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Applications of Dynamic Programming to Agriculture, Forestry and Fisheries : Review and Prognosis

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Farmers are faced with many decision problems in crop and livestock production which are multistage and stochastic. There have been many applications of dynamic programming (DP) to such decision problems, many primarily for illustrative purposes. It is argued that the advent of farmer access to computers will lead to on-farm use of DP. Applications of DP to forestry, fisheries and agricultural policy are also reviewed. The scope and limitations of DP are discussed, and the close relationship between DP and control theory is examined.

1 Introduction

Dynamic programming (DP) is a technique ideally suited for use in finding the optimal sequencing of injection of inputs and harvesting of outputs in many types of agricultural production. It has ready application to cropping decisions involving irrigation and the use of fertilizers and pesticides. It can be used in livestock decision making for determining optimal feeding and replacement strategies. To date DP has been little used for such decision making in an operational way on the farm, but various recent developments indicate that the decision-making environment on farms in the future may be more amenable to the use of DP than in the past. The purpose of this article is to comment on these developments, to explain the method of DP, and to examine the scope of DP for aiding in the management of agricultural and other growing resources, with reference to some of the many research applications.

2 Recent Developments Favouring DP Applications

Many of the concepts of modern decision theory now firmly established in the literature of agricultural decision making (see, *e.g.*, Halter and Dean 1971; Agrawal and Heady 1972; Anderson *et al.* 1977) are easily incorporated in the DP approach. Such concepts include decision trees with stochastic events, revision of parameter estimates of probability distributions based on operating experience, and the more general idea of maximising utility instead of profit over time. An understanding of these concepts at the farm level is becoming more prevalent as more farmers and farm managers are exposed to them through extension, management literature and formal education in agricultural colleges.

The use of such decision aids, including DP, increases the demand for the acquisition and handling of data on the environment in which the farm operates, technical relationships and prices. As computing power in the form of either periodic centralised processing, on-line processing or on-farm microprocessing becomes more readily available to farmers, the scope for the use of decision aids for optimal control of farm production increases. The potential of on-farm computers has been discussed recently by various authors (*e.g.*, Blackie and Dent 1979; Nix 1979; Nuthall 1979; Sargent 1980). However, a conclusion from their views must be that whilst the potential exists, it exists in the long run rather than the immediate future, and coexists

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with the potential for many failures through applications lacking farm relevance or based on inadequate data.

DP problems are particularly suited to solution by computer. Whilst the DP algorithm typically requires a vast number of calculations, the computer programme is usually concise because of the repetitive nature of the algorithm. For example, in the typical DP problem, the same optimization procedure is followed across all states for all stages of the system. However, the fact that a complex farm DP problem necessitates the writing of a computer programme has probably been one of the stumbling-blocks to the on-farm use of DP. Until recently there were no readily available general-purpose DP computer programmes as there are for linear programming.[1] Even so, the number of students enrolled in management courses who also learn a computer programming language such as FORTRAN continues to expand, and the widespread use of simple languages such as BASIC makes computer programming easier in any case. However, there now exist several computer packages for solving a wide range of DP problems, including stochastic and infinite-stage problems. One package is known as DYNACODE (Hastings 1975) and has already been applied in agriculture to the marketing and grazing of beef cattle (Clark and Kumar 1978). Hazen and Morin (1980) describe other packages.

Another development which increases the potential of DP applications to farm management is the interest in planning coupled with control, or the continual monitoring of planned versus actual results (see, *e.g.*, Kennedy 1973 and forthcoming; Barnard and Nix 1979; Blackie and Dent 1979, Ch. 4). If explicit recognition is to be given to the planning environment being uncertain then stochastic or adaptive DP formulations of the farm problem are relevant. However, even if the DP formulation is deterministic, information from the DP solutions can be used in a planning and control framework. Embedded in the solution to a particular problem with a particular initial state and time horizon are the solutions to the same type of problem but with other initial states and time horizons. If unexpected factors lead to things not going according to plan, the new optimal plan is available as part of the original solution. Indeed, for those problems satisfying the conditions for the application of the certainty equivalence proposition of Theil (1958), the implementation each period of the optimal first-period decision rule for the deterministic model with stochastic variables set at their mean values is an optimal strategy under uncertainty. For first-period certainty equivalence to apply, the objective function must be quadratic, the state transformation function linear, and random disturbances additive (see Rausser and Hochman 1979, Ch. 4).

3 The Method Of DP

Bellman is credited with the formal conceptualizing of what he termed 'dynamic programming'. His many papers and books (*e.g.*, Bellman 1957 and 1961; Bellman and Dreyfus 1962; Bellman and Kalaba 1965; Bellman *et al.* 1970) have done much to point out the very great practical scope of the technique and to place the DP approach on a rigorous mathematical basis. Initial DP applications were to inventory control in the 1950s. Since then DP has been an established part of the tool kit of operations research (OR). Early examples of DP applications to agriculture are storage policies for U.S. grains (Gustafson 1958) and replacement policies for egg-laying flocks (White 1959). Most OR books devote a chapter or two to DP. Examples of specialist texts with an economics or management orientation are Hadley (1964), Nemhauser (1966), Jacobs (1967), Kaufmann and Cruon (1967), Beckmann (1968), White (1969), Gluss (1972), Hastings (1973), Norman (1975), Bertsekas (1976), Dore (1977), Dreyfus and Law (1977), Hastings and Mello (1978) and White (1978). The interested reader requiring a full account of the technique is referred to these texts. An introduction to

[1] Note, however, footnote 4.

DP with agricultural examples is given by Throsby (1964*a*), Agrawal and Heady (1972) and Hanf and Schiefer (forthcoming). See also Burt and Allison (1963) and Burt (1965) for other early considerations of DP applications in farm management. An insight into the basic ideas of DP is given below.

Nemhauser (1966) and Jacobs (1967) have pointed out that the term 'dynamic programming' is not the best name for describing the concept and relevance of DP. Although DP is usually applied to a problem of sequential decision making through time, it can be applied to static allocation problems. Instances of applications to allocation problems in agriculture are land allocated to pasture improvement (Throsby 1964*b*) and investment in methods of flood control (Thampapillai 1980). As well, DP is not a programming algorithm for solving a specific type of problem, but rather an approach to solving a multistage decision problem by converting it to a problem requiring the solution of sequential single-period problems. A more revealing term for the technique is recursive optimisation (Nemhauser 1966) or the theory of multistage decision processes (Jacobs 1967).[2]

3.1 Bellman's Principal of Optimality

The fundamental concept which is the basis of DP formulations is the Principle of Optimality formulated by Bellman (1957, p.83): 'An optimal policy has the property that, whatever the initial state and optimal first decision may be, the remaining decisions constitute an optimal policy with regard to the state resulting from the first decision'.

At first sight the principle appears to be a recursive truism, and yet it is only a step away from being a guide as to how to solve efficiently multistage decision problems. A simple illustration may help. Suppose we can describe our present state by *A*, and have to reach one of three destinations, *H*, *I* or *J* at least cost. Decisions must be made, one after the other, at stages 1, 2 and 3. Each decision consists of selecting one of three alternative states to go to for the following stage, the decision incurring a cost dependent on the two states connected. The number of feasible decision paths amongst which to choose the optimal path is 3^3 or 27. A schematic representation of this decision problem is shown in Figure 1. Suppose the least-cost path over all stages turns out to be *ABFJ*. Bellman's Principle of Optimality states that if *ABFJ* is the optimal path with respect to *A*, then so also must *BFJ* be an optimal path with respect to *B*, and so also *FJ* with respect to *F*.

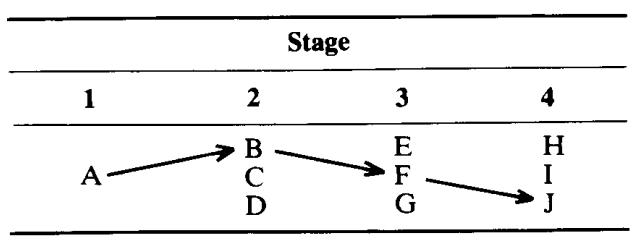

Figure 1: A three-stage problem

By reversing the order of analysis of the optimal path, the original three-stage problem can be converted to a sequence of three single-stage problems. Consider the states at the penultimate stage represented by the points *E*, *F* and *G*. For each point, the least-cost decision out of three possible decisions can be made. Moving back a stage towards *A*, the desirability of moves from *B*, *C* and *D* to *E*, *F* or *G* depend on the subsequent costs of moving from *E*, *F* and *G* just noted, and the state transition costs from stage 2 to stage 3.

[2] A commentator on this article has pointed out that neither of these terms is without problems. "Recursive optimization" could be confused with "recursive programming". "The theory of multistage processes" is inappropriate in that not all multistage decision processes are amenable to solution by DP.

Moving back to stage 1 at A, the solution to the problem may now be completed by selecting the best move to B, C or D. Minimum costs subsequent to B, C or D are already known, so to these all that have to be added are the transition costs between stages 1 and 2.

The problem could have been solved by calculating the costs of all 27 possible paths and identifying the least cost path. Using backward induction, however, required consideration of 3 + 9 + 9 = 21 single-stage paths. A comparison between the calculations involved in total enumeration as against those in the DP approach appears more dramatic as the number of stages in the decision sequence is increased. For example, if two more decision stages were added to the above problem, the DP approach would require consideration of only 39 single-stage paths compared with the 243 possible multistage paths. As a rule of thumb, the amount of calculation involved in solving a multistage decision problem increases linearly with number of stages if DP is used, exponentially if total enumeration is used.

Whilst the DP approach generally saves a substantial amount of computation compared with total enumeration, there is nevertheless inevitably much additional information generated which may be considered redundant to the original problem. This is because as we proceed with the process of backward induction, an optimal path is discovered for each possible state of the system but only up to that stage. We have no way of knowing which of these paths will be incorporated in the optimal paths for the complete decision process until we have moved all the way back to the initial stage. A DP strategy keeps all possible options open until a decision must be made. However, the additional information generated may not be redundant, if we are interested in an analysis of the sensitivity of the objective function to alternative initial states of the system, or use DP in a control framework as mentioned earlier.

3.2 The Recursion Equations

The DP approach of backward induction applied to an N-stage decision process can be formally specified by a recursion equation. Because backward induction is normally used, it is usual to have stage subscripts denote the number of stages remaining in the process. To be applicable, the decision process must have the following three properties. First, at any stage n, a decision d_n is to be made, which together with the current state of the system, x_n, determines x_{n-1}, the state of the system at the next stage. In other words, there is a transformation function, $x_{n-1}=t_{n-1}(x_n,d_n)$. Secondly, the decision d_n and state x_n also determine the stage return r_n. A typical objective of the N-stage process is to maximise $\sum_{n=1}^{n} r_n(x_n,d_n)$ with respect to d_n, $n=1,\ldots,N$. Thirdly, the objective function must be separable, and is usually additive. An example of a multiplicative function is given by Hall and Butcher (1968), which enables overall crop yield to be zero if crop production fails at any stage. Another agricultural example of a non-additive function is in a multistage stochastic utility analysis by Hardaker (1979).

An important point is that the process must be fully described at any stage n by x_n. That is, x_{n-1} and r_n must only depend on d_n, x_n and any exogenous variables. Only to this extent must the behaviour of the system be dependent on the history of the system prior to stage n. In other words, the process must possess the Markov property. In principle, any process may be made to satisfy the Markov property by suitable definition of state variables. Often the DP practitioner has to settle for formalising a process which has the Markov property to a reasonable degree of approximation. Otherwise the number of possible states the system may assume may be too large for practical solution. Bellman terms this problem "the curse of dimensionality" and is referred to again in section 5.1.

Basic to the recursion equation is the optimal return function, $f_n(x_n)$, defined as the optimal return from following the optimal sequence of decisions from the current stage n to the final stage 1. The recursion equation is:

$$f_n(x_n) = \max_{d_n} \left(r_n(x_n,d_n) + f_{n-1}(x_{n-1})\right) \quad n = 1, \ldots, N \tag{1}$$

subject to $x_{n-1} = t_{n-1}(x_n,d_n)$ and known $f_0(x_0)$.

If the DP problem is to be solved numerically, a discrete range of values must be chosen for x_n and d_n. Working through equation (1) recursively from $n = 1$ to $n = N$ gives an optimal return value f_n and an optimal decision d_n^* for every state of every stage. This information is required for determining the optimal path $d_n^*,\ldots,d_1^*$ given any initial state of the system, x_N.

For the backward induction process, knowledge of the terminal return function, $f_0(x_0)$, is essential to start the recursive process. Equation (1) may be solved for $n=1$, giving $f_1(x_1)$, which in turn gives $f_2(x_2)$, and so on $f_N(x_N)$. For some deterministic problems an alternative procedure is forward induction. It is useful if the time horizon for the problem is unknown but is dependent on the state of the system, for example replacement of a capital item. It may be necessary for the solution of the multistage control of flow processes which involve feedback or feedforward flows between stages. The recursion equation (1) still applies with a change in the interpretation of stage subscripts. Subscripts denote stage number in normal temporal order. The initial return function is $f_0(x_0)$. Instead of determining at each stage n the decision d_n, the optimal state to go to, the d_n refers to the optimal state to have come from. For the forward induction process the inverse transformation function is required, but may not always be readily computed. For most DP applications backward induction is used. A forestry example of the use of forward induction is Risvand (1969).

3.3 Extensions

The basic DP method can be extended to the following more complex multistage decision processes: processes for which the stage return and the transformation depend not only on x_n and d_n, but also the occurrence of the random event k_n with known probability p_k (the stochastic case); stochastic processes for which the probability distribution of k_n is unknown (the adaptive case); and processes consisting of a large number of stages, and for which at some stage the optimal decision $d_n(x_n)$ is a steady-state decision $d(x)$ (the infinite-stage case). These extensions are briefly taken up in turn.

3.3.1 Stochastic DP

Reference has already been made in section 2 to how the certainty equivalence theorem may be exploited for solving a certain class of stochastic problem. The method relies on a deterministic DP formulation. The more general type of stochastic DP problem is formulated with the value of the objective function an expected value, and a recursion equation

$$f_n(x_n) = \max_{d_n} \left(\sum_{k=1}^{m} p_k(r_n(x_n,d_n,k) + f_{n-1}(t(x_n,d_n,k))) \right) \quad n=1, \ldots, N \tag{2}$$

subject to $\sum_{k=1}^{m} p_k = 1$,

showing the return and transformations to depend on the random event k, which could also be shown with a stage subscript. In the stochastic case, an optimal path for the multistage process cannot be specified because it depends on the k which eventuate at each stage. Only optimal one-stage decision rules dependent on x_n and k can be given.

3.3.2 Adaptive DP

The probability distribution of k may be unknown, but if the class of the distribution is known and is defined by say one sufficient statistic θ (as in the case of the Poisson distribution) then the adaptive case may be formulated by introducing θ as a state variable. In the adaptive case θ is not known with certainty, but is thought to be one of a discrete range of possibilities. At each subsequent stage the estimate of θ is revised to take account of the most recently revealed values of the stochastic element. The recursion equation for the adaptive case is equation (2) modified to show the optimal return function as $f_n(x_n, \theta_n)$ and the probability of the k-th random event as $p(\theta_n)_k$. For all stages up to the last stage $(n = 1)$ the optimal single period stage for each state must be determined for all possible values of θ, the range of θ reflecting all possible combinations through time of values of the random element. The revision of the estimate of at each stage may be based on statistical procedures, such as Bayes' principle or exponential weighting. θ may be an m-element vector if m sufficient statistics describe the probability distribution of k. In practice DP formulations for $m > 2$ are rarely contemplated because the resulting increase in dimensionality places solution beyond present computing bounds.

3.3.3 Steady-state DP

For a steady-state DP problem the stage return and transformation functions must be stationary or independent of time. For example, in agricultural applications, constant real prices and costs, and technology, must be assumed. For sufficiently large n, $f_n(x_n)$ is identical to $f_{n-1}(x_{n-1})$, and the relevant recursion equation is equation (1) without stage subscripts. In this case $f(x)$ appears on both sides of the recursion equation, but generally it is difficult or impossible to solve for $f(x)$ analytically. Burt (1964*b*) has suggested an analytical method of obtaining approximate solutions for $f(x)$ which is discussed further in section 5.2. In the remainder of this subsection we consider methods of successive approximations used for obtaining numerical solutions.

The formulation of the infinite-stage problem is dependent on whether, as $n \rightarrow \infty$, $f_n(x)$ tends towards a finite value or towards infinity. The limit will be a finite magnitude for most cases in which stage returns are discounted. In this case the problem can be formulated as one of maximizing $f_n(x)$, $n \rightarrow \infty$. There are two basic methods of successive approximation. One is approximation in return or function space. Equation (1) is solved iteratively, using trial values of $f_{n-1}(x_{n-1})$ on the right hand side. The procedure is initiated by guessing $f_0(x_0)$. After any iteration, $f_n(x_n)$ becomes $f_{n-1}(x_{n-1})$ for the following iteration. The procedure continues until values of $f_n(x_n)$-$f_{n-1}(x_{n-1})$ are sufficiently small. The alternative method is approximation in policy space. Given any policy $d(x)$ it is possible to find $f(x)$ by solving a set of simultaneous equations. Values for $f(x)$ may be substituted for values of $f_{n-1}(x_{n-1})$ in equation (1) at the n-th iteration, and a revised optimal policy, $d_n(x)$ determined. The policy is used to find the revised return function, and the process is repeated. The process is initiated using a guess at the optimal policy, $d_0(x)$. The optimal policy has been found once there is no change in policy from one iteration to the next. The method of approximation in policy space is often preferred because the optimal solution is usually found after a few iterations, and the optimal return function is found exactly compared with only to some degree of approximation when approximation in function space is employed. The main drawback of the method compared with approximation in return space appears for problems with a large number of discrete states. The number of simultaneous equations to be solved in such problems is likewise large, and may be computationally prohibitive.

Burt and Allison (1963) and Burt (1966) favour the combined use of both methods: iteration in return space is pursued until optimal policies at each successive iteration do not change and it appears that the optimal steady-state policy has been found; the policy-iteration equation is then used to check if this is actually the case by finding the present values of pursuing the test policy over an infinite number of stages.

The present values are used in another round of the return-iteration method. If this round results in the same recommended policy, the policy is the optimal steady-state policy. If not, further iteration in return space is pursued, and the whole process repeated. The main advantage of this approach over the use of policy iteration alone is for those problems in which over an initial planning horizon the problem is not stationary, so that return iteration is required at least for the initial planning horizon.

Many dynamic agricultural problems require discounting, so that $f(x)$ is finite for the infinite-stage problem. However, in some cases, discounting can be ignored. For example, if the problem were one of determining the optimal daily feed rations for laying hens over a year, and the biological transformations and prices could be assumed constant from day to day, then the problem might be more satisfactorily viewed as an infinite-stage problem without discounting. For such cases $f(x)$ cannot be defined. Instead the problem is formulated in terms of maximizing a linear function $v(x) + ng$, where g is the average gain per stage once an optimal repetitive decision rule has been reached. Any system with a large number of stages, n , and finite stage possibilities for x, must have a repetitive optimal policy. The function $v(x)$ represents the returns from starting at state x before reaching the optimal cycle. Recursive equations can be formulated which involve just $v(x)$, g and $r(x,d)$, and these can be solved by various methods of successive approximation. Stochastic infinite — stage problems can be solved by this method with various extensions (see in particular Howard 1960; and extensions by Hastings 1973; White 1978).

4 Applications

A categorized list of many of the DP applications to agriculture, forestry and fisheries may be found in the Appendix, Table 1. Farm management applications are listed by enterprise type, and inventory applications by product stored. Studies are listed under the following headings in chronological order. Numbers of studies reported are shown in brackets below.

A)	*Biological systems*	
(i)	beef and/or sheep grazing	(5)
(ii)	beef feedlot or feedlot/grazing	(5)
(iii)	dairy	(6)
(iv)	broilers	(2)
(v)	laying hens	(3)
(vi)	cropping	(9)
(vii)	water management	(16)
(viii)	pesticide management	(2)
(ix)	forestry	(11)
(x)	fisheries	(3)
B)	*Farm machinery*	(8)
C)	*Financial*	(4)
D)	*Storage — regional, national or world levels*	
(i)	grains	(8)
(ii)	wool	(1)
(iii)	butter	(1)
(iv)	fodder	(1)
(v)	apples	(1)

Before referring further to Table 1 it is useful to delineate two types of application of DP to agriculture, the distinction being the nature of the transformation function, $t(x,d)$. Often the transformation function, in deterministic cases, can be specified from first principles or is axiomatic. That is, the state of the system at the next stage follows as a matter of logic from the current state and decision. For example, in inventory problems, the stock at the next stage equals stock at this stage plus any incomings, less any outgoings between this stage and the next. In replacement problems, a current

decision to keep or replace an asset automatically specifies the age of the asset at the next stage. Problems with axiomatic transformations are the traditional DP problems dealt with in OR texts. There are many examples in the literature of DP applications in agriculture to problems with axiomatic transformations, ranging from farm to sector level. Some of these are reviewed below. The early agricultural DP applications referred to above (Gustafson 1958; White 1959) were inventory and replacement problems respectively. However, there are many other agricultural applications in which the transformation is less transparent and depends on a biological process. For example, empirical information is required to predict the biomass of a crop at the next stage given current biomass and a decision to irrigate.

DP applications involving biological transformations are generally at the farm level in agriculture, but may be at a more aggregate level as in the case of forest or fishery management. We now turn to DP applications involving biological transformations.

4.1 DP Applications with Biological Transformations

Because agriculture is the management of biological processes for gain to humans, and humans are generally interested in efficient or optimal management, the discipline of agriculture provides fertile ground for the marrying of the studies of biological response and OR techniques. However, the decision-making environment is generally complex. Typically, the agricultural process involves decisions on the timing and composition of one or more input injections and one or more subsequent harvests. The response of yield to input may be non-linear, dependent on the time interval between injection and harvest, and also dependent on time of the year. Yield functions are generally stochastic: there is considerable variation in input-output relationships between plants and animals of the same species; and control is imperfect — livestock exercise choice in their intake, and plant growth depends on many factors which may be uncontrollable such as the incidence of solar energy. Prices of outputs and inputs may not be constant, but may be stochastic and discontinuous functions of time, subject to seasonal and other variations.

Optimal sequencing of inputs for deterministic, continuous biological processes can be determined by various methods. Dillon (1977, Ch. 3) reviews a range of methods, including DP, but concentrates on the calculus approach for exposition. A further approach, which has received relatively little attention in the agricultural economics literature on biological management, is optimal control theory[3] (but see Rausser and Hochman 1979). Control theory approaches have been more widely considered in analysing problems in fisheries (*e.g.*, Clark 1976; Anderson 1977), forestry (*e.g.*, Schreuder 1971) and natural resources (*e.g.*, Smith 1977*a*, *b*). A simple example of a control theory formulation for livestock production is given here. The formulation leads on naturally to the DP approach, a numerical solution method often resorted to for solving control theory problems, particularly problems involving stochastic and discontinuous functions.

Suppose the aim is to feed an animal from time $t = 0$ to $t = T$ so as to maximize the present value of net returns. The animal is to be sold at $t = T$ for a per-unit liveweight return of P_T. The state of the system through time is represented by liveweight x, and the control variable through time by feed level, u. Let $g(x,u,t)$ be the liveweight gain function, c be the per unit cost of feed, and ρ be the continuous rate of discount. The control problem is to maximize with respect to u the function:

$$Z = \int_0^T (-c.u + P_T.g(x,u,t))e^{-\rho t}dt + P_T.x_0.e^{-\rho T} \tag{3}$$

subject to $\dot{x} = g(x,u,t)$

where x_0 is initial liveweight.

[3] The term 'control theory' refers in this article to the use of the maximum principle to solve continuous time problems. It is sometimes, perhaps more correctly, used in a generic sense to include DP and other methods.

Necessary conditions for a local maximum can be obtained using the maximum principle (Intriligator 1971, Ch.14) by formulating the current-value Hamiltonian

$$H = -c.u + P_T.g(x,u,t) + \lambda.g(x,u,t) \tag{4}$$

where λ is referred to as a costate variable or a dynamic Lagrange multiplier.

The necessary conditions for an interior solution are

$$\partial H/\partial u = 0 \tag{5}$$
$$\partial H/\partial x = -\dot{\lambda}. \tag{6}$$

The first two terms of the Hamiltonian in (4) describe the *current* contribution of x and u to net return. The final term is the value of the *future* contribution of x and u to net return, λ indicating by how much the present value of the animal would increase if liveweight increased by an additional unit. Condition (5) stipulates that at any point in time, feeding should continue to the point at which an extra unit of feeding would add less to the net return $(P_T.\partial g/\partial u - c)$ than would be gained from the future management of a heavier animal $(\lambda.\partial g/\partial u)$. Condition (6) stipulates that at any point in time liveweight should be brought to the point beyond which a one-unit increase in liveweight would bring returns exactly offset by the fall in value of the marginal unit of liveweight. Whilst liveweight x is not directly a control variable, it is indirectly in that it is a function of the history of the control variable u.

A useful way of viewing the results from control theory is as a limiting case of the results obtained for discrete-time problems with Lagrange multipliers (see, *e.g.*, Dorfman 1969; Benavie 1970). In turn, DP can be viewed as closely related to the control theory approach. Intriligator (1971, Ch. 14) discusses the relationship. For the discrete DP formulation of the liveweight gain example, we can keep the previous notation with the following changes: r replaces ρ and is the discrete rate of interest for discounting; subscripts denote number of stages to go to the beginning of period T; and $f_n(x_n)$ is the return from following the optimal policy over the next n stages given that current liveweight is x_n. The recursion equation is

$$f_n(x_n) = \max_{u_n} [-c_n.u_n + (1+r)^{-1}.f_{n-1}(x_n+g(x_n,u_n,n))] \quad n = 1, \ldots, N \tag{7}$$

with $f_0(x_0) = p_0x_0$.

Similarities can be seen between (7) and maximizing H in equation (4) with respect to u. In both cases, assuming an interior solution, maximizing entails selecting u to find a balance between marginal current return and the marginal future return of opposite sign. Note that $\partial f_n(x)/\partial x$ is the counterpart of λ in the control theory framework. Burt and Cummings (1977) suggest that these terms be referred to as the marginal present value of current resource stocks (here, current liveweight).

The problem just considered involves the optimal sequencing of inputs to promote growth for harvesting at a predetermined date. Calculus, control theory and DP are all suitable techniques to use for solving such a problem. The comparative strength of DP as a solution technique becomes evident when the more realistic problems encountered in practice are considered. The harvesting date is generally a variable to be determined, depending on input and output prices which may vary discontinuously with time. The harvesting process may be followed by subsequent harvesting from other biological units (as in beef and crop production) in which case the problem can be viewed as an optimal input-sequencing problem superimposed on an optimal replacement problem. Alternatively, the problem may be one of finding optimal input-sequencing within production periods, together with the optimal number of harvests from the same biological unit, before replacement of that unit (as in milk and fruit production). Growth rate is generally a stochastic function of inputs.

Mortality is always a possibility for a living system. Future output and input prices are seldom known with certainty.

Such real-world complications beyond the simple input-sequencing problem can be handled relatively easily in a DP framework compared to calculus or control theory frameworks. We now look at some of the ways DP models have been used for the management of living systems, as well as for replacement and inventory problems.

4.2 The Range of DP Applications

Delineation of suitable stages, states, decisions and return functions are basic to the formulation of any DP model, and for this reason a brief description of these is given in Table 1 for each DP study. Such information together with comments are meant to give some idea of the scope of each study. Remarks here are confined mainly to a description of the range of DP applications reported.

Some DP applications at the farm level determine the optimal sequencing of inputs and harvesting of output only, and do not consider the wider problem of optimal sequencing of inputs and outputs together with optimal replacement of a biological or capital unit. In such applications the planning horizon may be reasonably constrained to the growing season. Examples are the determination of optimal irrigation and pesticide management, and maintenance operations on farm machinery. DP may also be used at the farm level as a network-analysis tool for finding the optimal sequencing of different production and marketing activities (Boyce *et al.* 1971).

Most farm enterprise and farm machinery applications of DP are at least partly replacement problems. Explicit recognition is given to the fact that optimal management of current biological or capital units is dependent on the timing and management of replacement units. For example, the following are interdependent: the thinning schedule for a forest and the rotation length; the maintenance schedule for a combine harvester and how long it is kept; and the feeding of cattle and the timing of their marketing.

Whilst the farmer provides feed in all livestock enterprises, in many DP applications to livestock management it is unnecessary to formulate the problem with feed as an input to be optimally sequenced. This is the case for *ad libitum* feeding applications in general, and grazing applications in particular. Such applications may still have to deal with the determination of best ration mixes or grazing strategies. However, in many intensive livestock production applications, such as in beef feedlotting or broiler production, next-period liveweight gain is a decision variable additional to the decision of whether to keep or replace. The one-period liveweight gain can generally be achieved through a range of possible mixes of feeds, which means there is scope for employing techniques such as linear programming (LP) for determining the best mix of feeds for a particular liveweight gain. Studies combining DP and LP models in this way have been mainly applied to beef production and are listed under A(ii) in Table 1. Meyer and Newett (1970) in an early and perhaps little noticed article state (p.419) "It should be recognised that this solution technique is a sophisticated mathematical tool, and represents a significant and, to date, the most inclusive contribution to feedlot enterprise optimization". The technique would have helped with some problems which have been reported difficult to solve using a calculus approach. For example, Heady *et al.* (1976) explain the problem of determining the optimal sequencing of rations with different possible protein rations in each of three weight gain intervals in pig management. The objective was to maximize profit per day, and the problem was simplified by specifying a required final liveweight. A solution was obtained by total enumeration. The same problem could have been solved by DP, but in this case, because of the small number of stages (3), without any saving in computation. However, it is possible that the authors were constrained in the formulation of their problem given their solution technique. Another example is a similar study by Melton *et al.* (1978) into feedlot rations for beef steers given

alternative objectives. One problem was that gain isoquants were not convex over the whole range of feed-input combinations, and another was how to maximize profit per unit of time given five weight-gain intervals. DP would probably have proved a more satisfactory technique than the approximation technique adopted.

A particular advantage of DP is that problems can be formulated to tackle stochastic processes with relative ease. In the case of livestock management, stochastic DP models have been used for determining: feeding strategies in droughts of unknown length (Toft and O'Hanlon 1979); replacement strategies for dairy herds in which progression to the subsequent lactation is uncertain (Jenkins and Halter 1963; Smith 1973; McArthur 1973; Stewart *et al.* 1977); and replacement timing when output prices are uncertain for beef production (Yager *et al.* 1980); for broiler production (Hochman and Lee 1972); and for egg production (Pouliquen 1970).

The group with the largest number of reported applications is water management. This probably reflects the importance of water as an agricultural input and the fact that water management involves high levels of capital investment for the benefit of many users. DP applications in this area may be treated as inventory, input-sequencing and multi-harvesting types of application. The DP problem may be subsumed within a hierarchy of decision levels. Other decision levels involve the optimal area to crop, the type of crops, and the long-run level of investment in reservoirs. Integrating a DP model into a hierarchy of decision levels poses interesting challenges (Burt 1964*a*; Dudley *et al.* 1971*a;* Dudley and Burt 1973).

Whereas there appears to have been greater emphasis in solving agricultural problems with numerical DP rather than with analytical DP or control theory, the reverse seems to be the case for fisheries. There may be two reasons for this. One may be the lack of quantitative data on populations, reproduction rates, migration rates, life expectancy and interaction between species which may be necessary for a numerical approach. Another reason may be that fishery authorities and other research bodies stimulate more interest in models predicting qualitative equilibria with implications for species survival than do commercial fishing enterprises primarily concerned with solving day-to-day management problems. Be that as it may, three numerical DP applications are referred to in Table 1. Lewis (1977) and Smith and Silvert (1977) describe how numerical DP may be applied to stochastic fishery problems. Sancho and Mitchell (1975) use DP to obtain an analytical solution to the problem of determining fishing effort subject to a fishing quota.

Another fertile ground for DP applications is the determination of storage policies for grains at the national and international levels, given the stochastic production and prices of grains. Applications were few after the pioneering work of Gustafson (1958) until the concern over the instability of grain prices and historically low grain reserves of the 1970s. The recent application of optimizing models seems to have been stimulated by earlier work based on qualitative analysis or simulation. Blandford and Lee (1979) conclude in their article on stabilization policies:

> Empirical modelling undoubtedly will continue to prove a major medium for the analysis of alternative stabilization policies in international commodity markets. In the past, simulation has proved the most popular technique, but optimal control theory seems to possess a number of distinct advantages. Most important is the way in which appropriate decision rules for market intervention are directly derived from underlying policy objectives, and the ability of the method to deal with multiple, and sometimes conflicting, aims.

A comprehensive treatment of DP applied to optimal policy for grain storage at the world and national levels may be found in the book by Gardner (1979). Extensions of the basic model are placed in a practical context. Particular consideration is given to the interaction between public and private stockpiling.

This brief overview of DP applications to the management of biological systems is suggestive of the technique's scope and potential. We conclude by considering some of the limitations of the technique, and likely developments in future applications.

5 Evaluation and Prognosis

Whilst the scope of DP has been examined in the previous section, its restriction to solving particular types of problems should be stressed as well. Though it is possible to solve problems with DP that are solvable with other techniques such as calculus, LP and quadratic programming (QP), the application of DP would often be relatively quite inefficient. DP gains a comparative advantage over other techniques whenever it can be used to solve multistage, stochastic, non-linear problems.[4] To this extent, DP assumes something of the status of "a technique of last resort" which applies to simulation. In practice the question of whether DP should be applied as against some other approach is often answered by considering the loss in the precision of results from using a less expensive, more powerful technique subject to more restrictive structural assumptions.

5.1 Limitations

Barnard and Nix (1979) introduce the topic of DP in their book on farm planning and control with the following: "While fascinating intellectually, dynamic programming has little significance for practical farm planning at the present time and so will be dealt with relatively briefly". If by "farm planning" is meant planning for the whole farm, we would have to agree that DP has little to contribute compared with other programming methods. This may be one reason why, apart from the study by Larson *et al.* (1974), there has been little use of DP for extending the theory of the firm or studying farm-firm growth as earlier envisaged by Johnson (1965), Minden (1968) and Throsby (1968). However, the potential of DP for aiding planning and control in single enterprises requiring intensive management should not be overlooked.

DP applications to feedlot management have already been described. However, a problem noted by several authors (*e.g.*, White 1959; Bonnieux 1969; Kennedy 1973; Smith 1973) is the variation between animals in response of liveweight gain to feeding. A separate feeding and marketing policy applied to each animal in a group is seldom likely to be economic, except perhaps for stud animals. Kennedy (1973) suggested a method of attempting to identify the gain potential of animals over time by comparing periodic liveweight gains with feed inputs, and on this basis assigning animals to one of three groups for separate management. An alternative put forward by Bonnieux (1969) is to specify the state of the livestock system by the number of animals in a range of liveweight categories, and to treat state transitions as a stochastic Markov process.

Usually numerical solution of DP problems involves the selection of a grid of discrete feasible values for the state variables which are in reality continuous variables. The use of a grid of discrete values makes it likely that the recommended DP path will be suboptimal to some extent compared to the true optimal path. This can be overcome at the expense of extra computation by expanding the grid, or by successive redefinition of the grid on the state variables based on what previous DP runs have shown to be the ranges of interest (Nemhauser 1966, Ch. 4).

A further approximation is introduced if the range of decisions from each state do not exactly reach the discrete values of the state variables at the next stage. Kao and Brodie (1979) overcame the resulting rounding error problem in a forestry application of DP by devising a system of "neighbourhood storage locations". The system is only applicable if forward recursion is used. Rather than exactly prespecifying the discrete values of states at each stage, feasible areas are specified around each discrete value or node. The best approach into the area from alternative states in the previous stage are

[4] This may not always be true. For example, Hadley (1964) and Kislev and Amiad (1968) have pointed out that infinite-planning-horizon Markov decision processes may be solved by LP as well as by DP. However the LP matrix would likely be large. One LP activity column is required for each feasible decision coupled with each state.

evaluated. The node is then defined as the state value exactly reached by the best approach.

An important restriction on problems that can be solved with DP is the requirement that the decision process possess the Markov property, referred to earlier. This is not a property which strictly holds for some formulations of DP problems. Consider the example of livestock feeding, in which the optimal sequence of liveweight gains is to be determined. At any stage it may be convenient but not strictly correct to assume that the rate of gain for the next period is dependent only on current liveweight and the feed decision, and that the feeding history of the animal is otherwise irrelevant. The type of prior feeding is likely to affect conversion of feed to gain in ways other than through its impact on current liveweight. The fat/lean-meat composition of any gain is likely to be a function not just of current liveweight but also of previous feeding, so that the return is not just a function of liveweight. Another example is applications in which price is a stochastic state variable, described by econometric equations with explanatory variables having lags of more than one time period. Conceptually such processes can be modelled so as to obey the Markov requirement, but at the expense of specifying additional state variables. This brings us to the much-quoted "curse of dimensionality". It is often pointed out that the capacity of current computers limits the number of state variables which can be handled in DP to a maximum of about three. The limitation on the number of state variables is especially restricting in fishery applications. In the case of long-lived migratory species it would be desirable to specify as state variables population, age, fishing ground and species. With some ingenuity, the curse can be partly exorcised. If the problem involves maximizing, for example, the sum of quadratic stage returns subject to equality constraints, then control theory solutions based on DP may be obtained, as Dalton (1976) demonstrated for a problem with 10 state variables. Details of the approach may be found in Chow (1975). For other problems it may be possible to solve approximately a DP problem with many state variables by using the results of a DP sub-problem with a subset of the state variables in an iterative process (see, *e.g.*, Burt *et al.* 1980, in which an approximate solution is obtained for a DP problem with 15 state variables). Morin and Esogbue (1974) have shown how computation can be reduced in some DP problems by exploiting discontinuities of the optimal return function. Storage of optimal returns and decisions can be reduced if certain dominance conditions hold (see Morin and Marsten 1976). Notwithstanding these possibilities, the statement by Throsby (1974) referring to DP requiring numerical solution that ". . . the technique stands to gain more than most from the arrival of new generations of bigger and faster computers" remains true today. However, not all DP problems need to be solved by a numerical procedure. Some can be solved using DP as an analytical device, in which case problem formulation may not be limited by the number of state variables. The analytical use of DP is further discussed in the next section.

5.2 New Horizons

Candler and Musgrave (1960) stated: "It is anticipated that just over the horizon there are a host of new applications of dynamic programming". There undoubtedly are still many now. We end with some thoughts on likely future developments.

At the farm level, three stimuli can be seen for farm-management applications. One is improved information on technical response functions in agriculture. For example, a limiting factor in applying DP to feedlot management has been the imprecise knowledge of the determinants of the appetite constraint of cattle. Another stimulus is the diffusion of knowledge of DP through courses on OR techniques in agriculture. Thirdly, there is the growing computing capacity on the farm.

Areas of DP application in farm management which appear to have received little attention to date are the feeding and marketing of pigs[5] and the storage of perishable

[5] However, Professor K. Riebe of the University of Kiel, West Germany, has achieved successful results in applying DP to hog fattening in Schleswig-Holstein (Personal communication, December 1979.)

commodities. Perhaps there could be more investigation of DP formulations of the complex of decisions involved in integrated pasture and livestock management, as identified, for example, by Dillon and Burley (1961). Another area which remains to be exploited at the farm level is adaptive DP. For example, adaptive DP could be applied to milk production. To date, the decision to be made in dairy applications has been limited to "keep or replace" (see Table 1, section A(iii)). Adaptive DP models could be formulated which included supplementary feeding decisions for individual cows, dependent on periodically-revised estimates of the milk-yield performance of each milking cow.

As Table 1 indicates, there have been numerous applications of DP to thinning and rotation decisions in forestry. Brodie and Kao (1979) enthusiastically endorse the potential of DP:

> The interaction of silviculture techniques — precommercial thinning, fertilization, commercial thinning and rotation — and size-dependent costs and revenues can be analyzed simultaneously, not just partially or singly for particular situations. Numerical analysis with dynamic programming is one of the simplest optimization techniques for nonlinear problems with complex cost and revenue functions. It will likely become the dominant tool for economic optimization of even-aged stands.

However, Kao and Brodie (1980) use another nonlinear programming technique, the modified flexible polyhedron method, for solving the same type of problem. They stress that the method has the advantage over numerical DP models of treating time and stocking variables as continuous. However, DP does not appear to have been much used for solving stochastic forestry problems. This is surprising because there are many uncertainties in forestry, such as future prices and costs over long planning horizons, weather and climate effects on growth, and the risks of attack by disease and fire.

At the agricultural sector level, besides scope for investigating stock-holding policies, there is much potential for applying control theory and adaptive control theory models to problems of agricultural policy. Possibilities in this area have been discussed by Burt (1969), Tintner (1969), Freebairn and Rausser (1974), Rausser (1978) and Rausser and Hochman (1979).

On the research front, continued developments can be seen in three areas. One is the interest in further reducing the dimensionality problem already referred to. One possibility which remains to be further exploited is the use of DP as an analytical device. For some multistage problems it is possible to solve the final stage analytically, and to use the final-stage solution to solve the penultimate stage analytically, and so on. It may further be possible to observe a pattern in the sequence of optimal decision rules for successive stages, and thus to determine the decision rule for any stage by induction. This approach was followed by Kennedy *et al.* (1973) and Kennedy (1981) in the case of the fertilizer decision for which fertilizer carryover is a relevant consideration. Any number of state variables, in this case previous periods of fertilizer application, could be incorporated. Another approach to the analytical use of DP has been suggested and tested by Burt (1964*b* and 1981) and Burt and Cummings (1977). It results in an approximately optimal decision rule for the problem described by the steady-state version of equation (1). The method involves approximating $f(t(x,d))$ on the right-hand side of equation (1) by the first few terms of the Taylor's series evaluated at x, and finding the conditions for optimality for the revised problem analytically. The optimality conditions for the revised problem have been found to give a decision rule which is a good approximation to the optimal decision rule for the original problem derived numerically, provided the state variables do not approach zero (Burt 1964*b* and 1981). The method has various advantages: analytical solutions can be derived for any number of state variables; and unlike the continuous control theory approach based on the maximum principle, the optimal decision path is defined

for discrete stages and is therefore operational, and is readily extended to stochastic problems.

A second area is the specification of objective functions in terms other than the present value of expected net revenue. Already there have been applications in which variance of net returns as well as their expected value have been treated as an additional indicator of performance (*e.g.*, Burt and Johnson 1967) or included in a utility function to be maximized (*e.g.*, Hudson 1976; Hardaker 1979). Objectives in terms of net social welfare and price variability have also been specified for inventory applications. However, we are likely to see in future the traditional goal in agricultural production of efficiency tempered by environmental and ecological considerations. For example, optimal forest rotations may be assessed by criteria other than maximization of net return, such as aesthetic or wildlife considerations. DP is an ideal technique for solving problems in which multiattribute utility functions, additive or multiplicative such as those suggested by Keeney and Raiffa (1976), have to be maximized. It is also suitable for determining efficient frontiers of tradeoff between objectives, an application termed multiobjective DP by Tauxe *et al.* (1979*a*,*b*).

A third area of likely further development is the integration of DP models with other models such as simulation, LP and QP. As Table 1 shows, there have been applications of stochastic DP to grazing, milk production, water management, fisheries and farm machinery in which transition probabilities have been estimated from the results of a simulation model. Kingma (1974) has incorporated a DP model for determining policy for harvesting a forest within a timber-livestock simulation model. Some applications employ linear or nonlinear programming for stage optimization within the DP framework (*e.g.*, Nelson 1969; Burt and Cummings 1970; Thampapillai 1980). Finally, the method suggested and employed by Sondakh and Hardaker (1981) of using parametric LP recursively to optimize at each stage across all states is likely to be further exploited. In the notation of equation (1), at each stage n a linear segmented approximation of $f_{n-1}(x_{n-1})$ is incorporated in a LP matrix, and $f_n(x_n)$ is obtained from the optimal values of the LP objective function for all values of x_n after parametizing x_n.

In summary, the decision environment at the farm and policy levels appears favourable for greater real-world use of DP in areas which research has shown to be amenable to the DP approach. As for further research, progress to date suggests that new methods of DP will continue to be applied to the management of agriculture and natural resources.

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Appendix

Table 1: Summary of DP Applications
(see key on p.173 for explanation)

Study Application/Author(s)	Interval between stages	No. of stages	State(s)	Decision(s)	Objective function maximized	Comments
A) BIOLOGICAL SYSTEMS						
A(i) *Beef and/or sheep grazing*						
Throsby (1964*a*)	1 year	∞	Average age of a group of animals	Keep or replace	NR	A simple numerical example in an expository article
Scobie (1967)	1 year	∞	Age of sheep	Keep or replace with choice of age	PV of NR	Applied to wether flocks
Fisher (1974)	1 month	∞	(i) Vegetation per paddock (*stoch.*) (ii) No. of sheep per paddock (*stoch.*)	Buy or sell sheep	ENR	State increment DP used for extending states to two types of vegetation and three age classes of sheep; transition probabilities obtained from simulation of grazing
Clark and Kumar (1978)	3 months	8	Liveweight (234-639 Kg)	Graze or replace	NR	Different cattle breeds
Toft and O'Hanlon (1979); Henderson and Toft (1979)	1 month	18	For each stock class: (i) No. previously sold (ii) No. previously agisted (iii) Upper limit on cash outlay (iv) Drought (*stoch.*)	(i) No. agisted (ii) No. sold	Minus EC	A drought tactic model for feeding cattle or sheep; subjective probability distribution of drought length elicited from the grazier
A(ii) *Beef-feedlot or feedlot/grazing*						
Bonnieux (1969, 1972)	14 days	Variable	No. of animals in each weight category (*stoch.*)	One ration from a range of alternatives	ENR	Management of a veal calf fattening operation; liveweight transition stochastic
Nelson (1969); Nelson and Eisgruber (1970)	1 month	36	(i) No. of cattle (ii) Average liveweight of cattle (180-540 Kg) — both by type and pen	(i) Liveweight gain (ii) Replace	PV of NR	Bayesian updating of cattle prices; feed costs from response surface fitted to LP solutions

Table 1: Summary of DP Applications (continued)

Study Application/Author(s)	Interval between stages	No. of stages	State(s)	Decision(s)	Objective function maximized	Comments
Meyer and Newett (1970)	10 days	4	Liveweight (230-360 Kg)	Liveweight gain	NR	Choice of 17 least cost rations determined by LP; model adaptable to feeding operations for any type of animal
Kennedy (1972)	28 days	12–24	Liveweight (100–500 Kg)	(i) Liveweight gain (ii) Replace	PV of NR	LP least cost feed rations; feedlot and/or grazing; see also Kennedy (forthcoming)
Yager *et al.* (1980)	30 days	12	(i) Liveweight (350-600 Kg) (ii) Cow price *(stoch.)*	(i) Liveweight gain (ii) Replace	PV of ENR	LP least cost feed rations; feed requirements depend on time of year
A(iii) *Dairy* Jenkins and Halter (1963)	1 year	∞ and 12	Lactation no. (12) *(stoch.)*	Keep or replace	ENR	Results given for (i) constant prices of cows, feed and milk and (ii) 1950-1961 actual prices
Giaever (1966)	variable: zero to 21 months *(stoch.)*	∞	(i) Lactation no. (5) *(stoch.)* (ii) Production history (7) *(stoch.)* (iii) Length of current calving interval (3) *(stoch.)*	Keep or replace	PV of ENR	Stage length is stochastic and depends on state and transition
Redman and Kuo (1969)	1 year	10	Lactation no (7) *(stoch.)*	Keep or replace with cows in selected lactation no.	PV of ENR	Applied to Holstein cows of three possible production levels; extensive sensitivity analysis
Smith (1971, 1973)	1 year	15	(i) Lactation no. (6) *(stoch.)* (ii) Calving interval (iii) Production levels for each of 2 immediately prior lactations (29)	Keep or replace	PV of ENR	
McArthur (1973)	1 year	∞	(i) No. of lactation records *(stoch.)* (ii) Production levels averaged over all lactations	Keep or replace	PV of ENR	DP policy evaluated against a Monte Carlo generation of lactation records with normal benchmark culling rules; DP policy showed no appreciable benefit over benchmark rules

Table 1: Summary of DP Applications (continued)

Study Application/Author(s)	Interval between stages	No. of stages	State(s)	Decision(s)	Objective function maximized	Comments
Stewart *et al.* (1977)	1 year	10	(i) Lactation no. (7) *(stoch.)* (ii) 305-day milk yield (11) (iii) 305-day milk fat % (7) (iv) Body weight (5)	Keep or replace	PV of ENR	Applied to Holstein cows; extensive sensitivity analysis
A(iv) *Broilers* Hochman and Lee (1972); Hochman (1973); Rausser and Hochman (1979)	1 week	52	(i) Age of flock (7-12 weeks) (ii) Selling price of broilers *(stoch.)*	Keep on *ad libitum* ration or replace	ENR	Seasonal variation in broiler prices allowed for — with prices (i) normally and (ii) uniformly distributed; returns from meat sales adjusted for meat quality
Kennedy *et al.* (1976)	1 week	∞	Bird weight per 100 birds surviving (ages 0–12 weeks)	(i) Liveweight gain (ii) Replace	PV of NR	Feed costs from linear interpolation of LP solutions; Lagrange multiplier used for also maximizing NR per unit of floor space
A(v) *Laying hens* White (1959); Halter and White (1962)	1 month	120	Age of flock	Keep or replace	NR	One of the earliest applications of DP to agriculture
Brookhouse and Low (1967)	4 weeks	13 and 130	Age of flock (0-64 weeks)	Keep or replace	PV of NR	Extension considered to deal with one rearing unit and more than one flock
Pouliquen (1970)	1 month	36	No. of months from start date to arrival of current batch	No. of months from start date to arrival of previous batch	NR	Egg prices seasonal and cyclical; profitability of forced moulting investigated
A(vi) *Cropping* Burt and Allison (1963)	1 year	∞	Initial soil moisture (1 of 5 levels) *(stoch.)*	Plant land to wheat or leave fallow	PV of ENR	Initial soil moisture depends on whether wheat was planted the previous year, and precipitation in the previous year

Table 1: Summary of DP Applications (continued)

Study Application/Author(s)	Interval between stages	No. of stages	State(s)	Decision(s)	Objective function maximized	Comments
Throsby (1964*a*)	1 year	6	Land available for allocation to different activities	No. of units of land allocated to different activities with different production periods	NR	A simple numerical example in an expository article
Throsby (1964*b*)	1 year	3	Land available for pasture improvement	No. of units of land allocated to pasture improvement	NR	A simple numerical example in an article which examines in detail how the qualitative nature of solutions depends on the form of the period return functions
Burt and Johnson (1967)	1 year	∞	Initial soil moisture (1 of 6 levels) *(stoch.)*	(i) Plant land to wheat or leave fallow (ii) Mix of wheat/fallow strategies	(i) PV of ENR (ii) Weighted sum of ENR and VNR	Extension of Burt and Allison (1963) — considers more empirical data, and trade-off possibilities between mean returns and variance of returns when mixed strategies are allowed
Burt (1971)	Irregular	∞	Length of previous renewal cycle *(stoch.)*	Length of current cycle	PV of ENR	Describes various models for determining the optimal cycle length for removing scrub from pasture, dependent on previous cycle length and time since last renewal
Iskakov and Finkelstejn (1973)	1 year	3	Crop at current stage	Crop at next stage	NR	A simple numerical example of crop rotation in an expository article
Kennedy *et al.* (1973)	4 months	∞	(i) Land fallow or crop age (ii) Season	Continue, plant or replace	PV of NR	Optimal fertilizer application when there is fertilizer carry-over is deduced by induction; investigates a cropping cycle for sorghum in the Northern Territory

Table 1: Summary of DP Applications (continued)

Study Application/Author(s)	Interval between stages	No. of stages	State(s)	Decision(s)	Objective function maximized	Comments
Stauber *et al.* (1975)	1 year	50	Residual nitrogen in the soil *(stoch.)*	Nitrogen applied to the crop	PV of ENR	Optimal fertilizer application when there is fertilizer carry-over is shown to be a special case of S,s inventory control problem; investigates fertilizer policies for seeded grasses, with fertilizer carryover deduced from yield
Burt (1981)	1 year	∞	Percentage organic matter in the upper six inches of soil	Percentage of land in wheat	PV of NR	DP results shown to be closely approximated by results from the method proposed by Burt and Cummings (1977)
A(vii) *Water Management* Burt (1964*a*, 1966)						
(a) Surface water	1 month	∞	Quantity of surface water *(stoch.)*	Water used from surface water stock	PV of ENR	Considers the integration of a range of models dealing with investment decisions, land allocation and water management policy
(b) Ground water	5 years	∞	Quantity of ground water *(stoch.)*	Water used from ground water stock	PV of ENR	
Flinn and Musgrave (1967)	30 days	8	Quantity of water available to be allocated	No. of irrigations (0 to 6)	NR	Model's use for pricing irrigation water also considered
Hall and Butcher (1968)	Unspecified	10	(i) Quantity of water available to be allocated (ii) Soil moisture level	Irrigation water applied	NR	Total return from the crop a multiplicative function of period returns
deLucia (1969)	1 week	22	(i) Quantity of surface water *(stoch.)* (ii) Streamflow during previous period *(stoch.)* (iii) Water in root-zone reservoir *(stoch.)*	(i) Water used from surface storage (ii) Water applied from aquifer	ENR	Random variables are net stream inflow into surface storage and effective rainfall

Table 1: Summary of DP Applications (continued)

Study Application/Author(s)	Interval between stages	No. of stages	State(s)	Decision(s)	Objective function maximized	Comments
Asopa (1971)	9 days	5	Crop condition *(stoch.)*	Irrigation water applied	ENR	No constraint on water available; there is a charge for water use
Burt and Stauber (1971)	10 days	5	(i) Crop condition *(stoch.)* (ii) Water in storage	Irrigation water applied	ENR	Rainfall stochastic; state variable (ii) eliminated by the use of a Lagrange multiplier
Dudley *et al.* (1971*a*)	15 days	6	(i) Soil moisture level *(stoch.)* (ii) Water in storage	Terminal soil moisture level maintained during the stage	ENR	State transition probabilities based on the results of a plant water simulation model; the way in which results could be used for the intermediate-run problem of area of crop to plant is discussed in Dudley (1969) and Dudley *et al.* (1971*b*); see also Dudley *et al* (1972)
O'Loughlin (1971)	1 month	∞	(i) Reservoir level at start of year *(stoch.)* (ii) Area of the irrigated crop	Release of water	PV of ENR	Optimal area of crop to be planted at the start of each growing season is determined; three models run end-to-end to model a changing crop sequence throughout the year
Dudley (1972)	1 year	20	Reservoir level at start of year *(stoch.)*	Irrigation water allocated to current season	PV of ENR	Used in conjunction with other models (Dudley *et al.* 1971*a* and *b*, 1972) to find PV of ENR a function of acreage developed for irrigation
Biere and Lee (1972)	4 months	∞	Surface reservoir level	Release of reservoir water to recharge valley aquifers	PV of reduction in expected pumping costs	Solved assuming a stationary process with respect to years, non-stationary with respect to seasons
Dudley and Burt (1973)	Irregular	7 within year; ∞ no. of years	(i) Percentage available soil moisture *(stoch.)* (ii) Reservoir level *(stoch.)* (iii) Acreage available for irrigation	(i) Seasonal irrigation (ii) Acreage irrigated	PV of ENR	State variable transition probabilities are calculated by simulation; variance of net benefits treated as a criterion for trade-off against expected value of net benefits

Table 1: Summary of DP Applications (continued)

Study Application/Author(s)	Interval between stages	No. of stages	State(s)	Decision(s)	Objective function maximized	Comments
Yaron and Olian (1973)	1 year	50	Soil salinity level *(stoch.)*	Quantity of water used for leaching before summer irrigation	PV of ENR	Estimates the value of water quality in irrigation
McFarland (1975)	1 year	50	(i) Soil salinity level (ii) Ground water stock	(i) Water applied during the growing season (ii) Water applied for leaching during the dormant season	PV of NR	Yearly cropping plan determined using LP
Matanga and Miguel (1979)	1 year	∞	Salinity in the root zone *(stoch.)*	(i) Water applied for leaching prior to the irrigation season (ii) Seasonal irrigation depth	PV of ENR	
Thampapillai (1980)	None: each stage is a reach	4	Expansion to capacity of structures in the previous stage and the next stage	Expansion to capacity of existing structures in that stage	ENG	Uses two-part returns — one from a stochastic geometric programming model minimizing expected social cost of investment and the other from a quadratic programming model maximizing a measure of social return
A (viii) *Pesticide management*						
Shoemaker (1973*a*, *b*, *c*)	Growing season divided by 9	9	(i) Pest density (ii) Parasite density	Dosage rate of pesticide	Minus sum of costs of pesticide application and pest damage	A hypothetical numerical application; considers extensions for dealing with many real world additional complexities
Raphael (1979)	Half-week	24	(i) Population density of 3 sizes of pest larvae (ii) Population density of pest's predator	Dosage rate of pesticide	Minus sum of costs of pesticide application and pest damage	Argues that further refinements must await results of more biological research
A(ix) *Forestry*						
Hool (1966)	2 years	8	(i) Stand volume *(stoch.)* (ii) Tree count	Thinning, harvest cutting or selection cutting	Expected merchantable yield	Policies of thinning from above compared with thinning from below

Table 1. Summary of DP Applications (continued)

Study Application/Author(s)	Interval between stages	No. of stages	State(s)	Decision(s)	Objective function maximized	Comments
Amidon and Akin (1968)	5 years	13 and others	Stand volume	Intensity of thinning	PV of NR	
Kilkki and Vaisanen (1969)	5 years	10	Stand volume	Intensity of thinning	PV of NR	
Risvand (1969)	4 years	11	(i) Stand volume per decare (ii) Mean diameter	Thinning grade	PV of NR	Forward recursive method used, giving plans for a range of time horizons
Kingma (1974)	1 year	∞	(i) Merchantable volume per tree of mean size (ii) Stock rates of trees	Proportion of stand cut	PV of NR	Solution obtained through value iteration and successively finer grids
Won (1974)	5 years	10	Volume of stumpage	No. of trees harvested per acre	PV of NR	
Murphy *et al.* (1977)	Rotation length	*n*	Years to end of the planning horizon	Rotation length	FV of NR	*n* set so that total planning horizon is less than 100 years; forward recursive method; results for perfect and imperfect capital markets
Brodie *et al.* (1978)	10 years	∞	Stand stocking	Intensity of thinning	PV of NR	Contrasts advantages of forward and backward recursion
Brodie and Kao (1979) Kao and Brodie (1979)	10 years 1 year	∞	(i) No. of trees (ii) Basal area of stand	Intensity of thinning	PV of NR	Allows for accelerated diameter growth as a result of thinning; "neighborhood storage locations" used to overcome rounding errors due to discrete state levels
Chen *et al.* (1980)	10 years	11	Basal area of stand	Amount of basal area removed	Physical harvest over one rotation	Calculus used for maximization across the continuous state variable

Table 1: Summary of DP Applications (continued)

Study Application/Author(s)	Interval between stages	No. of stages	State(s)	Decision(s)	Objective function maximized	Comments
A(x) *Fisheries* Lewis (1975)	1 year	∞	Population of tuna *(stoch.)*	Fish catch	ENG	Emphasis on uncertainty about size of current stock, yield and price applied to East Pacific Yellowfin Tuna fishery; see also Lewis (1977)
Anderson and Ben-Israel (1979)	1 year	25	(i) Species (ii) Population level	Fishing effort for each fleet (fishing days per year)	PV of NR to fishermen plus PV of consumer surplus	Optimal allocation of effort amongst fleets determined by LP subroutine; applied to the US cod, haddock and flounder fisheries.
Dudley and Waugh (1980)	1 month	12	Population of prawns *(stoch.)*	Fish catch	ENR	Investigates optimal management and investment policies for the prawn fishery in Exmouth Gulf; population change is simulated stochastically
B) FARM MACHINERY Sowell (1967)	1 year	25 and ∞	Machine age	Keep or replace with a similar or alternative machine	Minus holding costs	Applied to self-propelled cotton pickers; expected losses due to machine breakdown simulated
Liang (1968)	1 day	80	(i) Day on which previous replacement failed (ii) Suitability of day for work *(stoch.)*	Carry out preventative maintenance or not	Minus expected operating cost	Transition probabilities derived from simulation models
Ambrosius (1970)	1 year	10	Farm tractor complements and combines by size, capacity and age	Keep or replace	PV of NR	Problem reduced to manageable size by applying DP only to a subset of feasible tractor-combine combinations selected at random
Boyce *et al.* (1971)	variable	6	The set of feasible production alternatives	Production alternative	Minus hours of labour input per ton of carrots produced and marketed	DP used to select the best path through a network of nodes

Table 1: Summary of DP Applications (continued)

Study Application/Author(s)	Interval between stages	No. of stages	State(s)	Decision(s)	Objective function maximized	Comments
Morey *et al.* (1972)	7 days	13	(i) Area to be harvested (ii) Days available for field work between stages *(stoch.)*	Hours of combine operation per day	ENR	Model is extended from the problem of harvesting 1 crop (corn) to 2 crops (corn and soybeans)
Corrie and Boyce (1972)	1 day	20	Day no.	Selectively harvest or not	NR	Cauliflowers harvested in any pass that would mature before the next pass
Hesselbach (1974)	1 year	10, 15 and 20	(i) No. of harvesters (ii) Age of harvesters (iii) Capacity of harvesters	Keep or replace	PV of NR	Applied to US corn harvesting; emphasis on sensitivity of results to length of planning horizon; see also Hesselbach and Wijnands (1977)
Kelly (1981)	1 year	15	(i) Tractor age (ii) Tractor capacity	Keep or replace with selected capacity	PV of NR	Simulates the tractor replacement decision within a recursive linear programming framework
C) FINANCIAL						
Hausman (1969)	about 6 weeks	5	Forecast of crop supply *(stoch.)*	Buy or sell some of the crop outside the contract to supply entered into before planting	Minus expected penalty costs in fulfilling supply contract	Period penalty costs increase (i) through time and (ii) to the square of any amounts bought or sold outside the supply contract; comment by Janssen (1971)
Hudson (1976)	30 days	11	(i) Liveweight (ii) Futures commitment–futures or cash strategy (iii) Cash price *(stoch.)* (iv) Cash/futures spread *(stoch.)*	(i) Keep or replace (ii) Futures commitment	PV of expected utility	Model determines how best to exploit cash and futures markets in the fattening of beef cattle
Hardaker (1979)	1 year	∞	(i) Stock of IEDs (ii) Pre-tax farm income *(stoch.)*	Purchase or sale of IEDs	(i) PV of expected post-tax income (ii) PV of future stream of certainty equivalent utilities	IED= Income Equalization Deposit; computes the value of use of IEDs for farmers with different levels of mean and variance of taxable income

Table 1: Summary of DP Applications (continued)

Study Application/Authors	Interval between stages	No. of stages	State(s)	Decision(s)	Objective function maximized	Comments
Sondakh and Hardaker (1981)	1 year	∞ and 8	Initial cash position	Vector of farm planning activity levels	PV of consumption above basic needs	Parametric LP used to maximize recursive equation at each stage across all initial cash positions
D) STORAGE — REGIONAL, NATIONAL, OR WORLD LEVELS						
D(i) *Grains*						
Gustafson (1958)	1 year	∞	(i) U.S. feed grain stocks (ii) U.S. feed grain production *(stoch.)*	Storage or release	PV of ENG	A pioneering application; many extensions considered, *e.g.* exports and multiregional production and consumption.
Browning (1970)	1 year	25	(i) U.S. wheat stocks (ii) U.S. wheat production *(stoch.)*	Storage or release	PV of ENG	Uses the framework developed by Gustafson (1958) to evaluate food aid, 1960–1985
Cochrane and Danin (1976)	1 year	10	(i) Price of grain *(stoch.)* (ii) Reserve stocks *(stoch.)*	Storage or release	Minus price variability	Model run for 1975–1985 for world grain market in total and for wheat, rice and coarse grains; grain production assumed to be stochastic and completely price inelastic
Johnson and Sumner (1976)	1 year	25	(i) Grain stocks (ii) Grain production *(stoch.)*	Storage or release	PV of ENG	Model run for 1948–1973; investigates the optimal carryover levels for three insurance programs guaranteeing deliveries in the event of production shortfalls; run for countries and regions
Gardner (1977)	1 year	∞	(i) Public wheat stocks (ii) Wheat production *(stoch.)*	Private storage or release	PV of ENR and PV of ENG	Investigates the effect of public on private grain storage; see also Gardner (1979)

Table 1: Summary of DP Applications (continued)

Study Application/Author(s)	Interval between stages	No. of stages	State(s)	Decision(s)	Objective function maximized	Comments
Alaouze *et al.* (1978, 1979)	1 year	24	(i) Australian wheat stocks (ii) Australian wheat production *(stoch.)*	Storage or release	PV of ENR	Assumes wheat sold is sold for export with demand for Australian wheat exports infinitely elastic; rules obtained for a range of simulated price sequences
Kennedy (1979)	1 year	∞	(i) World wheat stocks (ii) World wheat production *(stoch.)*	Storage or release	PV of ENR and PV of ENG	Contrasts storage policies pursued by a monopoly storage agency and under perfect competition; a cobweb production process is allowed for
Burt *et al.* (1980)	1 year	20	(i) Average price of wheat during preceding crop year *(stoch.)* (ii) Wheat stocks	U.S. wheat exports	PV of ENG for (i) U.S. (ii) World	Results were also obtained taking account of lagged variables in the supply and demand equations; an iterative proximate solution method was used to solve a problem with many more state variables than can be normally handled in DP; see also Koo and Burt (1980) for results of similar model but with a penalty added to the objective function for price variability
D(ii) *Wool* Dalton (1976)	3 months	20	(i) Previous 8 periods' wool prices *(stoch.)* (ii) Wool production *(stoch.)* (iii) Wool stockpile	Wool sold to trade	Minus wool price variation	An optimal control theory formulation with a quadratic objective function and linear demand and supply functions stipulated as constraints
D(iii) *Butter* Townsley (1964)	1 month	12	Level of New Zealand butter in store	New Zealand butter sales on the U.K. market	NR	Total New Zealand sales and other sales for each month assumed known
D(iv) *Fodder* Throsby (1964*a*)	1 month	5	Stock of fodder *(stoch.)*	Stock held during the month	ENR	A simple numerical example in an expository article
D(v) *Apples* Kenyon and Carman (1971)	Variable 1-3 months	5	Stock of apples	Sales of apples	PV of NR	Determines monopoly marketing rules for Californian apples

Key to Table 1

State(s)

(stoch.) — State next period partly determined by uncertain exogenous variables

Objective function maximized

EC — Expected cost
ENG — Expected net gain (to society)
ENR — Expected net revenue
FV — Future value
NR — Net revenue
VNR — Variance of net revenue