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Price elasticities, joint products, and international trade

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This paper extends the basic results of Houck's insight for derived demand elasticities for the case of joint products by allowing for the possibility of the joint and raw products being traded. Theoretical relationships between individual demands for a set of jointly-produced commodities that are traded and composite demand for the raw product from which the joint products originate are derived. It is shown that while the derived price elasticity of domestic demand retains the same form as Houck's original formula, the relevant price elasticities of demand to include in the formula are elasticities of total demand instead of domestic demand elasticities. Using the USA soybean industry as an example, this generalised formula that takes into account trade is implemented to calculate the elasticity of total demand for USA soybeans. The usefulness of this formula for policy-makers to trace out the impacts of changes in market conditions and trade policy in the joint-products, and how it will impact the price elasticity of domestic and total demand for the raw product, is demonstrated.

1. Introduction

The purpose of the present paper was to derive theoretical relationships between individual demands for a set of jointly-produced commodities that are traded, and the composite demand for the raw product from which the joint products originate. In particular, this paper extends the basic results of Houck's insight for derived demand elasticities for the case of joint products by allowing for the possibility of the joint and raw products being traded.

Examples of joint products being produced from a basic agricultural commodity are common. Examples include meat and wool from lambs, ribs and roasts from beef cattle, and meal and oil from soybeans. The phenomena of joint products is less common in other industries such as manufacturing where usually multiple inputs are combined to make up a single output. The economics of joint products and the property that these products cannot be produced separately has implications for the price elasticity of demand for the underlying basic commodity. This relationship was recognised by Houck

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(1964) who showed the price elasticity of demand for the basic commodity (raw farm product) is a weighted harmonic average of the price elasticities of the joint products (if all coefficients are computed at the same level of marketing). However, Houck's work did not consider the case where the joint and raw products are traded which is often the case with agricultural commodities. The present paper is concerned with generalising Houck's work to allow for this possibility and to demonstrate the usefulness of this formula for analysts to trace out the impacts of changes in market conditions and trade policy in the joint-products and how it impacts the price elasticity of demand for the raw product.

2. Joint products and international trade

In deriving the relationship we could follow Houck's general approach and use the case of a basic commodity X that gives rise to joint products, X_1 and X_2. But to make the exercise more interesting we depict the case of the USA soybean industry, in-line with our empirical example and discussions in relation to trade policy in the subsequent sections.[1] Choosing a specific industry but keeping the model simple does not detract from the generality of our results; we could have also derived the relationship in terms of any industry that exhibits joint products.

Consider the following simplified structure of the USA soybean sector represented by

$$M^d = D_{md}(P_m) \quad \text{[Soybean meal domestic demand]} \tag{1}$$

$$M^x = D_{mx}(P_m^f) \quad \text{[Soybean meal export demand]} \tag{2}$$

$$O^d = D_{od}(P_o) \quad \text{[Soybean oil domestic demand]} \tag{3}$$

$$O^x = D_{ox}(P_o^f) \quad \text{[Soybean oil export demand]} \tag{4}$$

$$M^s = \alpha B^d \quad \text{[Soybean meal supply]} \tag{5}$$

$$O^s = \beta B^d \quad \text{[Soybean oil supply]} \tag{6}$$

$$P_b = P_m\left(\frac{M^s}{B^d}\right) + P_o\left(\frac{O^s}{B^d}\right) - MM \quad \text{[Farm-level price of soybeans]} \tag{7}$$

$$B^s = S_b(P_b) \quad \text{[Supply of soybeans]} \tag{8}$$

$$B^x = D_{bx}(P_b^f) \quad \text{[Soybean export demand]} \tag{9}$$

$$M^d = M^s - M^x \quad \text{[Market-clearing for soybean meal]} \tag{10}$$

[1] It is important to stress that at the time Houck developed this derived price elasticity of demand formula that international trade in the soybean complex was trivial. Thus, it was not necessary at that time to account for trade in the joint and raw products.

$$O^d = O^s - O^x \quad \text{[Market-clearing for soybean oil]} \tag{11}$$

$$B^d = B^s - B^x \quad \text{[Market-clearing for soybeans]} \tag{12}$$

$$P_m^f = g(P_m) \quad \text{[Foreign soybean meal price]} \tag{13}$$

$$P_o^f = h(P_o) \quad \text{[Foreign soybean oil price]} \tag{14}$$

$$P_b^f = k(P_b) \quad \text{[Foreign soybean price]} \tag{15}$$

where M^d, M^s, and M^x are quantities of soybeans demanded, supplied, and exported; O^d, O^s, and O^x are quantities of soybean oil demanded, supplied, and exported; B^d, B^s, and B^x are quantities of soybeans demanded, supplied, and exported; α and β are the fixed yields of soybean meal and soybean oil supplied per unit of soybeans demanded; MM is the marketing margin (per unit of soybeans demanded); P_m, P_o, and P_b are domestic prices of soybean meal, soybean oil, and soybeans, respectively; and P_m^f, P_o^f, and P_b^f are the foreign prices of soybean meal, soybean oil, and soybeans, respectively.

This fifteen-equation model characterises competitive equilibrium in the USA soybean market. As the model indicates, the soybean market is characterised by joint products and international trade in both the jointly-produced soybean products and the raw product. Further maintained assumptions include homogenous products internationally and less than perfectly competitive markets with elasticities of price transmission for soybean meal, soybean oil, and soybeans not necessarily being equal to 1. Following Houck, we assume that the farm price of soybeans can be represented as a weighted average of returns from soybean meal and soybean oil less the processing (crushing) margin, equation (7). As in Houck, we assume fixed yields for soybean meal and soybean oil so upon substituting for M^s and O^s from (5) and (6) we obtain

$$P_b = \alpha P_m + \beta P_o - MM \tag{16}$$

Because our interest is in the expression for derived total demand for the raw product (i.e., soybeans) in the presence of international trade in the jointly-produced products (soybean meal and soybean oil) and the raw product (soybeans), equation (16) is the focus of attention for the rest of the present paper. We only consider in this paper the case of a constant marketing margin. The case in which the margin is variable (depending upon either output prices or quantity of the raw material) can be analysed along the lines suggested by Houck.

3. Derived elasticity of demand with international trade

As in Houck, we consider the experiment where the price of the raw product, P_b, is changed exogenously and ask: What is the effect on total quantity

demanded of a given change in soybean price? In regard to equations (1)–(15) this means that this comparative static result can be derived by eliminating equation (8) from the set of equations (taking P_b as exogenous) and evaluating how the quantity of the raw product (the sum of the quantity demanded domestically (B^d) and and the quantity exported (B^x)) changes as the domestic and export markets for soybean meal and soybean oil move to their new equilibrium levels in response to a change in the price of soybeans. In an analogous fashion to Houck, this comparative static result can be characterised through implicit differentiation of the farm price equation. Implicit differentiation of equation (16) with respect to P_b yields

$$1 = \alpha\left(\frac{\partial P_m}{\partial M^d}\right)\left(\frac{\partial M^d}{\partial P_b}\right) + \beta\left(\frac{\partial P_o}{\partial O^d}\right)\left(\frac{\partial O^d}{\partial P_b}\right). \tag{17}$$

But from (10) we obtain

$$\frac{\partial M^d}{\partial P_b} = \frac{\partial M^s}{\partial P_b} - \frac{\partial M^x}{\partial P_b} = \left(\frac{\partial M^s}{\partial B^d}\right)\left(\frac{\partial B^d}{\partial P_b}\right) - \left(\frac{\partial M^x}{\partial P_m^f}\right)\left(\frac{\partial P_m^f}{\partial P_m}\right)\left(\frac{\partial P_m}{\partial M^d}\right)\left(\frac{\partial M^d}{\partial P_b}\right) \tag{18}$$

or

$$\frac{\partial M^d}{\partial P_b} = \frac{\left(\frac{\partial M^s}{\partial B^d}\right)\left(\frac{\partial B^d}{\partial P_b}\right)}{\left[1 + \left(\frac{\partial M^x}{\partial P_m^f}\right)\left(\frac{\partial P_m^f}{\partial P_m}\right)\left(\frac{\partial P_m}{\partial M^d}\right)\right]}. \tag{19}$$

Likewise, from (11) we obtain

$$\frac{\partial O^d}{\partial P_b} = \frac{\left(\frac{\partial O^s}{\partial B^d}\right)\left(\frac{\partial B^d}{\partial P_b}\right)}{\left[1 + \left(\frac{\partial O^x}{\partial P_o^f}\right)\left(\frac{\partial P_o^f}{\partial P_o}\right)\left(\frac{\partial P_o}{\partial O^d}\right)\right]}. \tag{20}$$

Therefore, after substituting (19) and (20) into (17), we obtain

$$1 = \alpha\left(\frac{\partial P_m}{\partial M^d}\right)\frac{\left(\frac{\partial M^s}{\partial B^d}\right)\left(\frac{\partial B^d}{\partial P_b}\right)}{\left[1 + \left(\frac{\partial M^x}{\partial P_m}\right)\left(\frac{\partial P_m^f}{\partial P_m}\right)\left(\frac{\partial P_m}{\partial M^d}\right)\right]} + \beta\left(\frac{\partial P_o}{\partial O^d}\right)\frac{\left(\frac{\partial O^s}{\partial B^d}\right)\left(\frac{\partial B^d}{\partial P_b}\right)}{\left[1 + \left(\frac{\partial O^x}{\partial P_o}\right)\left(\frac{\partial P_o^f}{\partial P_o}\right)\left(\frac{\partial P_o}{\partial O^d}\right)\right]}. \tag{21}$$

This expression can be converted to elasticities by noting that the price elasticity of domestic demand for soybeans and the domestic and export demand elasticities for soybean meal and soybean oil can be expressed as follows: $\eta_{bb} = \left(\frac{\partial B^d}{\partial P_b}\right)\left(\frac{P_b}{B^d}\right)$, $\eta_{mm} = \left(\frac{\partial M^d}{\partial P_m}\right)\left(\frac{P_m}{M^d}\right)$, $\eta_{xm} = \left(\frac{\partial M^x}{\partial P_m}\right)\left(\frac{P_m}{M^x}\right)$, $\eta_{oo} = \left(\frac{\partial O^d}{\partial P_o}\right)\left(\frac{P_o}{O^d}\right)$, and $\eta_{xo} = \left(\frac{\partial O^x}{\partial P_o}\right)\left(\frac{P_o}{O^x}\right)$. The elasticities of price transmission for soybean meal and soybean oil can also be written as $\epsilon_{fm} = \left(\frac{\partial P_m^f}{\partial P_m}\right)\left(\frac{P_m}{P_m^f}\right)$ and $\epsilon_{fo} = \left(\frac{\partial P_o^f}{\partial P_o}\right)\left(\frac{P_o}{P_o^f}\right)$, respectively.

Making use of these definitions of elasticities, the inverse function rule $\left(\text{i.e., } \frac{\partial M^d}{\partial P_m} = \frac{1}{\left(\frac{\partial P_m}{\partial M^d}\right)}\right)$, $\frac{\partial M^s}{\partial B^d} = \alpha$, $\frac{\partial O^s}{\partial B^d} = \beta$, substituting into (21), and re-arranging yields

$$1 = \left(\frac{B^d}{P_b}\right)\eta_{bb}\left[\frac{\alpha^2 P_m}{\left(M^d\eta_{mm} + M^x\eta_{xm}\epsilon_{fm}\right)} + \frac{\beta^2 P_o}{\left(O^d\eta_{oo} + O^x\eta_{xo}\epsilon_{fo}\right)}\right]. \tag{22}$$

Noting from (5) and (6) that $\alpha = \frac{M^s}{B^d}$ and $\beta = \frac{O^s}{B^d}$ and solving for η_{bb} we obtain

$$\eta_{bb} = \frac{P_b}{\left[\frac{\alpha P_m M^s}{\left(M^d\eta_{mm} + M^x\eta_{xm}\epsilon_{fm}\right)} + \frac{\beta P_o O^s}{\left(O^d\eta_{oo} + O^x\eta_{xo}\epsilon_{fo}\right)}\right]}. \tag{23}$$

Finally, noting that the elasticities of total demand for soybean meal and soybean oil can be expressed as

$$\eta_m = \left(\frac{M^d}{M^s}\right)\eta_{mm} + \left(\frac{M^x}{M^s}\right)\eta_{xm}\epsilon_{fm} = \frac{M^d\eta_{mm} + M^x\eta_{xm}\epsilon_{fm}}{M^s}$$

$$\eta_o = \left(\frac{O^d}{O^s}\right)\eta_{oo} + \left(\frac{O^x}{O^s}\right)\eta_{xo}\epsilon_{fo} = \frac{O^d\eta_{oo} + O^x\eta_{xo}\epsilon_{fo}}{O^s}$$

respectively, equation (23) can be expressed as

$$\eta_{bb} = \frac{P_b}{\left(\frac{\alpha P_m}{\eta_m} + \frac{\beta P_o}{\eta_o}\right)}. \tag{24}$$

While the derived price elasticity of domestic demand in equation (24) is shown to retain the same form as Houck's original formula, there is an important distinction in that the relevant price elasticities of demand for the

joint products to include in the formula are the elasticities of total demand. That is, instead of using the domestic demand elasticities for the jointly-produced products – which is what Houck's original formula suggests – we should use the elasticities of total demand which are weighted averages of domestic demand elasticities and the product of the elasticity of export demand and the elasticity of price transmission when the products are exported. This more general formula that accommodates trade also reverts (or nests) Houck's original formula in the case where there is no trade in the joint products making $M^x = O^x = 0$, meaning that $\eta_m = \eta_{mm}$ and $\eta_o = \eta_{oo}$.

One could also obtain these results by reverting to the original set of equations, substituting (13) into (2), (14) into (4), adding (1) and (2) together to obtain total soybean meal demand as a reduced-form function of the domestic soybean meal price, and adding (3) and (4) together to obtain total soybean oil demand. In this form, this new set of equations represents the exact same structure for the joint products as considered by Houck, so that his basic formula still applies with domestic elasticities for the jointly produced products replaced with total demand elasticities.[2]

There will also be instances where the appropriate policy variable will be the elasticity of total demand for soybeans which, in the presence of trade in soybeans, can be shown using the result from (24) to be of the following form

$$\eta_b = s^d \cdot \left[\frac{P_b}{\left(\frac{\alpha P_m}{\eta_m} + \frac{\beta P_o}{\eta_o} \right)} \right] + (1 - s^d) \cdot \eta_{xb} \epsilon_{fb}, \tag{25}$$

where s^d is the share of domestic disappearance of soybeans $s^d = \left(\frac{B^d}{B^s} \right)$, η_{xb} is the elasticity of export demand for soybeans, and ϵ_{fb} is the elasticity of price transmission for soybean price. This formula shows how trade in the joint products and trade in the raw product affect the elasticity of total demand for the raw product.

4. Empirical example

To provide an example of implementing this formula and to investigate what the distinction of allowing for trade means empirically, we continue with our soybean illustration and calculate the derived elasticity of domestic and total

[2] While this approach may seem obvious, especially after the fact, it was not all that transparent until the result was derived using the approach of Houck. As the ensuing analysis shows, deriving the elasticity of total demand for the raw product using the general structure laid out in equations (1)–(15) allows us to model specific policy interventions in trade and changes in market conditions of the joint products.

demand for USA soybeans. We use elasticity estimates and price and quantity data from Jiang *et al.* (2001) who developed a model of the world soybean complex that consists of four regions: USA, China, European Union (EU), and Rest of the World (ROW). For the empirical illustration we specifically need estimates of the elasticities of domestic demand for soybean meal and soybean oil in each region which are as follows:

Country	η_{mm}	η_{oo}
USA	−0.13	−0.18
China	−0.60	−0.33
EU	−0.58	−0.22
ROW	−0.04	−0.09

Based on averages taken over the period 1996–1999, the USA has been a net exporter of both soybean meal and soybean oil, China a net importer of both soybean meal and soybean oil, the EU a net importer of soybean meal but a net exporter of soybean oil, and the ROW a net exporter of soybean meal but a net importer of soybean oil. In the short run, the elasticity of export demand for soybean meal facing USA producers can be calculated as an import-share weighted sum of the import demand elasticities of soybean meal for China and EU. Assuming no time for soybean meal supply to respond to a price change, the elasticity of import demand is simply the country's elasticity of domestic demand divided by its imports as a share of total consumption.[3] China's soybean meal imports over the period 1996–1999 were about 20.7 per cent of total consumption and the EU's imports were about 51.5 per cent. Imports from China accounted for 15.9 per cent of all exports and imports from the EU accounted for 84.1 per cent.[4] Therefore, the elasticity of export demand for USA soybean meal is estimated to be −1.41.[5] In the case of soybean oil, China imports about 32.8 per cent of its

[3] The reason meal and soybean oil supply in other countries would likely be completely inelastic in the short run (within one year time period) is because they are closely related to their domestic supplies of soybeans, which likely would not respond immediately to a change in current year price. The longer the time period allowed for adjustment, the more likely soybean meal and soybean oil supply would respond to a change in price. This means that the export demand elasticities we calculate are best viewed as lower bound estimates to the true estimates.

[4] Without detailed information of exports by destination, we make the simplifying assumption that world shares of exports is representative of the breakdown of shares for USA exports by destination.

[5] The elasticity of export demand for USA soybean meal is calculated as follows:

$$\eta_{xm} = (0.159)(-0.60/0.207) + (0.841)(-0.58/0.515) = -1.41.$$

soybean oil and the ROW imports about 10.2 per cent. Imports from China accounted for 44.7 per cent of all exports and imports from the ROW accounted for 55.3 per cent. Therefore, the elasticity of export demand for USA soybean oil is estimated to be -0.94.[6]

The formula in (24) also requires estimates of the elasticities of price transmission for soybean meal and soybean oil. Using the Rotterdam CIF price as representative of the foreign price, we regressed this price on the USA price for each of the markets. Based on these regressions the following estimates of the elasticities of price transmission were obtained: $\epsilon_{fm} = 0.89$ and $\epsilon_{fo} = 0.97$.[7] Using these elasticities of export demand and price transmission, together with elasticities of domestic demand, and the respective shares of exports for soybean meal and soybean oil in the elasticity of total demand formulas, we obtain:

$$\eta_m = 0.794(-0.13) + 0.206(0.89)(-1.41) = -0.36$$

$$\eta_o = 0.870(-0.18) + 0.130(0.97)(-0.94) = -0.27.$$

Over the period 1996–1999, the USA soybean price was approximately \$215/MT (or \$5.85/bu), the soybean meal price was \$210/MT (or \$190.51/short ton), the soybean oil price was \$466/MT (or 21.14 cents/lb), and the yields of soybean meal (α) and soybean oil (β) were 0.79 and 0.17, respectively. Substituting these numbers into equation (24) we calculate that the price elasticity of domestic demand for USA soybeans is

$$\eta_{bb} = \frac{215}{\left[\dfrac{0.79(210)}{-0.37} + \dfrac{0.17(466)}{-0.27}\right]} = -0.29.^{8}$$

For comparison sake, to gauge what impact trade in the joint products has on the price elasticity of domestic demand of the basic commodity, we can

[6] The elasticity of export demand for USA soybean meal is calculated as follows:

$$\eta_{xo} = (0.447)(-0.33/0.328) + (0.553)(-0.09/0.102) = -0.94.$$

[7] The following linear regressions were used to estimate elasticities of price transmission using price data from the *Oilseeds: World Markets and Trade* (USDA, FAS) over the period 1974/1975–2001/2002:

$$P^f_{m,t} = 24.052 + 0.944 * P_{m,t}, \text{ with } \epsilon_{fm} = 0.89 \text{ (calculated at the sample mean)}$$

$$P^f_{o,t} = 15.483 + 0.996 * P_{o,t}, \text{ with } \epsilon_{fo} = 0.97 \text{ (calculated at the sample mean)}$$

where $P^f_{m,t}$ and $P^f_{o,t}$ are the annual average Rotterdam CIF prices for soybean meal and soybean oil and $P_{m,t}$ and $P_{o,t}$ are the annual average USA price for soybean meal and oil, respectively.

[8] In light of the discussion above in note 3, because the export demand elasticities are lower bound estimates of the true estimates then it is reasonable to expect that the estimate of price elasticity of domestic demand for soybeans is even larger (in absolute value) than 0.29.

use the domestic own-price elasticities of demand for soybean meal and soybean oil instead of the total elasticities of the joint products. This would imply a price elasticity of domestic demand for USA soybeans of

$$\eta_{bb} = \frac{215}{\left[\frac{0.79(210)}{-0.13} + \frac{0.17(466)}{-0.18}\right]} = -0.13.$$

Therefore, the correct elasticity that takes account of the trade in joint products using the elasticities of total demand for soybean meal and soybean oil results in an estimated elasticity that is more than double – a magnitude of 2.2 times larger. Clearly, trade in the joint products can have an important impact on the underlying responsiveness of the basic commodity to changes in price.

Finally, taking account of trade in the soybeans we can calculate the elasticity of total demand for USA soybeans using (25). This requires estimates of the elasticity of export demand for USA soybeans and the elasticity of price transmission for soybean price. Using two additional auxillary regressions the elasticity of export demand for USA soybeans was estimated to be -0.63 and the soybean elasticity of price transmission was estimated to be 0.88.[9] Substituting these numbers into equation (25) we calculate that the price elasticity of total demand for USA soybeans is

$$\eta_b = 0.656(-0.29) + 0.344(-0.63)0.88 = -0.38.$$

This empirical example illustrates a well understood phenomenon that allowing for trade results in a more elastic demand. The magnitude of the increase in the responsiveness of demand, however, is relatively small with the elasticity of total demand only being 1.3 times larger than the elasticity of domestic demand. Interestingly, taking account of trade in the joint products has a more profound impact on the elasticity of total demand than does taking account of trade in the raw product. This result stems from the significant increase in the elasticity of domestic demand when trade in the

[9] To estimate the elasticity of export demand for USA soybeans the following regression was estimated using annual data over the period 1974/75–1999/2000:

$$B_t^x = -2047.430 - 39.331 * t - 1.781 * P_{b,t}^f + 0.036 * wgdp_t - 1.113 * stham_t + 486.474 * B_{t-1}^x,$$
with $\eta_{bx} = -0.63$ (calculated at the sample means)

where B_t^x is exports of USA soybeans (millions of bushels) (WASDE, USDA) t is a linear trend; $P_{b,t}^f$ is the Rotterdam, CIF price (\$/MMT) (USDA, FAS); $wgdf_t$ is per capita world GDP (current \$US) (World Bank 2002); $stham_t$ is the sum of annual production in Brazil and Argentina (MMT) (FAO 2002). The elasticity of price transmission was estimated from the following linear regression using price data from the *Oilseeds: World Markets and Trade* (USDA,FAS) over the period 1974/1975–2001/2002:

$$P_{b,t}^f = 29.582 + 1.0163 * P_{b,t}, \text{ with } \epsilon_{fb} = 0.88 \text{ (calculated at the sample mean)}$$

where $P_{b,t}^f$ is the annual average Rotterdam CIF prices for soybeans and $P_{b,t}$ is the annual average USA price for soybeans.

joint products is taken into account (2.3 times larger) and the significant share of domestic demand ($s^d = 0.656$), compared with an elasticity of export demand that is 2.2 times larger in magnitude than the elasticity of domestic demand with a less significant share of export demand ($(1 - s^d) = 0.344$).

5. Implications of generalised derived elasticity of total demand for trade policy

This generalised derived elasticity of total demand for soybeans serves as a useful tool to analyse changes in market conditions and trade policy in the soybean complex on the total demand response for soybeans. In the case of the raw product, the impacts of alternative scenarios is already well understood and straightforward. For example, if the USA becomes a small-country in the international market for soybeans (i.e. $\eta_{xb} \to \infty$), the elasticity of total demand becomes perfectly elastic (i.e. $\eta_b \to \infty$). In this extreme case, the size of the elasticity of domestic demand and whether one allows for trade in the joint products becomes irrelevant. Furthermore, trade liberalisation in foreign soybean markets will cause the elasticity of price transmission to become larger, and in the case of complete liberalisation, this elasticity will approach 1 (i.e. $\epsilon_{fb} \to 1$) making total demand more elastic. Alternatively, trade restrictions in foreign soybean markets will cause the elasticity of price transmission to become smaller and, in the case of complete isolation, this elasticity will approach 0 (i.e. $\epsilon_{fb} \to 0$) making the total demand elasticity equal to the elasticity of domestic demand. What is less transparent is how changes in trade policy and other trade-related scenarios for the joint products will impact the domestic and total demand for soybeans – the focus of the remainder of this section. This problem can be simplified by limiting attention to the impacts on the domestic demand for soybeans using (24). The second term in (25), $(1 - s^d) \cdot \eta_{xb}\epsilon_{fb}$, remains unaffected by changes in trade-policy and other related trade scenarios concerning the joint products, so the impacts on total demand can easily be inferred from how the domestic demand for soybeans is impacted. More specifically, it can be shown that

$$\frac{\partial \eta_b}{\partial z} = s^d \cdot \frac{\partial \eta_{bb}}{\partial z} \tag{26}$$

where z is either an elasticity of export demand or price transmission for one of the joint products (i.e. η_{xm}, η_{xo}, ϵ_{fm} or ϵ_{fo}) that can be used to depict changes in market conditions and trade policy for the joint products.

5.1 Small-country case

Recent trade statistics in the world soybean meal market from the USDA,ERS (July 2002) reveal that the USA is losing market share as

production in other areas expands (i.e. South America). World exports of soybean meal increased from 2.826 MMT in 1964 to 44.468 MMT in 2001. Over this period the USA share shrunk from 0.65 in 1964 to 0.16 in 2001 compared to South America whose share increased from 0.04 (negligible) to 0.60 in 2001. Over the last decade USA exports have remained relatively stagnant at around 6.390 MMT whereas South America exports have almost doubled from 14.366 MMT in 1991 to 26.867 MMT in 2001. One possible scenario, if predictions of further expansion of South American soybean production and crushing capacity transpire, is that the USA would become a small-country in this international market. The extreme case is that the elasticity of USA foreign demand for soybean meal goes to infinity (i.e. $\eta_{xm} \to \infty$). While one might expect this to cause derived domestic demand for soybeans to become infinite, (24) shows that the derived elasticity of domestic demand becomes proportional to the elasticity of total demand for oil:

$$\eta_{bb} = \left(\frac{\eta_o \cdot P_b}{\beta P_o}\right) \text{ when } \eta_{xm} \to \infty.$$

Using the same empirical estimates as previously this would imply

$$\eta_{bb} = \left(\frac{-0.27 \cdot 215}{0.17 \cdot 466}\right) = -0.75 \text{ and } \eta_b = -0.68.$$

The derived domestic demand for soybeans becomes more elastic, increasing in magnitude by more than two-fold, thereby increasing the elasticity of total demand by s^d of this increase according to (26).[10] The reverse is also true. If the USA became a small-country in the soybean oil market, η_{bb} would be proportional to the elasticity of total demand for soybean meal. It also follows that in the case where a country has small-country status in trade for both joint products, the elasticity of domestic demand for the raw product approaches infinity (i.e. if $\eta_{xm} \to \infty$ and $\eta_{x0} \to \infty$ then $\eta_{bb} \to \infty$.) Thus, the single-product result of total demand being perfectly elastic in the 'small-country' case is also true when there are multiple joint-products *all* with small-country status in trade.

[10] While this formula appears to be the same as the derived elasticity of domestic demand for the single-product case (with fixed input proportions) where the derived elasticity of domestic demand equals the input cost share times the elasticity of total demand for the product, this is not the case. This is because there are still two joint products being produced from the same raw product making the cost share $\frac{P_b B^d}{P_o Q^s} = \frac{P_b}{\beta P_o}$ actually larger than 1 so that the derived elasticity of domestic demand (η_{bb}) actually is larger than the elasticity of total demand for the product (η_o) as the empirical illustration shows.

5.2 Trade liberalisation in the importing country

China's recent accession into the World Trade Organisation (WTO) requires trade liberalisation in both the soybean meal and soybean oil markets. This liberalisation includes binding tariffs for soybean meal at 5 per cent and reducing tariffs on soybean oil from 13 to 9 per cent upon accession. In addition Tariff Rate Quota (TRQ) and state trading in the soybean oil market must be eliminated by 2006. These liberalisations will cause the elasticities of price transmission for soybean meal and soybean oil to increase and, in the extreme case of complete liberalisation, these elasticities would approach 1 (i.e. $\epsilon_{fm} \to 1$ and $\epsilon_{fo} \to 1$).

Limiting attention to the effect of liberalisation in the soybean oil market, differentiating (24) with respect to ϵ_{fo} results in

$$\frac{\partial \eta_{bb}}{\partial \epsilon_{fo}} = \left(\frac{P_b \alpha^2 {P_o}^2}{{\eta_o}^2}\right)\left(\frac{O^x}{O^s}\right)\eta_{xo} < 0 \text{ and } \frac{\partial \eta_b}{\partial \epsilon_{fo}} = s^d \cdot \frac{\partial \eta_{bb}}{\partial \epsilon_{fo}} < 0.$$

This comparative static result shows that liberalisation in the importing country for one of the joint products causes the elasticity of domestic demand for the raw product to be more elastic. It follows that the elasticity of total demand will also be more elastic, with the magnitude of the increase tempered by the importance of domestic demand in total disappearance, s^d. A greater elasticity of total demand for USA soybeans has implications for government payments under the current farm program (the provision for Loan Deficiency Payments (LDP)) which is of interest to policy-makers. When the posted county price (PCP) is below the loan rate, a more elastic total demand for soybeans results in a smaller (larger) LDP payment per bushel in the presence of an outward (inward) shift in supply, all else being equal. Thus trade liberalisation in the joint-products can result in a reduced (increased) cost of the farm-program for the raw product in the presence of an outward (inward) shift in USA supply.

5.3 Trade restrictions in the importing country

Finally, another scenario of interest is that of import restrictions on one of the joint products that is exported. This restriction would be reflected in lower elasticities of price transmission. One such example might include restrictions on USA soybean meal imports due to concerns about GMO's. Using (24) we can deduce that this would cause the derived price elasticity of domestic demand for soybeans to become more inelastic. In the extreme example of a complete ban, the elasticity of price transmission would equal zero (i.e. $\epsilon_{fm} = 0$), transforming (24) to

$$\eta_{bb} = \frac{P_b}{\left(\dfrac{\alpha P_m M^s}{M^d \eta_{mm}} + \dfrac{\beta P_o}{\eta_o}\right)}.$$

This result shows that insulation from importing countries domestic soybean meal market causes the derived price elasticity of domestic demand for soybeans to be a weighted harmonic average of the price elasticity of total demand for soybean oil and the elasticity of domestic demand for soybean meal.

Using the same empirical estimates as previously this would imply

$$\eta_{bb} = \frac{215}{\left(\dfrac{0.79 \cdot 210 \cdot 33,688}{26,734 \cdot -0.13} + \dfrac{0.17 \cdot 466}{-0.27}\right)} = -0.11 \text{ and } \eta_b = -0.26.$$

Thus, the derived price elasticity of domestic demand for soybeans is reduced by more than 50 per cent. The elasticity of total demand is smaller by a magnitude of around 30 per cent with, once again, the relative importance of domestic demand in total disappearance dampening this effect on total demand. The reverse is also true: if the USA instead became insulated from other countries' domestic soybean oil market, the formula for η_{bb} would be a weighted harmonic average of the elasticity of total demand for soybean meal and the elasticity of domestic demand for oil.

6. Concluding remarks

The purpose of the present paper was to extend Houck's derived price elasticity of demand formula for jointly-produced commodities to the case involving international trade in the commodities produced as well as the raw product. The main finding is that the formula for domestic demand is shown to hold, but with the requirement that the price elasticities of the jointly-produced commodities be elasticities of total demand (weighted averages of domestic and export elasticities) rather than price elasticities of domestic demand for the jointly-produced commodities. Using the USA soybean industry as an example, this generalised formula that takes into account trade is implemented to calculate the elasticity of total demand for USA soybeans. Interestingly, in this empirical application it was determined that taking account of trade in the joint products is found to have a more profound impact on the responsiveness of total demand than taking account of trade in the raw product. The usefulness of this formula for analysts to trace out the impacts of changes in market conditions and trade policy in the joint-products case, and how it will impact the elasticity of domestic demand and total demand for the raw product, is demonstrated. The relationship among

elasticities of joint products and the raw product is not so straightforward when these products are traded internationally and when trade interventions occur.

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