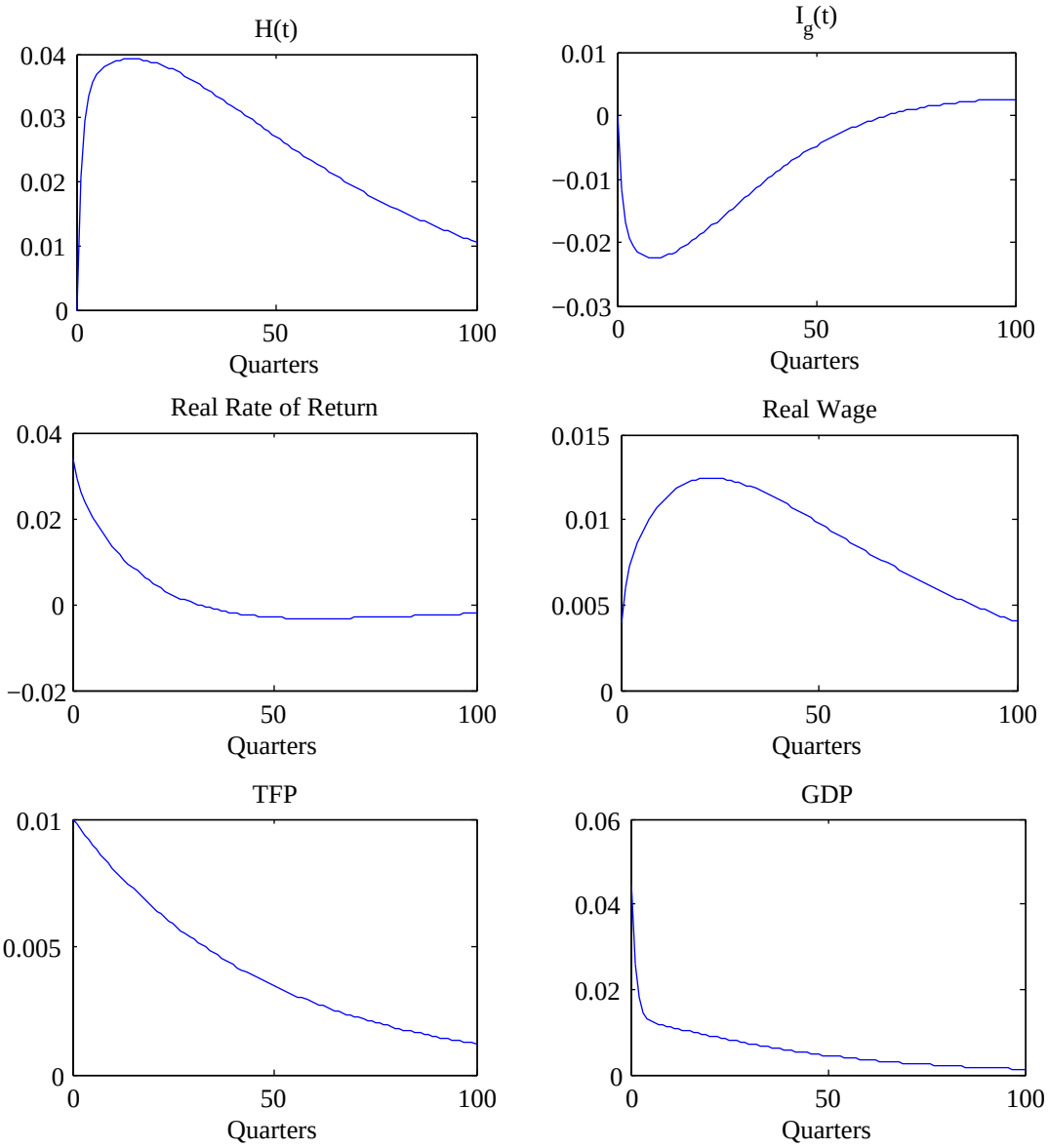


Figure 5

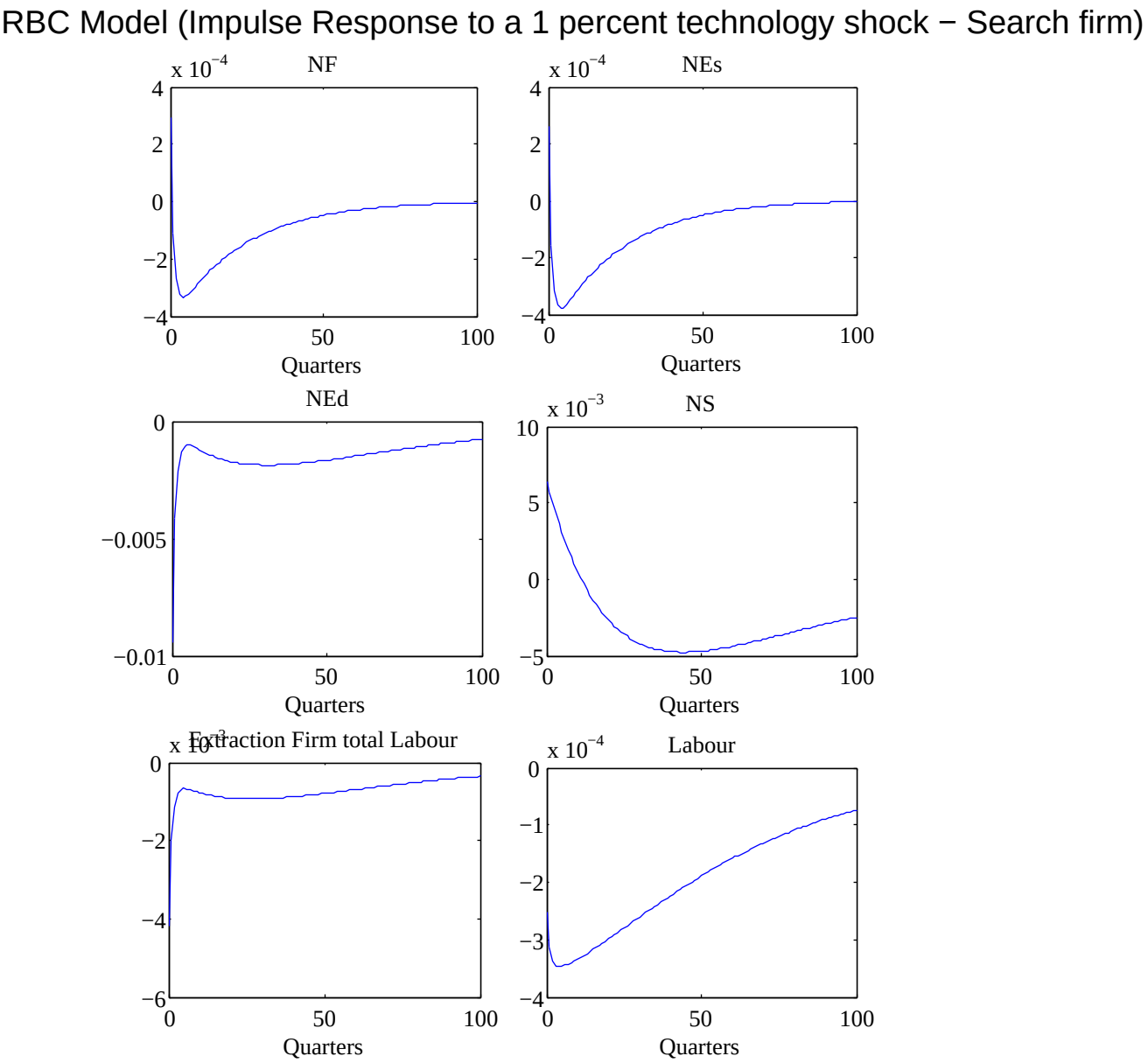

RBC Model (Impulse Response to a 1 percent technology shock − Search firm)
NF
x 10^−4
NEs
x 10^−4
NEd
NS
x 10^−3
Extraction Firm total Labour
x 10^−3
Labour
x 10^−4
Quarters

Figure 5 cont.

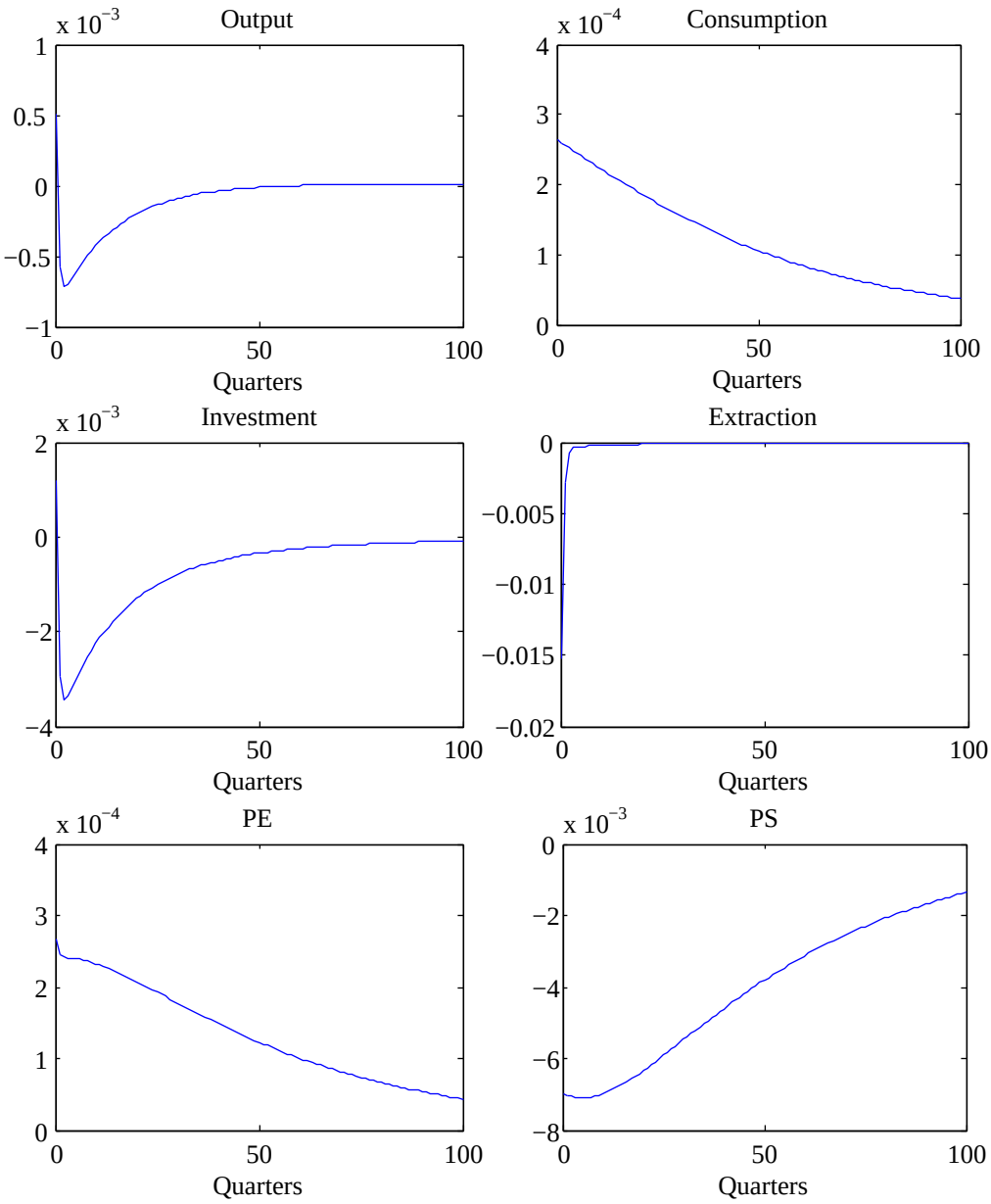


Figure 5 cont.

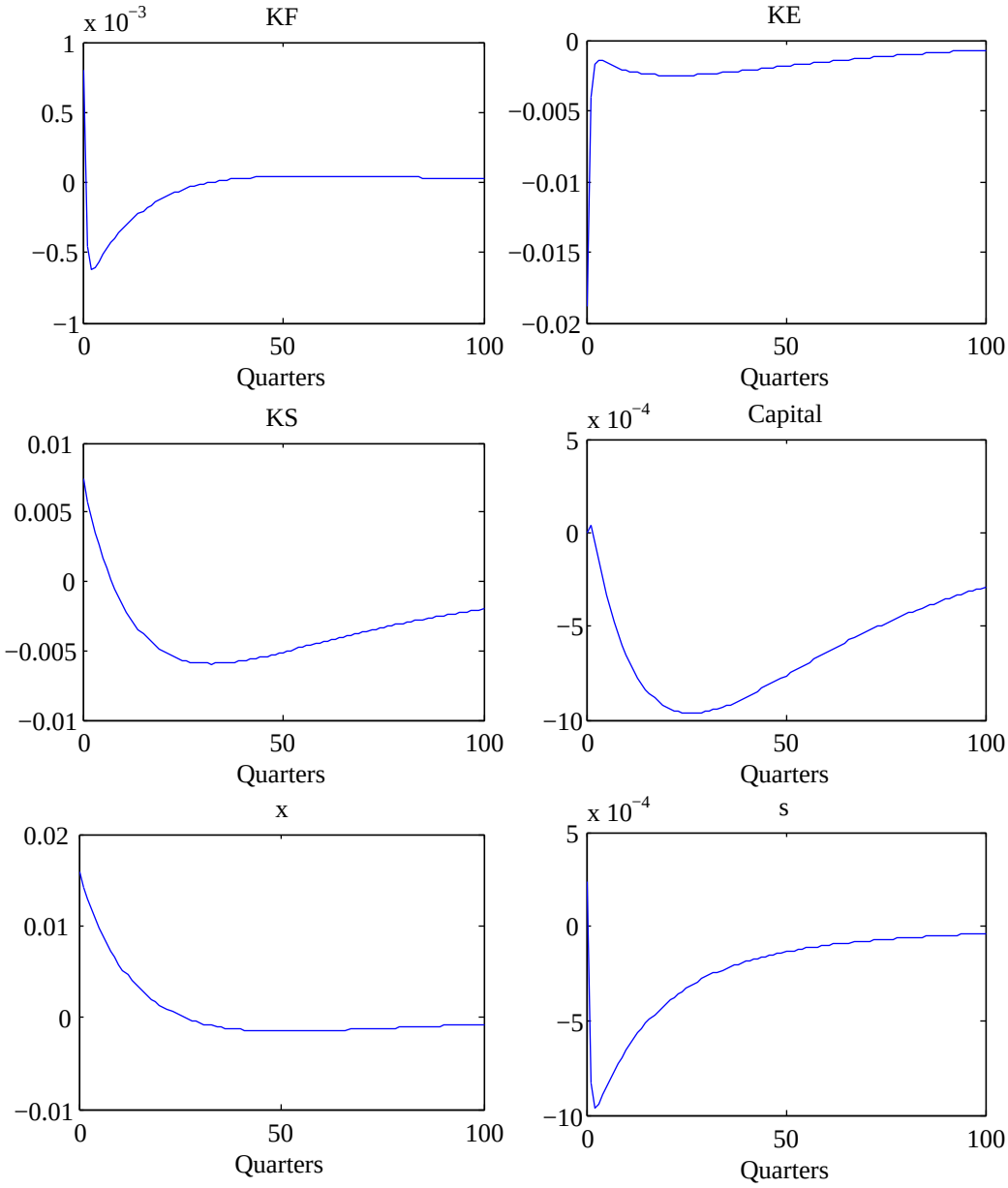


Figure 5 cont.

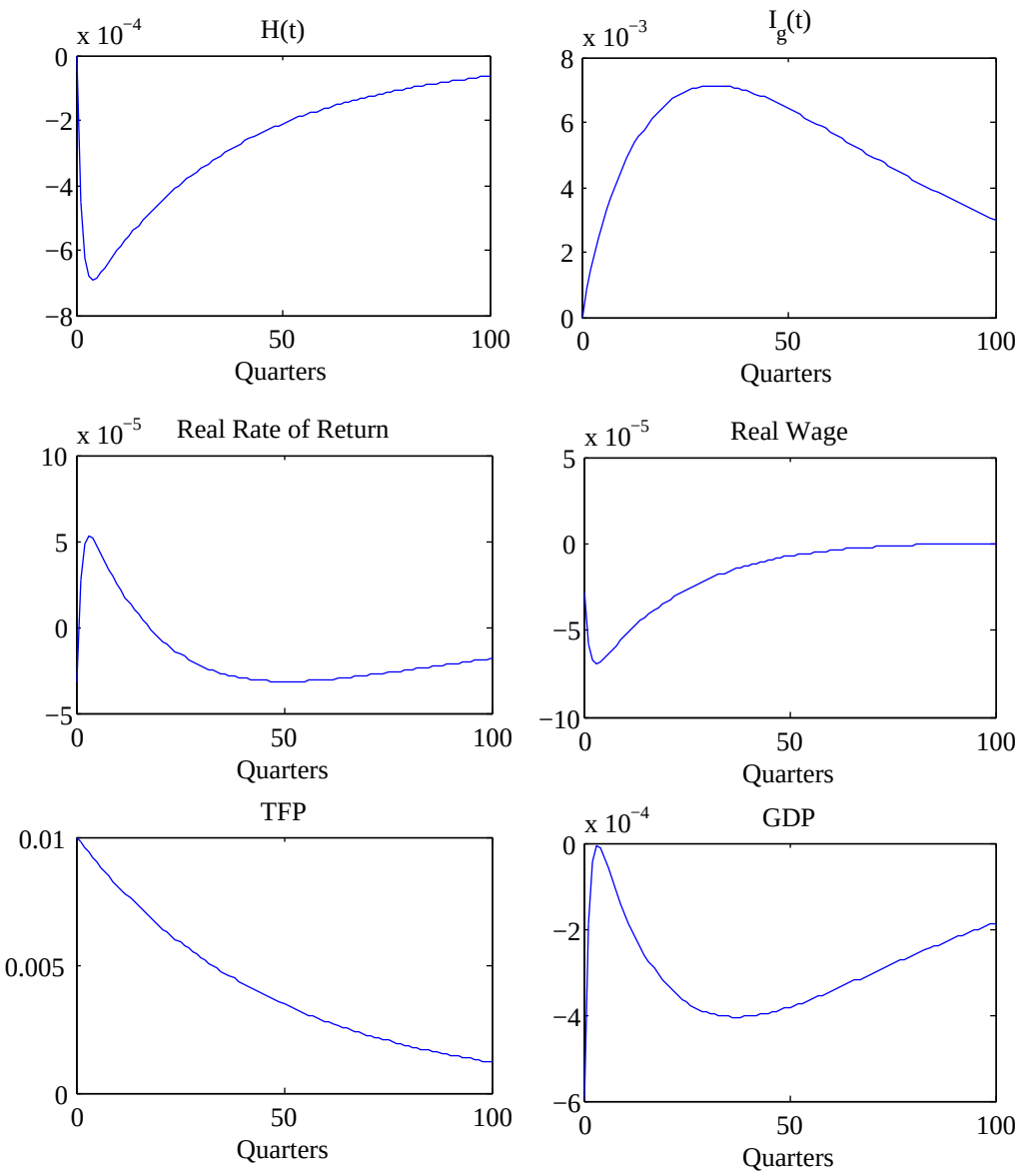


Figure 6

RBC Model (Impulse Response to a 1 percent technology shock − Final goods firm)

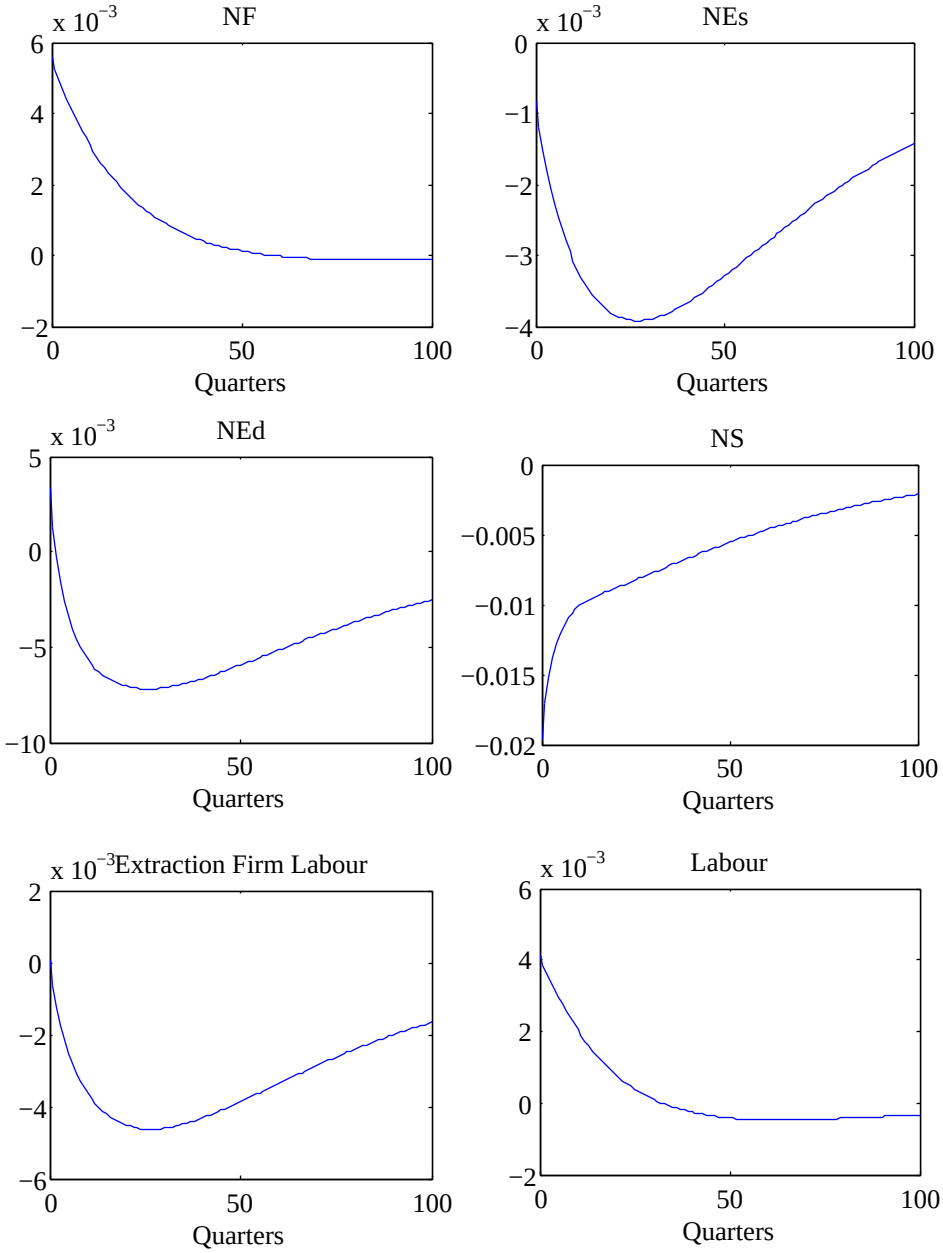


Figure 6 cont.

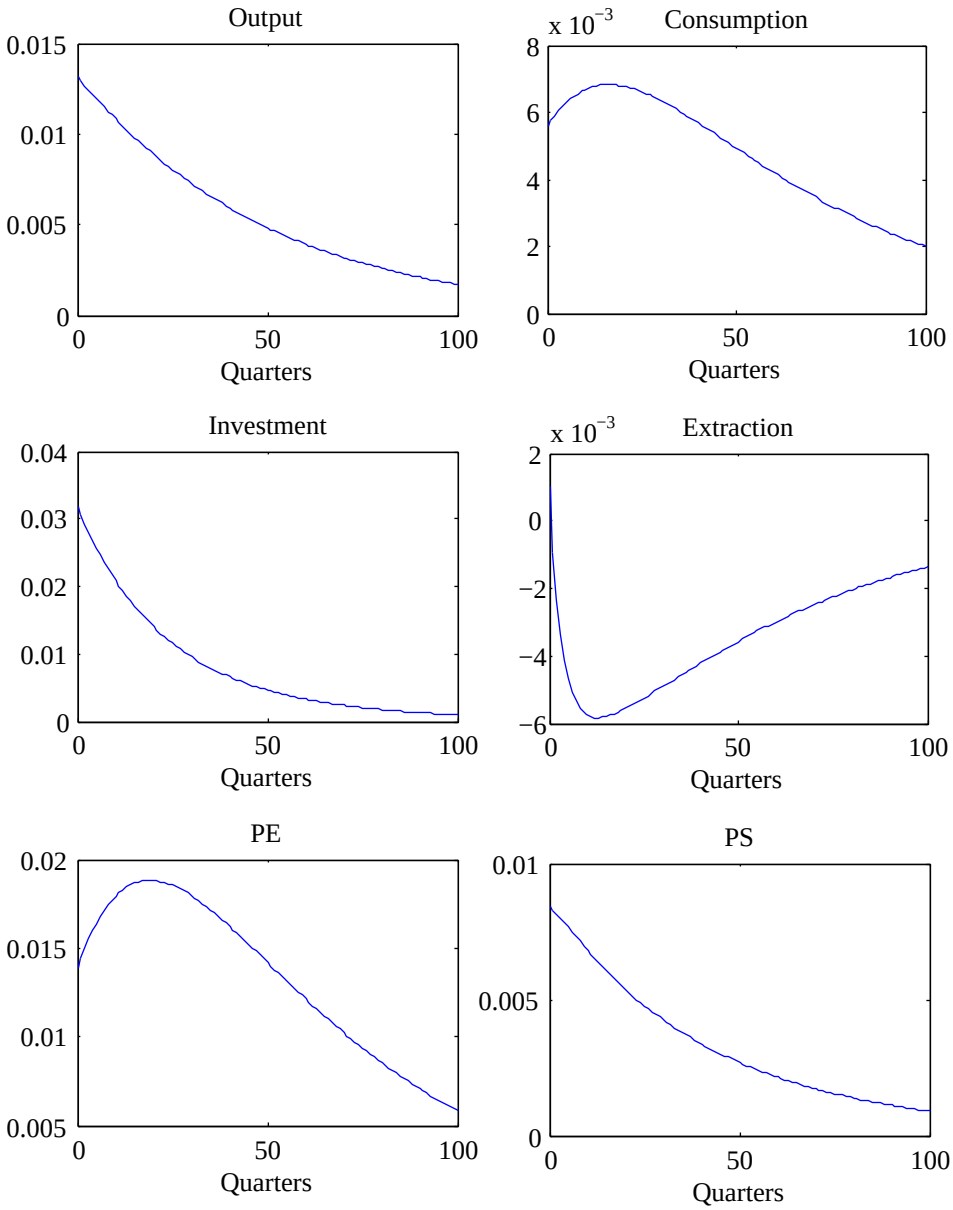


Figure 6 cont.

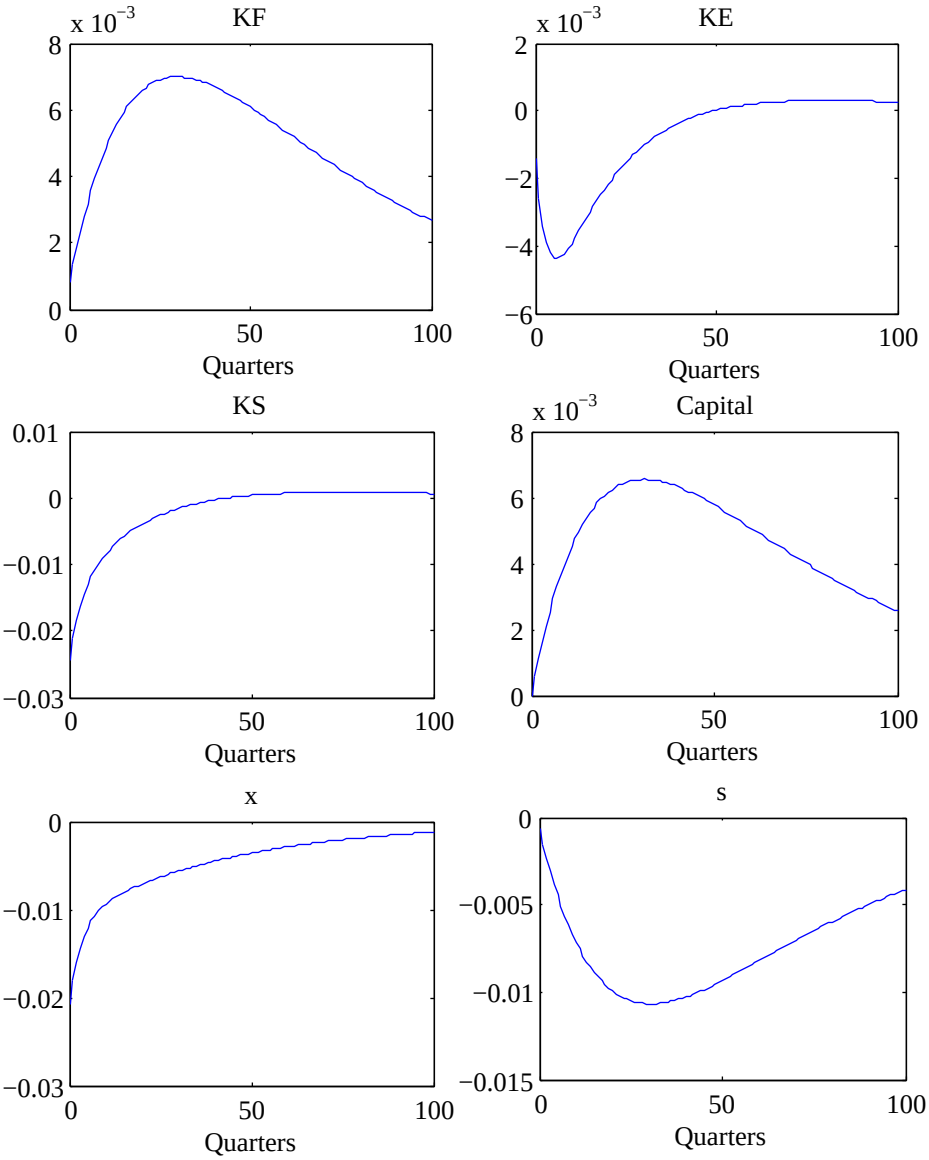


Figure 6 cont.

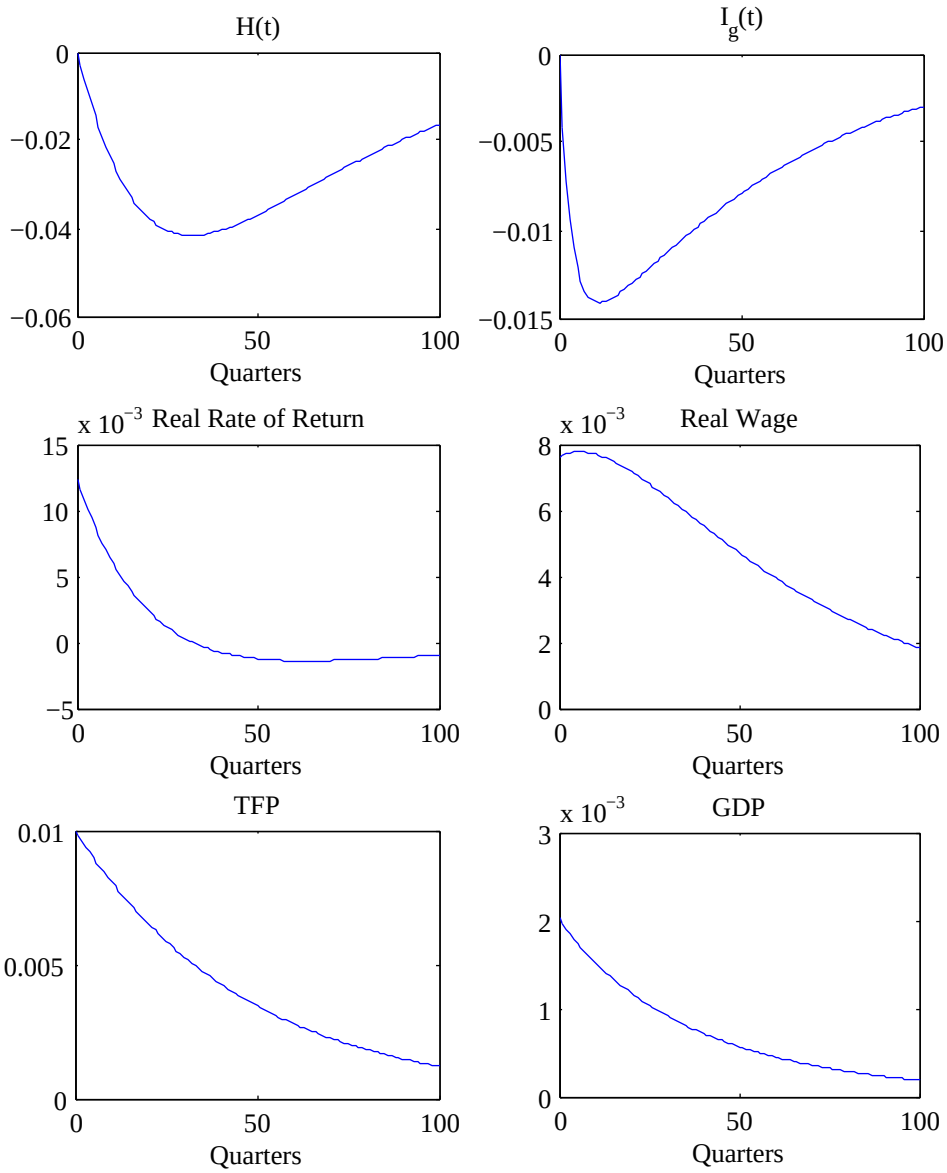


Figure 7

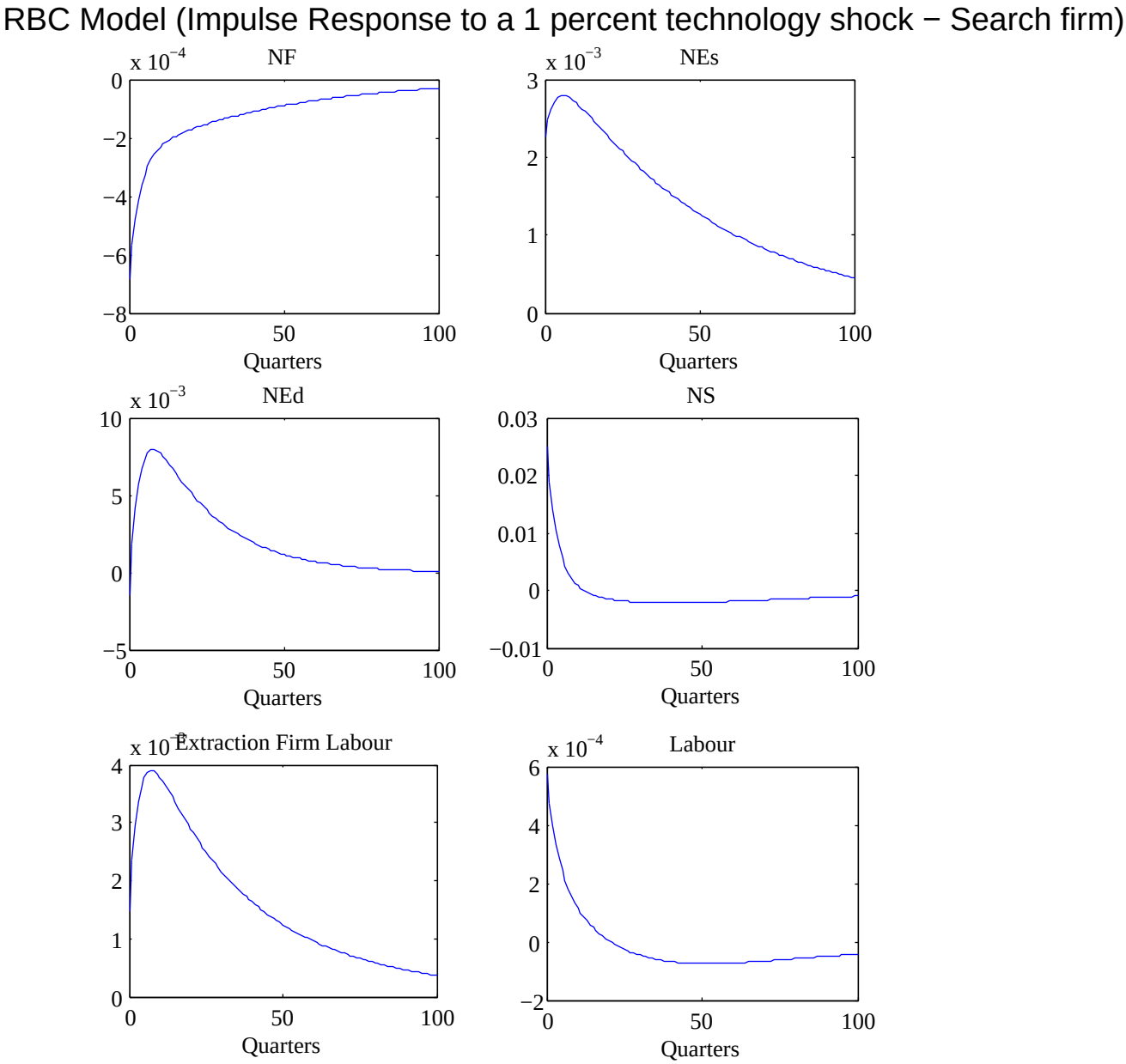

RBC Model (Impulse Response to a 1 percent technology shock − Search firm)
NF
x 10^−4
NEs
x 10^−3
NEd
x 10^−3
NS
Extraction Firm Labour
x 10^−3
Labour
x 10^−4
Quarters

Figure 7 cont.

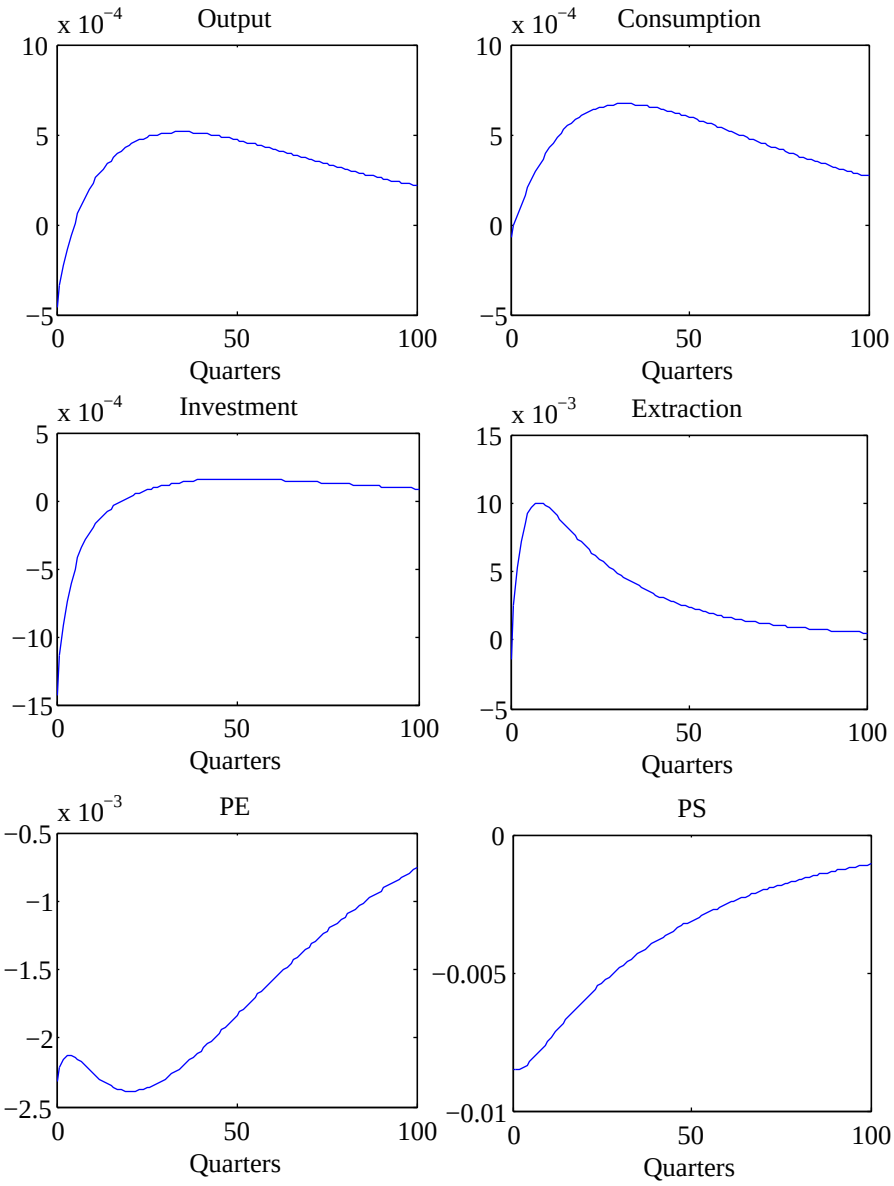


Figure 7 cont.

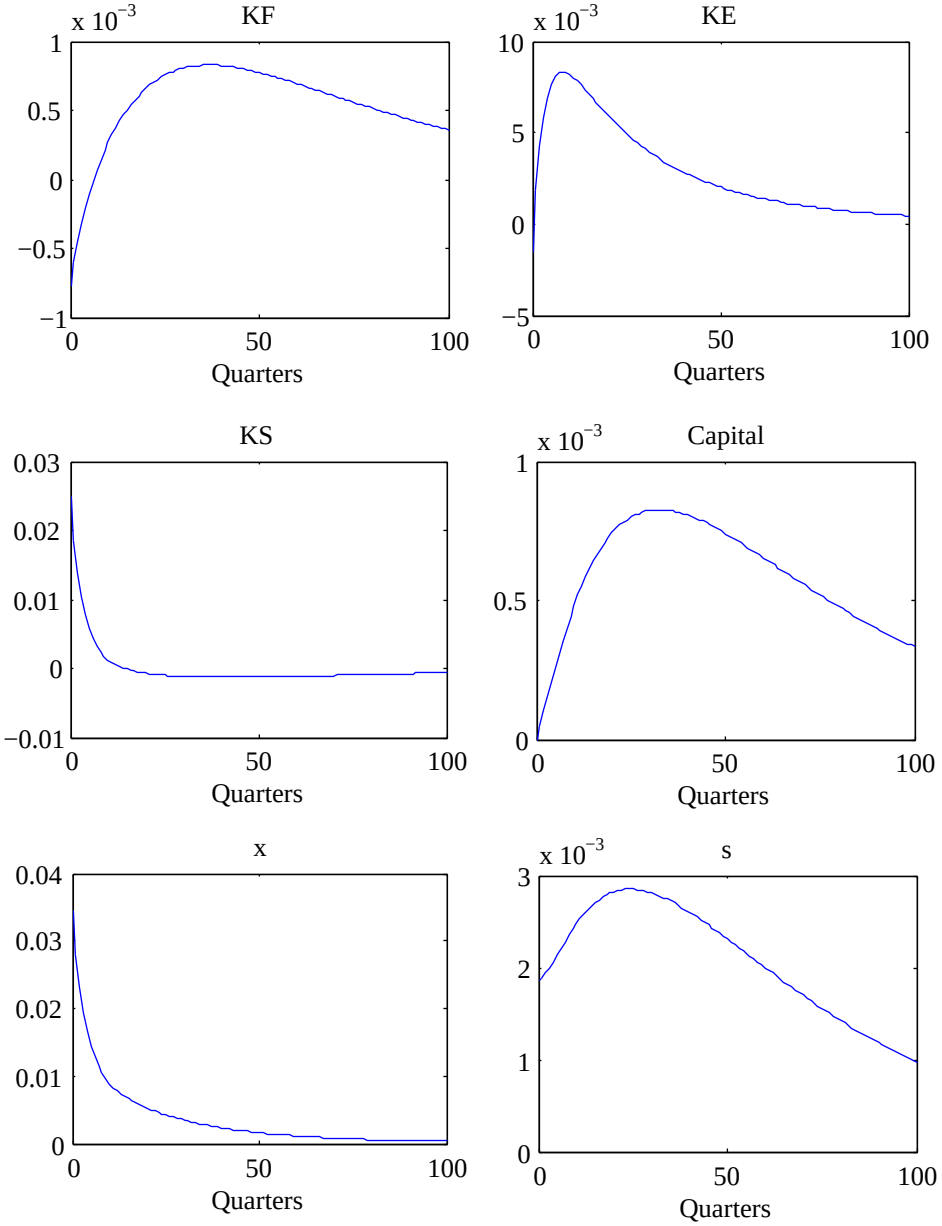


Figure 7 cont.

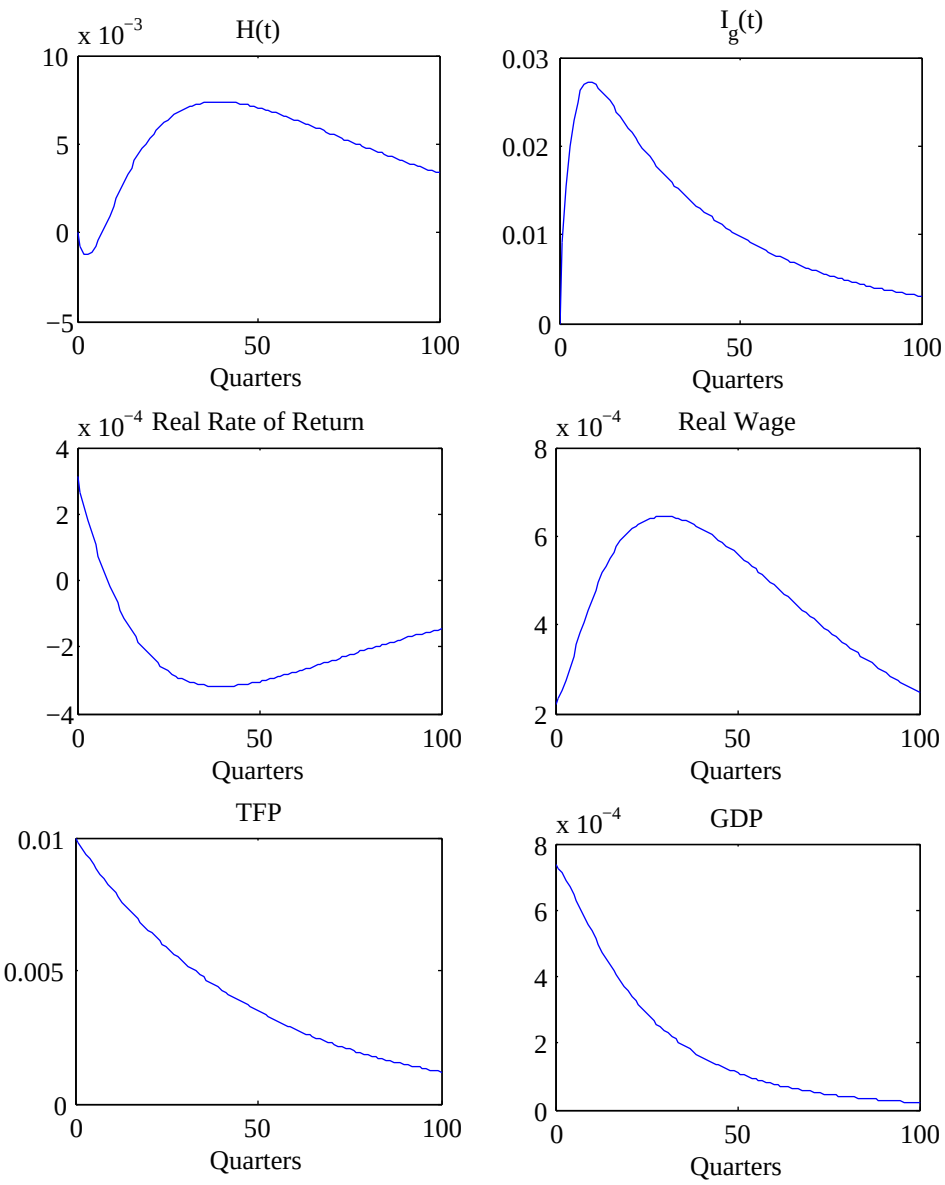


Figure 8: Results - Parameter Set 1

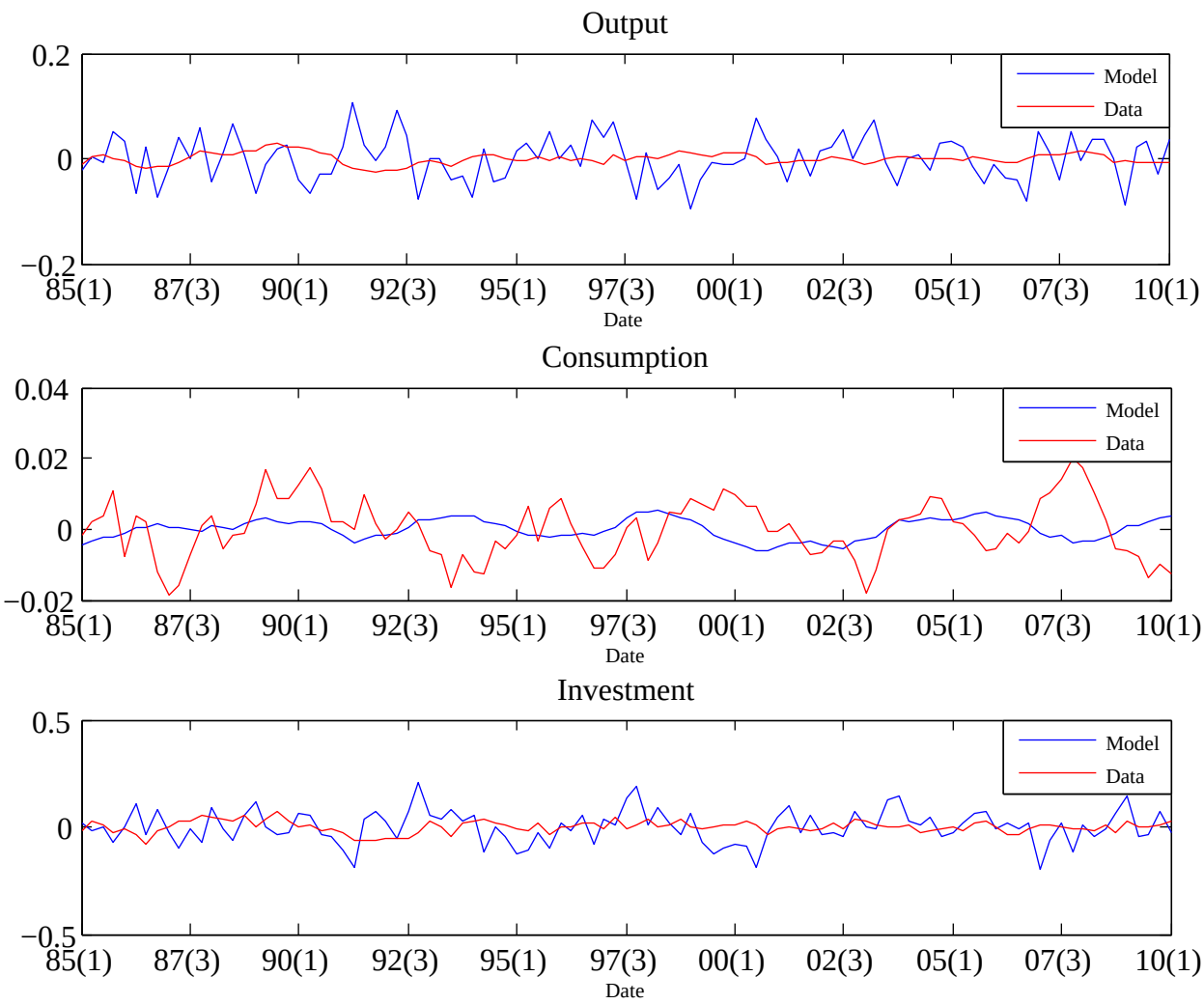


Figure 8 cont.

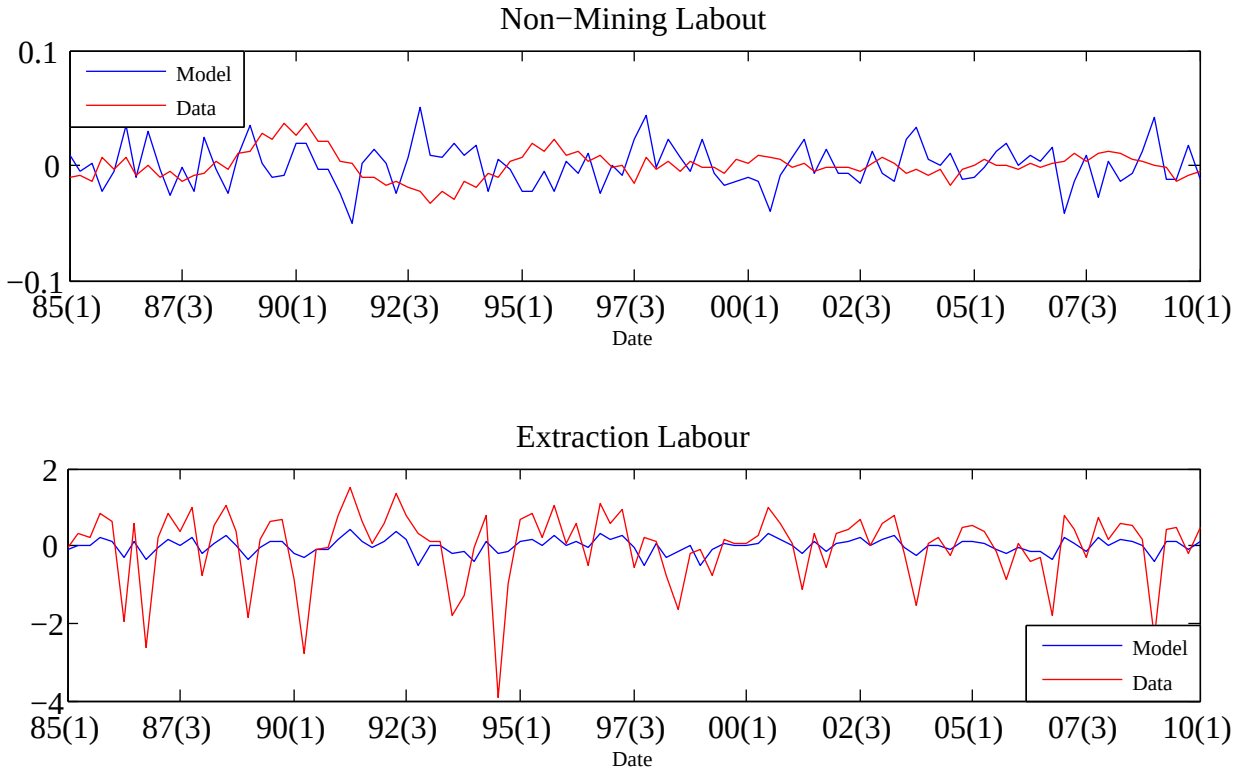


Figure 8 cont.

Figure 9: Results - Parameter Set 2

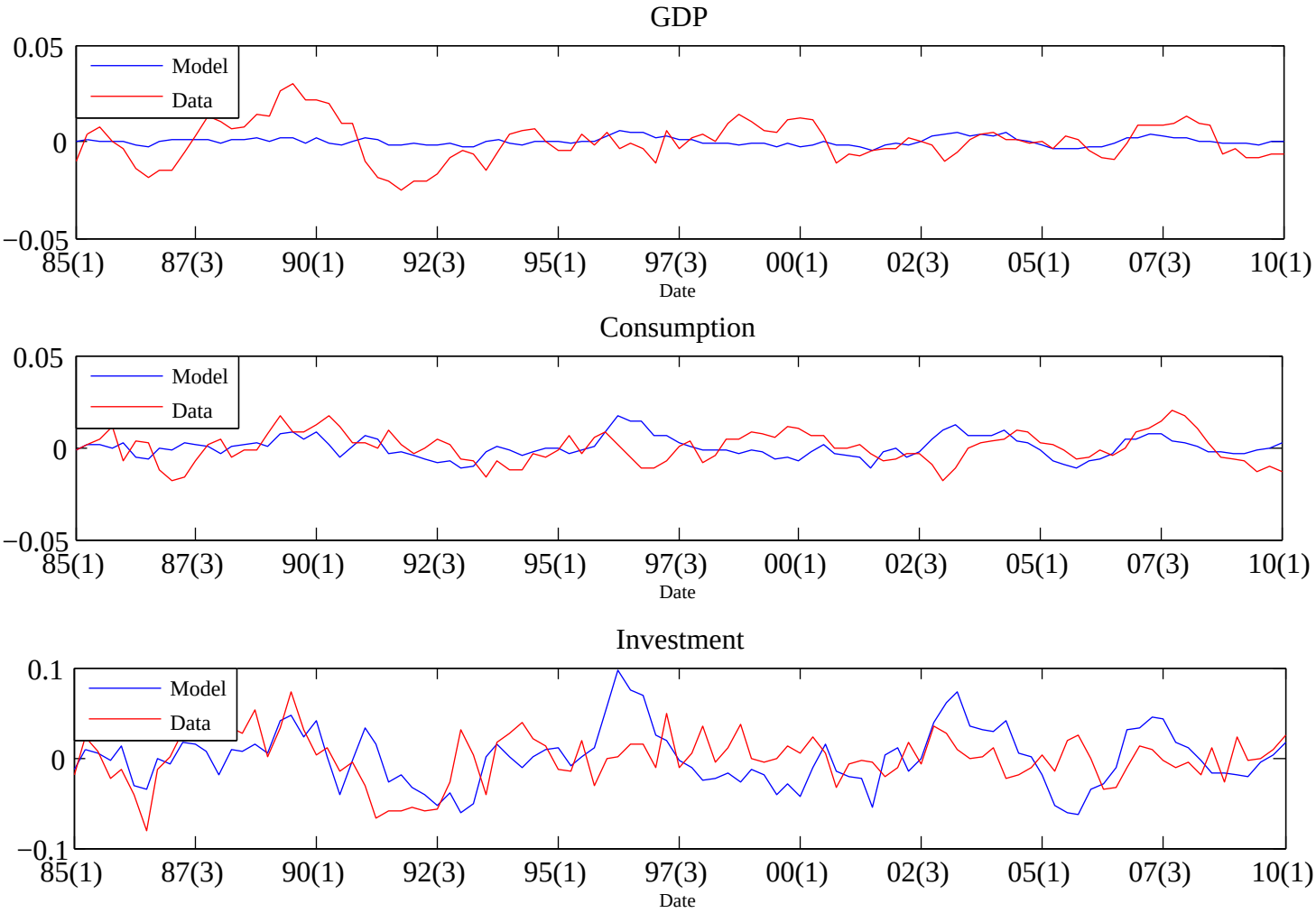
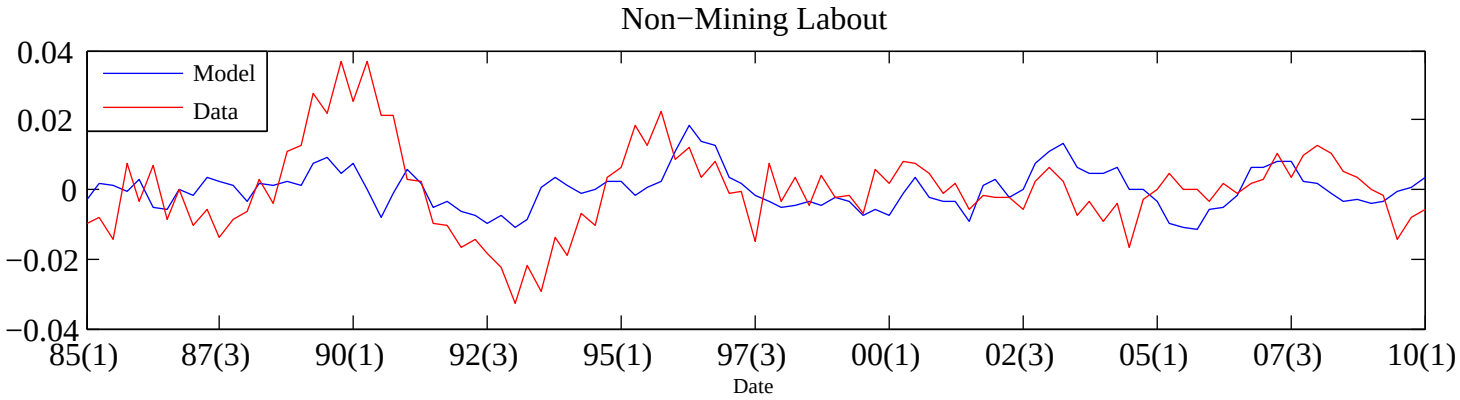


Figure 9 cont.



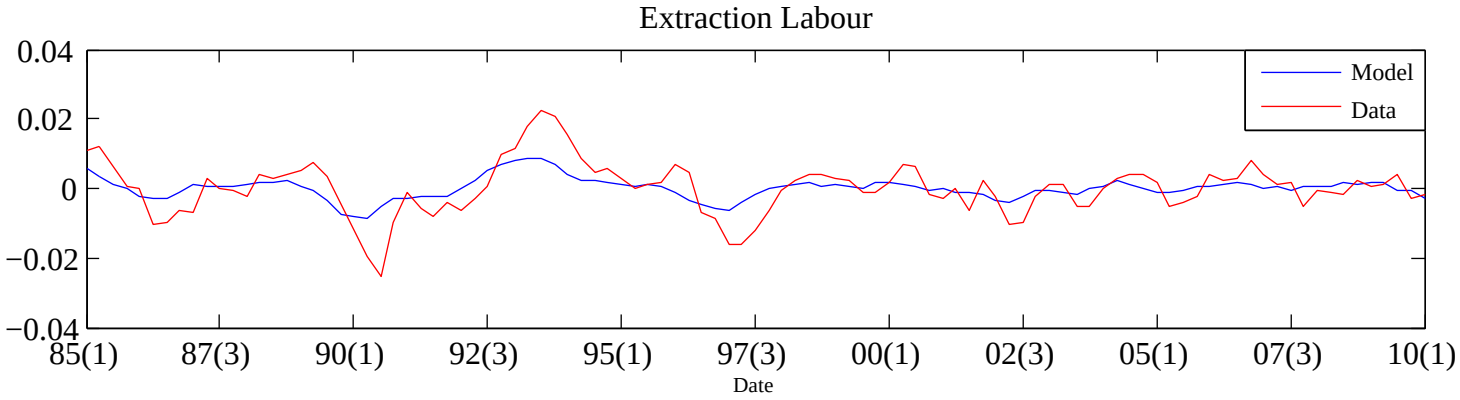
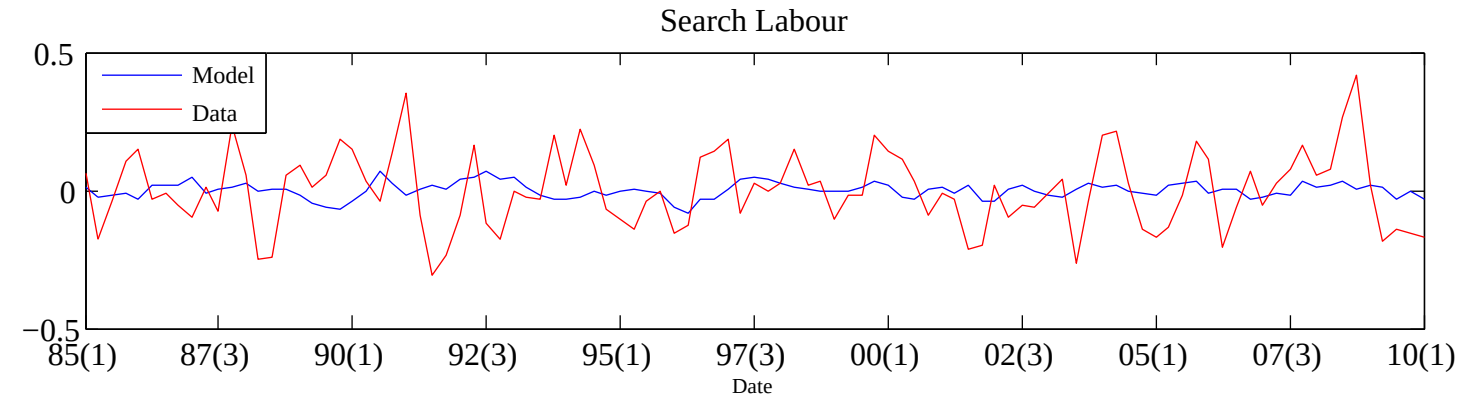
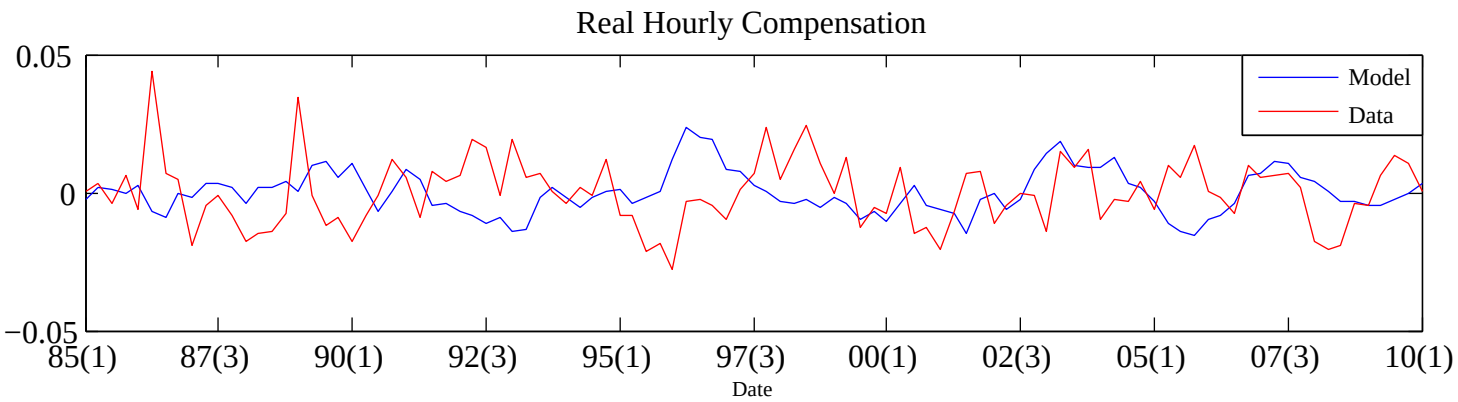


Figure 9 cont.





B Appendix B: Derivation of log-linearised equations

To log-linearise the model a first-order Taylor expansion of each equilibrium equation is taken around the steady state. Taking the total derivative around the steady state of the household's intertemporal consumption trade off with capital for the final goods firm gives

$$-\frac{1}{c^2}dc_t = \frac{\beta}{\gamma_z}E_t\left\{\frac{1}{c}dR_{F,t+1} - (1+R_F-\delta)\frac{1}{c^2}dc_{t+1}\right\}$$

this can then be rewritten as

$$-\frac{1}{c}\frac{dc_t}{c} = \frac{\beta}{\gamma_z}E_t\left\{(1+R_F-\delta)\frac{1}{c}\left(\frac{R_F}{1+R_F-\delta}\right)\frac{dR_{F,t+1}}{R_F} - (1+R_F-\delta)\frac{1}{c}\frac{dc_{t+1}}{c}\right\}.$$

Denoting $\hat{x}_t = \frac{dx_t}{x}$ as the percentage deviation of x_t from the steady state value, x, the household's intertemporal consumption trade off can be rewritten as

$$-\frac{1}{c}\hat{c}_t = \frac{\beta}{\gamma_z}E_t\left\{(1+R_F-\delta)\frac{1}{c}\left(\frac{R_F}{1+R_F-\delta}\right)\hat{R}_{F,t+1} - (1+R_F-\delta)\frac{1}{c}\hat{c}_{t+1}\right\}.$$

In the steady state $\frac{1}{c} = \frac{\beta}{\gamma_z}(1+R_F-\delta)\frac{1}{c}$, therefore the log-linearised equation becomes,

$$-\hat{c}_t = E_t\left\{\left(\frac{R_F}{1+R_F-\delta}\right)\hat{R}_{F,t+1} - \hat{c}_{t+1}\right\}. \tag{B.1}$$

As the real rates of return on capital are equal, only one of the household's intertemporal consumption trade-offs need to be log-linearised for the system. Therefore,

$$\hat{R}_{F,t} = \hat{R}_{E,t} \tag{B.2}$$

$$\hat{R}_{F,t} = \hat{R}_{S,t} \tag{B.3}$$

must then be added to the system to enforce equality. Log-linearising the household's final goods labour supply equation gives

$$
\begin{aligned}
dw_{f,t} &= \frac{\theta dc_t}{(1-(N_F+N_{Es}+N_{Ed}+N_S))^{\eta}} + \\
&\frac{\theta c\eta(1-(N_F+N_{Es}+N_{Ed}+N_S))^{\eta-1}}{[(1-(N_F+N_{Es}+N_{Ed}+N_S))^{\eta}]^2} dN_{F,t} + \\
&\frac{\theta c\eta(1-(N_F+N_{Es}+N_{Ed}+N_S))^{\eta-1}}{[(1-(N_F+N_{Es}+N_{Ed}+N_S))^{\eta}]^2} dN_{Ed,t} + \\
&\frac{\theta c\eta(1-(N_F+N_{Es}+N_{Ed}+N_S))^{\eta-1}}{[(1-(N_F+N_{Es}+N_{Ed}+N_S))^{\eta}]^2} dN_{Es,t} + \\
&\frac{\theta c\eta(1-(N_F+N_{Es}+N_{Ed}+N_S))^{\eta-1}}{[(1-(N_F+N_{Es}+N_{Ed}+N_S))^{\eta}]^2} dN_{S,t}
\end{aligned}
$$

$$
\begin{aligned}
w_f\hat{w}_{ft} &= \frac{\theta c}{1-(N_F+N_{Es}+N_{Ed}+N_S)^{\eta}}\hat{c}_t + \\
&\frac{\theta c\eta(1-(N_F+N_{Es}+N_{Ed}+N_S))^{\eta-1}}{[(1-(N_F+N_{Es}+N_{Ed}+N_S))^{\eta}]^2} N_F\hat{N}_{F,t} + \\
&\frac{\theta c\eta(1-(N_F+N_{E,s}+N_{E,d}+N_S))^{\eta-1}}{[(1-(N_F+N_{Es}+N_{Ed}+N_S))^{\eta}]^2} N_{Ed}\hat{N}_{Ed,t} + \\
&\frac{\theta c\eta(1-(N_F+N_{Es}+N_{Ed}+N_S))^{\eta-1}}{[(1-(N_F+N_{Es}+N_{Ed}+N_S))^{\eta}]^2} N_{Es}\hat{N}_{Es,t} + \\
&\frac{\theta c\eta(1-(N_F+N_{Es}+N_{Ed}+N_S))^{\eta-1}}{[(1-(N_F+N_{Es}+N_{Ed}+N_S))^{\eta}]^2} N_S\hat{N}_{S,t}
\end{aligned}
$$

$$
\begin{aligned}
w_f\hat{w}_{f,t} &= \frac{\theta c}{(1-(N_F+N_{Es}+N_{Ed}+N_S))^{\eta}} \\
&\{\hat{c}_t + \frac{\eta(1-(N_F+N_{Es}+N_{Ed}+N_S))^{\eta-1}}{(1-(N_F+N_{Es}+N_{Ed}+N_S))^{\eta}} N_F\hat{N}_{F,t} + \\
&\frac{\eta(1-(N_F+N_{Es}+N_{Ed}+N_S))^{\eta-1}}{(1-(N_F+N_{Es}+N_{Ed}+N_S))^{\eta}} N_{Ed}\hat{N}_{Ed,t} + \\
&\frac{\eta(1-(N_F+N_{Es}+N_{Ed}+N_S))^{\eta-1}}{(1-(N_F+N_{Es}+N_{Ed}+N_S))^{\eta}} N_{Es}\hat{N}_{Es,t} + \\
&\frac{\eta(1-(N_F+N_{Es}+N_{Ed}+N_S))^{\eta-1}}{(1-(N_F+N_{Es}+N_{Ed}+N_S))^{\eta}} N_S\hat{N}_{S,t}\}
\end{aligned}
$$

$$
\begin{aligned}
\hat{w}_{f,t} &= \hat{c}_t + \frac{\eta N_F(1-(N_F+N_{Es}+N_{Ed}+N_S))^{\eta-1}}{(1-(N_F+N_{Es}+N_{Ed}+N_S))^{\eta}}\hat{N}_{F,t} + \\
&\frac{\eta N_{Ed}(1-(N_F+N_{Es}+N_{Ed}+N_S))^{\eta-1}}{(1-(N_F+N_{Es}+N_{Ed}+N_S))^{\eta}}\hat{N}_{Ed,t} + \\
&\frac{\eta N_{Es}(1-(N_F+N_{Es}+N_{Ed}+N_S))^{\eta-1}}{(1-(N_F+N_{Es}+N_{Ed}+N_S))^{\eta}}\hat{N}_{Es,t} + \\
&\frac{\eta N_S(1-(N_F+N_{Es}+N_{Ed}+N_S))^{\eta-1}}{(1-(N_F+N_{Es}+N_{Ed}+N_S))^{\eta}}\hat{N}_{S,t}
\end{aligned}
$$

$$
\begin{aligned}
\hat{w}_{f,t} &= \hat{c}_t + \frac{\eta N_F}{1-(N_F+N_{Es}+N_{Ed}+N_S)}\hat{N}_{F,t} + \\
&\frac{\eta N_{Ed}}{1-(N_F+N_{Es}+N_{Ed}+N_S)}\hat{N}_{Ed,t} + \\
&\frac{\eta N_{Es}}{1-(N_F+N_{Es}+N_{Ed}+N_S)}\hat{N}_{Es,t} + \\
&\frac{\eta N_S}{1-(N_F+N_{Es}+N_{Ed}+N_S)}\hat{N}_{St}
\end{aligned} \tag{B.4}
$$

From the other labour supply equations from the household

$$\begin{aligned}
\hat{w}_{ed,t} &= \hat{c}_t + \frac{\eta N_F}{1-(N_F+N_{Es}+N_{Ed}+N_S)}\hat{N}_{F,t} + \\
&\frac{\eta N_{Ed}}{1-(N_F+N_{Es}+N_{Ed}+N_S)}\hat{N}_{Ed,t} + \\
&\frac{\eta N_{Es}}{1-(N_F+N_{Es}+N_{Ed}+N_S)}\hat{N}_{Es,t} + \\
&\frac{\eta N_S}{1-(N_F+N_{Es}+N_{Ed}+N_S)}\hat{N}_{St} \qquad \text{(B.5)} \\
\hat{w}_{es,t} &= \hat{c}_t + \frac{\eta N_F}{1-(N_F+N_{Es}+N_{Ed}+N_S)}\hat{N}_{F,t} + \\
&\frac{\eta N_{Ed}}{1-(N_F+N_{Es}+N_{Ed}+N_S)}\hat{N}_{Ed,t} + \\
&\frac{\eta N_{Es}}{1-(N_F+N_{Es}+N_{Ed}+N_S)}\hat{N}_{Es,t} + \\
&\frac{\eta N_S}{1-(N_F+N_{Es}+N_{Ed}+N_S)}\hat{N}_{S,t} \qquad \text{(B.6)} \\
\hat{w}_{s,t} &= \hat{c}_t + \frac{\eta N_F}{1-(N_F+N_{Es}+N_{Ed}+N_S)}\hat{N}_{F,t} + \\
&\frac{\eta N_{Ed}}{1-(N_F+N_{Es}+N_{Ed}+N_S)}\hat{N}_{Ed,t} + \\
&\frac{\eta N_{Es}}{1-(N_F+N_{Es}+N_{Ed}+N_S)}\hat{N}_{Es,t} + \\
&\frac{\eta N_S}{1-(N_F+N_{Es}+N_{Ed}+N_S)}\hat{N}_{S,t}. \qquad \text{(B.7)}
\end{aligned}$$

Log-linearising the household's budget contraint around the steady state gives

$$\begin{aligned}
\gamma_z k\hat{k}_{t+1} &= A_F(k_f^{\alpha_1}N_F^{\alpha_2}s^{1-\alpha_1-\alpha_2})\hat{A}_{F,t} + \alpha_1 A_F(k_f^{\alpha_1}N_F^{\alpha_2}s^{1-\alpha_1-\alpha_2})\hat{k}_{f,t} \\
&+\alpha_2 A_F(k_f^{\alpha_1}N_F^{\alpha_2}s^{1-\alpha_1-\alpha_2})\hat{N}_{F,t} + (1-\alpha_1-\alpha_2)A_F(kf^{\alpha_1}N_F^{\alpha_2}s^{1-\alpha_1-\alpha_2})\hat{s}_t \\
&+(1-\delta)k\hat{k}_t - c\hat{c}_t.
\end{aligned}$$

The equation stating that total capital stock is equal to the capital stock which goes to the three firms must also be log-linearised.

$$k\hat{k}_t = k_f\hat{k}_{f,t} + k_e\hat{k}_{e,t} + k_s\hat{k}_{s,t}. \qquad \text{(B.8)}$$

Log-linearising the final goods firm's labour, capital and non-renewable resource demands respectively gives

$$\begin{aligned}
\hat{w}_{f,t} &= \hat{A}_{F,t} + \alpha_1 \hat{k}_{f,t} + (\alpha_2 - 1)\hat{N}_{F,t} + (1 - \alpha_1 - \alpha_2)\hat{s}_t \quad \text{(B.9)} \\
\hat{R}_{F,t} &= \hat{A}_{F,t} + (\alpha_1 - 1)\hat{k}_{f,t} + \alpha_2 \hat{N}_{F,t} + (1 - \alpha_1 - \alpha_2)\hat{s}_t \quad \text{(B.10)} \\
\hat{P}_{E,t} &= \hat{A}_{F,t} + \alpha_1 \hat{k}_{f,t} + \alpha_2 \hat{N}_{F,t} + (-\alpha_1 - \alpha_2)\hat{s}_t \quad \text{(B.11)}
\end{aligned}$$

From the exogenous process for the Hicks-neutral technology

$$\begin{aligned}
\ln(A_{F,t+1}) &= (1 - \rho_f)\ln(A_F) + \rho_f \ln(A_{F,t}) + \epsilon_{t+1} \\
\hat{A}_{F,t+1} &= \rho_f \hat{A}_{F,t} + \epsilon_{t+1} \quad \text{(B.12)}
\end{aligned}$$

where $\ln(\frac{A_{F,t+1}}{A_F}) = \hat{A}_{F,t+1}$ when $\frac{A_{F,t+1}}{A_F}$ is close to one. From the extraction firm, the log-linearised capital demand equation is

$$\begin{aligned}
R_E \hat{R}_{E,t} &= P_E \zeta_1 A_E k_e^{\zeta_1 - 1} N_{Ed}^{\zeta_2} ig^{1-\zeta_1-\zeta_2}\, \hat{P}_{E,t} - P_S \zeta_1 A_E k_e^{\zeta_1 - 1} N_{Ed}^{\zeta_2} ig^{1-\zeta_1-\zeta_2}\, \hat{P}_{S,t} \\
&\quad + D\, \zeta_1(\zeta_1 - 1) A_E k_e^{\zeta_1 - 1} N_{Ed}^{\zeta_2} ig^{1-\zeta_1-\zeta_2}\, \hat{k}_{e,t} \\
&\quad + D\, \zeta_1 \zeta_2 A_E k_e^{\zeta_1 - 1} N_{Ed}^{\zeta_2} ig^{1-\zeta_1-\zeta_2}\, \hat{N}_{Ed,t} \\
&\quad + D\, \zeta_1(1 - \zeta_1 - \zeta_2) A_E k_e^{\zeta_1 - 1} N_{Ed}^{\zeta_2} ig^{1-\zeta_1-\zeta_2}\, \hat{ig}_t \\
&\quad - \frac{w_{es}}{(1-\nu) A_H h^\nu N_{Es}^{-\nu}} \zeta_1 A_E k_e^{\zeta_1 - 1} N_{Ed}^{\zeta_2} ig^{1-\zeta_1-\zeta_2}\, \hat{w}_{es,t} \\
&\quad - \frac{\nu w_{es}}{(1-\nu) A_H h^\nu N_{Es}^{-\nu}} \zeta_1 A_E k_e^{\zeta_1 - 1} N_{Ed}^{\zeta_2} ig^{1-\zeta_1-\zeta_2}\, \hat{N}_{Es,t} \\
&\quad + \frac{\nu w_{es}}{(1-\nu) A_H h^\nu N_{Es}^{-\nu}} \zeta_1 A_E k_e^{\zeta_1 - 1} N_{Ed}^{\zeta_2} ig^{1-\zeta_1-\zeta_2}\, \hat{h}_t. \quad \text{(B.13)}
\end{aligned}$$

From the digging labour demand equation

$$\begin{aligned}
w_{ed} \hat{w}_{ed,t} &= P_E \zeta_2 A_E k_e^{\zeta_1} N_{Ed}^{\zeta_2 - 1} ig^{1-\zeta_1-\zeta_2}\, \hat{P}_{E,t} - P_S \zeta_2 A_E k_e^{\zeta_1} N_{Ed}^{\zeta_2 - 1} ig^{1-\zeta_1-\zeta_2}\, \hat{P}_{S,t} \\
&\quad + D\, \zeta_2 \zeta_1 A_E k_e^{\zeta_1} N_{Ed}^{\zeta_2 - 1} ig^{1-\zeta_1-\zeta_2}\, \hat{k}_{e,t} \\
&\quad + D\, \zeta_2(\zeta_2 - 1) A_E k_e^{\zeta_1} N_{Ed}^{\zeta_2 - 1} ig^{1-\zeta_1-\zeta_2}\, \hat{N}_{Ed,t} \\
&\quad + D\, \zeta_2(1 - \zeta_1 - \zeta_2) A_E k_e^{\zeta_1} N_{Ed}^{\zeta_2 - 1} ig^{1-\zeta_1-\zeta_2}\, \hat{ig}_t \\
&\quad - \frac{w_{es}}{(1-\nu) A_H h^\nu N_{Es}^{-\nu}} \zeta_2 A_E k_e^{\zeta_1} N_{Ed}^{\zeta_2 - 1} ig^{1-\zeta_1-\zeta_2}\, \hat{w}_{es,t} \\
&\quad - \frac{\nu w_{es}}{(1-\nu) A_H h^\nu N_{Es}^{-\nu}} \zeta_2 A_E k_e^{\zeta_1} N_{Ed}^{\zeta_2 - 1} ig^{1-\zeta_1-\zeta_2}\, \hat{N}_{Es,t} \\
&\quad + \frac{\nu w_{es}}{(1-\nu) A_H h^\nu N_{Es}^{-\nu}} \zeta_2 A_E k_e^{\zeta_1} N_{Ed}^{\zeta_2 - 1} ig^{1-\zeta_1-\zeta_2}\, \hat{h}_t. \quad \text{(B.14)}
\end{aligned}$$

where $D = \left[P_E - \frac{w_{es}}{(1-\nu)A_H h^\nu N_{Es}^{-\nu}} - P_S\right]$. Log-linearising the extraction firm's intertemporal trade-off for above-ground inventory gives

$$P_E\hat{P}_{E,t} - \frac{w_{es}}{(1-\nu)A_H(h^\nu N_{Es}^{-\nu})}\hat{w}_{es,t} + \frac{\nu w_{es}}{(1-\nu)A_H(h^\nu N_{Es}^{-\nu})}\hat{h}_t -$$
$$\frac{\nu w_{es}}{(1-\nu)A_H(h^\nu N_{Es}^{-\nu})}\hat{N}_{Es,t} = \frac{\beta}{\gamma_z}\{\left[\frac{\nu w_{es}N_{Es}}{(1-\nu)h} - \frac{w_{es}}{(1-\nu)A_H(h^\nu N_{Es}^{-\nu})}\right]\hat{w}_{es,t+1}$$
$$\left[-\frac{\nu w_{es}N_{Es}}{(1-\nu)h} + \frac{\nu w_{es}}{(1-\nu)A_H(h^\nu N_{Es}^{-\nu})}\right]\hat{h}_{t+1}$$
$$+\left[\frac{\nu w_{es}N_{Es}}{(1-\nu)h} - \frac{\nu w_{es}}{(1-\nu)A_H(h^\nu N_{Es}^{-\nu})}\right]\hat{N}_{Es,t+1} + P_E\hat{P}_{E,t+1}\} \tag{B.15}$$

From the intertemporal trade-off for in-ground inventory gives

$$\begin{aligned}
\gamma_z P_S\hat{P}_{S,t} &= \beta\{(1-(1-\zeta_1-\zeta_2)A_E k_e^{\zeta_1}N_{Ed}^{\zeta_2}ig^{-\zeta_1-\zeta_2})P_S\hat{P}_{S,t+1} \\
&\quad +P_E(1-\zeta_1-\zeta_2)A_E k_e^{\zeta_1}N_{Ed}^{\zeta_2}ig^{-\zeta_1-\zeta_2}\hat{P}_{E,t+1} \\
&\quad -\frac{w_{es}}{(1-\nu)A_H(h^\nu N_{Es}^{-\nu})P_S}(1-\zeta_1-\zeta_2)A_E(k_e^{\zeta_1}N_{Ed}^{\zeta_2}ig^{-\zeta_1-\zeta_2})\hat{w}_{es,t+1} \\
&\quad +\frac{\nu w_{es}}{(1-\nu)A_H(h^\nu N_{Es}^{-\nu})P_S}(1-\zeta_1-\zeta_2)A_E(k_e^{\zeta_1}N_{Ed}^{\zeta_2}ig^{-\zeta_1-\zeta_2})\hat{h}_{t+1} \\
&\quad -\frac{\nu w_{es}}{(1-\nu)A_H(h^\nu N_{Es}^{-\nu})P_S}(1-\zeta_1-\zeta_2)A_E(k_e^{\zeta_1}N_{Ed}^{\zeta_2}ig^{-\zeta_1-\zeta_2})\hat{N}_{Es,t+1} \\
&\quad +D\,\zeta_1(1-\zeta_1-\zeta_2)A_E(k_e^{\zeta_1}N_{Ed}^{\zeta_2}ig^{-\zeta_1-\zeta_2})\hat{k}_{e,t+1} \\
&\quad +D\,\zeta_2(1-\zeta_1-\zeta_2)A_E(k_e^{\zeta_1}N_{Ed}^{\zeta_2}ig^{-\zeta_1-\zeta_2})\hat{N}_{Ed,t+1} \\
&\quad +D\,(-\zeta_1-\zeta_2)(1-\zeta_1-\zeta_2)A_E(k_e^{\zeta_1}N_{Ed}^{\zeta_2}ig^{-\zeta_1-\zeta_2})\hat{Ig}_{t+1}\}.
\end{aligned} \tag{B.16}$$

where $D = \left[P_E - \frac{w_{es}}{(1-\nu)A_H h^\nu N_{Es}^{-\nu}} - P_S\right]$. From the extraction firm's constraints

$$\begin{aligned}
\gamma_z h\hat{h}_{t+1} &= h\hat{h}_t - s\hat{s}_t + \zeta_1 A_E(k_e^{\zeta_1}N_{dE}^{\zeta_2}ig^{1-\zeta_1-\zeta_2})\hat{k}_{e,t} \\
&\quad +\zeta_2 A_E(k_e^{\zeta_1}N_{Ed}^{\zeta_2}ig^{1-\zeta_1-\zeta_2})\hat{N}_{Ed,t} \\
&\quad +(1-\zeta_1-\zeta_2)A_E(k_e^{\zeta_1}N_{Ed}^{\zeta_2}ig^{1-\zeta_1-\zeta_2})\hat{ig}_t \qquad \text{(B.17)} \\
\gamma_z ig\hat{ig}_{t+1} &= ig\hat{ig}_t + x\hat{x}_t - \zeta_1 A_E(k_e^{\zeta_1}N_{Ed}^{\zeta_2}ig^{1-\zeta_1-\zeta_2})\hat{k}_{e,t} - \\
&\quad \zeta_2 A_E(k_e^{\zeta_1}N_{Ed}^{\zeta_2}ig^{1-\zeta_1-\zeta_2})\hat{N}_{Ed,t} - \\
&\quad (1-\zeta_1-\zeta_2)A_E(k_e^{\zeta_1}N_{Ed}^{\zeta_2}ig^{1-\zeta_1-\zeta_2})\hat{ig}_t \qquad \text{(B.18)} \\
\hat{s}_t &= \nu\hat{h}_t + (1-\nu)\hat{N}_{Es,t} \qquad \text{(B.19)}
\end{aligned}$$

Log-linearising the search firm's labour and capital demand equations gives

$$\begin{aligned}\hat{w}_{s,t} &= \hat{P}_{S,t} + (1 - A_S(k_s^{\kappa} N_S^{1-\kappa}))\hat{A}_{S,t} + (\kappa - \kappa A_S(k_s^{\kappa} N_S^{1-\kappa}))\hat{k}_{s,t} \\ &\quad -(\kappa + (1-\kappa)A_S(k_s^{\kappa} N_S^{1-\kappa}))\hat{N}_{S,t} \qquad \text{(B.20)} \\ \hat{R}_{S,t} &= \hat{P}_{S,t} + (1 - A_S(k_s^{\kappa} N_S^{1-\kappa}))\hat{A}_{S,t} + ((\kappa - 1) - \kappa A_S(k_s^{\kappa} N_S^{1-\kappa}))\hat{k}_{s,t} \\ &\quad +((1-\kappa) - (1-\kappa)A_S(k_s^{\kappa} N_S^{1-\kappa}))\hat{N}_{S,t}. \qquad \text{(B.21)}\end{aligned}$$

The search firm's exogenous process for the Hicks-neutral technology is

$$\hat{A}_{S,t+1} = \rho_s \hat{A}_{S,t} + \epsilon_{t+1}. \qquad \text{(B.22)}$$

Finally the log-linearised equation for the market clearing condition for the deposits of the non-renewable resource is

$$\begin{aligned}x\hat{x}_t &= e^{-A_S k_s^{\kappa} N_S^{1-\kappa}} \mu A_S(k_s^{\kappa} N_S^{1-\kappa})\hat{A}_{S,t} + e^{-A_S k_s^{\kappa} N_S^{1-\kappa}} \mu\kappa A_S(k_s^{\kappa} N_S^{1-\kappa})\hat{k}_{s,t} \\ &\quad + e^{-A_S k_s^{\kappa} N_S^{1-\kappa}} \mu(1-\kappa) A_S(k_s^{\kappa} N_S^{1-\kappa})\hat{N}_{S,t} \qquad \text{(B.23)}\end{aligned}$$

C Appendix C: Matrices

The following equations are used to solve the system by the Blanchard-Kahn method,

$$\begin{aligned} M_{cc}Y_t &= M_{cs}S_t \\ M_{ss0}E_tS_{t+1} + M_{ss1}S_t &= M_{sc0}E_tY_{t+1} + M_{cs1}Y_t + M_{se}\epsilon_{t+1} \end{aligned}$$

where the first equation is the observation equation and the second is the state equation. $Y_t = [\hat{w}_{f,t}, \hat{w}_{ed,t}, \hat{w}_{es,t}, \hat{w}_{s,t}, \hat{N}_{F,t}, \hat{N}_{Ed,t}, \hat{N}_{Es,t}, \hat{N}_{S,t}, \hat{R}_{F,t}, \hat{R}_{E,t}, \hat{R}_{S,t}, \hat{x}_t, \hat{s}_t, \hat{k}_{f,t}, \hat{k}_{e,t}, \hat{k}_{s,t}]'$ and $S_t = [\hat{ig}_t, \hat{h}_t, \hat{k}_{f,t}, \hat{k}_{e,t}, \hat{k}_{s,t}, \hat{A}_{F,t}, \hat{A}_{S,t}, \hat{c}_t, \hat{P}_{E,t}, \hat{P}_{S,t}]'$. Stacking (B.4),(B.5), (B.6), (B.7), (B.10), (B.9), (B.11), (B.21), (B.21), (B.14), (B.13), (B.23), (B.19), (B.8), (B.2) and (B.3) (from appendix B)into the static equation in matrix form (due to the size of the matrices they needed to be spilt onto multiple lines) gives

$$\begin{pmatrix} -1 & 0 & 0 & 0 & 0 & 0 & 0 & C & D & E \\ 0 & -1 & 0 & 0 & 0 & 0 & 0 & C & D & E \\ 0 & 0 & -1 & 0 & 0 & 0 & 0 & C & D & E \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 & C & D & E \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1-\alpha_2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & -\alpha_2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\alpha_2 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & w_{ed} & N*H & 0 & 0 & 0 & 0 & 0 & 1-\zeta_2*H*J & \nu N*H*J \\ 0 & 0 & N*I & 0 & 0 & R_E & 0 & 0 & -\zeta_2*I*J & \nu N*I*J \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -(1-\nu) \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & R_F & -R_E & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & R_F & 0 & -R_S & 0 & 0 & 0 \end{pmatrix}$$

$$
\begin{pmatrix}
F & 0 & 0 & 0 & 0 & 0 \\
F & 0 & 0 & 0 & 0 & 0 \\
F & 0 & 0 & 0 & 0 & 0 \\
F & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & -(1-\alpha_1-\alpha_2) & -\alpha_1 & 0 & 0 \\
0 & 0 & -(1-\alpha_1-\alpha_2) & 1-\alpha_1 & 0 & 0 \\
0 & 0 & \alpha_1+\alpha_2 & -\alpha_1 & 0 & 0 \\
\kappa+(1-\kappa)G & 0 & 0 & 0 & 0 & -\kappa+\kappa G \\
(\kappa-1)+(1-\kappa)G & 0 & 0 & 0 & 0 & (1-\kappa)+\kappa G \\
0 & 0 & 0 & 0 & -\zeta_1 * H * j & 0 \\
0 & 0 & 0 & 0 & (1-\zeta_1) * I * J & 0 \\
B(1-\kappa) & x & 0 & 0 & 0 & B\kappa \\
0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & k_f & k_e & k_s \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0
\end{pmatrix}
\begin{pmatrix}
\hat{w}_{f,t} \\
\hat{w}_{ed,t} \\
\hat{w}_{es,t} \\
\hat{w}_{s,t} \\
\hat{R}_{F,t} \\
\hat{R}_{E,t} \\
\hat{R}_{S,t} \\
\hat{N}_{F,t} \\
\hat{N}_{Ed,t} \\
\hat{N}_{Es,t} \\
\hat{N}_{S,t} \\
\hat{x}_t \\
\hat{s}_t \\
\hat{k}_{f,t} \\
\hat{k}_{e,t} \\
\hat{k}_{s,t}
\end{pmatrix}
$$

$$=$$

$$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 1-G & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1-G & 0 & 0 & 1 \\ (1-\zeta_1-\zeta_2)H*J & \nu N*H & 0 & 0 & 0 & 0 & P_E*H & -P_S*H \\ (1-\zeta_1-\zeta_2)I*J & \nu N*I & 0 & 0 & 0 & 0 & P_E*I & -P_S*I \\ 0 & 0 & 0 & 0 & B & 0 & 0 & 0 \\ 0 & \nu & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & k & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} \hat{ig}_t \\ \hat{h}_t \\ \hat{k}_t \\ \hat{A}_{F,t} \\ \hat{A}_{S,t} \\ \hat{c}_t \\ \hat{P}_{E,t} \\ \hat{P}_{S,t} \end{pmatrix}$$

To solve for the state equation, (B.19), (B.18), (B.8), (B.12), (B.22), (B.1), (B.15) and (B.16) are stacked into the following matrix formations

$$\begin{pmatrix} \gamma_z ig & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \gamma_z h & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \gamma_z k & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & \frac{\beta}{\gamma_z}(P-\nu N) & 0 & 0 & 0 & 0 & -\frac{\beta}{\gamma_z}P_E & 0 \\ \beta(-\zeta_1-\zeta_2)J*L & \frac{\beta}{\gamma_z}\nu N*L & 0 & 0 & 0 & 0 & \beta P_E*L & \beta(P_S-P_S*L) \end{pmatrix}$$

$$\begin{pmatrix} \hat{ig}_{t+1} \\ \hat{h}_{t+1} \\ \hat{k}_{t+1} \\ \hat{A}_{F,t+1} \\ \hat{A}_{S,t+1} \\ E_t\hat{c}_{t+1} \\ E_t\hat{P}_{E,t+1} \\ E_t\hat{P}_{S,t+1} \end{pmatrix}$$

$$+$$

$$\begin{pmatrix} -ig+(1-\zeta_1-\zeta_2)M & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -(1-\zeta_1-\zeta_2)M & -h & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -(1-\delta)k & -A & 0 & c & 0 & 0 \\ 0 & 0 & 0 & -\rho_f & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -\rho_s & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & \nu N & 0 & 0 & 0 & 0 & P_E & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\gamma_z P_S \end{pmatrix}$$

$$\begin{pmatrix} \hat{ig}_t \\ \hat{h}_t \\ \hat{k}_t \\ \hat{A}_{F,t} \\ \hat{A}_{S,t} \\ \hat{c}_t \\ \hat{P}_{E,t} \\ \hat{P}_{S,t} \end{pmatrix}$$

$$=$$

$$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{R_F}{1+R_F-\delta} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{\beta}{\gamma_z}(P-N) & 0 & 0 & 0 & 0 & 0 & 0 & \frac{\beta}{\gamma_z}(P-\nu N) \\ 0 & 0 & \beta N * L & 0 & 0 & 0 & 0 & 0 & -\beta\zeta_2 J * L & \beta\nu N * L \end{pmatrix}$$

$$
\begin{matrix}
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & -\beta\zeta_1 J * L & 0
\end{matrix}
\begin{pmatrix}
E_t\hat{w}_{f,t+1} \\
E_t\hat{w}_{ed,t+1} \\
E_t\hat{w}_{es,t+1} \\
E_t\hat{w}_{s,t+1} \\
E_t\hat{R}_{F,t+1} \\
E_t\hat{R}_{E,t+1} \\
E_t\hat{R}_{S,t+1} \\
E_t\hat{N}_{F,t+1} \\
E_t\hat{N}_{Ed,t+1} \\
E_t\hat{N}_{Es,t+1} \\
E_t\hat{N}_{S,t+1} \\
E_t\hat{x}_{t+1} \\
E_t\hat{s}_{t+1} \\
E_t\hat{k}_{f,t+1} \\
E_t\hat{k}_{e,t+1} \\
E_t\hat{k}_{s,t+1}
\end{pmatrix}
$$

$+$

$$
\begin{pmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\zeta_2 M & 0 & 0 & x & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \zeta_2 M & 0 & 0 & 0 & -s \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & \alpha_2 A & 0 & 0 & 0 & 0 & (1-\alpha_1-\alpha_2)A \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & T & 0 & 0 & 0 & 0 & 0 & 0 & \nu N & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{pmatrix}
$$

$$
\begin{pmatrix}
0 & -\zeta_1 M & 0 \\
0 & \zeta_1 M & 0 \\
\alpha_1 A & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & 0
\end{pmatrix}
\begin{pmatrix}
\hat{w}_{f,t} \\
\hat{w}_{ed,t} \\
\hat{w}_{es,t} \\
\hat{w}_{s,t} \\
\hat{R}_{F,t} \\
\hat{R}_{E,t} \\
\hat{R}_{S,t} \\
\hat{N}_{F,t} \\
\hat{N}_{Ed,t} \\
\hat{N}_{Es,t} \\
\hat{N}_{S,t} \\
\hat{x}_t \\
\hat{s}_t \\
\hat{k}_{f,t} \\
\hat{k}_{e,t} \\
\hat{k}_{s,t}
\end{pmatrix}
$$

$$
+
\begin{pmatrix}
0 & 0 \\
0 & 0 \\
0 & 0 \\
1 & 0 \\
0 & 1 \\
0 & 0 \\
0 & 0 \\
0 & 0
\end{pmatrix}
\begin{pmatrix}
\epsilon_{f,t+1} \\
\epsilon_{s,t+1}
\end{pmatrix}
$$

where

$$
\begin{aligned}
A &= A_F(k_f^{\alpha_1} N_F^{\alpha_2} s^{1-\alpha_1-\alpha_2}) \\
B &= e^{-A_S k_s^{\kappa} N_S^{1-\kappa}} \mu A_S k_s^{\kappa} N_S^{1-\kappa} \\
C &= \frac{\eta N_F}{(1-(N_F+N_{Ed}+N_{Es}+N_S))} \\
D &= \frac{\eta N_{Ed}}{(1-(N_F+N_{Ed}+N_{Es}+N_S))} \\
E &= \frac{\eta N_{Es}}{(1-(N_F+N_{Ed}+N_{Es}+N_S))} \\
F &= \frac{\eta N_S}{(1-(N_F+N_{Ed}+N_{Es}+N_S))} \\
G &= A_S(k_s^{\kappa} N_S^{1-\kappa}) \\
H &= \zeta_2 A_E(k_e^{\zeta_1} N_{Ed}^{\zeta_2-1} i g^{1-\zeta_1-\zeta_2}) \\
I &= \zeta_1 A_E(k_e^{\zeta_1-1} N_{Ed}^{\zeta_2} i g^{1-\zeta_1-\zeta_2}) \\
J &= \left[P_E - \frac{w_{es}}{(1-\nu)A_H h^{\nu} N_{Es}^{-\nu}} - P_S\right] \\
L &= (1-\zeta_1-\zeta_2)A_E(k_e^{\zeta_1} N_{Ed}^{\zeta_2} i g^{-\zeta_1-\zeta_2}) \\
M &= A_E(k_e^{\zeta_1} N_{Ed}^{\zeta_2} i g^{1-\zeta_1-\zeta_2}) \\
N &= \frac{w_{es}}{(1-\nu)A_H(h^{\nu} N_{Es}^{-\nu})} \\
P &= \frac{\nu w_{es} N_{Es}}{(1-\nu)h}
\end{aligned}
$$

D Appendix D: Data Sources

All data, with the exception of the data constructed for the measure of capital, are quarterly figures, from 1985(1) to 2010(1). All data was sourced from the Australian Bureau of Statistics (ABS) excluding the real rate of return, which was provided by the Reserve Bank of Australia (RBA).

Output (Y) - Gross domestic product, seasonally adjusted (ABS Cat. No. 5206.0, Table 1).

Consumption (C) - All sectors, final consumption expenditure, seasonally adjusted (ABS Cat. No. 5206.0, Table 2).

Investment (I) - All sectors, gross fixed capital formation + changes in inventories + exports of goods and services - imports of goods and services, seasonally adjusted (ABS, Cat. No. 5206.0 Table 2).

Capital (K) - All industries, net capital stock - current prices (ABS Cat. No. 5204.0, Table 63) + private non - farm inventory levels (ABS Cat. No. 5206.0 Table 30).

Non-Mining Labour (N_F) - Employed; total; Australia; total - Employed; total; Australia; mining (ABS Cat. No. 6291.0.55.003, Table 5).

Extraction Labour (N_E) - Employed; total; Australia; mining (ABS Cat. No. 6291.0.55.003, Table 5) - Employed; total; Australia; Exploration and other support services (ABS data sourced from Datastream 100900559)

Search Labour (N_S) - Employed; total; Australia; Exploration and other support services(ABS data sourced from Datastream 100900559)

Real Hourly Compensation (W) - Constructed from average weekly earnings, Australia (ABS Cat. No. 6302.0, Table 2), average actual hours worked; Total(Actual hours worked);Total(Status in Employment);Persons (ABS Cat. No. 6291.0.55.003, Table 13)

and Consumer Price Index (CPI)- Index numbers; all groups; Australia (ABS Cat. No. 6401.0, Table 7).

Real Rate of Return (R) - Cash rate and inflation rate (rba.gov.au) .

Average hours worked - Average weekly actual hours worked; Total (Actual hours worked); Total (Status in Employment); Persons (ABS Cat. No. 6291.0.55.003, Table 13).

Gross domestic product, current prices, percentage change, seasonally adjusted (ABS, Cat. No. 5206.0 Table 2).