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Investment planning under uncertainty and flexibility: the case of a purchasable sales contract*

Oliver Musshoff and Norbert Hirschauer†

Investment decisions are not only characterised by irreversibility and uncertainty but also by flexibility with regard to the timing of the investment. This paper describes how stochastic simulation can be successfully integrated into a backward recursive programming approach in the context of flexible investment planning. We apply this hybrid approach to a marketing question from primary production which can be viewed as an investment problem: should grain farmers purchase sales contracts which guarantee fixed product prices over the next 10 years? The model results support the conclusion from dynamic investment theory that it is essential to take simultaneously account of uncertainty and flexibility.

Key words: dynamic programming, flexibility, investment, sales contract, stochastic simulation, uncertainty.

1. Introduction

In its conventional form, the net present value criterion does not account for entrepreneurial flexibility with regard to the timing of an investment. Taking account of temporal flexibility implies that a deferrable investment opportunity (option) should only be carried out (exercised) if the profitability of the immediate investment is higher than the profitability of the deferred investment (cf. Jorgensen 1963; Dixit and Pindyck 1994, p. 138). Viewing the problem as a time-interdependent (dynamic) decision problem is equivalent to considering opportunity costs over time. That is, the critical exercise value (investment trigger) may be increased compared to a situation without flexibility. If one additionally takes uncertainty into consideration the effect of temporal opportunity costs will be even more pronounced (cf. Myers 1977; Dixit and Pindyck 1994; Pietola and Myers 2000; Carey and Zilberman 2002) because, in the course of time, more information will be available.

The weakness of the conventional net present value criterion is a substantial drawback in many areas of agro-economic decision-making which are often

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† Dr Oliver Musshoff (email: oliver.musshoff@agr.uni-goettingen.de), Georg-August-Universität Göttingen, Platz der Göttinger Sieben 5, D-37073 Göttingen, Germany. Dr Norbert Hirschauer (email: norbert.hirschauer@landw.uni-halle.de), Martin-Luther-Universität Halle-Wittenberg, Ludwig-Wucherer-Str. 2, D-06108 Halle (Saale), Germany.

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characterised by irreversibility *and* uncertainty *and* flexibility. Taking account of these characteristics in the decision process and trying to improve performance warrants increased planning efforts especially in large farming companies which face increasing risks in competitive and liberalised markets such as in Australia or Canada. This applies to most examples of 'classical' investments in buildings and animal production facilities even though they have not been analysed as flexible investments in the past (cf. Pietola and Wang 2000). Another relevant example is the switching decision between conventional and organic farming (cf. Kuminoff and Wossink 2005; Mußhoff and Hirschauer 2008).

The applicability of different approaches to stochastic dynamic decision problems (i.e. problems that entail choices at various stages and uncertain events embedded between these stages) depends on a number of contingencies: analytical solutions are only available for a subset of stochastic processes and for time-continuous exercise options with infinite lifetime (cf. McDonald and Siegel 1986). Finite difference methods (e.g. Brennan and Schwartz 1977) and numerical integration (e.g. Parkinson 1977) are able to solve problems with discrete exercise options and finite lifetime, but they are still restricted to a subset of stochastic processes. Stochastic simulation (cf. Boyle 1977), in contrast, represents a powerful method of representing probabilistic information. Nevertheless, Monte Carlo simulation has scarcely been applied to stochastic dynamic decision problems, in general, and flexible farm investment problems, in particular. The reason is that a stand-alone simulation does not allow for the consideration of flexibility and the solution of time-interdependent decision problems (cf., e.g. Hull 2000). Therefore, stochastic dynamic decision problems such as the optimal exercise strategy of American-type options or analogous decision problems have been frequently solved in the past by using stochastic decision trees (e.g. Magee 1964; Trigeorgis 1996; Hull 2000).

However, the use of decision trees is limited to certain types of time-interdependencies. Binomial trees (cf., e.g. Cox *et al.* 1979; Jarrow and Rudd 1983), and sometimes trinomial trees (Omberg 1988), are used to model Brownian motion. Going beyond equal jump processes, Nelson and Ramaswamy (1990) demonstrate that binomial trees can even be used to model the stationary Ornstein–Uhlenbeck process. Boyle (1988) demonstrates how to include two correlated random walk variables in a lattice framework. While allowing for a discrete approximation of Markov processes (and especially equal jump processes), decision trees are not suited if the time-series analysis reveals that the 'correct' process is non-Markov. The Markov property is the precondition for a recombining stochastic tree. Non-recombining trees, the nodes of which grow exponentially with each additional time step, become unmanageable 'bushy messes' (Hardaker *et al.* 2004, p. 203) even with a comparatively low number of time steps. In line with Briys *et al.* (1998), Hull (2000) and Longstaff and Schwartz (2001) we may conclude that, even though the increased computing power of modern PCs has improved the capacity of decision tree analysis over the last years, more complex problems that include non-Markov

processes and/or multiple and correlated variables exhibiting diverse processes cannot – at least not practically – be handled by means of decision trees.

In practice we do not know the results of the statistical analysis in advance (Laughton and Jacoby 1995; Spahr and Schwebach 1998; Odening *et al.* 2007). Hence, for solving stochastic dynamic decision problems, we need a generally applicable approach with the capacity to process probabilistic information independent of the type of process and the number of stochastic variables. In the past decade, a number of simulation-based procedures have been developed which are capable of handling the wide diversity of stochastic processes, including non-Markov ones (e.g. Grant *et al.* 1997; Glasserman 2004; Ibanez and Zapatero 2004).

We are the first, to our knowledge, to apply such an approach to a flexible contract problem in primary production. We examine whether the stochastic dynamic decision model provides an economic rationale for the observed behaviour of German grain farmers who did not buy sales contracts which had been offered to them by a large grain dealer. The application resorts to a simulation-based backward recursive procedure originally developed to price American-type financial options (cf. Grant *et al.* 1997). The Grant-procedure exploits the fact that the most flexible method for modelling random variables is stochastic simulation. It furthermore accounts for the limitations of a standard simulation which does not include an optimisation algorithm for time-interdependent decision problems. In our application, minor modifications and refinements are made to the Grant-approach. Emphasising its characteristics, the modified approach is labelled 'bounded recursive stochastic simulation' (BRSS).

The rest of this paper is organised as follows: In Section 2, we describe the decision situation of the grain producers in detail. This includes the description of the investment problem (subsection 2.1), the formulation of the investment model (subsection 2.2), and the description of the time-series model (subsection 2.3) which is used to represent the stochastic market price of rye. In Section 3, we explain the hybrid approach BRSS which is applied to the specific flexible investment problem of the grain growers. We present the results of our analysis in Section 4. The paper ends with concluding comments (Section 5).

2. A stochastic dynamic decision problem from primary production

2.1 Description of the investment problem

In 2004, the EU stopped rye market intervention. Therefore, rye producing farmers were considerably concerned both about the level and the fluctuations of future rye prices. In the years 2003–05, relating to these concerns, a large grain dealer in the new federal states of Germany offered a sales contract to grain farmers. Buying a contract would have required an immediate payment of €250. It would also have obliged the contracting farmer to sell five metric

tons of rye annually to the grain dealer for the next 10 years. In exchange, the grain dealer would have fixed the price at €90 per ton of rye, independent of future market price volatility. The contract furthermore specified that the dealer has the right to charge the farmer for 'cover-purchases' at market prices whenever the farmer does not supply the contracted amount of rye.

The farmers' decision problem can be seen as an investment problem: the costs of the contract which amount to €250 per five tons (€50 per ton) are viewed as investment costs. The useful lifetime of the investment is 10 years, and the differences between the contracted price and the volatile market prices represent the uncertain future investment cash flows. Contrary to conventional investment applications (e.g. purchase of capital goods), the profitability of this 'contractual investment' increases with a decreasing product price.

We assume that the farmers considered this investment as both risky and flexible. That is to say, the essential characteristics of dynamic investments are to be found in this contract problem: sunk costs, uncertainty of future cash flows and flexibility with regard to the timing of the investment. The deferrable investment opportunity to purchase the contract offered by the grain dealer can thus be seen an option which is analogous to an American-type call option with a strike price of €250 (relating to a contract of five tons). It is to be noted that the contract itself represents a futures contract (or more precise: a package of 10 annual futures) certifying the right *and* the obligation to sell the specified amount of five tons of rye at the price of €90 per ton in each of the 10 years following the conclusion of the contract.

The stochastic dynamic investment model which we consequently use to represent the farmers' decision situation is based on the following assumptions and considerations:

1. The rye prices on the market for food grains are uncertain. We assume that the stochastic process, as derived from the time series of rye prices according to statistical tests (see below), represents the best model to forecast the future rye market prices (see subsection 2.3).
2. The individual risk aversion of farmers is unknown to us due to the general problem of eliciting subjective risk attitudes. This is why – starting from the risk-free interest rate – we carry out variant calculations regarding the risk-adjusted discount rate (see subsection 2.2).
3. We assume that the contract clause regarding cover purchases makes the contract complete in that it makes any breaches unattractive to farmers. We also assume that farmers, who buy sales contracts, allow for risky yields by contracting only quantities they are certain to be able to supply even in years with adverse weather conditions.
4. Looking at the grain dealer, we assume that he will not breach the contract either because he would experience high economic losses, partly due to a deterioration of individual reputation. We furthermore do not consider the farmers' credit risk arising from an (unlikely) insolvency of the grain dealer.

5. No additional contract advantages – such as exclusive options to buy contracts in the future that might be linked to the conclusion of the present contract – are considered. The existence of such follow-up options would make the immediate investment comparably more competitive and reduce the investment trigger.
6. No additional contract disadvantages – such as a loss of flexibility regarding future land use – are considered. Having in mind that in the considered region with poor soils and low rainfall much rye is produced due to its draught resistance, we consider only the partial planning problem of whether to market rye with or without the contract.[1]
7. Not only the market price of rye, but also the contract terms that will be offered to the farmer in the future are uncertain. First of all, this regards the question over which period the sales contract will be offered in principle. This is equivalent with asking how long the decision to buy the contract can be deferred. Besides the question of how long the contract will be offered, there is also uncertainty regarding the question of whether, and eventually how, the grain dealer will adjust the contract terms (e.g. the guarantee price) in the future.

The important question for the farmer is whether and when to sign the contract. Given the stochastic development of rye prices, we modelled farmers who faced the question which observed rye market price (critical price) should trigger them to buy the contract. Aiming to account for the farmer's uncertainty regarding the contract terms we distinguish two scenarios. Within the *constant-contract-term scenario 1*, we assume that the grain dealer will offer the contract at constant terms for a certain period. Since farmers are uncertain about this contract-offer period, we carry out variant analyses for periods from zero to five years and for varying risk attitudes. Within the *constant-contract-value scenario 2*, we maintain the variant calculations regarding the contract-offer period over which the decision to buy the contract can be deferred and regarding the risk attitudes. Since farmers are also uncertain about the terms they get in the case of a future contract conclusion, we now assume that the grain dealer tries to maintain a constant-contract-value offer. To be more precise, we assume that, within the time he upholds the contract offer in principle, he will annually adjust the guarantee price according to the drift rate found for the stochastic rye market price.

[1] We aim at finding a plausible economic rationale for the contract refusal of farmers in the new federal states of Germany who produce rye at present because they face both low rainfalls and quickly draining sandy soils. Climatic change makes us expect rather more precarious rainfalls in these areas in the future (Lasch *et al*. 1999). It seems thus plausible to assume that the relative competitiveness of rye will even increase. This justifies the a priori assumption that rye will be produced in the future and thus a partial approach that searches only for the best utilisation (marketing) strategy. In other words, we argue that the expected future opportunity costs of entering the contract (i.e. the costs caused by the fact of not being able to produce something else if something else becomes more profitable) are negligible.

2.2 Formulation of the investment model

In the model, buying the contract is seen as an additional investment into a running business. Contrary to isolated investments, this requires the quantification of the *incremental* benefits which accrue from the investment. The net present value NPV_t of the contract investment which can be carried out within the time span T in different years t ($t = 0, 1, \ldots, T$) is to be calculated as follows:

$$\begin{aligned} &\mathrm{NPV}_t = V_t - I = \mathrm{PV}_t^G - \mathrm{PV}_t - I \text{ with} \\ &\mathrm{PV}_t^G = \sum_{\omega=t+1}^{t+Z} P_t^G \cdot e^{-(\rho-p)\cdot(\omega-t)} \text{ and } \mathrm{PV}_t = \sum_{\omega=t+1}^{t+Z} E(P_\omega)\cdot e^{-\rho\cdot(\omega-t)} \end{aligned} \tag{1}$$

V_t describes the expected present value of future investment cash flows, and I (= €50 per ton) the investment cost. Z (= 10 years) denotes the useful lifetime of the investment, that is, the contracted supply period. The *certain contract cash flows* P_t^G result from the fixed rye price which is guaranteed for Z years if the farmer signs the contract in time t. $E(P_\omega)$, with $E(\cdot)$ as the expectation value operator, designates the expectation values of the *uncertain without-contract cash flows* (i.e. the uncertain market prices per ton of rye) for the production periods ω, $\omega = t + 1, t + 2, \ldots, t + Z$. These expectation values depend on the stochastic process identified for the rye price. The additional symbol ω is needed because t itself varies depending on when the option is exercised. The contract investment generates a positive return in a time-period ω if the market price falls below the guarantee price. PV_t^G is the present value of future cash flows for one ton of rye, if the farmer buys the contract. PV_t is the expected present value of cash flows without the investment, that is, if the farmer continues to sell rye on the spot market. ρ describes the risk-adjusted discount rate, and p is the risk premium which is added to the risk-free interest rate r. The following relationship is valid (cf. Hull 2000, p. 502):

$$\rho = r + p \tag{2}$$

For discounting the certain contract cash flows P_t^G, the risk-free interest rate $r = \rho - p$ is adequate, independent of the risk attitudes of the decision-maker. However, utilising the risk-free interest rate r for discounting the volatile without-contract cash flows P_ω is only justified in the case of risk-neutral decision-makers. A risk-averse decision-maker requires an additional risk-premium p for discounting the uncertain market prices of rye.

Being used to discount inflation-adjusted prices, the risk-free interest rate r is derived as the inflation-adjusted average return on German Federal bonds with remaining lives of 15–30 years, the assumption being that there is no risk involved in these bonds. The nominal (time continuous) average return of German Federal bonds for the period of 1988–2003 amounts to 6.3 per cent per annum. In the same period, the (continuous) inflation rate amounts to

2.0 per cent (Deutsche Bundesbank 2006). We use the resulting real interest rate of 4.3 per cent as proxy for the (continuous) risk-free interest rate.

The subjective risk attitudes of decision-makers remain inherently uncertain unless reliable empirical evidence is provided through adequate survey work. Due to the well-known empirical problems of quantifying subjective risk attitudes (cf. Hudson *et al.* 2005, for an overview), risk premiums required by decision-makers are often assumed for modelling purposes (cf., e.g. Gebremedhin and Gebrelul 1992; Berg 2003). Frequently, additional sensitivity analysis is used to mitigate the problem. The range of values explored in this study is supported by the literature. Gebremedhin and Gebrelul (1992), for example, use six different discount rates ranging from 0 to 12 per cent in their study of meat goat enterprises. Marchant *et al.* (2004) choose three discount rates, namely 3, 5 and 10 per cent per annum. We carry out a sensitivity analysis which varies the risk premium p in four increasing steps of 2.5 per cent, thus covering a range of risk-adjusted discount rates from 4.3 to 14.3 per cent per annum.

If the investment decision is made immediately, a non-negative net present value is realised which hereafter will be referred to as 'intrinsic value'.[2] The non-negativity of this value is due to the fact that no investment obligation is involved. Formally, the intrinsic value can be calculated as follows:

$$i_t = \max(0, \mathrm{NPV}_t) \tag{3}$$

If the investment decision is postponed, the 'living' opportunity to invest has a continuation value f_t which describes the value of the optimal future investment decision:

$$f_t = E(F_{t+1}) \cdot e^{-\rho} \tag{4}$$

F_{t+1} is the value of the investment opportunity in $t + 1$. The value of F_{t+1} is defined by the optimal future decision strategy and the development pattern of the stochastic variable. Investing immediately implies that one realises the intrinsic value by eliminating the continuation value. A profit maximising investor will invest only if the intrinsic value exceeds the continuation value. Otherwise, 'wait and see' is a better strategy. At any one point in time, the value of an investment opportunity F_t equals the maximum of the intrinsic value i_t (of the immediate investment) and the continuation value f_t (of the postponed investment):

$$F_t = \max(i_t, f_t) \tag{5}$$

Equation (5) is equivalent to the Bellman equation (cf. Bellman 1957). It can be shown that under certain regularity assumptions stopping region and continuation region are separated by an unambiguous critical value P^* or V^* (see Dixit and Pindyck 1994, p. 129).

[2] This term is taken from the domain of option pricing theory where similar decision problems such as the determination of the early exercise frontier for American-type options on stocks have to be solved.

2.3 The time-series model

In Equation (1), the rye market price P_t represents a stochastic variable.[3] Due to the fact that EU-intervention on the rye market has come to an end, empirical time series of rye prices from Germany do not contain much useful information with regard to the stochastic pattern that is to be expected in the future. Assuming that future rye prices will follow the price pattern of the world market, we resort to a time-series analysis of Canadian price data. Canada is a country with significant rye production and largely liberalised agricultural markets. While realising that even Canadian rye prices may be affected by state intervention (e.g. by the price of wheat which, in turn, is partially influenced by the Canadian Wheat Board), they seem to be the best available proxy for rye world market prices.[4] To be more specific: we use – in accordance with the real interest rate – inflation-adjusted average annual Canadian rye prices from 1988 to 2003 (Agriculture and Agri-Food Canada 2003) in order to determine the pattern of plausible future price fluctuations in Germany.

The identification of the most adequate stochastic process requires, first of all, a test of stationarity. According to the Dickey–Fuller test (Dickey and Fuller 1981), the rye price series is non-stationary at a significance level of 5 per cent. Geometric Brownian motion (GBM) is a non-stationary Markov process that is well suited to represent price dynamics because it excludes negative (price) values. The time discrete and state continuous version of GBM, which is required for simulation, is to be formalised as follows (cf. Luenberger 1998, p. 311):

$$P_t = P_{t-\Delta t} \cdot e^{\left[\left(\alpha - \frac{\sigma^2}{2}\right) \cdot \Delta t + \sigma \cdot \sqrt{\Delta t} \cdot \varepsilon_t\right]} \tag{6}$$

α denotes the drift rate and σ the standard deviation of the logarithmic relative changes in rye prices (cf. Campbell *et al.* 1997, p. 363). ε_t describes a standard normally distributed random number (white noise), and Δt denotes the length of a time interval (here: one year). Given Equation (6), $E(P_t) = P_{t-\Delta t} \cdot e^{\alpha \cdot \Delta t}$ denotes the expectation value for the rye price which is needed to calculate the present value of cash flows without contract investment according to Equation (1). Calculations based on Canadian rye prices yield a drift rate $\alpha = 1.24$ per cent, and a standard deviation $\sigma = 28.10$ per cent per annum.

Note that farmers cannot exit the contract after having signed. Thus, the relevant annual payoffs of signing the contract (= incremental benefits) as well as their present value V_t (= $\mathrm{PV}_t^G - \mathrm{PV}_t$) can be negative. That is, while both depend

[3] Structural models representing causal relationships are necessarily larger and more complex than time-series models which mirror the stochastic pattern of relevant random variables. In line with many other contributions on investment analysis under uncertainty (cf. e.g. Pietola and Wang 2000; Odening *et al.* 2005) we have decided to estimate a time-series model for forecasting.

[4] Despite potential price effects of an increasing bio-energetic use of agricultural crops, we furthermore assume that a time-series analysis of (past) rye prices reveals relevant probabilistic information for a future-orientated decision-making (time stability hypothesis).

Table 1 Decision model parameters and data

Investment cost I	€50 per ton of rye
Useful life of the investment Z	10 years
Assumed contract-offer period	Systematically varied between zero and five years
Risk-free interest rate r	4.3% per annum
Risk premium p	Systematically varied between 0.0% and 10.0% per annum
Range of risk-adjusted discount rate ρ	Between 4.3% and 14.3% per annum
Stochastic process of the rye market price P_t	Geometric Brownian motion (GBM)
Parameters of the stochastic process	
Drift rate α	1.24%
Standard deviation σ	28.10%
Guaranteed rye price at present P_0^G	€90.0 per ton
Corresponding present value PV_0^G	€715.8 per ton
$\mathrm{PV}_0^G = \sum_{\omega=1}^{Z} P_0^G \cdot e^{-(\rho-p)\cdot\omega} = P_0^G \cdot \left(\frac{1-e^{-(\rho-p)\cdot Z}}{e^{\rho-p}-1}\right)$	
Initial rye market price P_0	€90.0 per ton
Corresponding present value PV_0	€763.4 per ton (for $\rho = 4.3\%$) €671.3 per ton (for $\rho = 6.8\%$) €593.2 per ton (for $\rho = 9.3\%$) €526.9 per ton (for $\rho = 11.8\%$) €470.3 per ton (for $\rho = 14.3\%$)
$\mathrm{PV}_0 = \sum_{\omega=1}^{Z} E(P_\omega)\cdot e^{-\rho\cdot\omega} = P_0 \cdot \left(\frac{1-e^{-(\rho-\alpha)\cdot Z}}{e^{\rho-\alpha}-1}\right)$	

on the stochastic variable rye price P_t which follows GBM, neither incremental investment benefits nor the present value V_t of incremental benefits follow GBM.

Data used in the decision model are summarised in Table 1.

3. The hybrid solution algorithm BRSS

Hereafter we illustrate the BRSS-method and apply it to the above-described contract problem from primary production. When describing the principal procedure we refer to an early exercise period of five years. While GBM turns out to be the incidental result of the statistical analysis in the exemplary case, we use the generally applicable BRSS procedure, thus taking into account that in practical applications the type of process is not known in advance. When describing the BRSS procedure, which facilitates the consideration of any probabilistic information, we refer to stochastic processes in general. It should be noted that – in order to economise on planning costs – one should always choose the method which presumably causes the least costs for the required performance. This includes learning costs.

Table 2 Summary of symbols used for the description of BRSS working steps

$t = 0, 1, \ldots, T = T-5, T-4, \ldots, T-1, T$	Early-exercise dates of the investment option (here: $T = 5$)
$V^*_{T-5}, V^*_{T-4}, \ldots, V^*_{T-1}, V^*_T$	Critical present value of investment cash flows at different exercise dates
$P^*_{T-5}, P^*_{T-4}, \ldots, P^*_{T-1}, P^*_T$	Critical market price of rye at different exercise dates
$n = 1, 2, \ldots, N$	Consecutive number of test values (here: $N = 10$)
$V(n)_{T-5}, V(n)_{T-4}, \ldots, V(n)_{T-1}$, $P(n)_{T-5}, P(n)_{T-4}, \ldots, P(n)_{T-1}$	Test values (present values of investment cash flows) and corresponding test prices used as starting point for the Monte Carlo simulation
$s = 1, 2, \ldots, S$	Consecutive number of simulation runs (here: $S = 50\,000$)
$f(n, s)_{T-5}, f(n, s)_{T-4}, \ldots, f(n, s)_{T-1}$	Continuation value at different exercise dates and depending on the nth test value in the sth simulation run

The BRSS-method is a hybrid approach comprising two elements: first, dynamic programming with its capacity for a simultaneous consideration of the uncertainty and the flexibility of investments, and second, Monte Carlo simulation with its nearly unlimited capacity for a modelling of stochastic variables (i.e. including non-Markov processes, multiple stochastic variables, correlations, etc.). Table 2 summarises the symbols that are used below when describing the BRSS procedure in detail.

$V^*_{T-5}, V^*_{T-4}, \ldots, V^*_{T-1}, V^*_T$ denote the early exercise strategy, that is, the critical present values of investment cash flows that are to be determined. At each early exercise date t ($t = T-5, T-4, \ldots, T-1, T$), it will be optimal to invest (buy the sales contract) whenever the corresponding critical present value of investment cash flows is reached. After some manipulation, this investment trigger can be alternatively expressed as critical rye price P^*_T. The general relationship between V_t and P_t (and thus V_t^* and P_t^*) is defined by the investment model as specified in Equation (1). The exact relationship between V_t and P_t, however, depends on the specific type of the stochastic process which is assumed for the future production and contract years.[5]

While determining the critical exercise values in the backward recursive manner of dynamic programming, we use Monte Carlo simulations to represent the

[5] If the stochastic market price of rye P_t follows a Markov process, every expected present value of the investment returns V_t corresponds to an unambiguous *initial* rye price since the expected future prices depend only on the current price level ($E(P_\omega|P_t)$). In the most simple case, if P_t follows random walk with zero drift, the price P_t corresponding to V_t is simply derived from Equation (1) by multiplying PV_t with the corresponding capital recovery factor for the useful life of the investment Z and the risk-adjusted interest rate ρ. In our case, that is, GBM with a non-zero drift α, the relevant rate to be used is $\rho - \alpha$ (cf. Table 1). In the non-Markov case, the expected present value V_t, still depends on the expected future prices. These, however, depend on the evolvement of the price to its current level, that is, on other past values ($E(P_\omega|P_t, P_{t-1}, P_{t-2}, \ldots)$). In other words, after identifying the stochastic process and given $P_{t-1}, P_{t-2}, \ldots$, for any given present value PV_t corresponding prices P_t can be constructed according to Equation (1).

identified stochastic process. At each early exercise date ($T-1, T-2, \ldots, T-5$), we choose N ($n = 1, 2, \ldots, N$) test present values of the investment cash flows ($V(n)_{T-1}$, $V(n)_{T-2}, \ldots,$ $V(n)_{T-5}$). The range which these test values are taken from is determined by using the known critical exercise value of the following period as a lower bound, and by making an educated guess at an upper bound, obtaining thereby an interval which is divided into $N-1$ equal subintervals deemed sufficiently small for subsequent interpolation. We then determine the corresponding test prices $P(n)_{T-1}$, ($P(n)_{T-2}, \ldots, P(n)_{T-5}$). These test prices are used as starting points for a Monte Carlo simulation comprising S ($s = 1, 2, \ldots S$) runs. Thus, for each of the test prices at each early exercise date a total of S 'path-dependent continuation values' $f(n, s)_{T-1}$, ($f(n, s)_{T-2}, \ldots, f(n, s)_{T-5}$) are determined.

3.1 Determination of the critical exercise value at the last potential exercise date

The critical exercise value at the termination date T is the starting point for the backward recursion. There is no temporal flexibility left. Hence, the investment should be carried out if we observe a market price of rye of P_T^* at which the expected present value of future investment cash flows covers the investment costs ($V_T^* = I$).

3.2 Determination of the critical exercise value at the penultimate potential exercise date

Using the critical exercise value of the following period (here: $V_T^* = 50$) as a lower bound and choosing an upper bound (here: €500) we generate a rather large interval to select the N (here: 10) test values from. Starting from each of the corresponding test prices $P(n)_{T-1}$, S (here: 50 000) simulation runs (paths) are simulated according to the given price process. Knowing V_T^* and P_T^*, we first calculate the path-dependent continuation values $f(n, s)_{T-1}$ for all paths and test prices:

$$f(n, s)_{T-1} = \max(0, V(n, s)_T - I) \cdot e^{-\rho} \tag{7}$$

After simulating enough paths[6] for an isomorphic representation of the stochastic process, the continuation value $f(n)_{T-1}$ for each test value is calculated as the average of all path-dependent continuation values for this test value:

$$f(n)_{T-1} = \sum_{s=1}^{S} f(n, s)_{T-1} \cdot S^{-1} \tag{8}$$

[6] For a technical description of how to use stochastic simulation to model a wide variety of stochastic processes with established software packages see Winston (1998, p. 325). Regarding the number of required simulation runs, Haug (1998, p. 40) stipulates that at least 10 000 runs should be carried out. Fortunately, with any given number of simulation runs, one can improve the stability of the solution by using so called variance reduction methods without a great increase of computational time. An overview of various variance reduction procedures can be found in Glasserman (2004, chs 4 and 5).

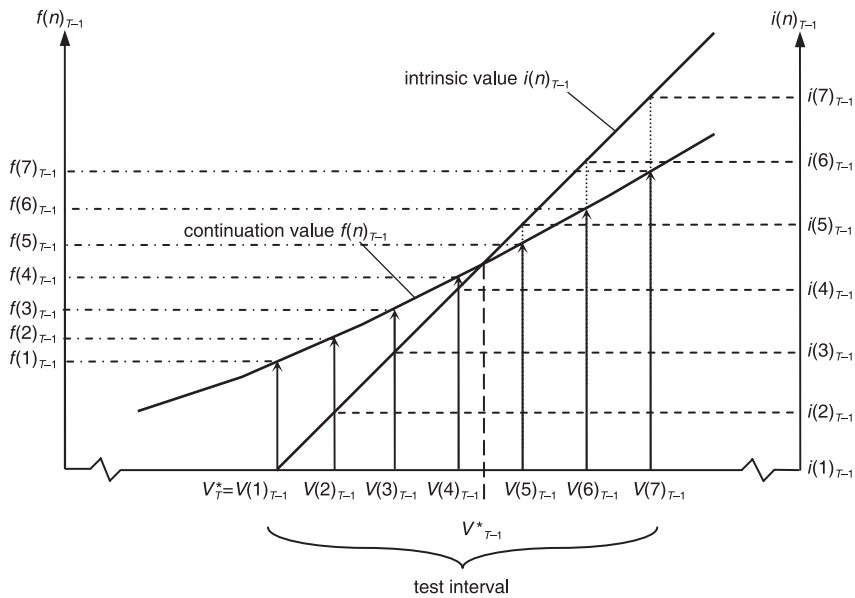

Figure 1 Determination of the critical present value V^*_{T-1} using the BRSS.

In order to compare the possible strategies 'invest' and 'wait', we additionally need to calculate the intrinsic value. The intrinsic value $i(n)_{T-1}$ for each test value $V(n)_{T-1}$ can be directly derived as:

$$i(n)_{T-1} = \max(0, V(n)_{T-1} - I) \tag{9}$$

It is very unlikely that the so-called identity condition where intrinsic value and the continuation value coincide will be met by one of the predefined test values. In most cases, the critical value will fall between two test values where a change of sign of the difference of intrinsic value and continuation value occurs. The critical present value V^*_{T-1} is then determined by means of linear interpolation. In the example presented in Figure 1, one needs to interpolate between the values $V(4)_{T-1}$ and $V(5)_{T-1}$.

Reducing the length of the initial interval and maintaining the number of subintervals improves the quality of the approximation because it shortens the subinterval on which one needs to interpolate. The initial interval must be enlarged if it did not yield a change of sign of the difference of intrinsic value and continuation value.

3.3 Determination of the critical exercise values at the remaining early exercise dates

The principal procedure described above is applied backwards until all critical early exercise values are known. However, whereas the determination of the

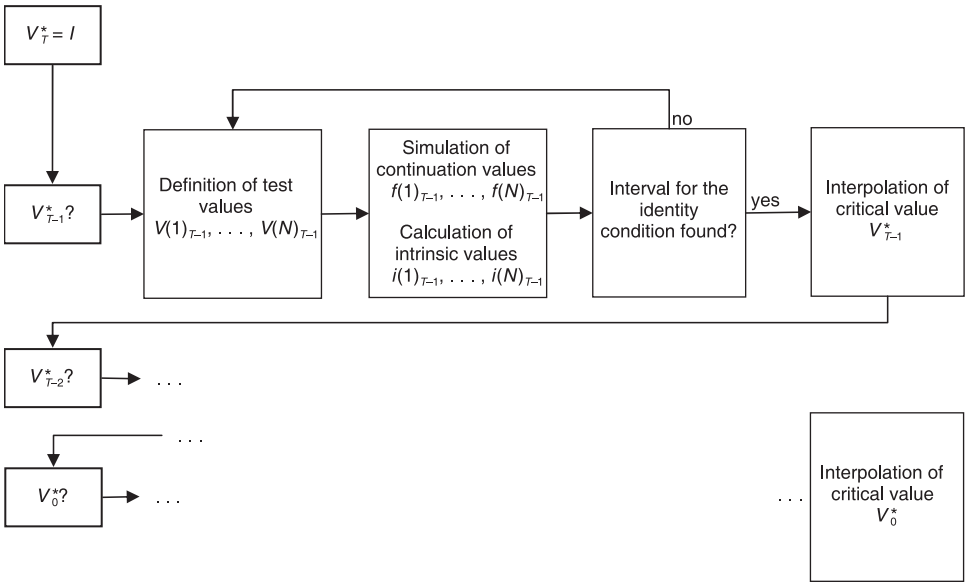

Figure 2 Procedural steps for determining the optimal investment strategy by means of BRSS.

continuation value at the pen-ultimate exercise date reflects the valuation of a European option, the determination of the continuation value at earlier exercise dates truly reflects the valuation of an American option. That is, when one determines, at any exercise date t, the critical exercise value V_t^* and P_t^* one has to take into account that a future investment may be carried out at time $t + 1$, $t + 2$ up to T. Again stochastic simulation can be used to determine the continuation values for a given set of test values $V(n)_t$, because the future exercise strategy is known. The procedure that is used to determine the remaining critical values is analogous to the one described above. Only the computation of the continuation value for each path in Equation (10) has to be modified according to the actual exercise time κ resulting in each path:

$$f(n, s)_t = \max(0, V(n, s)_\kappa - I) \cdot e^{-\rho \cdot (\kappa - t)} \tag{10}$$

$$\text{with } \kappa = \begin{Bmatrix} t+1, & \text{if } V(n,s)_{t+1} \geq V^*_{t+1} \\ t+2, & \text{if } V(n,s)_{t+2} \geq V^*_{t+2} \wedge V(n,s)_{t+1} < V^*_{t+1} \\ \vdots & \\ T, & \text{otherwise} \end{Bmatrix}$$

Figure 2 provides an overview of the working steps. They can be easily programmed in spreadsheet software such as MS-EXCEL which include a random number generator.

The minor, but effective modifications which we have made to improve the approach of Grant *et al.* (1997) can be summarised as follows. At a given early exercise date, our first working step is to predefine N equally distant test

prices. We then use the same sequence of random numbers for all simulations starting from these test-values. This saves a great deal of effort and time compared to the original procedure which required the repeated generation of random numbers for subsequent simulations of price paths starting from different test-values. Furthermore, using a predefined sequence of test-values facilitates the technical automation of consecutive work steps whereas the original procedure required an intervention after each simulation in that either a lower or a higher test value for the next simulation had to be explicitly chosen by the analyst according to the result of the previous simulation.[7]

4. Results

Table 3 shows the critical exercise prices and critical present values of investment returns for $t = 0$, depending on the risk-adjusted discount rate and the period

Table 3 Constant-contract-term scenario 1: critical exercise values (in € per ton) for different risk-adjusted discount rates ρ and different contract-offer periods*

	Critical rye prices in $t = 0$					Critical present values in $t = 0$				
	1	2	3	4	5	6	7	8	9	10
ρ (%)	4.3	6.8	9.3	11.8	14.3	4.3	6.8	9.3	11.8	14.3
Contract-offer period										
0 years	78.5	89.3	101.0	113.7	127.4	50.0	50.0	50.0	50.0	50.0
1 year	58.6	69.0	79.8	91.5	104.2	218.7	201.2	189.8	180.1	171.3
2 years	53.2	63.7	74.6	86.4	99.0	264.5	240.7	224.1	210.0	198.5
3 years	50.3	60.8	71.9	83.7	96.4	289.1	262.3	241.9	225.8	212.1
4 years	48.2	59.0	70.2	82.2	95.0	307.0	275.8	253.1	234.6	219.4
5 years	46.8	57.8	69.1	81.2	93.9	318.8	284.7	260.3	240.4	225.1

*The critical price P_0^* is to be derived as: $P_0^* = (\mathrm{PV}_0^G - V_0^*)[(e^{\rho-\alpha} - 1)/(1 - e^{-(\rho-\alpha)\cdot Z})]$ (cf. Table 1).

[7] Using the above-described hybrid procedure, the solution to the decision problem is straightforward as long as one deals with a single stochastic state variable (such as the rye price) following a Markov-process. In this simple case, one needs, at each time instant, to determine one critical exercise value forming, in turn, a two-dimensional early exercise strategy over time. In the case of multiple stochastic state variables, one needs, at each time instant, to determine critical combinations of values for different stochastic variables (an early exercise function) forming, in turn, a multidimensional early exercise strategy over time (cf. Ibanez and Zapatero 2004). The same applies if the stochastic variable follows a more complex stochastic process such as a non-Markov process. Because the regularity assumptions are not met we are not able to express the optimal exercise strategy at any one early exercise date as one critical value. Instead, it needs to be expressed as critical combinations of all those values that determine the forecast of the stochastic variable. If we assume, for example, an autoregressive process of the second order, we need to determine a critical value in period t for a given value of this variable in the previous period $t - 1$. By a subsequent systematic parameterisation of this previous value, we are able to determine critical combinations. With regard to the BRSS procedure this implies that, at each early exercise date, we replicate the process of determining test prices (corresponding to the test present values) as well as the simulation starting from these test prices for each of the parameterised previous values.

over which the contract offer is presumably maintained. This contract-offer period was varied from zero (no flexibility to defer) to five years.[8] In the considered scenario 1, the contract terms are presumed to be constant within this contract-offer period.

The first value in the first column of Table 3 reveals that, if the contract offer is not maintained (i.e. if the farmer faces again a now or never decision), the critical rye price amounts to €78.5 per ton for a risk-neutral farmer. In column 6, the corresponding critical present value of future investment cash flows is shown. It is equivalent to the costs of the contract investment of €50.0 per ton. Given the presently observed price of €90.0 per ton (which results in a present value of future cash flows of –€47.6 per ton if the farmer exercises the investment), a risk-neutral farmer should not buy the contract. He would realise a negative net present value of the contract investment of –€97.6 per ton.

Farmers are even less inclined to buy the sales contract if they assume that they can defer the investment decision because the contract offer is maintained for a certain period. The five subsequent rows of columns 1 and 6 in Table 3, headed by the presumed contract-offer period, depict the corresponding results. If a risk-neutral farmer believes, that he can defer the contract investment for one year, he should only buy the sales contract immediately if the presently observed market price for rye (present value of future investment cash flows) was below €58.6 (above €218.7) per ton.[9] With increasing time to defer, the critical price (present value) decreases (increases). With many future investment dates left, chances are high that a delayed investment will be more profitable than an immediate purchase of the contract. We could also say that the opportunity costs of investing immediately are higher if there is a lot of flexibility left. If there is no flexibility, temporal opportunity costs are down to zero and the critical present value is down to investment costs.

The opposite behaviour of the two criteria 'critical price' and 'critical present value' is due to the fact that the profitability of the considered contract investment increases with a decreasing product price (cf. Equation (1)). However, the important drop of the critical rye price comes with the first years of flexibility. In other words, the critical values are especially sensitive depending on whether zero, one, or two years of flexibility are assumed. In this context

[8] In the case considered here (i.e. with the assumption of constant contract terms), the critical values that are shown for $t = 0$ and for the presumed contract-offer periods of zero (1, 2, 3, 4, 5) years coincide with the critical early exercise values in the dates T ($T-1$, $T-2$, $T-3$, $T-4$, $T-5$) for an exercise period of five years. The value 79.8 (see column 3, row 2), for instance, indicates the critical price at which a farmer who uses a risk-adjusted discount rate of 9.3 per cent should exercise at the present date if he presumes that the contract offer will remain constant for one year. It also indicates the penultimate early exercise value in his early exercise strategy if he assumed an early exercise period of five years.

[9] It can be shown that the binomial-method, which also could be used because we incidentally found GBM, provides nearly identical results to the BRSS-method. In the considered case, using time steps of 0.05 years to model the stochastic variable, the value of the flexible contract investment opportunity, for instance, deviates only by 0.1 per cent.

it is interesting to note that the grain dealer had indeed maintained his offer for two years. The critical values do not change significantly if longer times to defer are assumed.

Columns 2–5 and columns 7–10 give the same sort of information for farmers with different risk-aversions. The variation of risk-adjusted discount rates shows that the more risk-averse a farmer, the more ready he is to insure against risk; that is, the critical market price the falling short of which makes him buy the fixed-price contract is higher than for a less risk-averse farmer. For any given flexibility, risk-averse decision-makers are more inclined to give up the chances of a market solution in favour of a reduction of price volatility. For a farmer with a risk-adjusted rate of 9.3 per cent (column 3), for instance, the critical price trigger is €101.0 per ton if there is no time to defer. If the farmer expects to be able to defer the investment for five years, the critical rye price amounts to €69.1 per ton. Looking at the corresponding critical present value, we find for this farmer (column 8) an increase of the critical value from €50.0 to €260.3 per ton. In this context, it is interesting to note that in the case of a now-or-never decision the critical present value of future cash flows is equivalent to the costs of the contract investment, independent of the risk attitude (see columns 6–10).

Table 4 shows the critical exercise prices and critical present values of investment returns for scenario 2. In this scenario, we maintain the variant calculations regarding the period over which the contract offer is presumed to be maintained. We now assume, however, that within this contract-offer period the grain dealer will, year by year, adjust the guarantee price he offers according to the drift rate of the stochastic rye market price.

A brief look at Table 4 shows that the systematic effects found in the previous Table 3 – that is, the impact of an increasing risk aversion as well as the impact of an increasing time to defer – are confirmed. In other words: both scenarios demonstrate that the critical price (present value) increases (decreases) with

Table 4 Constant-contract-value scenario 2: critical exercise values (in € per ton) for different risk-adjusted discount rates ρ and different contract-offer periods*

	Critical rye prices in $t = 0$					Critical present values in $t = 0$				
	1	2	3	4	5	6	7	8	9	10
ρ (%)	4.3	6.8	9.3	11.8	14.3	4.3	6.8	9.3	11.8	14.3
Contract-offer period										
0 years	78.5	89.3	101.0	113.7	127.4	50.0	50.0	50.0	50.0	50.0
1 year	53.5	64.8	76.0	88.3	100.9	262.0	232.5	214.8	198.8	188.6
2 years	47.4	58.9	70.3	82.4	95.1	313.7	276.5	252.4	233.4	218.9
3 years	43.9	55.5	67.3	79.3	92.2	343.4	301.9	272.2	251.5	234.0
4 years	41.4	53.4	65.3	77.6	90.4	364.6	317.5	285.4	261.5	243.4
5 years	39.7	52.1	64.0	76.4	89.5	379.1	327.2	293.9	268.5	248.1

*The critical price P_0^* is to be derived as: $P_0^* = (\mathrm{PV}_0^G - V_0^*)[(e^{\rho-\alpha} - 1)/(1 - e^{-(\rho-\alpha)\cdot Z})]$ (cf. Table 1).

Table 5 Critical guarantee prices and investment costs (in € per ton)*

	Scenario 1 (constant-contract-term)		Scenario 2 (constant-contract-value)	
	1	2	3	4
Contract offer period	P^{G}*	I*	P^{G}*	I*
2	107.4	–88.3	113.4	–124.7
1	100.8	–35.9	105.3	–65.3
0	80.9	122.6	80.9	122.6

*ρ = 9.3%, P_0 = €90 per ton.

increasing risk aversion. The critical price (present value) decreases (increases) with an increasing time to defer.

A brief comparison of both tables show that – for any one of the combinations of risk aversion and flexibility – the critical prices (present values) are lower (higher) in Table 4 compared to Table 3. These results can be interpreted as a lower inclination of the farmer to buy the contract. This is due to the assumption made in scenario 2 that the grain dealer will, year by year, increase the prices that are guaranteed in the contracts concluded in the future. From the farmer's point of view, this generates higher opportunity costs of the immediate investment.

Additional variant calculations reveal that – given a time to defer of at least one year, an observed rye price of €90 per ton, a stochastic future rye price development according to the specified time-series model, and the offered contract terms – the immediate purchase of the sales contract is not preferable for moderately risk averse farmers with a risk-adjusted discount rate of up to ρ = 11.5 per cent (scenario 1) or ρ = 12.2 per cent (scenario 2). In contrast, if one assumed that there is no flexibility at all, moderately risk-averse farmers should have bought the contract. This applies to both scenarios. While our stochastic dynamic decision model thus provides an economic rationale for the observed investment reluctance, it is interesting to know how the grain dealer would have to adjust the terms of the contract in order to make it attractive to farmers. Based on the fact that the presently observed rye market price is a 'given' for both market partners, Table 5 depicts the ceteris paribus critical guarantee price P^{G}* and the critical investment costs I* from the viewpoint of a farmer who uses a risk-adjusted discount rate of 9.3 per cent and who assumes a contract-offer period between zero and two years.

Scenario 1 shows, for instance, that the grain dealer has to guarantee a price of more than €107.4 (€100.8) per ton in order to make the contract offer attractive for a farmer who uses a risk-adjusted discount rate of 9.3 per cent and who expects to be able to defer the investment for two years (one year). The critical guarantee price decreases with the contract-offer period. The grain dealer could also adjust the investment costs in order to make his offer attractive to the farmer. Given the overall constellation of the other contract

terms, the grain dealer would ceteris paribus even have to pay €88.3 (€35.9) per ton in order to make his offer attractive to the farmer who uses a risk-adjusted discount rate of 9.3 per cent and who expects to be able to defer the investment for two years (one year). The critical investment costs increase with a decrease of the flexibility. Scenario 2 confirms these systematic effects. Due to its assumption of an increasing guarantee price, the farmers' refusal to buy the contract is all the more explained by the model in scenario 2.

5. Concluding comments

Traditional methods of investment appraisal exhibit major deficiencies with regard to a simultaneous consideration of the uncertainty of future cash flows and of the flexibility of investment timing under many circumstances. For instance, decision trees are not suited to represent problems that include stochastic variables which follow non-Markov processes. They are cumbersome, at least, in the case of multiple stochastic variables and non-Brownian Markov processes. We have refined and described a numerical procedure originally developed to price American-type options which is clearly superior under these circumstances. The approach, labelled BRSS, is a hybrid method which combines two essential advantages of conventional techniques: the power of stochastic simulation with respect to the modelling of all types of stochastic processes, and the capacity of dynamic programming for solving dynamic problems.

We have demonstrated the suitability of the hybrid approach for stochastic dynamic decision problems with an application to a decision problem of German grain farmers who were offered a sales contract by their grain dealer that would have fixed the price of rye independent of future market price developments. Given an opportunity to defer the decision, the farmers' decision situation represents a flexible investment planning problem. Considering uncertainty, irreversibility and flexibility, we find that it would only make sense to buy the contract immediately for very risk-averse farmers. We may thus conclude that our stochastic dynamic decision model provides both a plausible economic rationale for the empirically observed behaviour of decision-makers and a valuable decision support tool.

Future applications aiming at providing better explanations for observed behaviour and better decision support could tackle other important decisions that are to be made under risk and flexibility. This refers to other contract decisions as well as 'regular' capital investments (e.g. in buildings or technical equipments) or the decision to convert from conventional to organic farming. Doing so, it will be important to systematically extract the relevant stochastic information from the available data (time-series analysis) and to process this information adequately in a stochastic dynamic decision model. Such a decision model needs to be flexible enough to handle the many different stochastic processes that might result for the random variables from the time-series analysis.

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