The Quality-Quantity Trade-off in the Principal-Agent Framework

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Abstract

This paper uses the principal-agent theory to analytically investigate the optimal incentive-based compensation contract that a processor should offer to a grower performing efforts in quantity and quality. In this process, we contribute to the substantive literature on multi-task principal-agent models by analyzing the quality-quantity trade-off and studying the implications of such a relationship in the principal-agent framework. One striking result of these effects is that, under appropriate incentive-based grower’s compensation, the processor may encourage grower’s effort in quality without crowding out grower’s effort in quantity.

Key words: quality, quantity, trade-off, incentive contract, principal-agent framework, uncertainty

JEL Classification: D86

Introduction

With the constant pressure to meet consumer demands and the need to be competitive, quality is becoming a central point in the agri-food industry. As a result, in most grower-processor relationships, the processor has to induce the grower to engage in several efforts simultaneously. This raises the question of what the appropriate compensation method should be used for the inputs provided by growers to processors.

The grower’s performance can often be measured fairly accurately in some efforts. In others, however, available performance measures that may be used to provide explicit incentives to the grower may not even exist. For example, a grower may have to produce a certain amount of input which is easily measurable, but he may also have to make sure that the quality of input is high, which may be more difficult to measure (Fehr and Schmidt, 2004). As a result, a grower that seeks to maximize his own income may not act in the best interest of the processor. This potential conflict of interests creates a moral hazard problem, which can be resolved or reduced through proper compensation methods. One appropriate methodology to address the moral hazard problem is the principal-agent theory (Stiglitz, 1974; Holmström, 1979; Shavell, 1979). In this paper, we construct a normative model using this theoretical framework in which the processor, acting as the principal, delegates the task of producing a quality-

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This paper has been supported by the research project MICINN-FEDER ECO2009-09623.
differentiated input to the grower.

In addition to the grower’s risk attitude and performance measurement problems, which are common features in agency theory, there are a few other factors unique to the grower’s compensation problem. First, the multiple efforts performed by the grower in quantity and quality act jointly to determine the outcome. This is in contrast to, for example, a multi-task employee whose tasks are often modelled as additively separable in their contributions to the outcome (Holmström and Milgrom, 1991). Second, the processor has to consider the trade-off, often present, between the quality of a good and the quantity produced, when decides the compensation scheme. In the standard vertical-product-differentiation model, it is usually implicitly assumed that quality and quantity are independent choices (for example, Gabszewicz and Thisse, 1979, 1980; Shaked and Sutton, 1982, 1983). That is, at any set level of quality, a grower is free to produce as much as he desires. However, in many goods, such as tobacco and grapes, production can only be increased at the expense of lower quality. Indeed, the quality-quantity trade-off is an important research issue that has received substantial attention (see, for example, Folwell et al., 2006; McCannon, 2008). It is the feature of inputs for the grower-processor relationship that is considered here.

This paper contributes directly to the substantive literature on multi-task principal-agent models. In a seminal paper, Holmström and Milgrom (1991) analyzed optimal incentive provision in a model in which there were multiple outputs that the principal cared about and each of these outputs were functions of a single and different input all of which are provided by the agent. Their results show that there are important interaction effects between the incentives given for one task and the agent’s incentives for engaging in other tasks. Later, Ghatak and Pandey (2000) developed a model with a single output which was a function of two inputs. They showed that sharecropping contracts emerge as a natural solution. In Holmström and Milgrom’s model, it is to encourage the agent to produce reasonable amount of all outputs the principal cares about. In contrast, in Ghatak and Pandey’s model, it is to encourage the agent to produce a single output but induce him to take actions that generate a more favourable probability distribution of this single output from the principal’s point of view. Both papers, however, are silent on the inclusion of a multiplicative effect of agent’s efforts on the principal’s outcome. By taking into account an interaction effect between the ad agency’s various tasks in a multiplicative way, Zhao (2005) takes a further step towards investigating the incentive-based advertising agency compensation. In direct contrast to the findings in the traditional agency literature, Zhao obtains that when the risk is moderate, the advertiser should offer a higher incentive rate as the risk increases.

Our paper extends the previous multiple-task literature to analyze the quality-quantity trade-off and to study the implications of such a relationship in the principal-agent framework. One striking finding of our model is that, under appropriate incentive-based grower’s compensation, the processor may encourage grower’s effort in quality without crowding out grower’s effort in quantity, in direct contrast to the findings in the traditional multi-task principal-agent literature, according to which incentives must be balanced across tasks (Dewatripont et al., 2000).

Our paper also contributes to the agency theory methodologically. As discussed earlier, in our model, we study a multi-effort agent, the grower, which is different from other multi-task agents (e.g., Holmström and Milgrom, 1991) in that there is an interac-
tion effect between the grower’s efforts (quantity and quality) in a multiplicative way, whereas the majority of multi-task models usually assume an additive functional form between tasks. An exception is Zhao (2005). Furthermore, we take into account the quality-quantity trade-off, which is not considered in the conventional principal-agent models. Why previous models have not included these aspects is likely attributable to analytical problems. As Robe (2001, p.1) stated, “the history of the principal-agent model is replete with frustrations in obtaining not only exact, closed-form, but also numerical, solutions”.

The rest of the paper is organized as follows. In the next section, we describe our model and explain the key assumptions. We then analyze the incentive-based compensation contract in the third section. We then carry out a simulation exercise to understand the effects of the previous contract in the fourth section. The fifth section concludes the paper with some discussions on future research.

Model

We analyse a vertical structure in which end-markets are differentiated by quality. A processor-distributor (the principal) engages a grower (the agent) to produce a good considered to be the processor’s input. We suppose that the processor is able to market a finished product of an equivalent quality to the grower’s. Likewise, one unit of input is needed to produce one unit of output and there is no other input. We further consider that there are no raw material processing costs. Although these unrealistic assumptions are made for the purpose of analytical simplification, they do not take away from the applicability and implications of the model.

We aggregate the grower’s efforts into two variables for analytical tractability: quantity, $q$, and quality, $s$. It is usually difficult for the processor to monitor the agent’s efforts either because it is too costly to do so or because the processor lacks the expertise to valuate the agent’s performance directly. While quantity ($q$) is perfectly measured to the processors, quality measurement by the processor is imperfect and processors’ inability to perfectly observe input quality results in a moral hazard problem. We denote by $\tilde{s}$ the level of quality measured by the processor, according to: $\tilde{s} = s + \mu_s$, where $\mu_s \sim N(0, \sigma_s)$. The noise term $\mu_s$ captures the underlying uncertainty about measurement errors.

We assume that the processor is risk-neutral and the grower is risk-averse for two reasons. First, the cost of bearing risk is generally relative less for the processor than for the independent grower, such as a grower or a farmer (Milgrom and Roberts, 1992). Therefore, it is reasonable to argue that the grower is less risk-averse than the processor, and our assumption is a simplification of this fact. Second, it conforms to the research tradition in the agency literature (e.g. Allen and Lueck, 1999; Huffman and Just, 2000; Dubois and Vukina, 2004).

To model imperfect quality measurement and the grower’s risk aversion, we define $f(y|q, \tilde{s})$ as the probability density of the final output, where $y$ is the monetary value of the outcome of the output, which depends not only on the finished product quantity, but also on the measured quality. Depending on the processor’s objective, $y$ can be meas-
ured in terms of revenues, profits, sales, etc. We assume that \( f_q / f(y | q, s) \) and \( f_s / f(y | q, s) \) are increasing in \( y \). This assumption is known as the monotone likelihood-ratio property (MLRP) of the output function.

In general, market prices are higher for high-quality than for lower quality goods. Likewise, prices and yields appear to be inversely related in the aggregate market. Then, we assume that \( P = f(q, s) \), which satisfies \( P_q < 0, P_s > 0 \). Our price function is supported by Beard and Thompson (2003).

We need to specify how grower’s efforts in quantity and quality interact to generate processor’s revenue. Obviously, the total revenue from the sale of a good is the selling price \( (P) \) multiplied by the quantity sold. This suggests a positive interaction between the two variables in producing processor’s revenue, which means that \( f_q / f(y | q, s) \) must satisfy \( v_q > 0 \). This is in contrast to the substituting effect between multiple tasks studied by Holmström and Milgrom (1991), who assume that \( f_q / f(y | q, s) + f_s = 0 \), so \( v_s = 0 \).

The grower’s cost associated with his decisions of \( (q, s) \) is \( c(q, s) \). There is a cost associated with effort because it is unpleasant and forgoes the opportunity to undertake other activities. The standard vertical-product-differentiation model assumes that the cost is increasing in both quality and quantity, convex in quality, and the marginal cost of production is independent of quality (McCannon, 2008). To introduce the trade-off between quality and quantity, assume instead that the cost of production varies quadratically in line with the given level of quality. That is, \( c_q > 0, c_{q*} = 0, c_s > 0, c_{s*} > 0 \) and \( c_{q,s} > 0 \). This cost function is supported by Champsaur and Rochet (1989) and Giraud-Héraud et al. (1999).

Although most of our analysis can be carried out by using the general functional forms for \( f(y | q, s) \) and \( c(q, s) \), the exposition is significantly improved if specific functional forms are used. Specifically, we assume that \( c(q, s) = (k/2)q^2 \) and \( P = \frac{1}{2} - bq \), where \( k \) and \( b \) are positive constants. These functional forms satisfy all the assumptions laid out earlier.

The game is played as follows. The processor moves first by offering an incentive-based compensation contract, \( w \), to the grower, which is contingent on the processor’s revenues, i.e., \( w \equiv w(y) \). The grower decides whether to accept the contract or not. If he accepts the contract, he supplies his efforts in quantity and quality, \( q \) and \( s \) respectively. The grower’s and the processor’s payoffs are, respectively, \( u = w(y) - c(q, s) \) and \( \pi = y - w(y) \). If the grower rejects the contract, he receives the reservation utility, \( U \), the minimum that induces him to work, and his payoff is therefore zero.

Rather than solving a particular class of contract, we focus on a particular class of linear contract: a fixed rent plus a share of the principal’s revenues in the following form:

\[
w(y) = \alpha + \beta y, \alpha \geq 0, 0 \leq \beta \leq 1
\] (1)

We study this class of compensation contract for two reasons. First, it is easy to implement managerially, and, indeed, much economic activity takes place within a framework of incentive contracts, such as managerial compensation (e.g., Lemmon et al.,
2000; Murphy, 1986), franchising (LaFontaine, 1992, LaFontaine and Slade, 1998), advertising agencies (Zhao, 2005) and particularly agriculture (Stiglitz, 1974; Newberry and Stiglitz, 1979; Otsuka et al., 1992; Lanjouw, 1999; Ackerberg and Botticini, 2002; Roumasset and Lee, 2003). Second, according to principal-agent theory, when contracting is repeated many times and the agent has discretion in his effort, the structure of the optimal pay scheme is linear in the principal’s observed payoff (Holmström and Milgrom, 1987). This implies a two-part compensation scheme consisting of a fixed rent, which is independent of the observed outcome and a share of the observed outcome.

Consistent with the mean-variance approach\(^3\), for a risk-averse grower, his net utility is given by \( U = -\exp(\rho (w(y) - c(q,s))) \), where \( \rho \) describes the grower’s risk aversion. It is easier to deal with his certainty equivalent\(^4\), CE, which is given by \( CE = \alpha + \beta E(y) - c(q,s) - \left( \rho \beta^2 q^2 s^2 \sigma^2 / 2 \right) \). The term \( \rho \beta^2 q^2 s^2 \sigma^2 / 2 \) is the risk premium that the grower demands to compensate for the risk he bears. Since the effects of the grower’s risk aversion \( \rho \) and the imperfect quality measurement represented by the standard deviation \( \sigma \) are not separable in their impacts on the risk premium, we use one parameter, \( R(\equiv \rho \sigma^2) \), called the risk parameter, to describe the combined effect of risk aversion and uncertainty of measurement in the market.

**Equilibrium analysis of the incentive-based compensation method**

When devising an incentive-based compensation contract, the processor needs to determine the fixed fee \( \alpha \) and the incentive rate \( \beta \) so that the grower will respond to this contract through his decisions on quantity and quality in such a way that the processor’s net payoff is maximized. Formally, the processor solves the following problem to maximize his payoff:

\[
\max_{\alpha, \beta} \left( (1 - \beta) y - \alpha \right)
\]

subject to

\[
q, s = \arg \max_{q, s} \left( \alpha + \beta E(y) - c(q,s) - \left( R \beta^2 q^2 s^2 \sigma^2 / 2 \right) \right)
\]

and

\[
\alpha + \beta E(y) - c(q,s) - R \beta^2 q^2 s^2 \geq 0
\]

Constraint (4) says that compensation to the grower after adjusting for the risk premium must be no less than its reservation utility to ensure his participation in the contract. If the grower accepts the contract \( (\alpha, \beta) \), he will determine \( q \) and \( s \) so that his utility is maximized, which is formally constraint (3).

With this incentive-based compensation contract, some notable results follow. First, the processor chooses an incentive scheme to maximize the joint payoff of the processor and the grower (the processor’s gross payoff minus the grower’s cost of effort and risk bearing) and both have an incentive to fulfil the contract. Second, the processor compensates the grower for his efforts at a rate that provides partial insurance against income risk. With imperfect measurement on quality, and hence, on grower’s effort in quality, the processor does not provide full-income insurance to the grower because that would provide weak incentives for effort, leading to shirking. Third, the fixed rent of
the grower is positively related his reservation utility, but his reservation utility has no impact on the incentive component.

In this sequential-move game, we use backward induction by solving the optimization problem in equations (2)-(4). First, we consider the grower’s maximization problem in equation (3),

\[ q, s = \arg \max_{q, s} \left[ \alpha + \beta q (s - bq) - \left( k/2 \right) q s^2 - \left( R \beta^2 q^2 s^2 / 2 \right) \right] \quad (5) \]

\[ \frac{\partial CE}{\partial q} = \beta (s - 2bq) - \left( k/2 \right) s^2 - R \beta^2 qs^2 = 0 \quad (6) \]

\[ \frac{\partial CE}{\partial s} = \beta q - kqs - R \beta^2 q^2 s = 0 \quad (7) \]

Substituting the values of \( q \) and \( s \) obtained in (6) and (7) in processor’s problem (eq. 2), and maximizing with respect to \( \beta \) the optimal incentive is obtained. Finally, we calculate \( q^* \) and \( s^* \) and substitute them into equation (4) to obtain the optimal fixed rent \( \alpha^* \).

A quantitative application of this principal-agent model requires a numerical simulation because it remains impossible to explicitly solve \( q \) and \( s \) in the equations (6) and (7). Thus, a numerical simulation exercise will be carried out in the following section.

Simulating and discussing results

As we mentioned earlier, the multiplicative functional form of \( y(q,s) \) precludes closed-form solutions so we resort to numerical techniques to gain insights.

In this section, we carry out a simulation exercise with a wide range of scenarios, and selected the examples below as being representative of the behaviour we found. We use Mathematica to solve the model, and use Excel to draw the graphs using the data produced by Mathematica. We initially choose the following parameters: \( b = 0.00001 \) and \( k=0.4 \). It should be noted that these initial values are used for convenience and has no special significance here and that simulation results do not change substantially if different values for \( b \) and \( k \) are used.

It will be seen below that this framework is able to provide a consistent explanation for many issues relating to share contracts. However, before proceeding, we should note the caveat that this simulation exercise uses restrictive assumptions about the shapes of price and cost functions. Although these seem highly plausible to us for most situations, there may be situations which are not covered by our simulations.

Essentially, the processor needs to consider three variables when devising the optimal incentive-based method: the grower’s risk attitude, over-all uncertainty in quality measurement, and the grower’s reservation utility. As discussed earlier, we set the reservation utility equal to zero. Likewise, since the effects of risk aversion \( \rho \) and uncertainty in quality measurement are not separable in our model, we simply use one parameter, the risk parameter, \( R \), to capture the impact of overall risks.

Thus, we have a unique free parameter in our model: the risk parameter, \( R \). For the purposes of the simulation, we consider that \( R \) varies from 0 to 0.001, in steps of 0.00005. Still larger values for \( R \) do not substantially affect the results of simulation.
exercise reported in this section.

We start by examining the impact of the risk parameter, $R$, on the optimal incentive share, $\beta^*$. When quality measurement is perfect (i.e., $R=0$), it is easy to check that $\beta^*=1$. That is, the contract would assign all the revenue to the grower. As the risk parameter increases, the optimal incentive share decreases, as shown in Fig. 1(a). The simulation shows that this general pattern holds for the full range of the risk parameter. Indeed, if the quality measurement is infinitely imperfect (i.e., $R=\infty$), then we obtain that $\beta^*=0$ and the optimal pay scheme is a fixed salary equal to the certainty-equivalent utility (i.e., $w=a^*=0$). In this case, the processor optimally bears all the risk because he is risk neutral and less risk averse than the grower. This finding of the incentive share corroborates the traditional agency literature (e.g., Hueth, 2000; Huffman and Just, 2000).

As the risk parameter increases, the effectiveness of incentives diminishes and hence, the grower is less motivated to make an effort in quantity. This can be seen from Fig. 1(b) that there is an inverse relationship between incentive share and quantity effort. And the result about the quality effort is qualitatively the same as in the previous one. That is, the grower’s effort moves in tandem with the incentive share, as shown in Fig. 1(c). Therefore, the optimal choice is to decrease the supply of both efforts when the risk parameter decreases.

A corollary to the results of this numerical simulation is that, with an incentive-based compensation contract, the incentive share is effective with both grower’s efforts.

We can further draw some conclusions regarding the impact of the risk parameter on the total certain equivalent. This can be seen from fig. 1(d) that an increase in the risk parameter results in a decrease in the total certainty equivalent.

The previous results suggest discussing the implications of our theoretical findings and comparing this incentive-based compensation method with the traditional multi-task principal-agent model.

It is well known that incentives are an effective means to address the moral hazard problem in a multi-task principal-agent relationship. However, incentives work in a much complex way in the grower-processor relationship in a differentiated market than in other relationships. In addition to the grower’s risk attitude, overall uncertainty (in this case, imperfect measurement) and several efforts, which are common to all multi-task agency problems, the interactive effect between quantity and quality and the trade-off between them are factors that will affect the efficacy of incentives. These latter two features are unique to the processor’s problem compensation when he is concerned about quality. Because of these new features, some established findings in the multi-task principal-agent literature do not necessarily hold in our grower’s compensation problem.

In particular, the standard result of multi-task principal-agent theory requires that the incentives must be balanced across tasks. For example, if the different tasks are complements at the margin in the principal’s payoff function, i.e., if it is important for the principal that the agent engages in all tasks rather than concentrating his efforts on a single task, it is optimal for the principal to use low-powered incentives on the well-measured task compared to a situation where the agent is engaged solely in this task. The reason is that if the principal offers high-powered incentives for a task that is easy to measure and low-powered incentives for a task where measurement is difficult, then
the agent will focus his efforts on the task that is rewarded and the other task for which only small incentives are offered will be neglected.

![Graphs showing compensation contract with respect to risk parameter](image)

**Figure 1.** Compensation contract with respect to risk parameter

However, for our grower’s compensation problem, we find that, a unique incentive share is sufficient to incentive the grower to allocate his effort in the two tasks, quantity and quality. Qualitatively, this finding could corroborate the result found by Laffont and Tirole (1990). These authors argue that one can find conditions under which a manager should be rewarded only on the basis of total profit or cost, even though more disaggregated information about profits or costs on various activities is available.

**Conclusions**

This paper uses the principal-agent theory to analytically investigate the optimal incentive-based compensation contract that a processor should offer to a grower performing efforts in quantity and quality. In this process, we contribute methodologically to the multi-task principal-agent literature by including a quality-quantity trade-off and an agent performing multiple efforts that produce outcomes in an interactive way.

When designing a compensation contract based on this model, the processor has to take into account the interaction effect of the quantity and quality on the outcome and the inverse relationship between quantity and quality. One striking result of these effects is that the processor should offer a contract with a unique incentive based on the observed outcome, in direct contrast to the findings in the traditional multi-task agency literature.

One of the implications of the incentive-based method is that, as grower’s income is
directly related to the outcome of the quality, it is reasonable to assume that the grower has a strong voice in how the quality effort is formulated and executed. The processor may be forced to give the grower free rein in determining how the quality should be handled. However, in many real-world contracts between processors and growers, outcome-based contracts are not used in isolation, but in combination with input-monitoring. A convincing evidence of this statement is present in viticulture, where many wineries exert a direct control over vineyard activities by specifying input use such as the choice of irrigation technology employed. Clearly, the less the winery imposes on the grower the more responsibility the grower has for the outcome of the quality (Fraser, 2003). Thus, it could be helpful to both the processor and the grower to put the compensation issue in the larger perspective of quality management of the relationship between the two.

The present study has certain limitations that need to be taken into account when considering the study and its contributions. However, some of these limitations can be seen as fruitful avenues for future research under the same theme.

The analysis presented here leaves unanswered an interesting question in contracting. Holmström (1979)´s classical work about imperfect information in a problem of moral hazard proves that “a signal is valuable if and only if it is informative”. According to this proposition, the uncertainty in quality measurement led us to choose a compensation scheme based on the principal’s revenue. But later papers have suggested that if some aspects of the agent’s performance cannot be contracted, relying on subjective performance evaluation and voluntary bonus payments might be optimal (for example, MacLeod and Malcomson, 1989; Baker, Gibbons and Murphy, 1994; Fehr and Schmidt, 2004). Hence, there would be the possibility that an input-based contract could be more efficient second-best contract compared to the outcome-based contract by using subjective quality evaluation.

Another primary limitation of this analysis is that it is not a dynamic analysis, although in practice, processors tend to contract repeatedly with the growers whom they rely on. Then, it does not consider the possibility of a relationship between principal and agent over time, and hence, it does not take into account reputation effects of insincere behaviour. However, previous literature has proved that reputation can be an added incentive mechanism to induce performance under a contract (King, Backus and Gaag, 2007). These limitations will be considered in future research efforts.

Notes
1 In this paper “quality” is not perceived quality but objective (technical) quality
2 When the principal has a greater potential to impact on retail demand due to branding revenue sharing contracts are better to provide appropriate incentives that profit sharing contracts (Rubin, 1978). In this case, as there are no processing costs, it is indifferent.
3 The linear mean-variance utility function is routinely used, especially in agriculture (e.g., Chavas and Holt, 1990; Pope and Just, 1991; Gaynor and Gertler, 1995; Allen and Lueck, 1999)
4 The optimal choice for a decision maker faced with uncertainty is the maximization of his expected utility where the expected profit and variance are the arguments of
utility. As the expected utility derived from variable profits is equal to the utility derived from the certainty equivalent, CE, the maximization problem can be mathematically written as \( CE = E(\pi) - \rho \sigma^2_{\pi} / 2 \), where the coefficient \( \rho \geq 0 \) measures the risk aversion and \( \sigma^2_{\pi} \) is the variance of profit (Robison & Barry, 1987).

Let the Nash equilibrium values be denoted as *.

In contrast, the additive functional form prevalent in the conventional multi-task principal-agent models will yield closed-form solutions.

The Mathematica commands are available from the authors on request.

The results of all the other values of the risk parameter proved are available from the authors on request.

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