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Some Further Discussion on the Price Index for The Almost Ideal Demand System: A Chain Price Index Approach

BY

Mark G. Brown – Senior Research Economist - FDOC
Jonq-Ying Lee – Courtesy Professor, Food and Resource Economics, University of Florida

FLORIDA DEPARTMENT OF CITRUS
Economic and Market Research Department
P.O. Box 110249
Gainesville, Florida 32611-2049 USA
Phone: 352-392-1874
Fax: 352-392-8634

www.floridajuice.com

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Abstract

The issue of identification of the parameter α_0 in the price index of the Almost Ideal Demand System (AIDS) is examined. In nearly all empirical studies, the model's likelihood function has been extremely flat in α_0, and this parameter has not been able to be estimated. Assumed values are often used. In this paper, an AIDS-like model is developed with an easy- to-calculate chain-price index that replaces the price index in the original AIDS. The model stands by itself with respect to consumer demand theory and flexibility. An empirical analysis of beverage demands gives merit to the specification.

Key Words: Demand, Almost Ideal Demand System, differential and level models, chain price index.

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This paper examines an aspect of the Almost Ideal Demand System (AIDS) (Deaton and Muellbauer 1980a, b) that has caused some difficulty in estimation of the model (this demand system will be called the original AIDS in the following discussion). The price index used in specifying the real income variable of the model includes a constant α_0 that has been problematic. Generally, this parameter cannot be estimated along with the other model parameters as the model's likelihood function tends to be extremely flat in α_0. From the outset, the problem was recognized by Deaton and Muellbauer (1980a) who suggested assigning some plausible value to α_0 or using the Stone price index in place of the model's original price index. For example, a value of zero is sometimes assigned to α_0 (e.g., Moschini, Moro, and Green 1994; Zhen, Wohlgenant, Karns and Kaufman 2011). Values for this parameter have also been based on the interpretation that α_0 reflects the outlay required for a minimal standard of living when prices are unity. Regardless, assigning a value to α_0 is a somewhat problematic and an arbitrary task. Similarly, there are problems with using the Stone price index in place of the original price index in the AIDS (linear approximate AIDS or LA/AIDS, Pashardes 1993; Wan 1998; Moschini 1995). This index, used more often when the AIDS was first introduced, has fallen from favor more recently, as a result of problems including endogeneity of the budget shares used in the index and the question of how well it actually reflects the original index in the model. Other approaches have also been suggested including the use of lagged budget shares in the Stone price index to avoid the endogeneity problem (e.g., Eales and Unnevehr 1988). Overall, regardless the approach taken to deal with the AIDS price index, empirical and/or theoretical concerns continue to exist, suggesting further consideration be given to this issue as is done in this paper.

In the next section, a model in the same category as the LA/AIDS, but standing alone as a theoretically sound specification, is developed with an alternative price index that is easy to calculate and with features that are more attractive than those for the Stone price index and perhaps setting α_0 to some value in the original index (Moschini, Moro, and Green 1994). Then, an empirical application with respect to demands for different beverages is discussed, followed by concluding comments.

Model

The model developed here is quite similar to the Almost Ideal Demand System, but the approach taken to get to it is somewhat different than that taken in the original development of the AIDS. Our model is based on a general, un-parameterized cost function, and the demand parameters are based on the derivatives of this function, as opposed to the specific parameterization for the original AIDS. Deaton and Muellbauer (1980a, b) obtained their model from an explicit cost function. Here we obtain a model based on the differential approach used to develop such models as the Rotterdam demand system (Barten 1966; Theil 1975, 1976, 1980). First, a differential AIDS-like model is developed based on the general form of the cost function, and then a corresponding levels model that is the same as the AIDS model except for its price index is found. Although somewhat indirect, this approach provides useful insights into the model's price index.

Consider the log of the general cost function

(1) $\log x = \log c(p, u)$,

where x is total expenditures or income, $c(p, u)$ is the cost function, $p = (p_1, \ldots, p_n)$ is a vector of prices for n goods and u is utility. The vector of quantities associated with prices is $q = (q_1, \ldots,$

q_n). The cost function is assumed to obey the usual properties: homogeneous of degree one in prices, increasing in u, concave in prices, continuous in p with first and second derivatives, and obeys Shephard's Lemma (e.g., Deaton and Muellbauer 1980b).

Based on Shephard's Lemma, the general form of the demand equations for cost function (1) can be written as

(2) $w_i = \partial \log c(p, u) / \partial(\log p_i)$,

where $w_i = p_i q_i/x$ is the budget share for good i.

The total differential of demand equation (2) is

(3) $dw_{it} = \sum \gamma_{ij} \, d\log p_{jt} + (\partial w_i / \partial u) \, du_t$,

where subscript t indicates time, $dw_{it} = w_{it} - w_{it-1}$, $\gamma_{ij} = \partial^2 \log c(p, u) / \partial(\log p_i) \, \partial(\log p_j)$, $d \log p_{jt} = \log p_{jt} - \log p_{jt-1}$, and $du_t = u_t - u_{t-1}$. As for the LA/AIDS, the parameter notation in equation (3) for the price effects follows that for the original AIDS, but the interpretation is different. Specifically, for the original AIDS cost function, $\log(x) = \alpha_0 + \sum \alpha_j \log p_j + \frac{1}{2} \sum\sum \gamma_{ij} \log p_i \log p_j + u\beta_0 \prod p_k^{\beta k}$, the term $\partial^2 \log c(p, u) / \partial(\log p_i) \, \partial(\log p_j) = \gamma_{ij} + \beta_i \beta_j \log(x/P)$. That is, the parameter γ_{ij} in equation (3) is equivalent to $\gamma_{ij} + \beta_i \beta_j \log(x/P)$ for the original AIDS cost function. Thus, the parameter γ_{ij} defined for the original AIDS has a different interpretation, except when x=P or the β_i's are zero, in which case the term $\beta_i \beta_j \log(x/P)$ vanishes. The parameterization chosen in model (3) allows us to tractably deal (below) with the price index problem.[1] The parameter γ_{ij} here can be interpreted as the budget share for good i times the elasticity of that budget share with respect to the price of good j. This is parallel to the Slutsky coefficient in the Rotterdam model (Theil 1975, 1976, 1980) which is the budget share times the quantity elasticity with respect to price, utility held constant. The sign of γ_{ij} in model (3), then indicates, of course, how the budget share changes with price j, utility held constant.[2]

Next, consider the term du. Total differentiation of cost function (1) results in

(4) $d\log x = \sum(\partial \log c(p, u) / \partial(\log p_j))\, d \log p_j + (\partial \log c(p, u) / \partial u)\, du$,

or, rearranging and using result (2),

(5) $du = (d\log x - \sum w_j d \log p_j) / (\partial \log c(p, u)/\partial u)$,

or, given $1/(\partial \log c(p, u)/\partial u) = 1/(\partial \log x/\partial u) = (\partial u/\partial x)x$,

(6) $du = (d\log x - \sum w_j d \log p_j)\ (\partial u/\partial x)\, x$.

Substituting the right-hand side of (6) for du in equation (3) results in

(7) $dw_{it} = \sum\gamma_{ij}\, d\log p_{jt} + \beta_i\, (d\log x_t - \sum w_{jt}\, d \log p_{jt})$,

where $\beta_i = (\partial w_i/ \partial u)\, (\partial u/\partial x)\, x = (\partial w_i/ \partial x)\, x = ((p_i/x)(\partial q_i/\partial x) - (p_i q_i/x^2))\, x = p_i(\partial q_i/\partial x) - w_i$, the marginal propensity to consume for good i minus its budget share. The term $\sum w_{jt}\, d \log p_{jt}$ is known as the Divisia price index and $d\log x_t - \sum w_{jt}\, d \log p_{jt}$ is a measure of the change in real income (Theil 1975, 1976, 1980). This measure of real income is also found in the Rotterdam model. For estimation, the budget shares in the Divisia price index are replaced by their averages over periods t and t-1, following the same practice for the Rotterdam model (Theil 1975, 1976, 1980); that is, $w_{jt}^* = (w_{jt} + w_{jt-1})/2$ replaces w_{jt}. The coefficients γ_{ij} and β_i are treated as constants to be estimated.

The parameter β_i has the same interpretation in both model (7) and the original AIDS. Also, β_i is the budget share for good i times the elasticity of that budget share with respect to income, parallel to the marginal-propensity-to-consume coefficient in the Rotterdam model which is the budget share times the quantity elasticity with respect to income. Positive, zero and negative values of β_i indicate a superior good (budget share increases with income), neutral good (budget share is unchanged with an income change) and normal to inferior good (budget share decreases with income).

Model (7) is discussed by Deaton and Muellbauer (1980a) in their original paper in comparing the first-differenced form of the AIDS and the Rotterdam model. No discussion, however, was provided there on the linkage between model (7) and the cost function or the underlying utility maximization problem; instead, the Rotterdam real income variable was simply substituted for the first-differenced real income variable for the AIDS model for comparison of the two models. It was noted that the Rotterdam model also has the same right hand side as in equation (7), except for interpretation of the coefficients. The only difference in the two models is the left hand side, where in the Rotterdam model the variable $w_i d\log q_i$ replaces dw_i in equation (7). Barten (1993) provides further discussion on the AIDS and Rotterdam models and develops various models combining the features of the two models.

The results here explicitly link model (7) with the cost function and, hence, the underlying utility maximization problem. In addition to motivating the price index developed in this paper, this linkage may be useful in extending the model to non-price, non-income variables such advertising, past consumption and demographic variables. Translation (Pollak and Wales 1992), scaling (Barten 1964; Deaton and Muellbauer 1980b) and the Tinter-Ichimura-Baseman relationship, invoked for the Rotterdam model (Theil 1980), but also perhaps useful in the AIDS model (Brown and Lee 2010), might be considered to obtain parsimonious specifications with respect to the parameter space of such variables.

The parameters for demand equation (7) obey the following restrictions:

(i) $\sum \beta_i = 0$ (adding up),

(ii) $\sum_i \gamma_{ij} = 0$ (adding up),

(iii) $\sum_j \gamma_{ij} = 0$ (homogeneity),

(iv) $\gamma_{ij} = \gamma_{ji}$ (symmetry).

Restriction (iv) $\gamma_{ij} = \gamma_{ji}$ holds given γ_{ij} and γ_{ji} are both equal to $\partial^2 \log c(p, u) / \partial(\log p_i) \partial(\log p_j)$ since the result does not depend on the order of differentiation with respect to the log prices.

Restriction (iii) is based on the property that the cost function c(p, u) is homogeneous of degree one in prices. Given this property, the derivative $q_i = \partial c(p, u)/ \partial p_i$ is homogeneous of degree zero in prices. It then follows that $w_i = p_i q_i /x$ is also homogeneous of degree zero in prices[3] or $\sum_j \gamma_{ij} = \sum_i \partial w_i/ \partial(\log p_j) = \sum_i (\partial w_i/ \partial p_j) p_j = 0$.

Restriction (ii) holds given restrictions (iii) and (iv). Restriction (i) holds given $\sum\beta_i = \sum(p_i(\partial q_i/\partial x) - w_i) = 1\text{-}1 = 0$, since $\sum w_i = 1$ and $\sum p_i(\partial q_i/\partial x) = 1$, where the latter equality is based on differentiation of the budget constraint, $\sum p_i q_i = x$, with respect to income x.

The negativity condition depends on the values of the Slutsky coefficients k_{ij} which for the model here can be written as

(v) $\quad k_{ij} = \gamma_{ij} - w_i \Delta_{ij} + w_i w_j$

where $\Delta_{ij} = 1$ if $i = j$ and zero otherwise. The negativity requires that the matrix K with elements k_{ij} is negative semidefinite. The Slutsky coefficients for a good are that good's budget share times its compensated price elasticities, and can be calculated as $w_i (\varepsilon_{ij} + \varepsilon_i w_i)$, where ε_i and ε_{ij} are the income and uncompensated price elasticity, respectively. The elasticities for the current model are below equation (13) of this paper. The Slutsky coefficients of the original AIDS differs from (v) by the term $\beta_i\beta_j \log(x/P)$. This result stems from the earlier discussion following equation (3) where it was shown that for the original AIDS cost function, the $\partial^2 \log c(p, u) / \partial(\log p_i) \partial(\log p_j) = \gamma_{ij} + \beta_i\beta_j \log(x/P)$. The negativity condition can be checked, but cannot be imposed by parameter restrictions, as in the case of the original AIDS.

Given time series data where the first observation is for period 1, equation (7) can be summed over periods 1 through t to obtain a levels version of the model. Note for any variable y that $\sum_{k=1 \text{ to } t} dy_k = \sum_{k=1 \text{ to } t} (y_k - y_{k-1}) = y_t - y_0$. This relationship holds for variables dw_{it}, $d\log p_{jt}$ and $d\log x_t$ but does not hold for $\sum w_{jt}^* \, d \log p_{jt}$ given the multiplication of the log price changes by the average budget shares. Thus, summing equation (7) over period 1 through t yields

$$(8)\ w_{it} - w_{i0} = \sum \gamma_{ij} (\log p_{jt} - \log p_{j0}) + \beta_i (\log x_t - \log x_0 - \textstyle\sum_j w_{j1}^* d \log p_{j1} - \sum_{k=2 \text{ to } t} \sum_j w_{jk}^* d \log p_{jk}),$$

or, rearranging,

$$(9)\ w_{it} = \alpha_i + \sum \gamma_{ij} \log p_{jt} + \beta_i (\log x_t - \log P^c), \quad t = 1, \ldots, T,$$

where $\alpha_i = w_{i0} - \sum \gamma_{ij} \log p_{j0} - \beta_i (\log x_0 - \sum_j w_{j1}^* d \log p_{j1})$; and $\log P^c = \sum_{k=2 \text{ to } t} \sum_j w_{jk}^* d \log p_{jk}$, which is a chain price index ($P^c = 1$ or $\log P^c = 0$, for t=1).[4] In defining α_i, it is assumed that the model's structure applies to the unobservable zero-period values. In a study of the impact of real income on the marginal utility of income, Theil and Brooks (1970-71) obtained a similar levels equivalent for the Divisia volume index, $\sum w_{jt} \, d \log q_{jt}$.[5]

Thus, the general approach taken here to derive a demand system where the dependent variables are budget shares and the logs of prices and income are used as explanatory variables leads us to a model similar to the original AIDS. The essential difference between model (9) and the AIDS is with respect to the term $\log P^c$. In the original AIDS, the latter term is replaced by $\log P = \alpha_0 + \sum \alpha_j \log p_j + \frac{1}{2} \sum\sum \gamma_{ij} \log p_i \log p_j$, while in LA/AIDS, it is replaced by the Stone price index $\log P^s = \sum w_j \log p_j$. As discussed earlier, however, the use of the original price index log P has been problematic as it is nearly impossible to estimate α_0, and, alternatively, using the Stone price index $\log P^s$ has its problems including questions on how good the approximation actually is, its theoretical underpinning and endogeneity of the budget shares, although the latter can be dealt with by using lagged budget shares in the price index.

Consider in more detail these alternative price indexes.[6] First, focusing on the price of product j, the chain price index in full can be written as

$$\log P^c = \textstyle\sum_j (w_{j2}^* \log(p_{j2} / p_{j1}) + w_{j3}^* \log(p_{j3} / p_{j2}) + \ldots), \tag{10}$$

or, taking the antilog,

$$P^c = \textstyle\prod_j (p_{j2}/ p_{j1})^{wj2^*} (p_{j3}/ p_{j2})^{wj3^*} \ldots \tag{11}$$

That is, the index is comprised of a sequence of price ratios for product j, each indicating the percentage change in price from one period to the next. The adjective "chain" is used to describe the index as price t appears twice in a chain fashion, first in the numerator and then in the denominator. The budget shares or weights for the price ratios indicate the importance of the price changes to the overall index. The larger a product's budget share, the larger is its weight. In general, the weights can be expected to change over time due to prices, income and other factors, resulting in varying product specific contributions to the overall index. For example, suppose in period t and t+k that the log percentage changes in the price of good j are the same, $\log(p_{jt} / p_{jt-1}) = \log(p_{jt+k} / p_{jt+k-1})$, but the budget share in period t+k is greater, $w_{jt+k}^* > w_{jt}^*$. Thus, product j's contribution to the overall price index increases from period t to t+k, given the increase in its budget share. In summary, $\log P^c$ captures three meaningful aspects of a price index, it directly measures the percentage change in a product price from one period to the next, it gives that percentage change more or less weight depending on the importance of the product as reflected by its budget share, and has a theoretical basis as found in deriving equation (9).

In contrast, consider the Stone price index which, although seemingly similar to the Divisia price index and its use in equation (10), is quite different. First, note only logs of prices are taken, not log differences. Consider product j's contribution to the overall index, and the change in this component over two periods, i.e., $w_{jt} \log p_{jt} - w_{jt-1} \log p_{jt-1}$ or, equivalently, w_{jt}

$\log (p_{jt} / p_{jt-1}) + (w_{jt} - w_{jt-1}) \log p_{jt-1}$. The first part of this change, $w_{jt} \log (p_{jt} / p_{jt-1})$, is similar to that in the chain index $\log P^c$, except the Stone index uses the actual budget share while the chain index here uses the average budget share over the two periods. The second component of the Stone index, $(w_{jt} - w_{jt-1}) \log p_{jt-1}$, measures the change in the budget share weighted by the lag of the price. This feature can result in the Stone price index changing in value even when prices are constant but the budget shares change, a feature that would not occur in the Divisia or chain price index, or the original AIDS price index (Changes in the budget shares, prices constant, may be due to non-price variables such as income or the error term that would be included in the model for empirical analysis.). This difficulty with the Stone price index means that, in general, it is not homogeneous of degree one with respect to prices in practice, as opposed to the original AIDS price index , the Divisia price index and associated chain price here, all of which are linear homogeneous in prices. As noted earlier, use of the Stone price index further results in an endogeneity problem, and underlies the LA/AIDS elasticity corrections discussed by Green and Alston (1990, 1991).

Lastly, the price index for the original AIDS can be written as $\log P = \alpha_0 + \sum_j (\alpha_j + ½\sum_i \gamma_{ij} \log p_i) \log p_j$, or, $\log P = \alpha_0 + \sum_j w_{jt}^{**} \log p_j$ where $w_{jt}^{**} = \alpha_j + ½\sum_i \gamma_{ij} \log p_i$. Given $\sum_i \gamma_{ij} = 0$, the weights are unchanged for proportional changes in all prices. Thus, from this view, the original index differs from the Stone index in that the weights for the original index depend only on prices. That is, the weights change only as a result of price changes, not changes in other factors, in contrast to what could occur in the Stone price index as discussed above.

In summary, both the chain price index and the original AIDS price index are well founded in consumer demand theory, based on the percentage changes in prices from one period to the next, and have meaningful weights. The chain price index, however, is easing to apply, while the original price index may be problematic in applications due to the difficulty in estimating α_0. In contrast, the Stone price index, although easy to apply and having some theoretical basis (Deaton and Muellbauer 1980b), has a weighting scheme that may not reflect that in the original or chain indexes. The weighting schemes of the different price indexes are, of course, important for measuring real income in empirical analysis. For example if all prices were doubled, the real income variables for the original AIDS and model (9), x/P and x/P^c, respectively, would be cut in half, while that for the Stone, x/P^s, may or may not be halved, depending on the budget share changes.

The elasticities for the model can be derived directly from equation (9) or from equation (7) by using the relationship that $dw_i = w_i \text{ dlog } p_i + w_i \text{ dlog } q_i - w_i \text{ dlog } x$.[7] In the case of equation (7), after substituting the latter expression for dw_i and rearranging terms, we find

(12) $$w_i \text{ dlog } q_i = - w_i \text{ dlog } p_i + w_i \text{ dlog } x + \sum\gamma_{ij} \text{ dlog } p_j + \beta_i (\text{dlog } x_t - \sum w_j \text{ d log } p_{jt}),$$

and, after dividing both side of equation (10) by w_i, we find

(13) $$\text{dlog } q_i = - \text{dlog } p_i + \text{dlog } x + \sum(\gamma_{ij/} w_i) \text{ dlog } p_j + (\beta_i /w_i) (\text{dlog } x_t - \sum w_j \text{ d log } p_{jt}).$$

Hence, from equation (10), the elasticities are[8]

$$\varepsilon_i = 1 + \beta_i /w_i \qquad \text{(income)}$$

$$\varepsilon_{ij} = -\Delta_{ij} + \gamma_{ij/} w_i - (\beta_i /w_i) w_j. \qquad \text{(prices, uncompensated)}$$

The above elasticities apply to the original AIDS model based log P, except for the price elasticity formulas where w_j is replaced by $\alpha_j + \sum_k \gamma_{jk} \log p_k$.

Application

A demand system for beverage products was studied based on the concept of two-stage budgeting,[9] where consumers first decide how much to spend on a group (first stage), and, then how to allocate this amount to the individual goods in the group (second stage). This paper focuses on the second stage: allocation of total beverage expenditures across individual beverage products. The second-stage demand equations for individual goods in the group, called conditional demands, are functions of the amount of income allocated to the group (group expenditures) and the prices of the goods in the group. Specification of these conditional demand equations follows the same general structure as the unconditional demands, except the income variable is group expenditures, the prices are those for the goods in the group, and the coefficients are conditional.

Conditional demands for beverages were studied using Nielsen data based on retail scanner sales for grocery stores, drug stores, mass merchandisers along with an estimate of Wal-Mart sales based on a consumer panel.[10] Twelve beverages were included in the model: 1) 100% orange juice, 2) 100% grapefruit juice, 3) 100% apple juice, 4) 100% grape juice, 5) remaining 100% juice, 6) vegetable juice, 7) less-than-100% juice drinks, 8) carbonated water, 9) water, 10) regular and diet soda, 11) liquid tea or tea for short, and 12) milk and shakes.

The data are for four-week periods, running from the four-week period ending August 11, 2007 through the four-week ending August 7, 2010 (40 four-weekly observations).[11] The raw data were comprised of gallon and dollar sales. Quantity demanded was measured by per capita gallon sales, obtained by dividing raw gallon sales by the U.S. population; prices were obtained by dividing dollar sales by gallon sales. Sample mean per capita gallon sales, prices and budget shares are shown in Table 1. The budget share for each beverage is conditional: that beverage's dollar sales divided by total dollar sales for the twelve beverages studies.

Beverage demands are subject to seasonality and two variables were included to account for seasonal changes in demand: s_1= sine($2\pi\alpha/52$) and s_2= cosine($2\pi\alpha/52$), respectively, with π =

3.14…, α indicates the 4-week period in a year (α =1, …., 13) (Brown and Lee, 2008). The coefficients on s_i (i = 1, 2) sum to zero across beverages to satisfy the adding-up property.

As the data add up by construction---the left-hand-side variables (budget shares) in the model sum over i to one---the error covariance matrix was singular and an arbitrary equation was excluded (the model estimates are invariant to the equation deleted as shown by Barten, 1969). The parameters of the excluded equation can be obtained from the adding-up conditions or by re-estimating the model omitting a different equation. The equation error terms were assumed to be contemporaneously correlated and the full information maximum likelihood procedure (TSP) was used to estimate the system of equations.

Two models were estimated---model (9), based on the chain price index, and the original model based on the assumption $\alpha_0 = 0$.[12] Symmetry and homogeneity were imposed on both models as part of the maintained hypothesis. As noted above, since the data add up by construction, the adding up conditions also hold. Given the sample specific data used, general conclusions cannot be made with regard to choosing one of these models. The sample specific results, however, provide some information on the merits of the models, and in particular, how well proposed model (9) explains the data compared to the original model under the assumption that $\alpha_0 = 0$.

Although the demand equations are estimated jointly based on the maximum likelihood procedure where individual equation R^2's are not maximized, the R^2's provide some measure of fit (Table 2). The results in the table show that the R^2's for model (9) and the original AIDS differ little across equation. An overall measure of fit for the entire set of demand equations is Theil's information inaccuracy (Theil 1967). The average information inaccuracy over the sample observations is defined as $I = \sum_t \sum_j w_{jt} (\log w_{jt} / w^p_{jt})/T$, where w_{jt} and w^p_{jt} are the observed and predicted values, respectively (e.g., Barten 1993). A smaller value of I indicates a better fit. The results in Table 2 show that model (9) performs somewhat better than the original AIDS, but the values of I are low for both models. Underlying these results, is a value of .96 for the simple correlation between the chain price index and the original price index with $\alpha_0 = 0$.[13] Overall, these fit measures offer little with respect to choosing between models, but they are supportive of the use of the chain price index in model (9), at least for this data set.

Each model has 110 parameters after imposing the restrictions noted above. In model (9), 52 percent of the parameters where statistically different from zero at the 10% level, while in the original AIDS 56 percent were statistically significant. The model income parameters, in combination with the budget shares, indicate deviation from unitary income elasticities, while the model own-price and cross price parameters, also in combination with the budget shares, as well as the income parameters, indicate deviation from unitary and zero price elasticities, respectively.

Income and own-price elasticities for the two models are shown in Table 3. The elasticities are relatively similar between the two models (the largest log difference in the income elasticities is 35.5%, while the largest log difference in the own-price elasticities is 8.2%). Both sets of elasticities seem plausible. For each model, most of the beverages are normal goods, with two, apple juice and grape juice, being neutral, based on the insignificance of their effects. The own-price elasticities for model (9) ranged from -.23 for milk and shakes to -2.08 for vegetable juice; the range for the original AIDS was similar from -.22 to -2.09 for the same beverages. The majority of cross-price elasticity estimates in both model were positive or insignificant, indicating substitute or neutral relationships.

Concluding Comments

Of the number of demand system models developed over the past decades, the AIDS clearly stands out. It is probably the most widely used demand system model for empirical analysis. Identification of the model's price index parameter α_0, however, has been problematic. It simply cannot be estimated for most data sets. Assumed values for α_0 are often used.

A model in the same category as the LA/AIDS is developed here, based on the differential approach used to develop the Rotterdam demand system. The approach is founded in demand theory, beginning with the cost function as in the case of the original AIDS. The model in this paper is based on a general cost function, while the original AIDS is based on a specific cost function. That difference leads us to a somewhat different but more tractable model than the original AIDS, with respect to the price index. A differential AIDS is first developed, followed by a levels version counterpart. The two step procedure reveals a chain price index as an alternative for the problematic AIDS price index. The model developed is not some ad hoc fix for the original AIDS, but stands by itself, comparable to the original AIDS as well as the Rotterdam model with respect to consumer demand theory and flexibility. The easy-to-calculate, chain-price index of the model, compared to the problematic original AIDS price index, is the primary advantage for the specification. Another advantage is that specification can be easily compared with the Rotterdam model and the AIDS-Rotterdam variants suggested by Barten (1993).

An empirical analysis of beverage demands indicates that the model based on the chain price index performs as well as the original AIDS with $\alpha_0 = 0$. The elasticities for both models appear to be reasonable. It should be recognized, however, that the results are data specific, and general conclusions cannot be drawn on model choice. Nevertheless, this initial empirical analysis is supportive of the AIDS-like demand system based on the chain-price-index developed here.

Footnotes

1. Choice of parameterization is an empirical issue, based on how well the model fits the data, as well as the parsimoniousness of the model with respect to its parameter space to be estimated in the first place. This issued is discussed by Barten (1993) with respect to the choice of various parameterizations involving the features of the AIDS and the Rotterdam model. For example, in the Rotterdam model, it is not the AIDS price coefficient γ_{ij} that is constant, but rather the Slutsky coefficient $k_{ij} = \gamma_{ij} + \beta_i\beta_j \log(x/P) - w_i \Delta_{ij} + w_i w_j$, where Δ_{ij} is the Kronecker delta, that is. Likewise, in the present model $\gamma_{ij} = \partial^2\log c(p, u) / \partial(\log p_i) \partial(\log p_j)$ is constant, not the original AIDS price coefficient.

2. This interpretation, however, does not apply to the original AIDS, since the partial derivative of budget share i with respect to the log of price j (utility constant) involves the term $\beta_i\beta_j \log(x/P)$.

3. For example, doubling all prices leaves q_i unchanged, and, hence, total expenditures x double, leaving p_i/x unchanged.

4. P^c is a special case of the chain geometric mean price index, In general, this chain price index tends to be better than binary price indices in measuring price changes over time (Forsyth and Fowler 1981; Silver 1984).

5. The procedure to obtain a levels version for equation (7) could also be used to obtain a levels version for the Rotterdam model.

6. See Moschini (1995) for other suggested alternative price indexes, as well as further discussion on the Stone price index.

7. $\log w_i = \log p_i + \log q_i - \log x$; thus, $d\log w_i = dw_i/w_i = d\log p_i + d\log q_i - d\log x$; and $dw_i = w_i\, d\log p_i + w_i\, d\log q_i - w_i\, d\log x$.

8. The budget shares in the Divisia and chain price indexes are treated as constants, following the Rotterdam model.

9. The demand system could be rationalized based on a multi-stage budgeting process as well. For example, the consumer budget problem might be viewed as first an allocation of total consumer expenditures or income to broad groups of commodities (e.g., food); then allocation of each broad group's portion of income to more specific subgroups of goods (e.g., beverages), and then allocation of each subgroup's portion to individual products in that subgroup (e.g., individual beverage products).

10. Data are for U.S. grocery stores doing \$2 million and greater annual sales, Wal-Mart stores excluding Sam's Clubs, mass-merchandisers, and drug stores doing \$1 million and greater annual sales.

11. Given the relatively short time period studied, impacts of past consumption reflecting product inventories held by consumers and habits were not considered. The four-weekly observations, however, may to some extent mitigate the impact of product inventories held by consumers. Also, given the limited data including lack of good instrumental variables, other demand issues such as independence of total expenditure on the 12 beverage categories (conditional income) and the error terms added to each beverage demand equation were not examined.

12. Differential model (7) could be estimated as well, and, for example, could be compared with the (differential) Rotterdam model or AIDS-Rotterdam-model combinations. In the present analysis, however, we focus on levels model (9) for comparison to the original AIDS in levels.

13. Given the high correlation between $\log P^c$ and $\log P$, as well as the difficulty in estimation of $\log P$ to begin with, it was not possible to estimate a model that included both price indices

together through an additional weighting coefficient.

References

Barten, A. P. (1964). "Family Composition, Prices and Expenditure Patterns." In P.E. Hart, G. Mills and J.K. Whittaker (eds.), *Econometric Analysis of National Economic Planning.* London: Butterworth, 277-292.

Barten, A. P. (1969). "Maximum Likelihood Estimation of a Complete System of Demand Equations." *European Economic Review* 1:7-73.

Barten, A.P. (1966). "Theorie en empirie van een volledig stelsel van vraegvergelijkingen." Doctoral Dissertation, Rotterdam: University of Rotterdam.

Barten, A. P. (1989). "Towards a Levels Version of the Rotterdam and Related Demand Systems." In B. Cornet and H. Tulkens (eds.), *Contributions to Operations Research and Econometrics: The Twentieth Anniversary of Core.* London: The MIT Press, 442-465.

Barten, A. P. (1993). "Consumer Allocation Models: Choice of Functional Form." *Empirical Economics* 18:129-158.

Basmann, R.L. (1956). "A Theory of Demand with Preference Variables." *Econometrica* 24:47-58.

Brown, M., and Lee, J. (2008). "Impacts of Income on Price and Income Responses in the Differential Demand System." *Journal of Agricultural and Applied Economics*, 40,2 (August):593-608.

Brown, M., and Lee, J. (2010). "Preference Variable Impacts in Direct and Inverse Differential Demand Systems." *American Journal of Agricultural Economics*, 40,2 (April):889-898.

Deaton, A.S. and J. Muellbauer. (1980a). "An Almost Ideal Demand System." *American Economic Review* 70:312-26.

Deaton, A.S. and J. Muellbauer. (1980b). *Economics and Consumer Behavior*. Cambridge, MA: Cambridge University Press.

Eales, J., and L. Unnevehr. (1988). “Demand for Beef and Chicken Products: Separability and Structural Change.” *American Journal of Agricultural Economics* 70 (3): 521-32.

Forsyth, F.G. and R.F. Fowler (1981). “The Theory and Practice of Chain Price Index Number,” *Journal of the Royal Statistical Society*, *series A (General)*, 144(2): 224-46.

Green, R. and J.M. Alston. (1990). “Elasticities in AIDS Models.” *American Journal of Agricultural Economics* 72,2:442-445.

Green, R. and J.M. Alston. (1991). “Elasticities in AIDS Models: A Clarification and Extension.” *American Journal of Agricultural Economics* 73, 3:874-875.

Ichimura, S. (1950-51). AA Critical Note on the Definition of Related Goods.@ *Review of Economic Studies* 18:179-183.

Moschini, G., Moro, and R. D. Green. (1994) “Maintaining and Testing Separability in Demand Systems,” *American Journal of Agricultural Economics*, 76: 61-73.

Moschini, G. (1995). "The Stone Index in Demand Estimation," *American Journal of Agricultural Economics*, 77: 63-68.

Pashardes, P. (1993). “Bias in Estimating the Almost Ideal Demand System with the Stone Index Approximation,” *The Economic Journal*, 103:908-15.

Pollak, R.A. and T.J. Wales, H. (1992*).* *Demand System Specification and Estimation*, New York: Oxford University Press.

Silver, M.S. (1984). “Criteria for Choosing between Alternative Consumer Price Index Number Formulae with Special Reference to Chained Indices,” *The Statisitician*, 33: 229-237.

Theil, H. (1967). *Economics and Information Theory*, Amsterdam: North Holland Publishing

Company.

Theil, H. (1971). *Principles of Econometrics*. New York: John Wiley & Sons, Inc. (North-Holland Publishing Company).

Theil, H. (1975). *Theory and Measurement of Consumer Demand*, Vol. I. Amsterdam: North-Holland Publishing Company.

Theil, H. (1976). *Theory and Measurement of Consumer Demand*, Vol. II. Amsterdam: North-Holland Publishing Company.

Theil, H. (1980). *The System-Wide Approach to Microeconomics*, Chicago: University of Chicago Press.

Theil, H. and R.B. Brooks (1970-71). "How Does the Marginal Utility of Income Change When Real Income Changes?" *European Economic Review*, 2:218-240..

Tintner, G. (1952). "Complementarity and Shifts in Demand." *Metroeconomica* 4:1-4.

Wan, G. H. (1998). "Linear Estimation of the Nonlinear Almost Ideal Demand System: A Monte Carlo Study," *Applied Economics Letters*, 5: 181-6.

Zhen, C., M. Wohlgenant, S. Karns and P. Kaufman. (2011). "Habit Formation and Demand for Sugar-Sweetened Beverages." *American Journal of Agricultural Economics* 93(11) January: 175-193.

Table 1. Descriptive Statistics of Beverage Sample, Week Ending 7/21/07 through 8/07/10.

Beverage	Per Capita Gallons/Week		Price: $/Gallon		Budget Share	
	Mean	Std Dev	Mean	Std Dev	Mean	Std Dev
Orange[1]	0.154	0.011	5.669	0.212	0.062	0.006
Grapefruit[1]	0.006	0.000	6.309	0.078	0.003	0.000
Apple[1]	0.059	0.007	4.451	0.268	0.019	0.003
Grape[1]	0.016	0.002	6.456	0.287	0.008	0.001
Remaining Fruit Juice[1]	0.056	0.004	7.211	0.182	0.029	0.002
Vegetable	0.033	0.003	7.332	0.177	0.017	0.002
Juice Drinks[2]	0.433	0.063	3.987	0.137	0.122	0.011
Carbonated Water	0.055	0.004	3.055	0.146	0.012	0.001
Water	0.901	0.129	1.671	0.057	0.107	0.012
Soda	1.496	0.114	3.113	0.117	0.331	0.017
Liquid Tea	0.122	0.021	3.696	0.090	0.032	0.004
Milk & Shakes	0.975	0.029	3.725	0.364	0.259	0.024

[1] 100% juice.

[2] Less than 100% juice.

Table 2. Model Goodness of Fit.

Beverage	Coefficients of Determination (R^2)	
	Model (9)[1]	Original AIDS[2]
Orange[3]	0.926	0.927
Grapefruit[3]	0.787	0.788
Apple[3]	0.820	0.822
Grape[3]	0.804	0.804
Remaining Fruit Juice[3]	0.878	0.876
Vegetable	0.626	0.625
Juice Drinks[4]	0.913	0.912
Carbonated Water	0.865	0.865
Water	0.810	0.812
Soda	0.727	0.729
Liquid Tea	0.937	0.938
Milk & Shakes	0.971	0.971
	Average Information Accuracy	
	7.76E-06	8.53E-06

[1] Based on chain price index.
[2] Based on $\alpha_0 = 0$.
[3] 100% juice.
[4] Less than 100% juice.

Table 3. Conditional Income and Uncompensated Own- Price Elasticities.

Beverage	Model (9)[1]		Original AIDS[2]		Model (9)[1]		Original AIDS[2]	
	Income Elasticity	Standard Error	Income Elasticity	Standard Error	Own-Price Elasticity	Standard Error	Own-Price Elasticity	Standard Error
Orange	0.996	0.230	1.006	0.239	-1.722	0.248	-1.718	0.250
Grapefruit	0.663	0.371	0.766	0.372	-1.502	0.248	-1.500	0.249
Apple	-0.599	0.578	-0.854	0.561	-1.008	0.149	-0.992	0.149
Grape	0.722	0.627	0.914	0.608	-0.610	0.307	-0.630	0.305
Remaining Fruit Juice	0.626	0.259	0.651	0.264	-1.455	0.235	-1.458	0.235
Vegetable	1.393	0.602	1.635	0.556	-2.076	0.347	-2.093	0.346
Juice Drinks	1.335	0.256	1.460	0.228	-1.018	0.144	-1.085	0.135
Carbonated Water	1.392	0.206	1.477	0.197	-1.393	0.113	-1.394	0.113
Water	1.096	0.452	1.263	0.412	-0.687	0.282	-0.746	0.283
Soda	1.424	0.253	1.317	0.247	-1.173	0.145	-1.179	0.152
Liquid Tea	0.829	0.321	0.952	0.295	-1.135	0.247	-1.180	0.249
Milk & Shakes	0.406	0.122	0.386	0.122	-0.234	0.047	-0.224	0.048

[1] Based on chain price index.

[2] Based on $\alpha_0 = 0$.