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Criteria for optimal production under uncertainty. The state-contingent approach*

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The state-contingent approach to production economics presented by Chambers and Quiggin provides a new basis for deriving optimality criteria for production under uncertainty. In the present paper criteria are formally derived for risk-averse producers. It is not possible to derive useful criteria for strictly risk-averse producers, but useful criteria for risk-neutral producers are presented for three different types of input. Based on a formal definition of 'good' and 'bad' states of nature, the use of inputs and levels of production of strictly risk-averse producers are compared to those of risk-neutral producers. Depending on the type of input, risk-averse producers may use more or less input than risk-neutral producers.

1. Introduction

While the subject of planning and decision making under uncertainty has been treated extensively in the published literature on agricultural economics (e.g., Robison and Barry 1987; Hardaker *et al.* 1997), there are in fact no prescriptive criteria for optimal production under uncertainty. The book 'Uncertainty, Production, Choice, and Agency – The State-Contingent Approach' written by Chambers and Quiggin (2000) provides a new theoretical basis for describing and analysing production under uncertainty. The purpose of the present paper is to use the theory and the concepts presented in this book to derive prescriptive criteria for optimal production under uncertainty.

The first real attempts to model uncertainty in relation to production were made by Magnusson (1969) and Sandmo (1971). Magnusson focused on production risks. Sandmo focused on uncertain product prices. Using the expected utility (EU) model, Sandmo showed that output under price

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uncertainty is smaller than when prices are certain. However, he did not provide any (prescriptive) criteria for optimising production that could be used by decision-makers when planning production under uncertainty. In fact, he explicitly stated that that while '… under certainty, the solution is characterised by equality between price and marginal cost, there is no obvious way of making such a comparison [under uncertainty]' (Sandmo 1971, p. 66).

As with the Sandmo approach, developments in the theory of economics under uncertainty have typically provided no formal criteria that could be used by decision-makers when planning production and input use. Instead, most of the effort has been directed towards discussing the utility function, especially the advantages and disadvantages of the EU model based on the von Neuman-Morgenstern utility function (von Neumann and Morgenstern 1944; Schoemaker 1982; Quiggin 1993). Related major issues have been estimating risk attitudes and parameters of the utility functions (Binswanger 1980; Antle 1987), and estimating variability of prices and yields in the form of parameters of the moments of the probability distribution (Anderson and Griffiths 1981; Rasmussen 1997). In relation to production decisions, Just and Pope (1978) provided a definition of risk-reducing versus risk-increasing inputs, and formulated conditions under which an input could be considered risk-reducing. However, this concept of risk-reducing inputs appears not to have provided any prescriptive power for decision-making. In 1983 Antle wrote: 'Agricultural economists have made little progress in analysing or measuring production risk in ways that provide useful information for farm management' (p. 1099). After studying dynamic models of production, Antle (1983) also found that farmers' optimal production decisions are affected by risk whether they are risk-neutral or risk-averse.

The published literature dealing with economic decisions under uncertainty (risk management) (e.g., Anderson *et al.* 1977; Barry 1984; Robison and Barry 1987; Hardaker *et al.* 1997) typically takes a 'defensive' approach in the sense that it describes how firms/producers should respond to risk or cope with risk. The major subjects are the sources of risk/uncertainty and how the firm may respond by performing risk management; that is, holding financial reserves, gathering information, forward contracting, buying insurance, diversification, etc. But very little is said about the criteria to be used when making the basic production decisions; that is, deciding how much input to apply and how much of a product to produce. The reason is that, with the theoretical foundation used so far, the well-known marginal principle used so successfully under certainty breaks down under uncertainty. Or, as Hardaker *et al.* (1997) put it: 'What happens to the prescriptions of the [production] theory when the prevalence of risk and the reality of widespread risk aversion are recognised? … The simple answer is that the prescriptions of optimality of conventional production theory are invalidated, or at least qualified. In principle, each production decision needs to be analysed using the SEU [Subjective Expected Utility] model' (p. 127).

The state-contingent approach presented by Chambers and Quiggin (2000) and in a number of articles (e.g., Chambers and Quiggin 2001, 2002) provides the basis for deriving criteria for optimising production under uncertainty using the well-known marginal principles from the theory of production under certainty. The approach is based on the 'simple' idea that goods are defined not only by type, place, and date, but also by a fourth dimension, the state of nature at the (future) time when the good will become available.

This idea is not new. As mentioned by Chambers and Quiggin (2000), the idea was first presented by Arrow in 1952 and Debreu further described the concept in 1959 (Debreu 1959). In 1965 Hirshleifer realised the close formal analogy with Fisher's model for riskless choice over time and used the approach to develop a theory of investment under uncertainty (Hirshleifer 1965). However, the real power of Chambers and Quiggin's book is that it combines the state-contingent approach (Hirshleifer and Riley 1992) with the modern (dual) approach to production economics presented by Chambers in his 1988 book.

In the following, the state-contingent approach is first presented, using the concepts and terminology of Hirshleifer and Riley (1992), and Chambers and Quiggin (2000). Conditions are presented under which the technology can be represented by a family of product transformation curves, one for each state of nature. The production technology under uncertainty is then described in the form of product transformation curves for three different types of input: state-general, state-specific, and state-allocable inputs. For each of these three types of inputs, optimality criteria for application are derived when producing one output under the general assumption that the decision-maker is risk-averse. It is shown that without specific assumptions concerning the functional form and parameters of the utility function, these (general) criteria have no prescriptive power. However, useful criteria are derived under the assumption that the utility function is linear (i.e., the decision-maker is risk-neutral).

With a view to comparing input-use of risk-neutral and (strictly) risk-averse producers, the concepts of 'good' and 'bad' states of nature are defined. Based on these definitions, conditions describing circumstances under which a (strictly) risk-averse decision-maker would use more or less inputs than a risk-neutral decision-maker are derived. It is concluded that the criteria describing optimal production under risk for a risk-neutral decision-maker are analogous to the criteria describing optimal production under certainty. It is not possible to state clearly whether a (strictly) risk-averse decision-maker will use more input or produce more output than a risk-neutral decision-maker. The answer will depend on the specific preferences of the decision-maker, partly described by what are considered 'good' and what are considered 'bad' states of nature from his or her point of view. It is finally concluded that the concepts of 'good' and 'bad' states of nature defined and presented in the present paper are useful and will probably prove useful in further analysis of economics of production under uncertainty.

2. Concepts and terminology

The state-contingent approach to describing production of an output z under uncertainty is based on the following concepts:

States of nature: $\Omega = \{1, 2, \dots, s, \dots, S\}$ (1)

Probabilities: $(\pi_1, \dots, \pi_s, \dots, \pi_S)$ (2)

Cost: $c = \mathbf{w}'\mathbf{x}$ (3)

Technology: $T(\mathbf{x}, \mathbf{z}) = 0$ (4)

Output: $z_s = \max\{z_s: T(\mathbf{x}, \mathbf{z}) \leq 0\} \quad \forall s \in \Omega$ (5)

Revenue: $r_s = z_s p_s \quad \forall s \in \Omega$ (6)

Net return: $y_s = r_s - c \quad \forall s \in \Omega$ (7)

Preferences: $W = W(y_1, \dots, y_S)$ (8)

We consider the production of only one output.[1] The uncertain production conditions are described in the form of a set of states Ω, from which 'nature' picks the state of nature independently of the decisions made by the decision maker. Nature picks the state of nature after the decision maker has made his production decision; that is, after he has decided how much input $\mathbf{x}$ to apply, where $\mathbf{x}$ is an input vector $\mathbf{x} = (x_1, \dots, x_N)$ with corresponding input prices $\mathbf{w} = (w_1, \dots, w_N)$. The decision-maker holds (subjective) probabilities π_s that nature will pick the s-th state of nature ($s = 1, \dots, S$). Depending on the state of nature, the output in state s (z_s) is determined by

[1] The concepts and derivations are easily generalised to more outputs as shown by Chambers and Quiggin (2000).

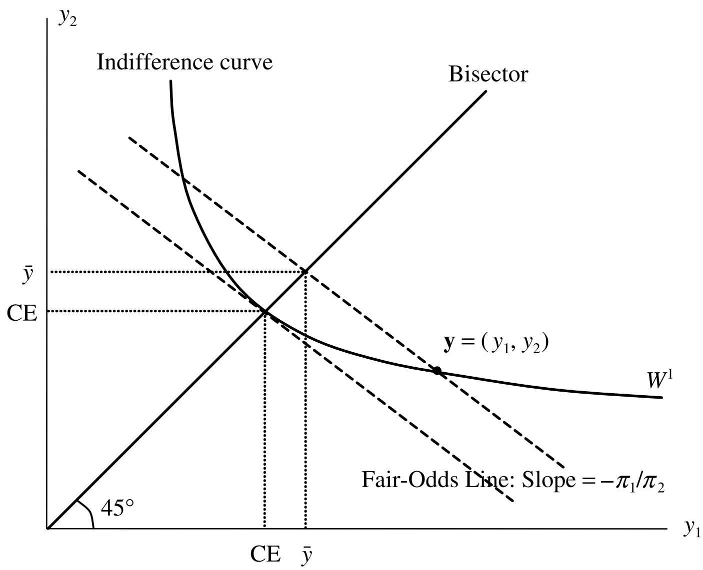

Figure 1 Key concepts related to the state-contingent production

a transformation function $T(\mathbf{x}, \mathbf{z}) = 0$ where $\mathbf{z}$ is a vector $(z_1, \dots, z_S)$ of state-contingent outputs. The product price is a state-contingent output price denoted $p_s(s = 1, \dots, S)$. Net return y_s in state s is revenue $z_s p_s$ minus cost c. The decision maker chooses the level of input $\mathbf{x}$ that maximises utility, assumed to be a non-decreasing function W of the state-contingent vector of net returns $\mathbf{y} = (y_1, \dots, y_S)$.

Chambers and Quiggin place relatively weak assumptions on the utility function W. The same is done here, the only assumption (apart from monotonicity, continuity and differentiability) being that the decision maker is risk-averse, defined by the condition that:

$$W(\bar{y}, \dots, \bar{y}) \geq W(y_1, \dots, y_S) \tag{9}$$

where $\bar{y}$ is the expected net return; that is,

$$\bar{y} = \pi_1 y_1 + \pi_2 y_2 + \dots + \pi_S y_S \tag{10}$$

and $(\bar{y}, \dots, \bar{y})$ is an S-dimensional vector of expected net returns. Figure 1 shows an indifference curve for a risk-averse decision-maker. Indifference curves for risk-averse decision-makers are convex. The figure illustrates that a production plan yielding a vector of state-contingent net returns $\mathbf{y} = (y_1, y_2)$ provides a utility of W^1. The expected net return is $\bar{y} = \pi_1 y_1 + \pi_2 y_2$ and the certainty equivalent (i.e., the certain return that provides the same utility as the uncertain project **y**) is CE. In figure 1 it is further illustrated that the slope of a tangent to the indifference curve where it crosses the bisector is equal to $-\pi_1/\pi_2$. A line with the slope of $-\pi_1/\pi_2$ is called the fair-odds line.[2] The bisector (or the certainty line, as it is called by Hirshleifer 1965, and Hirshleifer and Riley 1992) illustrates certainty because along the bisector the net return is the same in all states of nature (see Hirshleifer and Riley 1992 for further explanation).

The fact that

$$\frac{\pi_1}{\pi_2} = \frac{W_1(y, y)}{W_2(y, y)} \tag{11}$$

or more generally that

$$\frac{\pi_s}{\pi_t} = \frac{W_s(y, \ldots, y)}{W_t(y, \ldots, y)} \quad (s, t \in \Omega) \tag{12}$$

where $y = y_1 = y_2 = \ldots = y_S$ is an arbitrary scalar ($y \geq 0$) and W_s is the derivative of W with respect to y_s, will be used intensively in the following. Figure 1 illustrates that at point **y**, the (absolute) slope of the indifference curve W^1 is less than the (absolute) slope of the fair-odds line and thus that:

$$\frac{\pi_1}{\pi_2} > \frac{W_1(y_1, y_2)}{W_2(y_1, y_2)}. \tag{13}$$

Although the same definitions and terms as in Chambers and Quiggin (2000) will generally be used throughout, it is convenient to make a slight variation. While Chambers and Quiggin consider risk neutrality as a special case of risk aversion, I shall use the term risk neutrality when equation (9) holds with strict equality, and use the term risk-aversion when equation (9) holds with strict inequality. Thus, in figure 1 a risk-neutral decision maker has indifference curves equal to the fair-odds lines and a utility function given by:

$$W^*(\mathbf{y}) = \pi_1 y_1 + \pi_2 y_2 + \ldots + \pi_S y_S. \tag{14}$$

[2] A formal derivation of the fact that the indifference curve always has the slope of $-\pi_1/\pi_2$ where it crosses the bisector, is given by Chambers and Quiggin (2000), (pp. 89–90). An informal explanation is that at points where the indifference curve crosses the bisector (i.e., where the net return y is certain) the marginal substitution between incomes in different states of nature (the slope of the indifference curve) depends only on the relative probabilities (π_1/π_2) (and not on specific preferences related to uncertainty as such).

3. Production technology – types of inputs

The production technology over which to optimise production may be illustrated using the production possibility set, the input set, the output set, or the corresponding outer boundary of these sets; that is, the production function, the isoquant, or the product transformation curve (see Chambers (1988), for definitions). Under certainty, optimal input use in a one-input one-output setting may be illustrated graphically by using the production function. The optimal combination of inputs to produce a certain amount of output may be illustrated using the isoquant; and the optimal combination of output for a given amount of input may be illustrated using the product transformation curve. The corresponding criteria for optimal production in the three cases are: $p_h \partial f(\mathbf{x})/\partial x_i = w_i$, $(\partial f(\mathbf{x})/\partial x_i)/(\partial f(\mathbf{x})/\partial x_j) = w_i/w_j$, and $(\partial f_h(\mathbf{x})/\partial x_i)/(\partial f_k(\mathbf{x})/\partial x_i) = p_k/p_h$, respectively, where $\partial f(\mathbf{x})/\partial x_i$ is the marginal product with respect to the *i*-th input, w_i is the price of input *i*, $\partial f_h(\mathbf{x})/\partial x_i$ is the marginal product of producing output *h* with respect to the *i*-th input, and p_h is the price of product *h*.

As emphasised by Chambers and Quiggin, using the state-contingent approach to analyse production under uncertainty in principle makes it possible to use the same tools as used under certainty. All that is needed is to expand the definition of a good (product) by including a fourth dimension; that is, state of nature – to the definition of a good. Thus, besides type, place, and time, the state of nature that will prevail at the future time (after commitment of input) when the good becomes available is the fourth dimension in defining a good. By including the state of nature in the definition of a good, otherwise identical goods that will be available in different states of nature are treated as different goods, with the possibility of different prices, etc., as under certainty.

The core of describing production under uncertainty is therefore related to the production in each state of nature, and – not least – the possibility to substitute (*ex ante*) production in one state of nature for (*ex ante*) production in other states of nature.

3.1 Technology

Production technology under uncertainty may be described in a fairly general way using the transformation function T given by:

$$T(\mathbf{x}, \mathbf{z}) = 0 \tag{15}$$

which is an implicit description of technologically efficient production plans, where $\mathbf{x}$ is the vector $(x_1, \dots, x_N)$ of inputs and $\mathbf{z}$ is the vector $(z_1, \dots, z_S)$

of state-contingent outputs of a single stochastic output z. Assuming differentiability equation (15) may be represented in an explicit form as:

$$z_1 = f_1(z_2, \ldots, z_S, x_1, \ldots, x_N) \tag{16}$$

where f_1 is an explicit (production) function describing the maximum amount of output in state 1 as a function of output in the other S-1 states of nature and the input vector $\mathbf{x}$.

If the state-contingent outputs are independent in the sense that the amount of output produced in state s does not influence (and is not influenced by) the amount of output produced in state t, the production is non-joint in inputs (Chambers 1988). Thus, the production z_s in state s only depends on the amount of input committed to production prior to revelation of this state of nature. This means that z_1 in equation (16) does not depend on $z_2, \ldots, z_S$, but only on the input vector $\mathbf{x}$.[3] In the present paper, it will be assumed that inputs are non-joint except where otherwise stated. When production is non-joint in inputs the production technology may be described by the following set of independent production functions:

$$z_s = f_s(x_1, \ldots, x_N) \quad s \in \Omega. \tag{17}$$

The condition for writing equation (16) in the form of equation (17) is that at the time when the state of nature reveals itself, the production decision has already been taken, and therefore a specific input vector $\mathbf{x}$ has already been committed. Thus, the input vector $\mathbf{x}$ will be the same whatever the subsequent state of nature may be.[4] Chambers and Quiggin (2000) use the term output-cubical to describe the technology in (17), and they provide (p. 54) a graphical representation of output-cubical technologies for three states of nature ($S = 3$). Figure 2 provides a graphical representation of this technology for $S = 2$. The efficient production in figure 2 is $[z_{1a}, z_{2a}]$ if the input vector $\mathbf{x}^a$ is committed and $[z_{1b}, z_{2b}]$ if the input vector $\mathbf{x}^b$ is committed. If input $\mathbf{x}^a$ is committed the production in state 2 is relatively high compared to the production in state 1. If instead input $\mathbf{x}^b$ is committed production in state 1 is relatively high compared to production in state 2. Figure 2 thus illustrates that it is possible to substitute between state-contingent outputs by choosing different input vectors. It should be noticed that in the cases

[3] At this stage it is assumed that there are no budget restrictions or physical restrictions attached to the availability of the N inputs $(x_1, \ldots, x_N)$.

[4] In some types of production it may be possible to make input adjustments after the state of nature has revealed itself. This type of (dynamic) optimisation is not considered in the present paper.

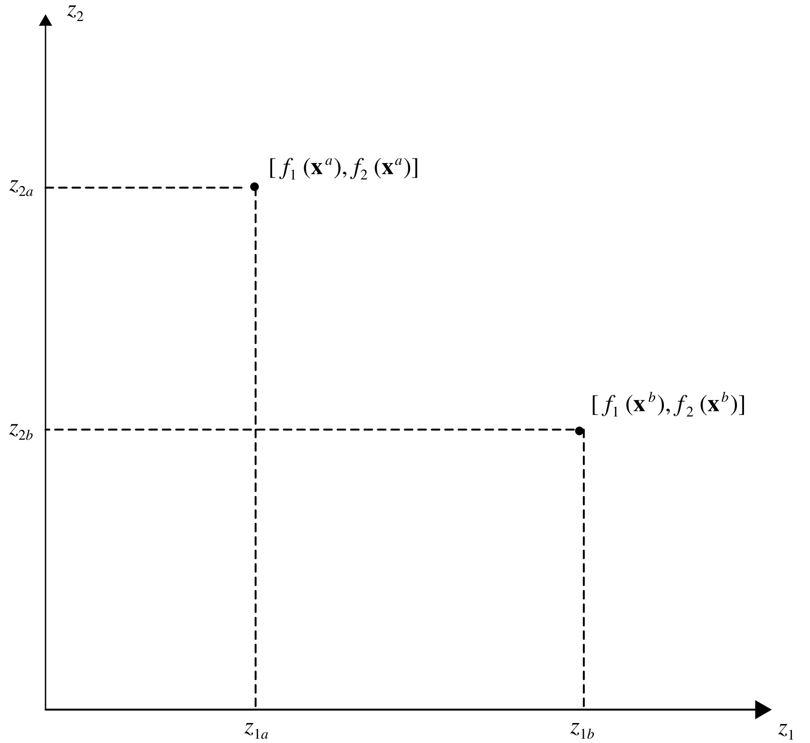

Figure 2 Output sets for state-contingent production

considered in this paper, the general condition for this kind of substitution is that the number of inputs is equal to or greater than the number of states.

Figure 3 further illustrates this substitutability, both between state-contingent outputs and between inputs. The input vector $\mathbf{x}^a$ produces z_{1a} in state 1 and z_{2a} in state 2. The input vector $\mathbf{x}^b$ produces z_{1b} in state 1 and z_{2b} in state 2. As illustrated by the isoquant z_{2a}, the output level z_{2a} in state 2 may be produced using different combinations of the two inputs x_1 and x_2, illustrated by the steep isoquant passing through $\mathbf{x}^a$ in figure 3. Correspondingly, the flat isoquant passing through $\mathbf{x}^a$ illustrates that the output level z_{1a} in state 1 may be produced using different combinations of the two inputs x_1 and x_2. Figure 3 also shows that if the decision maker chooses an input bundle with more x_2 and less x_1 (i.e., $\mathbf{x}^b$) then more output is produced in state 1 (z_{1b}) and less in state 2 (z_{2b}). Thus, the decision maker has the opportunity to control the uncertain output by controlling the input vector. This is the essence of the state-contingent approach because it shows that the decision maker may take an 'offensive' approach instead of a 'defensive' approach to planning under uncertainty.

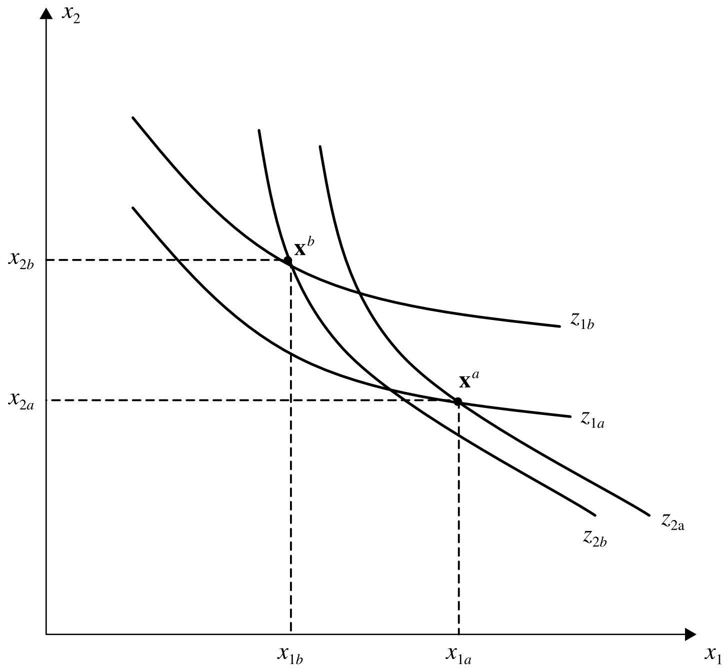

Figure 3 Substitution between input and states of nature

The normal properties attached to a production function f are that it is a positive, non-decreasing function with decreasing first-order derivatives, and this applies to f_s ($s = 1, \ldots, S$) in the following analysis. However, under uncertainty it is appropriate to consider the possibility that in some states of nature the optimal amount of input may be at a stage of the production function where output is decreasing in input (input has a poisoning effect). Accordingly, the property non-decreasing is not maintained, and I will consider both the increasing and the (possible) decreasing part of the production function, and therefore allow both $\partial f_s / \partial x \geq 0$ and $\partial f_s / \partial x < 0$.

As the technology considered in the paper is a special case of the more general technology considered by Chambers and Quiggin, each type of input has to be considered explicitly to gain tractable results.

3.2 Types of input

The crucial characteristic of production under uncertainty is that a specific input may yield different responses in different states of nature. Some inputs may influence production in some or all states of nature (state-general input). Some inputs may influence production in only one state of nature (state-specific input). In addition, some inputs may be allocated between

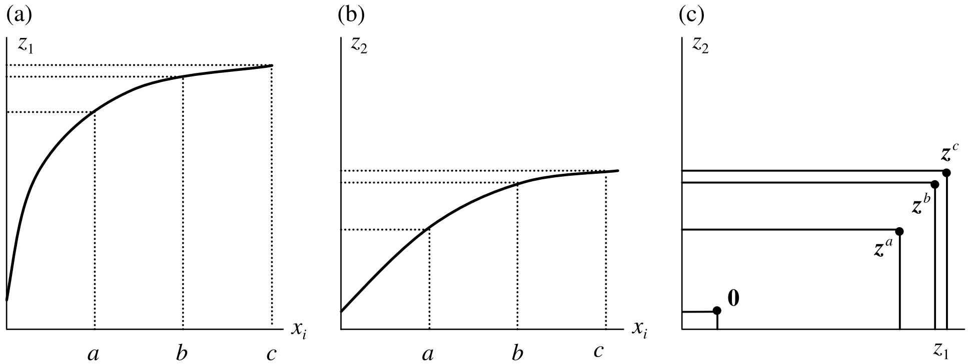

Figure 4 Derivation of product transformation curve for state-general input

different states of nature (state-allocable input). Finally, the availability of inputs may be limited by budgetary restrictions or physical restrictions.

3.2.1 State-general inputs

In the most general case, no specific assumptions are placed on the input. Accordingly, a state-general input is defined as an input that influences production in one or more (possibly all) states of nature.[5] The formal definition of a state-general input x_n is that

$$\frac{\partial f_s(\mathbf{x})}{\partial x_n} \neq 0 \quad \text{for one or more states } s \in \Omega \text{ for some (relevant) level of } x_n. \quad (18)$$

A state-general input is illustrated graphically in figure 4. Figure 4 assumes that there are only two states of nature. Figure 4(a) illustrates the production function in state 1 ($z_1 = f_1(x_i)$)[6] and figure 4(b) the production in state 2 ($z_2 = f_2(x_i)$). Corresponding to the four different levels of input (0, a, b, and c), there are four combinations of state-contingent output ($\mathbf{0}$, $\mathbf{z}^{\mathrm{a}}$, $\mathbf{z}^{\mathrm{b}}$ and $\mathbf{z}^{\mathrm{c}}$), as shown in figure 4(c).

Assuming free disposability of output (see Chambers 1988), the product transformation curves for each of the four input levels are illustrated as the right-angled curves with the state-contingent output $\mathbf{0}$, $\mathbf{z}^{\mathrm{a}}$, $\mathbf{z}^{\mathrm{b}}$ and $\mathbf{z}^{\mathrm{c}}$ just described as the corner points.

An example of a state-general input is the use of fertilisers in grain production. If the uncertain event is the weather during the growing season,

[5] Chambers and Quiggin (2000) use the term completely non-state-specific inputs to describe what is here called state-general inputs (p. 37).

[6] All other inputs are assumed fixed.

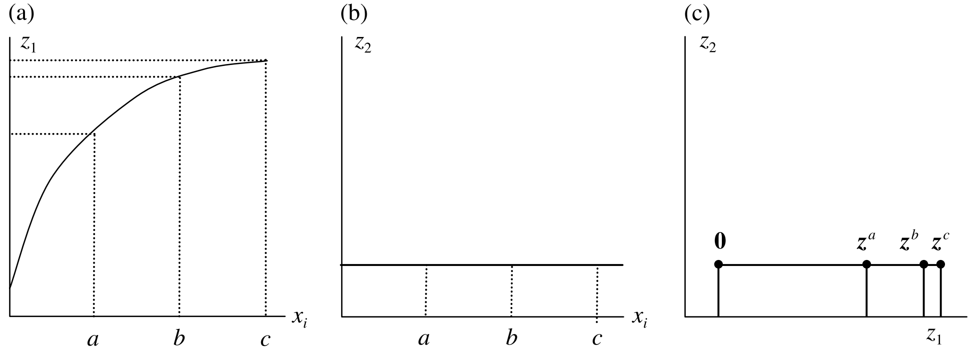

Figure 5 Derivation of the product transformation curve for state-specific input

the production z_1 in a 'wet' season (state 1) is determined by $f_1(x_n)$, and in a 'dry' season (state 2) the production z_2 is determined by $f_2(x_n)$. The concept non-joint in inputs means that even if one were willing to sacrifice some of the output in one state of nature it is not possible to influence the yield in the other state of nature. Yield when the season is 'wet' is completely independent of yield when the season is 'dry', and depends only on the amount of fertiliser x_n applied.

3.2.2 State-specific inputs

A state-specific input is a special case of a state-general input. A state-specific input is defined as an input that influences production in only one state of nature. Thus, the formal definition of a state-specific input x_n is that

$$\frac{\partial f_t(\mathbf{x})}{\partial x_n} > 0 \text{ and } \frac{\partial f_s(\mathbf{x})}{\partial x_n} = 0 \text{ for } s \neq t \text{ for some (relevant) level of } x_n \ (t, s \in \Omega). \tag{19}$$

A state-specific input may be illustrated as shown in figure 5. As in figure 4 there are two states of nature. However, only in one state (here state 1) has the input any influence on production. In state 2, production is a constant, independent of the application of the input x_i considered.

An example of a state-specific input is a pesticide that is only effective under certain weather conditions. Consider again production of grain under uncertain weather conditions where (z_1) is the output when the weather is 'wet' and (z_2) is the output when the weather is 'dry'. If a certain pesticide is effective only when the weather after pesticide application becomes 'wet' and has no effect when the weather afterwards becomes 'dry', then this input x_n is a state-specific input – specific to (and only to) the 'wet' state of nature.

The product transformation curves in figure 5(c) are, as before, right angles with corners corresponding to the four combinations of state-contingent output ($\mathbf{0}$, $\mathbf{z}^a$, $\mathbf{z}^b$ and $\mathbf{z}^c$).

3.2.3 State-allocable inputs

A state-allocable input[7] is defined as an input that may influence output in two or more states of nature, and which may be allocated (*ex ante*) to different states of nature. Therefore, the formal definition of a state-allocable input x_n is that

$$\frac{\partial f_s(\mathbf{x})}{\partial x_{ns}} > 0 \quad \text{for two or more states } s \in \Omega \text{ for some (relevant) level of } x_{ns} \qquad (20)$$

where x_{ns} is the amount of input x_n allocated (*ex ante*) to the s-th state of nature.

As long as there are no physical restrictions on the amount of input x_n available, a state-allocable input may be considered within the same framework as a state-general or state-specific input. The 'trick' is to consider x_{ns} as an independent input – an input that may itself be a state-general or a state-specific input. Thus, instead of considering the input vector $\mathbf{x} = (x_1, \ldots, x_n, \ldots, x_N)$ we consider the expanded input vector $\mathbf{x'} = (x_1, \ldots, x_{n1}, \ldots, x_{ns}, \ldots, x_{NS}, \ldots, x_N)$ and treat each x_{ns} as a normal variable input.

An example may clarify this.[8] Assume that the weather is uncertain. Assume further that labour (x_n) may be used either to build a dam that prevents flooding if the weather becomes 'wet' (state 1), or to improve the irrigation system that prevents drought if the weather becomes 'dry' (state 2). Then x_{n1} is the input 'labour used for building a dam' and x_{n2} is the input 'labour used for improving the irrigation system'.

In this example the two 'new' inputs (x_{n1} and x_{n2}) are state-specific inputs (they only influence production in one state of nature). In the following such inputs are called strictly state-allocable inputs. There may be examples of state-allocable inputs which are not strictly state-allocable. Consider for instance the input 'fertilisers'. Assume that application of fertilisers will increase production no matter what the weather in the subsequent growing season will be. However, there are different types of fertilisers, some of which will have a higher efficiency under certain weather conditions than others. If there are K such types of weather specific fertilisers then the relevant

[7] Chambers and Quiggin (2000) also use this term (p. 39).

[8] The example is taken from Chambers and Quiggin (2000, p. 38).

decision variable is no longer just the amount of fertiliser (x_n), but the amount of each type of fertiliser (x_{nk}, $k = 1, \ldots, K$). These 'new' inputs are in this case themselves state-general inputs.

If there is only a limited amount $\bar{x}_n$ of the state-allocable input available, then the allocation must comply with the following restriction:

$$\bar{x}_n \geq x_{n1} + x_{n2} + \ldots + x_{nS}. \tag{21}$$

Further, if each of the S 'new' inputs in equation (21) are state-specific (x_n strictly state-allocable) then the technology for a state-allocable input may be illustrated as shown in figure 6 for $S = 2$.

In figure 6 the available amount of input is c. With increasing amount of input allocated to state 1, output increases as illustrated in figure 6(a) (output in the other state is unaffected (figure 6(b)). With increasing amount of input allocated to state 2, output increases as illustrated in figure 6(d) (output in the other state is unaffected (figure 6c)). With some input allocated to state 1 (for instance a) and the rest (b) to state 2, the vector of state-contingent outputs will be as indicated by point P_{ab} in figure 6(e), where the points P_{c0} and P_{0c} correspond to the allocation of all input to state 1 and all input allocated to state 2, respectively.

Other combinations of input allocation are possible. If input allocation is allowed to change continuously, the result is a product transformation curve $P_{c0}P_{ba}P_{ab}P_{0c}$ as shown in figure 6(e). This curve resembles the 'normal' product transformation curve known from optimisation under certainty.

While the concept of a state-allocable input may be useful in a verbal description of planning problems under uncertainty, the concept is in fact redundant from an analytical point of view. State-allocable inputs may be fully described by using the concepts of state-general or state-specific inputs.

4. Criteria for optimal use of input

In the following, criteria for optimal use of inputs for each of the three types of input mentioned are derived. As one would expect, it is difficult to derive useful criteria for optimal production without knowing the exact form of the utility function. For a risk-neutral decision-maker with a linear utility function as in equation (14), useful criteria are relatively easy to derive. For a general risk-averse decision-maker it is not possible to provide specific criteria. However, it is possible to derive conditions that describe whether a risk-averse decision-maker will use more or less input than a risk-neutral decision-maker. This last part of the analysis depends heavily on the notion of 'good' and 'bad' states of nature. These two terms will therefore be defined first.

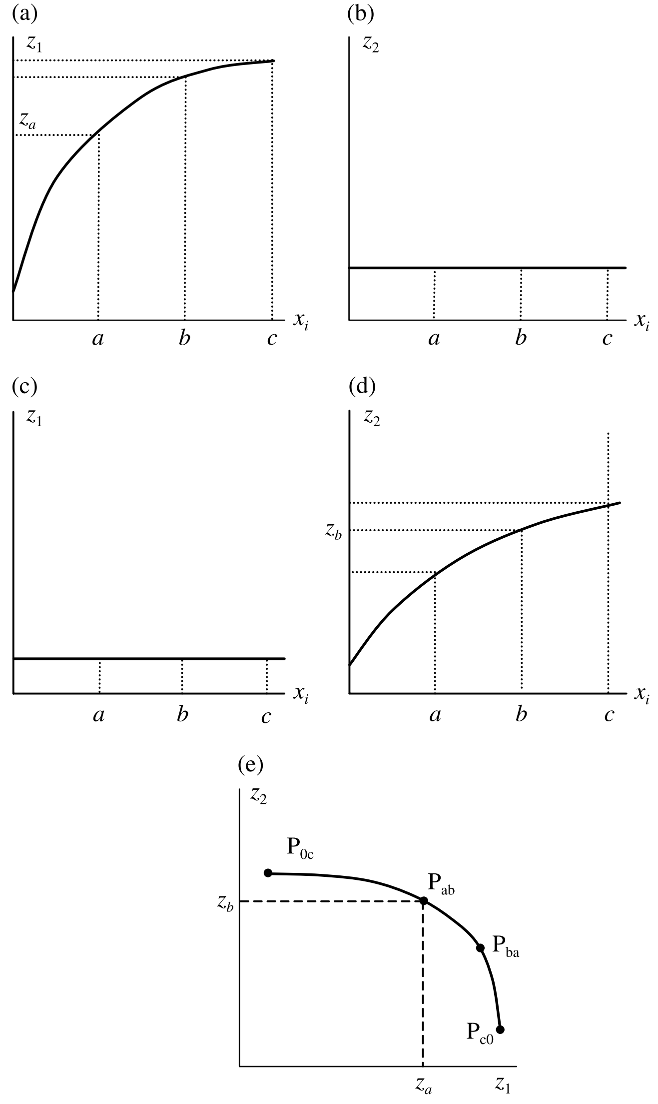

Figure 6 Derivation of the product transformation for state-allocatable input

4.1 'Good' and 'bad' states of nature

The terms 'good' and 'bad' are subjective concepts. What may be considered a 'good' and what may be considered 'bad' state of nature depends on the preferences of the decision maker. The benchmark therefore has to be carefully defined. Because the objective is to compare risk-neutral and risk-averse decision-makers, it is useful to take the conditions of the risk-neutral decision-maker as the benchmark.

Consider a risk-neutral decision-maker who optimises production using the input vector $\mathbf{x}^n$. The outcome is a state-contingent vector of net returns $(y_1(\mathbf{x}^n), \dots, y_S(\mathbf{x}^n))$ yielding a utility of $W^*(y_1(\mathbf{x}^n), \dots, y_S(\mathbf{x}^n)) = \pi_1 y_1(\mathbf{x}^n) + \pi_2 y_2(\mathbf{x}^n) + \dots + \pi_S y_S(\mathbf{x}^n)$.

Consider also a risk-averse decision-maker with a general utility function $W(y_1, \dots, y_S)$. As the scale of the utility function is arbitrary, it may be rescaled so that:

$$\sum_{s=1}^{S} W_s\{y_1(\mathbf{x}^n), \dots, y_S(\mathbf{x}^n)\} \equiv 1 \tag{22}$$

that is, the sum of the derivatives of $W(\mathbf{y})$ with respect to y_s ($s = 1, \dots, S$), W_s, at the point $\mathbf{y}(\mathbf{x}^n)$ is equal to 1.

Based on this scaling of the utility function, I define the following: to a risk-averse decision maker, a 'relatively good' (or just 'good') state of nature is at state s where:

$$W_s(y_1(\mathbf{x}^n), \dots, y_S(\mathbf{x}^n)) < \pi_s \tag{23}$$

and a 'relatively bad' (or just 'bad') state of nature is a state s where:

$$W_s(y_1(\mathbf{x}^n), \dots, y_S(\mathbf{x}^n)) > \pi_s \tag{24}$$

where $y_s(\mathbf{x}^n)$ is the state-contingent net return in state s for a risk-neutral decision-maker using the optimal amount of input $\mathbf{x}^n$. States where:

$$W_s(y_1(\mathbf{x}^n), \dots, y_S(\mathbf{x}^n)) = \pi_s \tag{25}$$

are defined 'neutral' states of nature.

Thus, a 'good' state is defined as a state where a state-contingent net income of \$1 gives a lower marginal utility than the probability of that state. Correspondingly, a 'bad' state is defined as a state, where a state-contingent net income of \$1 gives a higher marginal utility than the probability of that state. Marginal income is measured using the vector of state-contingent net incomes for a risk-neutral decision-maker optimising the use of input as the benchmark, and marginal utility is measured on a utility function that is locally (at point $\mathbf{y}(\mathbf{x}^n)$) scaled so that the sum of marginal utilities over the S states of nature is equal to 1.

It is immediately clear that if W is linear (as W^* in equation (14)), the scaling equation (22) has already been made, because the sum of the probabilities is 1. Also in this case there are no 'good' or 'bad' states of nature, just 'neutral' states of nature because neither equations (23) nor (24) are valid.

The definitions presented here are a generalisation of those informally given by Chambers and Quiggin (2000) for $S = 2$. To use the term 'bad' to describe a state where \$1 is worth more than the probability of that state, and 'good' to describe a state where \$1 is worth less than the probability of that state, is reasonable from the point of view that this definition complies with the view that a 'bad' state is something to be avoided, and that therefore one more dollar provides higher utility in a 'bad' state than in a 'good' state. Also, defining 'good' and 'bad' in relation to the state-contingent net returns of a risk-neutral decision-maker is appropriate, as will be shown in the following.

4.2 Criteria for state-general inputs

With inputs defined as state-general inputs the optimisation problem is given by the following:

$$\underset{\mathbf{x}}{\text{Max}}\, W(y_1, \dots, y_S) \tag{26}$$

where

$$y_s = f_s(\mathbf{x})p_s - \mathbf{wx} \quad (s \in \Omega). \tag{27}$$

The condition for optimal use of input x_n ($n = 1 \dots N$) is determined by setting the derivatives of (26) with respect to x_n equal to zero yielding:

$$\frac{\partial W}{\partial x_n} = \sum_{s=1}^{S} W_s(\mathbf{y})\left(p_s \frac{\partial f_s}{\partial x_n} - w_n \right) = 0 \quad (n = 1, \dots, N). \tag{28}$$

If the utility function W is linear (risk-neutrality) (28) reduces to:

$$\frac{\partial W}{\partial x_n} = \sum_{s=1}^{S} \pi_s \left(p_s \frac{\partial f_s}{\partial x_n} - w_n \right) = 0 \quad (n = 1, \dots, N) \tag{29}$$

which further reduces to:

$$E\left(p \frac{\partial f(x)}{\partial x_n} \right) = w_n \quad (n = 1, \dots, N) \tag{30}$$

where E is the expectation operator. Thus, a risk neutral decision-maker optimises the application of a state-general input x_i by increasing the application as long as the expected value of the marginal product is larger than the input price.

As the production function $f_s(\mathbf{x})$ and the product price p_s vary over states of nature the typical case is that:

$$\left(p_s \frac{\partial f_s(\mathbf{x})}{\partial x_n} - w_n\right) \neq \left(p_t \frac{\partial f_t(\mathbf{x})}{\partial x_n} - w_n\right) \quad (s, t \in \Omega) \tag{31}$$

that is, that the marginal net return in state s is different from the marginal net return in state t for some given amount of input x_n. According to equations (28) and (29) this means that for an optimal solution, the marginal net return will be positive in some states and negative in others. This means that at the optimal application of a state-general input the application would be too high in some states and too low in other states compared to the application if one knew in advance what state of nature would prevail. This is the case for both risk-neutral and risk-averse decision-makers.

The interesting question is whether the optimal application for a risk-averse decision-maker who optimises production according to equation (28) is higher or lower than the optimal application for a risk-neutral decision-maker who optimises production according to equation (29).

If all inputs are considered variable (substitution between inputs allowed) it is not possible to give a general answer. However, if all inputs except x_n are considered fixed inputs, then a risk-averse decision-maker would use more of input x_n than a risk-neutral decision maker if

$$\left.\frac{\partial W(y_1(\mathbf{x}), \ldots, y_S(\mathbf{x}))}{\partial x_n}\right|_{\mathbf{x}=\mathbf{x}^n} > 0 \tag{32}$$

that is, if the marginal utility is positive. As before, $\mathbf{x}^n$ is the optimal application of input for a risk-neutral decision-maker and $y_s(\mathbf{x}) = p_s f_s(\mathbf{x}) - \mathbf{w}\mathbf{x} (s = 1, \ldots, S)$.

Performing the differentiation in (32) yields:

$$\left.\frac{\partial W}{\partial x_n}\right|_{\mathbf{x}=\mathbf{x}^n} = \sum_{s=1}^{S} W_s \left.\left(p_s \frac{\partial f_s(\mathbf{x})}{\partial x_n} - w_n\right)\right|_{\mathbf{x}=\mathbf{x}^n} > 0. \tag{33}$$

It is not easy to interpret this condition. However, one would expect that a risk-averse decision-maker would use more input than a risk-neutral decision-maker if the input in question particularly improves the net return in those states of nature that are 'unpleasant' or 'bad' from the point of view of the decision maker, and vice versa.

Using the definition of 'good' and 'bad' states from equations (23) and (24), this is in fact the result obtained when interpreting condition (33). To see why, assume that the S states of nature are ranked so that the first t of these states are 'bad', and the remaining S-t states are 'good'. Further, assume that the marginal net return ($p_s \partial f_s(\mathbf{x}^n)/\partial x_n - w_n$ in the parenthesis in (33)) for a risk-averse decision-maker using the same amount of input as a risk-neutral decision-maker optimising production is positive in all these first t bad states of nature. In that case condition (33) applies, because high values of W_s ($W_s > \pi_s$) are combined with positive values of marginal net returns, and a risk-averse decision-maker would therefore use more input than a risk-neutral decision-maker.

The conclusion is that if the marginal net return from applying more input than x_i^n is positive in those states that are 'bad' to a risk-averse decision-maker, then it is optimal for a risk-averse decision-maker to use more of this input than a risk-neutral decision-maker. This also means that if the marginal net return by applying more input than x_i^n is negative in those states that are 'bad' to a risk-averse decision-maker, then it is optimal for a risk-averse decision-maker to use less of this input than a risk-neutral decision-maker. Because of the conditions that have been applied, the criterion seems weak. However, the criterion indicates what the important parameters are when comparing the use of a state-general input by risk-averse and risk-neutral decision-makers.

With only two states of nature the criterion may be illustrated graphically as shown in figures 7 and 8. In figure 7 it is shown how the net return curve in the bottom part of the figure is derived from state-contingent revenue and cost in the top of the figure. The net returns in state 1 and 2, respectively, illustrated in the middle part of figure 7, are derived as the difference between the revenue curve and the factor cost curves just above. With an increase in the amount of input x_i, the output in state 1 first increases (until a) and then decreases. In state 2, an increase in the amount of (the same) input x_i, first makes output increase (until b) and then decrease. Notice that because b is larger than a, the net return in state 2 still increases after the net return in state 1 has started decreasing. The net return curve at the bottom of figure 7 summarises how the state-contingent net returns in the two states develop when input x_i is increased. The arrow at the end of the curve indicates the direction in which input increases.

In figure 8 a net return curve K is combined with indifference curves to illustrate how the optimal amounts of input for a risk-neutral decision-maker (x_i^n) and a risk-averse decision-maker (x_i^a) are derived as the points of tangency between the net return curve and the indifference curve (straight line for risk-neutral decision-maker). In the example, $x_i^a >, x_i^n$, that is, a risk-averse decision-maker would use more input than a risk-neutral decision-maker. This result also applies using the criterion developed above. First notice that at the point x_i^n, $W_1/W_2 < \pi_1/\pi_2$. Therefore, due to

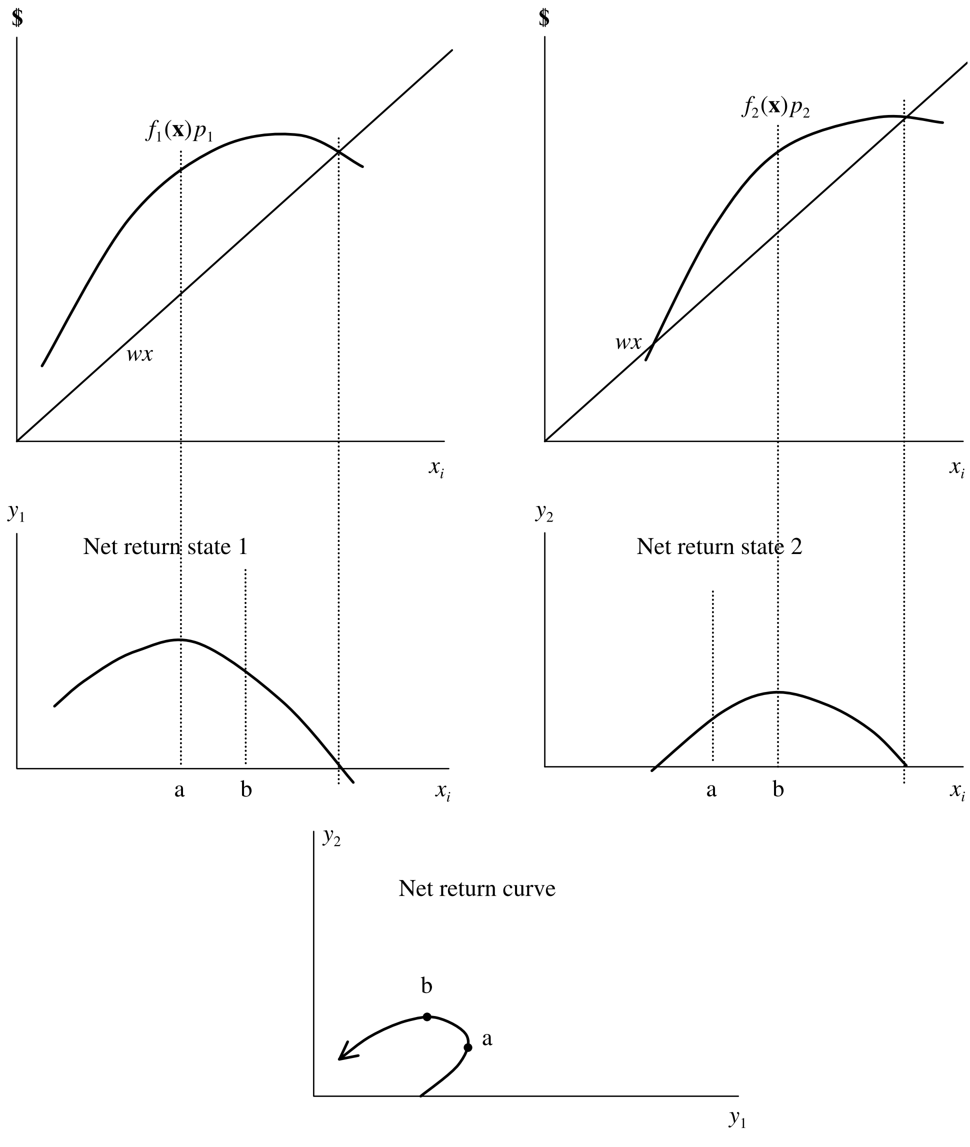

Figure 7 Derivation of the net return curve

the scaling in equation (22), $W_1 < \pi_1$ and $W_2 > \pi_2$. According to (23) and (24) this means that state 1 is 'good', and state 2 is 'bad'. As the marginal net return at the point x_i^n is negative in state 1 (the 'good' state) and positive in state 2 (the 'bad' state), the criterion $x_i^a > x_i^n$ applies.

Figure 8 also illustrates how difficult it is to compare the optimal input use of a risk-averse and a risk-neutral decision-maker. The result of the

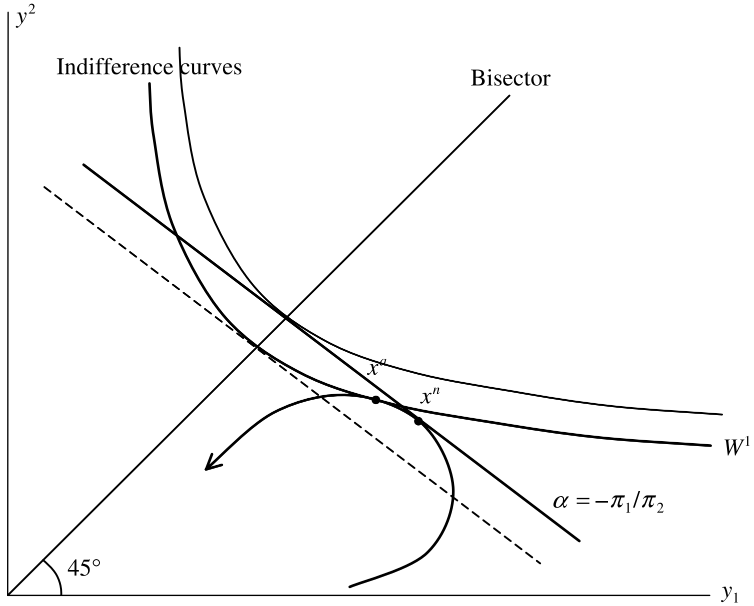

Figure 8 Derivation of optimal production for risk-averse (xa) and risk-neutral (xn) producer

comparison depends first of all on what is considered the 'good' state of nature and what is considered the 'bad' state of nature (i.e., whether the net return curve is over or under the bisector). Further, the result depends on whether the marginal income after having applied x_i^n units of input is positive or negative in the 'good'/'bad' state of nature (what direction the arrow points). If the arrow in figure 8 pointed the other way, then $x_i^a <, x_i^n$, and a risk-averse decision-maker would use more input than a risk-neutral decision-maker, because the point of tangency with the indifference curve of the risk-averse decision maker in this case is reached first.

4.3 Criteria for state-specific inputs

State-specific inputs are effective in only one state of nature. The optimisation problem is therefore given by the following:

$$\operatorname*{Max}_{\mathbf{x}} W(y_1, \ldots, y_S) \tag{34}$$

where

$$y_t = f_t(\mathbf{x})p_t - \mathbf{wx} \tag{35}$$

$$y_s = k_s - \mathbf{wx} \quad \text{for } s \neq t \tag{36}$$

and k_s is a constant.

The condition for optimal use of inputs **x** is determined by setting the derivatives of equation (34) with respect to x_n ($n = 1, \ldots, N$) equal to zero yielding:

$$W_t p_t \frac{\partial f_t(\mathbf{x})}{\partial x_n} = w_n \sum_{s=1}^{S} W_s \quad (n = 1, \ldots, N). \tag{37}$$

If the decision-maker is risk-neutral (37) reduces to:

$$\pi_t \left(p_t \frac{\partial f_t(\mathbf{x})}{\partial x_n} \right) = w_n \quad (n = 1, \ldots, N). \tag{38}$$

This means that a risk-neutral decision-maker should apply a state-specific input as long as the value of the marginal product multiplied by the probability of getting the state of nature where the state-specific input is active, is larger than or equal to the input price w_n.[9]

A risk-averse decision-maker uses more input x_n than a risk-neutral decision-maker if:

$$\left. \frac{\partial W(y_1(\mathbf{x}), \ldots, y_S(\mathbf{x}))}{\partial x_n} \right|_{\mathbf{x}=\mathbf{x}^n} > 0 \tag{39}$$

where, as before, x_n^n is the optimal application of input x_n for a risk-neutral decision-maker (other inputs considered fixed), and $y_s(\mathbf{x})$ is given by equations (35) and (36).

Performing the differentiation in (39) and using the value of $p_t \partial f_t(\mathbf{x}^n)/\partial x_n = w_n/\pi_t$, from (38), the condition (39) is equivalent to:

$$W_t > \pi_t \sum_{s=1}^{S} W_s \bigg|_{\mathbf{x}=\mathbf{x}^n} . \tag{40}$$

As the sum at the right hand side of equation (40) is equal to 1 (one) (see (22)), the condition in equation (40) indicates that state t is a 'bad' state (see (24)). It is therefore concluded that if the state-specific input x_n is directed towards a 'bad' state of nature, then a risk-averse decision-maker will use more input than a risk-neutral decision-maker, and vice versa.

[9] Notice that because $\partial f_s(\mathbf{x})/\partial x_n = 0$ for $s \neq t$, the condition (38) is in fact equivalent to (30).

This conclusion shows that, as for the state-general input, there is no unique answer to whether a risk-neutral decision-maker will use more or less state-contingent input than a risk-averse decision-maker. It depends on the type of state the input is directed towards.

4.4 Criteria for state-allocable inputs

A state-allocable input is characterised by the possibility of making an (*ex ante*) allocation of the input to whatever state (s) of nature is judged to be best. The optimisation problem for a state-allocable input x_n is given by the following:

$$\underset{x_{n1},\ldots,x_{nS}}{\text{Max}} \; W(y_1, \ldots, y_S) \tag{41}$$

where

$$y_s = f_s(\mathbf{x})p_s - w_n(x_{n1} + \ldots + x_{nS}) - \mathbf{w'x'} \quad (s = 1, \ldots, S) \tag{42}$$

and where $x_{ns} \geq 0$ is the application of input x_n with a view to increasing production in state s. The term $\mathbf{w'x'}$ is the (fixed) cost of buying the other $N - 1$ inputs.

In the following it is assumed that x_n is strictly state-allocable (each x_{nt} is state-specific). In that case the optimal application of input x_{nt} is determined by the same conditions as for state-specific inputs, that is,

$$W_t p_t \frac{\partial f_t(\mathbf{x})}{\partial x_{nt}} = w_n \sum_{s=1}^{S} W_s \quad (t = 1, \ldots, S). \tag{43}$$

If the decision-maker is risk neutral then, because of the scaling in (22), (43) may be written:

$$W_t p_t \frac{\partial f_t(\mathbf{x})}{\partial x_{nt}} = w_n \quad (t = 1, \ldots, S) \tag{44}$$

which further reduces to:

$$\pi_t \left(p_t \frac{\partial f_t(\mathbf{x})}{\partial x_{nt}} \right) = w_n \quad (t = 1, \ldots, S). \tag{45}$$

This means that a risk-neutral decision-maker should increase the application of a state-allocable input to state t as long as the value of the marginal product multiplied by the probability of getting state t is larger than the input price w_n.[10]

The question is whether a risk-averse decision-maker uses more or less state-allocable input than a risk-neutral decision-maker. The answer is given by comparing equations (44) and (45). The result is identical to the result concerning state-specific inputs. Thus, if a strictly state-allocable input is allocated to a 'bad' state of nature, then the risk-averse decision-maker will use more input directed to that state than a risk neutral decision-maker, and vice versa.

If the amount of input is restricted as in equation (21), the condition (43) changes to:

$$W_t p_t \frac{\partial f_t(\mathbf{x})}{\partial x_{nt}} = w_n \sum_{s=1}^{S} W_s + \lambda \quad (t = 1, \ldots, S) \tag{46}$$

where λ is the Lagrange multiplier of restriction (21).

If the decision-maker is risk neutral then the sum at the right hand side is 1 and $W_t = \pi_t$. This means that the limited amount of input x_n should be allocated between state s and state t so that:

$$\pi_t \frac{\partial f_t}{\partial x_{nt}} p_t = \pi_s \frac{\partial f_s}{\partial x_{ns}} p_s \quad (s, t \in \Omega) \tag{47}$$

that is, the value of the marginal product of input x_n in state t multiplied by the probability of getting state t should be equal to the marginal product of input x_n in state s multiplied by the probability of getting state s.

4.5 Criteria for optimal production of output

As in the case of certainty, one may want to consider production from the output side and to ask the question: What is the criterion for determining the optimal amount of output under uncertainty?

Under certainty the optimal production of a particular product (a commodity) is determined by the criterion $MC = p$, where MC is marginal cost and p is the product price. Under uncertainty it is not sufficient just to consider a particular type of product (a commodity). A commodity is not just one product. It is in fact S different (state-contingent) products, because a commodity that will be produced in state s is different from a commodity that will be produced in state t. (This type of statement is quite similar to the statement that a commodity that becomes available in one years time is

[10] Note that because $\partial f_s(\mathbf{x})/\partial x_{nt} = 0$ for $t \neq s$ the condition (37) is in fact equivalent to (30).

a different product compared to the same commodity that becomes available in two years time).

The optimisation problem under uncertainty therefore involves optimisation of production over the S states of nature as follows:

$$\operatorname*{Max}_{z_1,\ldots,z_S} W(y_1, \ldots, y_S) \tag{48}$$

where:

$$y_s = p_s z_s - c_s(\mathbf{w}, z_s) \tag{49}$$

and $c_s(\mathbf{w}, z_s)$ is the cost function:

$$c_s(\mathbf{w}, z_s) \equiv \min\{\mathbf{wx}: f_s(\mathbf{x}) \geq z_s,\ s = 1, \ldots, S\}. \tag{50}$$

As shown by Chambers and Quiggin (2000), the minimum cost of producing the entire vector of state-contingent outputs when the technology is output-cubical (state-general inputs) has to satisfy:

$$c(\mathbf{w}, \mathbf{z}) \geq \max_{s\in\Omega}\{c_s(\mathbf{w}, z_s)\}. \tag{51}$$

As state-specific inputs and state-allocable inputs are just special cases of state-general inputs, the condition in equation (51) may be considered a general condition for the (minimum) cost of producing an output vector $\mathbf{z}$.

Even in the case where equation (51) holds with strict equality (51) is not in general differentiable in z_s. Therefore it is not possible to solve equation (48) by using the derivatives. However, consider the special case where all inputs $(x_1, \ldots, x_N)$ are state-specific inputs; that is, each input influences production in only one state of nature. In this case the cost function $c_s(\mathbf{w}, z_s)$ is

$$c_s(\mathbf{w}, z_s) = \min\{\mathbf{w}^s\mathbf{x}^s: f_s(\mathbf{x}^s) \geq z_s,\ s = 1, \ldots, S\} = c_s(\mathbf{w}^s, z_s) \tag{52}$$

where $\mathbf{x}^s$ is the (sub)vector of inputs that are specific to state s and $\mathbf{w}^s$ is the corresponding (sub)vector of input prices. Because production in state s is completely independent of the input applied to state t and vice versa, the cost function for the entire vector of state-contingent outputs now has the simple additive form

$$c(\mathbf{w}, \mathbf{z}) = \sum_{s=1}^{S} c_s(\mathbf{w}^s, z_s) \tag{53}$$

and because equation (52) is differentiable, so is the cost function (53).

Under the condition that inputs are state-specific it is therefore possible to differentiate equation (48) with respect to **z**; that is, with respect to each of the elements ($z_1, \ldots, z_S$). Setting each derivative equal to zero leads to the following S optimality conditions:

$$W_s p_s - \sum_{t=1}^{S} W_t MC_s = 0 \quad \forall s \in \Omega \tag{54}$$

where MC_s is the marginal cost with respect to z_s; that is the derivative of the cost function c in equation (53) with respect to z_s. Notice that weighting MC_s by the marginal utility in all states of nature is a result of the fact that the cost of producing a marginal unit of output in state s is the same no matter what state of nature occurs. However, the marginal income (p_s) only occurs in state s. For a risk-neutral decision-maker (54) reduces to:

$$\pi_s p_s = MC_s \quad \forall s \in \Omega. \tag{55}$$

Therefore, a risk-neutral decision-maker should increase production of state s output as long as the marginal cost is lower than the product price in state s multiplied by the probability of state s.

This criterion is quite similar to the criterion for optimal production under certainty, the 'only' difference being that production under each state of nature is considered separately, and the product price is multiplied by the probability of being in the specified state of nature.

Taking the sum over states of nature on each side of equation (55) the following somewhat weaker condition appears:

$$\sum_{s=1}^{S} \pi_s p_s = \sum_{s=1}^{S} MC_s(\mathbf{w}^s, z_s) \tag{56}$$

which reduces to

$$E(p) = \sum_{s=1}^{S} MC_s(\mathbf{w}^s, z_s) \tag{57}$$

where $E(p)$ is the expected product price, and the sum on the right-hand side is the cost of producing one more unit in all states of nature; that is, the cost of producing one more unit of output with certainty. Thus, a risk-neutral decision-maker would produce one more unit of output as long as the cost of doing so is lower than the expected output price.

A risk-averse decision-maker will produce more in state s than a risk-neutral decision-maker if the following condition applies:

$$\frac{\partial W(y_1(\mathbf{z}^n), \ldots, y_S(\mathbf{z}^n))}{\partial z_s} > 0 \tag{58}$$

where

$$\mathbf{z}^n = (z_1^n, z_2^n, \ldots, z_S^n) \tag{59}$$

is the vector of optimal production in each state of nature for a risk-neutral decision-maker. Performing the differentiation in equation (58) and inserting the value of MC_s from equation (55), the following condition applies if it is optimal for a risk-averse decision-maker to produce more than a risk-neutral decision-maker:

$$p_s(W_s - \pi_s)\sum_{t=1}^{S} W_t > 0 \tag{60}$$

which is valid when $W_s > \pi_s$. But according to equation (24) this is just the definition of a 'bad' state of nature. It may therefore be concluded that if state s is a 'bad' state of nature then a risk-averse decision-maker will produce more in state s than a risk-neutral decision-maker. Correspondingly, if s is a 'good' state of nature, a risk-averse decision-maker will produce less in state s than a risk-neutral decision-maker.

Concerning the relationship between output in state s and in state t, the following relation is derived from equation (54):

$$\frac{W_s p_s}{W_t p_t} = \frac{MC_s}{MC_t}. \tag{61}$$

Under the condition that $p_s = p_t = p$ (i.e., no price uncertainty (only production uncertainty)), equation (61) reduces to:

$$\frac{W_s}{W_t} = \frac{MC_s}{MC_t} \tag{62}$$

which means that for optimal production the absolute value of the slope of the indifference curve is equal to the ratio of the marginal cost of producing in state s to the marginal cost of producing in state t.

Assume that the state-specific inputs considered are the amounts of a strictly state-allocable input being allocated to the S states of nature. In that case condition (62) is useful in determining the optimal allocation of a limited amount of input on the different states of nature for a risk neutral decision-maker. The interpretation of equation (62) in this context is that if a decision-maker is risk-neutral and there is no price uncertainty (only production uncertainty), then a limited amount of strictly state-allocable input should be divided between the different states of nature in such a way that the ratio of the marginal costs in each state of nature is equal to the ratio of the probabilities of being in those states of nature.

5. Conclusion

The objective of the present paper has been to derive useful criteria for optimal production under uncertainty using the state-contingent approach described by Chambers and Quiggin (2000). Compared to earlier attempts to derive criteria for optimal production under uncertainty, the state-contingent approach applied in the present paper has the merit of being based on the same marginal principles and optimisation tools as known so well from the general theory of production under certainty.

The analysis has been based on the general assumption that decision-makers are risk-averse. Under such a general assumption it is not possible to derive criteria that are applicable in a general decision-making context. Only when the decision-maker is risk-neutral, and the utility function therefore is linear, may useful general criteria be derived. However, it has been possible to derive conditions describing under which circumstances risk-averse decision-makers use more or less input than risk-neutral decision-makers. This information will prove useful both from a normative and from a descriptive point of view.

When the decision-maker is risk-neutral, the criteria for optimal production are analogous to the criteria for optimal production under certainty. From the input point of view the optimal application of input under certainty is determined by the criterion that the application of input should increase as long as the value of the marginal product exceeds the input price. Under uncertainty the general result is that the application of a state-general input should be increased as long as the expected value of the marginal product exceeds the input price. As state-specific and state-allocable inputs are special cases of state-general inputs, the same conditions apply to these inputs, although more specific criteria have been derived. From the output point of view the optimal production under certainty is determined by the criterion that production should expand as long as marginal cost is less than the product price. Under uncertainty it is not possible to derive criteria for optimal production in the general case (state-general inputs) because the cost function is not differentiable. However, when inputs are state-specific (including strictly state-allocable), optimal production is determined by the criterion that production in a specific state of nature should be increased as long as the marginal cost of increasing production in that specific state of nature is less than the product price in the particular state of nature multiplied by the probability of that state of nature.

When the decision-maker is risk-averse, it is optimal to use more input than used by a risk-neutral decision-maker if the input in question especially improves production in 'bad' states of nature. If the input in question especially improves production in 'good' states of nature, a risk-averse decision-maker will use less input. Thus, it is not possible to make general statements concerning who will use more input or who will produce more output: the risk-neutral decision-maker or the risk-averse decision-maker. The answer will depend on the specific preferences, partly described by what are considered 'good' and what are considered 'bad' states of nature from the point of view of the decision-maker.

The definition of 'good' and 'bad' states of nature provided in the present paper has proved useful and will probably also be helpful in further analysis of the economics of production under uncertainty.

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