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Integration, social distress, and policy formation

Bonn, January 2012
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**ISSN: 1436-9931**

**Published by:**
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This paper is a revised and condensed version of the text of keynote addresses that the author delivered at the European Meeting of the Public Choice Society, Athens, April 2009, and at the International Workshop on Regional Competitiveness and International Factor Movements, Orléans, March 2011.
Abstract

I study the integration of regions in the form of a merger of populations, which I interpret as a revision of people’s social space and their comparison set; I illustrate the way in which a merger can aggravate social distress; and I consider policy responses. Specifically, I view the merger of populations as a merger of income vectors; I measure social distress by aggregate relative deprivation; I demonstrate that a merger increases aggregate relative deprivation; and I show that a social planner is able to reverse this increase by means of least-cost, post-merger increases in individual incomes, but is unable to counter it by relying exclusively on a self-contained income redistribution that retains individual levels of wellbeing at their pre-merger levels.

Keywords: Integration of regions; Merger of populations; Revision of social space; Aggregate relative deprivation; Social distress; Policy responses

JEL classification: D04; D63; F55; H53; P51
1. A brief introduction

In this paper, I review the integration of economies as a merger of populations. When economies merge, a variety of benefits are anticipated: denser markets, increased efficiency and productivity brought about by scale effects, and the like. Classical trade theory has it that integration liberalizes trade and smoothes labor and financial flows. Denser and larger markets improve resource allocation and the distribution of final products. As a result, the welfare of the integrating populations is bound to rise. The picture may not be so bright, however. The merger of groups of people revises their social space and their comparison set. Belonging to a larger society can increase the aggregate level of distress. If so, then, the process of integration may not only enhance welfare but also chip away at the sense of wellbeing.

Using a robust example, I derive four results:

1. I show that the social distress of a merged population is higher than the sum of the levels of social distress of the constituent populations when apart.

2. I calculate the minimal cost that a social planner has to incur who seeks to reverse the adverse repercussion of the merger on aggregate social distress.

3. I calculate the minimal cost that a social planner has to incur who seeks to see to it that no individual ends up being subjected to post-merger distress that is greater than his pre-merger distress.

4. I show that it is impossible to escape the need to appeal to the social planner for funds to help address the post-merger reduction in the individuals’ wellbeing even when there are gainers from the merger who could in principle be taxed to facilitate income transfers to losers; apparently there is not enough of a gain to placate the losers.

In the short concluding section, I remark on how general these four results are.

2. Measuring regional distress

I measure the distress of a population by the sum of the levels of distress experienced by the individuals who constitute the population. I refer to this sum as the aggregate relative deprivation \((ARD)\) of the population. I measure the distress of an individual by the extra income units earned by others in the population, I sum up these excesses, and I normalize by the size of the population. This procedure tracks the seminal work of Runciman (1966) and its
articulation by Yitzhaki (1979) and Hey and Lambert (1980); a detailed description of the measure and of its derivation is in Stark and Hyll (2011). In my definition of relative deprivation, I resort to income-based comparisons, namely, an individual feels relatively deprived when others in his comparison group earn more than he does. I assume that the comparison group of an individual consists of all members of his population.

For an ordered vector of incomes in population $P$ of size $n$, $x=(x_1,\ldots,x_n)$, where $x_1 \leq x_2 \leq \ldots \leq x_n$, I define the relative deprivation of the $i$-th individual whose income is $x_i$, $i=1,2,\ldots,n$, as

$$RD(x_i,x) = \frac{1}{n} \sum_{j=i}^{n} (x_j - x_i).$$

To ease the analysis that follows, an alternative representation of the relative deprivation measure is helpful.

Let $F(x_i)$ be the fraction of those in population $P$ whose incomes are smaller than or equal to $x_i$. The relative deprivation of an individual earning $x_i$ in population $P$ with an income vector $x=(x_1,\ldots,x_n)$ is equal to the fraction of those whose incomes are higher than $x_i$ times their mean excess income; namely,

$$RD(x_i,x) = [1-F(x_i)] \cdot E(x-x_i \mid x>x_i).$$

To obtain Eq. (2) from Eq. (1), I multiply $\frac{1}{n}$ in (1) by the number of the individuals who earn more than $x_i$, and I divide $\sum_{j=i}^{n} (x_j - x_i)$ in (1) by this same number. I then obtain two ratios: the first is the fraction of the population that earns more than the individual, namely, $[1-F(x_i)]$; the second is mean excess income, namely, $E(x-x_i \mid x>x_i)$.

The aggregate relative deprivation is, in turn, the sum of the individual levels of relative deprivation

$$ARD(x) = \sum_{i=1}^{n} RD(x_i,x) = \sum_{i=1}^{n} \left( \frac{1}{n} \sum_{j=i}^{n} (x_j - x_i) \right).$$
ARD(x) is my index of the level of “distress” of population P. (For several usages of this measure in recent related work see Stark, 2010; Stark and Fan, 2011; Stark and Hyll, 2011; Fan and Stark, 2011; Stark et al., forthcoming; and Stark et al., 2012.)

In sections 3-5, the individuals’ incomes are not allowed to decline. I impose this constraint because I do not know the marginal rate of substitution between a fall in relative deprivation and a fall in income, and consequently and for example, I do not know how much income I could take away from an individual whose relative deprivation falls in the wake of the merger, and transfer that income to other individual(s) whose relative deprivation increases in the wake of the merger. Therefore, to guarantee that the wellbeing of an individual will not be reduced in the process, I impose the requirement that incomes cannot be reduced. In section 6, I use a wellbeing function that enables me to trade off lower income for lower relative deprivation.

3. Pre-merger and post-merger distress

I consider the merger of populations 1 and 2 with income vectors (1,2) and (3,4), respectively. Recalling from Eq. (1) that the distress of an individual is measured by the extra income units earned by others in his population (here, in population 1, it is 2-1), the summing up of these excesses (here, this is 1), and normalizing by dividing by the size of the population (here, this is 2), the pre-merger aggregate relative deprivation of population 1 is \( \text{ARD}(1,2) = 1/2 \). The pre-merger level of the aggregate relative deprivation of population 2 is \( \text{ARD}(3,4) = 1/2 \). In the merged population with income vector (1,2,3,4), the aggregate relative deprivation is \( \text{ARD}(1,2,3,4) = 5/2 > 1 = \text{ARD}(1,2) + \text{ARD}(3,4) \).

The result, that a merger entails an increase in aggregate relative deprivation, is neither trivial nor is it to be intuitively anticipated, the reason being that upon integration, the members of the poorer population (“1” and “2”) are subjected to more relative deprivation, whereas a member of the richer population (“3”) is subjected to less relative deprivation. Since one constituent population experiences an increase of its \( \text{ARD} \) while the other experiences a decrease, whether the \( \text{ARD} \) of the merged population is higher than the sum of the \( \text{ARDs} \) of the constituent populations cannot be predetermined. Put differently, in a setting in which others could only bring negative externalities, a smaller population will always experience less aggregate relative deprivation. But in a setting such as ours when others
joining in can confer both negative externalities (of “3” and “4” upon “1” and “2”) and positive externalities (of “1” and “2” upon “3”), it is impossible intuitively to foresee whether the expansion of a population will entail a reduction in aggregate relative deprivation or an increase.\textsuperscript{1}

To reiterate: in the example, incomes are held constant; the incomes of members of a constituent population are not affected by its merger with another population. I assume that a merger changes the social comparisons space that governs the sensing and calculation of relative income (relative deprivation), but that it leaves absolute incomes intact.

I next ask how a social planner who is concerned about the increase of the aggregate level of social distress will be able to respond in a cost-effective manner. Since the task is to maintain the aggregate, not the individual pre-merger level of distress, individuals’ relative deprivation may fall, rise, or stay the same.

4. Bringing down cost-effectively the aggregate level of relative deprivation to a level equal to the sum of the pre-merger levels of aggregate relative deprivation in the two populations when apart

Ordering the individuals by their incomes, I start “pumping” incomes from the bottom, and simultaneously gauge the aggregate relative deprivation response. The two processes move in tandem, and in opposite directions. The pumping from below is ratcheted up the hierarchy of the individuals, the set of the individuals to be “treated” is expanded a step at a time if the aggregate relative deprivation is still higher than it was prior to the merger, and the pumping ceases when the aggregate relative deprivation reaches its pre-merger level. Thus, I start by giving the individual earning 1 an additional unit of income; this is insufficient to bring down aggregate relative deprivation to its pre-merger level. I therefore add the next individual (the individual whose pre-merger income was 2) and proceed to increase the incomes of each of these two individuals whose incomes, for now, are 2 each. At the point where these two incomes are elevated to $11/4$ each, I obtain income distribution $(11/4,11/4,3,4)$ with $ARD(11/4,11/4,3,4) = 1$. Thus, in order to bring the aggregate relative deprivation in the merged population to the sum of the pre-merger levels, a social planner has to transfer $7/4$ units of income to the individual earning 1, and $3/4$ units of income to the individual earning

\textsuperscript{1}To see the variation in the externality repercussion even more starkly, note that when “3” joins “1” and “1,” he confers a negative externality on the incumbents; when “3” joins “5” and “5” he confers neither a negative externality nor a positive externality on the incumbents; and when “3” joins “4” and “5,” he confers a positive externality on incumbent “4.”
2, which gives $10/4$ as the total cost of implementing the policy. This cost can be conceived as a lower bound on the productivity and efficiency gains that the merger of the two regions has to yield to retain a measure of the population’s wellbeing at its pre-merger level.

5. Ensuring, cost-effectively, that no individual in the merged population senses higher relative deprivation than the relative deprivation that he had sensed prior to the merger

Here I ask how a social planner who is concerned that the individuals’ pre-merger level of distress should not increase will respond in a cost-effective manner.

I order the individuals by their pre-merger levels of relative deprivation, starting from the right; if individuals have the same pre-merger level of relative deprivation, I place leftmost the one with the lower income. I first raise the incomes at the top of the constructed hierarchy of the levels of relative deprivation. I do so in order to equate the levels of relative deprivations of the top-income individuals with their pre-merger levels of relative deprivation. Then, because the comparisons that yield relative deprivation are with incomes on the right in the income hierarchy, the changes made at the top determine how much incomes farther down the hierarchy have to be raised as I move leftwards. Thus, in the merged population with income vector $(1,2,3,4)$, the ordering according to the descending pre-merger levels of relative deprivation (with the lower of the two incomes associated with the same level of relative deprivation placed leftmost) is $(1,3,2,4)$. I pick first “for treatment” the individual with income 4. Noting that his relative deprivation was not increased as a result of the merger, I leave his income as before and thus, $(1,3,2,4)$ is the resulting income distribution. Moving leftwards, I next attend to the individual with income 2. Since his $RD$ now is $3/4$ whereas prior to the merger it was 0, I need to raise his income to 4. Consequently, I obtain income distribution $(1,3,4,4)$. Proceeding further leftwards to “3,” I see that now, as prior to the merger, the $RD$ of “3” is 1/2, so no increase in income is needed in this case. Thus, I obtain the income distribution $(1,3,4,4)$. Because the remaining individual with income 1 has now $RD$ of 2 whereas prior to the merger he had $RD$ of 1/2, I need to increase his income to 3 because then his $RD$ will be 1/2. Thus, the final income vector is $(3,3,4,4)$, which gives 4 as the total cost of implementing the policy.

Not surprisingly, since the constraint on implementing this policy is stricter than the constraint on implementing the preceding policy, enacting the latter policy is costlier $(4 > 2.5)$. As in the preceding case, this cost can be interpreted as a lower bound on
the productivity and efficiency gains that the merger has to yield in order to retain a measure of the population’s wellbeing at its pre-merger level.

6. The financial unfeasibility of a self-contained, non-publicly-financed policy aimed at retaining individuals’ levels of wellbeing at their pre-merger levels

I now relax the assumption that individuals’ incomes cannot be reduced. I assume that the wellbeing of individual $i$, $i = 1,2,...,n$ is a function of his absolute income and of his relative deprivation, such that the preferences of an individual in population $P$ with an ordered income vector $x$ are

$$u_i = u(x_i, x) = \alpha x_i - (1 - \alpha) RD(x_i, x)$$

(4)

where $0 < \alpha < 1$.

As already noted, when a population with income vector $(1,2)$ joins a population with income vector $(3,4)$, the relative deprivation of “3” falls. Thus, his wellbeing, as measured by equation (4), rises. The levels of relative deprivation of “1” and “2” increase, so their wellbeing takes a beating. It is tempting then to skim off income from “3,” who gains as a consequence of the merger, and disburse that income to those who experience a loss as a consequence of the merger, such that following the merger no individual will be worse off in terms of wellbeing as defined in equation (4). The maximal income that can be taken away from “3” is a little less than 1. The reason is that if a full income unit were to be taken away, “3” will experience relative deprivation of 1/2 - just as prior to the merger - and his income will be lower than prior to the merger. With the same relative deprivation and a lower income, the wellbeing of “3” is reduced. But imagine for a minute that one income unit (rather than a little less than that) is indeed taken away; to yield the maximal gain to $(1,2)$ that a unit of income could confer, the unit income has to be given to “1” such that the (hypothetical) income distribution will become $(2,2,2,4)$. In this case, the relative deprivation of “1” will be restored to 1/2. However, “2” (the second individual in this last income vector) is exposed to more relative deprivation than prior to the merger ($RD(2,(2,2,2,4)) = 1/2 > RD(2,(1,2)) = 0$), and there is no income around that can be taken away to appease this individual. But of course, and as already noted, we cannot even take away from “3” a full unit of income; we can at most take a little less than one unit, and therefore, a “tax and transfer” scheme cannot achieve its aim because there is not enough of a gain to compensate the losers while still keeping the gainer as well off as prior to the merger.
7. Generalizations and conclusion

In related writings (Stark, 2010; Stark et al., 2011) and in work in progress (Stark and Jakubek, 2011) it is shown that all four results hold or apply when each of the merged populations is of any size, when the populations overlap, when incomes are not (pair-wise) distinct, and when there are several gainers rather than just one. The only case in which a merger does not exacerbate aggregate deprivation is when the income distribution of the two populations is identical.

An increase in aggregate relative deprivation is a downside of the integration of regions. It puts a strain on the individuals in the merged population, casting a shadow on the production and trade (scale and scope) benefits anticipated from the integration. As already mentioned, the integration of regions is expected to increase efficiency. When the possibility of a merger is contemplated, an interesting question to address would then be whether the anticipated boost in productivity will suffice to pay for the cost of the policies discussed above.

The result that the ARD of two merged regions is higher than the sum of the ARDs of the constituent regions when apart carries through to the case of the merger of three (or more) regions. This follows straightforwardly from the following consideration. Suppose that there are three regions with populations $P_1$, $P_2$, and $P_3$ whose income vectors are (1,2), (3,4), and (5,6), respectively. In section 3, I already showed that $ARD(1,2,3,4) > ARD(1,2) + ARD(3,4)$. By the same reasoning, $ARD(1,2,3,4,5,6) > ARD(1,2,3,4) + ARD(5,6)$. And by transitivity, it follows that $ARD(1,2,3,4,5,6) > ARD(1,2) + ARD(3,4) + ARD(5,6)$. (This type of reasoning can be replicated for the case of the merger of any number of regions.) A different way of expressing this result is that an increase in the number of merged regions cannot reverse the ARD repercussion of the merger of fewer regions.
References


