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Dynamic adaptation to resource scarcity and backstop availability: theory and application to groundwater*

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In this paper we analyse the optimal management of a renewable resource (groundwater) with stock-dependent extraction cost and a backstop substitute, facing two-sector linear demands. Application to the Kiti region in Cyprus demonstrates the model's performance and is used to test for the difference between optimal and myopic behaviour. It is found that the presence of a backstop resource diminishes the importance of optimal dynamic behaviour, whereas in the absence of backstop the optimal control solution yields a value for social welfare significantly larger than the myopic policy.

Key words: backstop technology, endogenous adaptation, Gisser–Sanchez effect, groundwater resource management, multistage optimal control.

1. Introduction

The paper presents a theoretical analysis of a renewable natural resource management problem in the face of a backstop technology with perfectly elastic supply. The model follows Kim *et al*. (1989) in that it models multiple and heterogeneous sectors and uses multistage dynamic optimal control to characterise the optima (with and without a backstop) and the associated decentralised open-access resource outcomes. The contribution of this paper is to consider in this environment the impact of the presence of a backstop resource upon the optimal and open-access solutions while providing a pertinent case study. (Kim *et al*. look at the adoption of alternative water-using technologies rather than alternative water resources, which is qualitatively different.) This entails characterising and comparing the optimal and decentralised timing of the adoption of backstop resources by the various sectors

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and ultimately the resulting state of the renewable resource and welfare. With the closed-form solution of the model impossible to obtain, the model is calibrated to the case of groundwater use in the Kiti area of Cyprus, with desalination as the backstop, and ultimately solved numerically.

The discussion takes place with the Gisser–Sanchez effect (GSE) as its backdrop. The GSE is a paradoxical empirical result, present and persisting in the dynamic solutions of groundwater exploitation, since first identified by Gisser and Sánchez (1980). Numerous consecutive modifications and refinements of the basic Gisser–Sanchez model did not change the essence of the result, which is, the benefits from managing groundwater extraction are numerically insignificant, even in water-scarce regions. Koundouri (2000, 2004a,b) provides a critical review of the relevant published work. In this paper, we show that this effect remains in the presence of backstop resources, such as the desalination described in the application. That is, there are relatively few gains to be made from optimal management of the groundwater resource in the presence of a backstop when compared with myopic groundwater use, which defines decentralised resource use. In particular, the empirical part of the paper is composed of two simulations, one in the presence of the backstop resource (from desalination) and the other in its absence. In the former, it is shown that the GSE remains, whereas in the latter the GSE is absent. Given that the GSE has been shown to be a phenomenon that results when groundwater resources are abundant (relative to demand), this result is not surprising. The difference here is that the case study in question shows that the GSE is also determined by the availability and cost of alternative water resources, not just groundwater.

In Section 2, we formulate and solve a multistage optimal control model. In Section 3, we apply the model and test the robustness of GSE, via simulation, with and without backstop availability. Section 4 concludes the paper.

2. The model

The model characterises the optimal control solution for a renewable resource with stock-dependent extraction cost and a backstop substitute, facing two-sector linear demands. The model is applied to groundwater resources, where users with distinct demand curves exist (residential and agricultural sectors) and different supply sources are available; the cheaper ones being exhaustible but the most expensive ones (typically desalination or possibly large-scale recycling of past consumption) providing unlimited amounts of the input in question. Following Kim *et al.* (1989), the optimal solution employs the technique of multistage optimal control, which we extend towards incorporation of a backstop substitute. The technique allows identification of dynamic endogenous adaptations to increasing resource scarcity and backstop technology.

Farmers sell their production in competitive markets so that the price of water is equal to the value of its marginal product. The agricultural production function is assumed to be constant returns to scale, whereas factors

other than water and land are optimised conditional on the rate of water extraction. Access to the aquifer is restricted by land ownership. Following Kim *et al.* (1989), model construction begins by stating disaggregate categories of sector-specific demand curves. Equation (1) represents the inverse demand function for water for each of the two sectors that demand water

$$p = \frac{a_i}{b_i} - \frac{1}{b_i}(w_i), \quad i = 1, 2 \tag{1}$$

where p is the groundwater price, w_i represents groundwater quantities demanded by each sector i, a_i and b_i are sector-specific (uncompensated) demand parameters, assumed time-invariant, and $b_i > 0$. Sector-specific inverse demand curves are ordered so that $a_1/b_1 < a_2/b_2$, which implies that as p increases over time due to decreasing groundwater availability, water demand for each of the two economic sectors reaches zero sequentially. In the absence of backstop resource and given that there exist two economic sectors demanding water, one endogenous switching time results from this series of choke prices. Thus, aggregate water demand appears as a piecewise linear function.

The marginal pumping cost function (MC) is

$$MC_G = c[h(t)] \tag{2}$$

where h is the head of the aquifer above sea level. At lower head levels, it is more costly to extract water because more and/or deeper wells must be drilled and the water must be pumped farther distances. As the aquifer nears exhaustion ($h = 0$) extraction cost rises rapidly. Thus, we model the average cost of extracting water from the aquifer as a positive, decreasing, convex function of the head, that is, $c(h) \geq 0$, $c'(h) < 0$, $c''(h) \geq 0$ and $\lim_{h \to 0} c(h) = \infty$. A second feature of the model is that we are dealing with a coastal aquifer. A distinguishing feature of such aquifers is the possibility of seawater desalination, which provides a potential backstop water source. Thus, the aquifer is not the only possible source of water. Following Nordhaus (1973, 1979) and Heal (1976), we consider a super-abundant resource to flow from a backstop technology with constant unit cost ($\bar{p}$).

The hydrological equation of motion (3) gives the change in the level of the head of the aquifer through time and defines the constraint of the system (Gisser and Sánchez 1980; Todd 1980)

$$\dot{h}(t) = \frac{R + (f - 1 - s)\sum_{i=1}^{2} q_{Gi}(t)}{A \cdot S} \tag{3}$$

where $\dot{h}$ is the time derivative of $h(t)$ and $h(t) \leq h(t)^e$, where $h(t)^e$ is the natural hydrologic equilibrium. The aquifer's recharge rate is represented by R and $f(0 \leq f \leq 1)$ is the return-flow coefficient of percolation back into the aquifer. Another special feature of coastal aquifers is that they are vulnerable to

seawater intrusion; that is, as the aquifer is emptied, saltwater intrudes into the aquifer in order to re-establish the hydrological balance, eventually making water unusable. This quality externality is captured by the salinity coefficient $s(0 \leq s \leq 1)$ and indicates the loss of effective quantity of groundwater available in the aquifer, because of poor quality (increased salinisation).[1] The size of the aquifer is measured by A, and S is the storativity coefficient (the average saturation of water in the aquifer). The formulation in (3) implicitly assumes that changes in the water level are transmitted instantaneously to all users (infinite hydraulic conductivity). This assumption clearly exaggerates the degree of common property in some aquifers.

The goal of the planning equilibrium is to maximise the present value of generated economic surplus from intertemporal water use and the production of water from desalination, subject to the hydrological constraint of the system. Social benefits from water use are given by

$$\begin{aligned} SB &= \sum_{i=1}^{2} \int_{0}^{q_{Gi}(t)+q_{Di}(t)} \left[\frac{a_i}{b_i} - \frac{1}{b_i}(q_{Gi}(t) + q_{Di}(t)) \right] d[q_{Gi}(t) + q_{Di}(t)] \\ &= \sum_{i=1}^{2} \left[\frac{a_i}{b_i}(q_{Gi}(t) + q_{Di}(t)) - \frac{1}{2b_i}(q_{Gi}(t) + q_{Di}(t))^2 \right] \end{aligned} \tag{4}$$

where $q_{Gi}(t) + q_{Di}(t)$ represents the total quantity of water used by the ith economic sector, and is equal to the sum of sector ith's consumption of groundwater $[q_{Gi}(t)]$ and water produced from desalination. Total production costs (TC) are

$$TC(t) = \sum_{i=1}^{2} [c[h(t)]q_{Gi}(t) + \bar{p}q_{Di}(t)] \tag{5}$$

Thus, net social benefits (NSB) at a given time are

$$\begin{aligned} NSB = &\sum_{i=1}^{2} \left[\frac{a_i}{b_i}(q_{Gi}(t) + q_{Di}(t)) - \frac{1}{2b_i}(q_{Gi}(t) + q_{Di}(t))^2 \right] \\ &- \sum_{i=1}^{2} [c[h(t)] \cdot q_{Gi}(t) + \bar{p} \cdot q_{Di}(t)] \end{aligned} \tag{6}$$

Section 2.1 formulates the model in a two-stage dynamic framework. The multistage formulation and solution is given in Appendix 1 (available from http://www.aueb.gr/deos/uk/koundouri.htm).

[1] Please see Appendix 2 (available from URL: http://www.aueb.gr/deos/uk/koundouri.htm) for the solution of the multistage optimal control problem with aquifer recharge and salinisation coefficients being a function of the head of the aquifer's water table (h_t), that is, $R(h_t)$ and $s(h_t)$.

2.1 Two-stage optimal control formulation

The ordering of groundwater sector-specific demands according to their price intercept ($a_1/b_1 < a_2/b_2$) together with the existence of the backstop technology, results in a natural partitioning of the problem into two stages, with the first stage representing both sectors demanding water and the second stage defined after the exit of one of the sectors, that is, the sector with smaller a_i/b_i. This assures an intertemporal depletion path that moves along the two-sector, piecewise linear aggregate demand curve. We assume that both groundwater reserves in the aquifer under consideration and the price of desalination, are high enough to ensure positive groundwater use in the economy in the first time period. Given a discount rate $r > 0$, the mathematical representation of the dynamic problem developed in Section 2 is

$$\text{Max}J = \sum_{i=1}^{k} J_j = \sum_{i=1}^{k} \int_{T_{j-1}}^{T_j} e^{-rt} \left[\sum_{i=j}^{n} \int_0^{q_{Gi}(t)+q_{Di}(t)} \left(\frac{a_i}{b_i} - \frac{1}{b_i}(q_{Gi}(t)+q_{Di}(t)) \right) d(q_{Gi}(t)) + (q_{Di}(t)) - \sum_{i=j}^{n} [c[h(t)]q_{Gi}(t) + \bar{p}q_{Di}(t)] \right] dt$$

$$T(t=0)=T_o, \quad T(t=k)=\infty, \quad j=1,k \quad \text{and} \quad k \le 2 \tag{7}$$

Equation (7) can be summarised as

$$\text{Max} \sum_{j=1}^{k} J_j = \sum_{j=1}^{k} \left\{ \int_{T_{j-1}}^{T_j} e^{-rt} [SB_i(t) - TC_i(t)] dt \right\}$$
$$T(t=0) = T_o, \quad T(t=k) = \infty, \quad j = 1, k \quad \text{and} \quad k \le 2 \tag{8}$$

The constraints of the model are the stage-specific hydrological equations of motion

$$\dot{h}_j(t) = \frac{R + (f - 1 - s)\sum_{i=j}^{2} q_{Gi}(t)}{A \cdot S}, \quad T_{j-1} \le t \le T_j \tag{9}$$

The initial conditions are

$$h(t=0) = h_o \tag{10}$$

$$q_{Di}(t=0) = 0, \quad i = 1, 2 \tag{11}$$

where e is the exponential function, J_j represents social benefits for the jth depletion stage ($j = 1, 2$), $T_j(j = k - 1)$ is the jth switch time, also representing the point of backstop adoption. Assuming that $p < \bar{p}$, initial condition (11) indicates $q_{Di}(t = 0) = 0$.

The current value Hamiltonian functions of the above j-stage optimal control problem are

$$\mathcal{H}_j[q_{Gi}(t), q_{Di}(t), h(t), \lambda(t), t] =$$
$$\sum_{i=j}^{2} \int_0^{q_{Gi}(t)+q_{Di}(t)} \left[\frac{a_i}{b_i} - \frac{1}{b_i}(q_{Gi}(t) + q_{Di}(t)) \right] d[q_{Gi}(t) + q_{Di}(t)] \tag{12}$$
$$- \sum_{i=j}^{2} [c[h(t)] \cdot q_{Gi}(t) + \bar{p} \cdot q_{Di}(t)] + \lambda_j(t) \left[\frac{R + (f - 1 - s) \sum_{i=j}^{2} q_{Gi}(t)}{A \cdot S} \right] j = 1, 2$$

where $q_{Gi}(t)$, $q_{Di}(t)$ are the control variables guiding inter-sector water allocation over time, $h_j(t)$ are the state variables representing the hydrological motion of water over time for the j stages, and $\lambda_j(t)$ are the adjoint variables for the j stages, which represent groundwater scarcity rents at each solution stage.

Necessary conditions for optimality are derived by using the Pontryagin principle

$$\dot{h}_j(t) = \frac{R + (f - 1 - s) \sum_{i=j}^{2} q_{Gi}(t)}{A \cdot S}, \quad j = 1, k \tag{13}$$

$$\dot{\lambda}_j(t) = r \cdot \lambda_j(t) - \frac{\partial \mathcal{H}_{jl}}{\partial h_j(t)} = r \cdot \lambda_j(t) + c'[h(t)] \sum_{i=j}^{2} q_{Gi}(t), \quad j = 1, k \tag{14}$$

$$\frac{\partial \mathcal{H}_{jl}}{\partial q_{Gi}(t)} = \frac{a_i}{b_i} - \frac{1}{b_i}(q_{Gi}(t) + q_{Di}(t)) - c[h(t)] - \lambda_j(t) \left(\frac{s + 1 - f}{AS} \right) \leq 0,$$
$$\text{for } i = 1, 2;\ j = 1, k;\ k \leq 2 \tag{15}$$
$$= 0 \text{ if } q_{Gi}(t) > 0 \text{ and } q_{Di}(t) = 0 \text{ for } j = 1, k$$

$$\frac{\partial \mathcal{H}_{jl}}{\partial q_{Gi}(t)} = \frac{a_i}{b_i} - \frac{1}{b_i}(q_{Gi}(t) + q_{Di}(t)) - \bar{p} \leq 0, \quad \text{for } i = 1, 2;\ j = 1, k;\ k \leq 2 \tag{16}$$
$$= 0 \text{ if } q_{Di}(t) > 0$$

$$\lim_{t \to \infty} p_{ij}(t) = \bar{p} \tag{17}$$

$$\lambda_j(T_j-) = \lambda_j(T_j+) \quad j = 1 \tag{18}$$

$$\lambda_j(T_{j-1}) = -\left(\frac{\partial J_j^*}{\partial h(T_{j-1})} \right) \quad j = 1 \tag{19}$$

where J_j^* represents the optimal value function at the switching time. Each of the j stages represents a control problem and has an adjoint variable $\lambda_j(t)$ associated with it. Equations of (13) are the stages' equations of motion. The adjoint equations are represented by (14). These equations demonstrate that groundwater pumping costs $[c'[h(t)]\sum_{i=j}^{n} q_{Gi}(t)]$ create the value associated with user cost. The equations of (15) assure that water use for a particular sector equates marginal benefit to marginal pumping cost plus marginal user cost (scarcity value), and guide water allocation among the economic sectors to equate their marginal value products. They are necessary for allocative efficiency of groundwater over time and across sectors. The incorporation of marginal user cost in the equations ensures representation of the scarcity value of groundwater.

Once desalination is introduced, Equation (16) guides intersectoral allocation of water by equating their marginal value products. At this time, the opportunity cost of current pumping becomes zero. Equation (18) is the conventional transversality condition, which sets the efficiency price of the resource $p_{ij}(t)$ (given by $[a_i/b_i - 1/b_i(q_{Gi}(t) + q_{Di}(t))]$, $\forall i, j, t$) equal to the backstop price ($\bar{p}$) and must hold in the limit as time approaches infinity. The remaining two equations have been introduced by Kim *et al.* (1989) for the solution of a multistage optimal control problem. In equations of (18), T_j- is the point in time just before the jth switch time and T_j+ is the point in time just after the jth switch time. That is, these equations state that the adjoint variable, which represents user cost, must be continuous at each switch time. The equations of (19) give an additional set of transversality conditions for the stages prior to the final stage of desalination. These simply represent a marginal condition that equates the benefit of marginal accretions to the groundwater stock between stages by setting equal the user cost of one stage and the derivative of the optimal value function of the next stage with respect to h, both evaluated at the switching time.

2.2 Solution of the multistage system given backstop availability: the mining era

To solve the system of Equations (13) to (19) given the initial conditions in Equations (10) and (11), it is useful to think in terms of the optimal price path in each stage of the system. The inverse demand curve for water for ith agent is given by

$$D_j^{-1}[q_{Gi}(t) + q_{Di}(t)] \equiv \frac{a_i}{b_i} - \frac{1}{b_i}[q_{Gij}(t) + q_{Dij}(t)] = p_{ij}(t) \quad \text{for } i = 1, 2 \text{ and } j = 1, k \tag{20}$$

We assume that the cost of desalination is high enough and the extraction cost of groundwater is sufficiently low that groundwater extraction is always economic in the first instance. Then condition (15) holds with equality. As

mentioned in Section 2.1, this condition guides water allocation among sectors for each stage of the system to equate their marginal value product. Hence, the price of water for the different sectors is equal at each stage (i.e., $p_{1j}(t) = p_{2j}(t) = p_j(t)$ for $j = 1, 2$). This gives

$$\lambda_j(t)\left(\frac{s+1-f}{A \cdot S}\right) = p_j(t) - c[h_j(t)] \tag{21}$$

Thus, the *in situ* scarcity value of water $[\lambda_j(t)(s+1-f)/(A \cdot S)]$ is equal to the royalty $\{p_j(t) - c[h_j(t)]\}$. Rearranging Equation (14) yields

$$r \cdot \lambda_j(t) = \dot{\lambda}_j(t) - c'[h(t)]\sum_{i=j}^{2} q_{Gi}(t) \tag{22}$$

Equation (22) is a general optimal condition that must hold for all t, for all j, whether or not desalination is being used. This is a modification of Hotelling's r-per cent rule to the case of cost-dependent extraction costs, that is, the rate of change in the asset value is less than the interest rate. The left-hand side is the foregone marginal benefit of extracting water in terms of pounds realised after one period. The right-hand side is the marginal benefit of conservation, that is, the interest on the resource royalty. The first term on the right is the foregone increase in value that would have been realised by conserving the marginal unit. The second term is the increase in future extraction cost due to extracting the marginal unit now instead of later. Thus, Equation (22) says that at the margin, the benefit of extracting water must equal the efficiency cost of extracting water.

Substituting Equation (20) into (16) gives

$$\begin{aligned} p_j(t) - \bar{p} &\leq 0 \\ &< \text{if } q_{Dij}(t) = 0 \end{aligned} \tag{23}$$

which says that if price is below the cost of desalination at any of the j stages, desalination is not used. The initial condition of the system given in Equation (11), indicates that we assume that at first desalination is not used $[q_{Di}(t=0) = 0]$ hence $p_j(t) < \bar{p}$. From condition (21) and its time derivative we get

$$\dot{\lambda}_j(t) = \frac{A \cdot S}{(s+1-f)}[\dot{p}_j(t) - (c'[h_j(t)] \cdot \dot{h}_j(t))] \tag{24}$$

Combining Equations (13), (14), (21), and (24) gives

$$\dot{p}_j(t) = \frac{r[p_j(t) - c[h_j(t)]]}{A \cdot S} + \frac{c'[h(t)] \cdot R}{A \cdot S} \tag{25}$$

The solution of the model for the j stages of the mining era (during which $p_j(t) < \bar{p}$) is governed by the system of differential Equations (8) and (25); these define the optimal trajectory of water price and aquifer head, respectively. Equation (8) of Section 2.1 gives the hydrological equations of motion for each of the j stages for $q_{Gij}(t) = a_i - b_i p_j(t)$. The switching times ($T_j$) are defined by either one of the following two conditions

$$q_{Gij}(t) = 0 \tag{26}$$

$$p_j(t) = \frac{a_i}{b_i} \tag{27}$$

Thus, to derive optimal switching times we need a differential equation showing the time path of groundwater extraction by each economic sector. Given that Equation (15) holds with equality when desalination is not used, we take the time derivative of this equation for $q_{Dij}(t) = 0$. This gives

$$\dot{\lambda}_{jt} = \left(\frac{AS}{s+1-f}\right)\left(-\frac{1}{b_i}\dot{q}_{Gij}(t) - (c'[h_j(t)]\dot{h}_j(t))\right) \tag{28}$$

Combining Equations (13), (14), (15), with $q_{Dij}(t) = 0$ and Equation (29) gives a differential equation for the quantity of groundwater demanded by each sector,

$$\dot{q}_{Gij}(t) = r[q_{Gi}(t) + b_i c[h_j(t)] - a_i] - \frac{c'[h(t)]b_i R}{A \cdot S} \tag{29}$$

Hence, the system of differential equations to be solved for the derivation of endogenous switches is given by Equations (8) and (29). A numerical solution is provided in Section 3.

2.3 Steady-state conditions: the desalination era

Once desalination begins, the system reaches a steady state, at which point desalination continues to be used, price is fixed at $\bar{p}$, and the aquifer head is maintained at h^*; hence $p_j(t) = \bar{p}$, $\dot{p}_j(t) = 0$, and $\dot{h}_j(t) = 0$. Substituting $\bar{p}$ into Equation (21) gives

$$\lambda_j(t) = (\bar{p} - c[h(t)])\left(\frac{A \cdot S}{s(h_j(t)) + 1 - f}\right) \tag{30}$$

Combining Equations (13), (14), (30) and the time derivative of (30) we get

$$\bar{p} - c[h_j(t)] = -c'[h_j(t)]\frac{R}{r(A \cdot S)} \tag{31}$$

which gives the optimal steady-state level of the head of the aquifer.

Because $c'[h(t)] < 0$, $R > 0$, $r > 0$ and $AS > 0$, Equation (31) says that $c[h_j(t)] < \bar{p}$ whenever desalination is being used; the cost of extracting water from the aquifer is less than the cost of desalinated water. This suggests that the effect of extraction costs is to drive a wedge between the price of the unextracted resource and the price of the extracted resource, and in particular the latter is higher by the amount of the resource royalty. This is because there are gains to be had in storing over and above capital appreciation. It is also worth noting that desalination may never come into play. This would occur if demand were completely satisfied by water from the aquifer without ever drawing the aquifer down to h^*; a possibility relevant for water-abundant regions.

2.4 Summary of solution stages

Solving the optimal control problem requires finding the initial general equilibrium scarcity value of *in situ* groundwater (λ_0) that will cause the efficiency price path to rise to the desalination price at exactly the same time that the aquifer head reaches (h^*). In summary, the solution of the model entails the following three eras:

1. *Initial conditions of the system – no desalination*: initially $p_j(t_0) < \bar{p}$ and below the marginal benefit of water use for each of the ith sectors in the economy. Each sector i optimally chooses $q_{Di}(t_0)$, given the costs of acquiring groundwater and h_0. No water is produced by desalination (i.e., $q_{Di}(t = 0) = 0$) and total demand of water in the economy is satisfied from the aquifer.
2. *Endogenous adaptation to increasing resource scarcity – no desalination*: $p_j(t) < \bar{p}$ holds, while the model exhibits an intertemporal depletion path that moves along the two-sector, piecewise linear aggregate demand curve. As depletion of groundwater continues, the efficiency price of water increases and one of the sectors (the one with the lowest WTP for groundwater) exits the market. The model endogenously identifies sector specific optimal exit times.
3. *Endogenous adaptation to steady-state conditions – desalination begins*: $p_j(t) = \bar{p}$, which initiates production of desalinated water. This defines the steady-state where water inflows are exactly equal to water outflows and Equation (3) balances at h^*, thus $\dot{h}(t) = 0$. Price changes cease ($\dot{p}_{ij} = 0$) and the balance of quantity demanded is supplied by desalination such that $q_{Dk}(t) = a_k - \bar{p}b_k - q_{Gk}(t)$. The disaggregated representation of groundwater demand allows identification of the intersectoral allocation of both desalinated and extracted water.

3. Empirical illustration

The empirical application of the model uses data from the Kiti agricultural region, located in the coastal southern part of the semiarid Mediterranean

Table 1 Economic and hydrologic parameters

Par.	Description	Parameter value
b_a	Absolute value of the slope of agricultural water demand	6 118 500 m^3/£†
b_d	Absolute value of the slope of domestic water demand	1 270 000 m^3/£
a_a	The intercept of agricultural water demand	9 436 500 m^3
a_d	The intercept of domestic water demand	4 048 000 m^3
ε_a	Price elasticity of agricultural water demand‡	0.48
ε_d	Price elasticity of domestic water demand‡	0.18
k_1	Cost of pumping per m of water per metre of lift	0.02 £/m^3
k_2	The intercept of the pumping cost equation	0.37 £/m^3
f	Return flow coefficient	0.05 pure number
A	Area of the aquifer	12 000 000 m^2
S	Storativity coefficient	0.70 pure number
s	Salinity coefficient	0.30 pure number
R	Recharge rate	4 000 000 m^3 p.a.
h_o	Initial elevation of water table	3.45 m
$\bar{p}$	Unit price of desalination§	0.50 £/m^3

† In 1998, £1 Cyprus was worth 2.92 Australian dollars or 1.15 UK pounds. ‡ In the relevant economic range. § Engineering estimate.

island of Cyprus. The aquifer in this region does not receive substantial natural recharge and is not interconnected with surface water. Moreover, for all practical purposes, it can be assumed to have infinite hydraulic conductivity (Dr George Socratous, pers. comm., 2000). The Kiti aquifer is seriously depleted and has a smaller water capacity than any other aquifer used to test the robustness of GSE. Seawater intrusion is pervasive in the area. Although the doctrine of absolute land ownership implies that ownership of the resource depends on ownership of land overlying the aquifer (thereby limiting access), in all other respects land owners own groundwater as a common property resource subject to the rule of capture.

Table 1 summarises the economic and hydrologic parameters for the region, supplied by the Water Development Department of Cyprus. There are two sectors in the region demanding groundwater: agricultural and domestic water users. Given the absence of observations over a wide range of prices, relevant derived demands were estimated by linear programming. As explained in Section 2, sequential exits from the market are defined by the relative magnitude of sector-specific demand parameter ratio (a_i/b_i). These equal 1.54 £/m^3 for the agricultural sector and 3.19 £/m^3 for the domestic sector. These and all prices and welfare measures in the paper are in Cyprus pounds. At these prices, the quantity demanded by the agricultural and domestic sectors becomes zero, respectively; hence, the agricultural sector should exit the market first. This result is in agreement with the empirical regularity identified by Gibbons (1986), which indicates that the marginal net benefits from agricultural uses of water are lower than marginal net benefits from domestic use.

Table 2 Steady-state conditions under optimal control

Price of water (desalination price)	0.50 £/m^3
Unit extraction cost	0.31 £/m^3
Optimal head (h^*)	2.88 m a.s.l.†
Initial marginal scarcity value (λ_0)	0.20 £/m^3
Last endogenous switch time (t_{k-1})	1999
Quantity demanded by domestic	3.41 Mm^3
Quantity demanded by agriculture	6.38 Mm^3
Quantity extracted from the aquifer	3.20 Mm^3
Quantity produced by desalination	6.59 Mm^3

† m a.s.l. stands for metres above sea level.

The explicit marginal cost function used in the solution of the system is

$$c[h(t)] = k_1 \cdot [SL - h(t)] = k_2 - k_1 \cdot h(t)$$

The difference ($SL - h$) measures pumping lift, the distance from the water table to the irrigation surface. This pumping cost function (a specific form of a general cost function) is very popular in published studies; for example, see Gisser and Mercado (1973), Kim *et al.* (1989). Its derivatives have the desirable properties: a positive partial derivative with respect to q_G and a negative cross-partial derivative between q_G and pumping lift. Another point to note from Table 1 is that the return flow coefficient f is lower compared with other groundwater studies. This is due to uniform adoption of sprinkler irrigation systems in agriculture, which results in approximate equalisation of f between the two sectors.

The solution method involves first using the steady-state head Equation (31) to calculate the final head, and solving for the backstop adoption time (t_{k-1}) such that the solution to the system of differential Equations (8) and (26) with boundary conditions $h(t_{k-1}) = h^*$ and $p(t_{k-1}) = \bar{p}$ results in $h(t_0) = h_0$. Simultaneously, we solve the system of differential equations that allows derivation of endogenous switches, given by (8) and (29). Substituting the relevant parameters from Table 1 in equation (31) gives the optimal steady-state level of the head at $h^* = 2.28$ (m). Programming results indicate that with a 5 per cent per annum interest rate and desalination price ($\bar{p}$) at £0.50 per m^3, the initial per cubic metre scarcity value of *in situ* groundwater is £0.20 per m^3. This is similar to groundwater scarcity rents simulated for aquifers located in Israel (Mordechai Shechter, pers. comm., 2000), another semiarid country facing acute water scarcity. Steady-state conditions are summarised in Table 2.

Desalination is introduced almost instantaneously (in year 1), before any sectoral exits occur. At steady state, the quantity of water demanded by the domestic and agricultural sectors is 3.41 Mm^3 (millions of cubic metres) and 6.38 Mm^3, respectively. Of the total quantity of water demanded, 3.20 Mm^3 will be extracted from the aquifer and the remaining 6.59 Mm^3 will be provided from desalination. Of the total quantity of water produced, 2.30 Mm^3 and 4.29 Mm^3 will be consumed by the domestic and agricultural sectors, respectively. Moreover, 1.12 Mm^3 will be extracted by the domestic sector

Table 3 Welfare under optimal control (with backstop)

Consumer surplus from agricultural uses	£3.32 million
Consumer surplus from domestic uses	£4.59 million
Producer surplus from water extracted	£0.61 million
Annual welfare	£8.52 million
Total welfare discounted at 5%	£170.36 million

Table 4 Welfare under myopic extraction

t	h_0 m a.s.l.†	$c(\cdot)$ £/m^3	p £m^3	q_A Mm^3	q_D Mm^3	h_c m a.s.l.	w_A £m	w_D £m
1	3.45	0.30	0.30	7.61	3.67	2.25	4.74	5.30
2	2.25	0.32	0.32	7.47	3.64	1.07	4.55	5.21
3	1.07	0.35	0.42	6.88	3.52	0.00	3.87	4.87

† m a.s.l. stands for metres above sea level.

and 2.08 Mm^3 by the agricultural sector. Although the empirical application of the model does not identify any sectoral exits, the disaggregated representation of the resource demand allows accurate estimation of the intersectoral allocation of water, as well as the steady-state quantity of the backstop. This enables precise welfare calculation.

The components of annual welfare derived from these steady-state conditions are shown in Table 3. These add up to £170.36 million of total annual welfare discounted indefinitely at a 5 per cent rate of interest.

To test the robustness of the GSE, we calculate welfare under competitive (myopic) conditions of groundwater extraction. Myopic time trajectories of extraction rates and aquifer's head, as well as respective endogenous switch times, are analytically derived in Appendix 1 (available from http://www.aueb.gr/deos/uk/koundouri.htm). Taking a discrete-time approximation to our model gives the results in Table 4.

The first column of Table 4 gives the year of extraction, the second the opening head of the aquifer, and the third the unit cost of extraction given the opening head for each year. Column four gives the price of groundwater under myopic extraction, which is usually equal to the unit extraction cost. Columns five and six give the quantities of groundwater demanded by the agricultural and domestic sectors, respectively. Column seven gives the closing head for each extraction year. Actual lifts and the head of the water table in the region, correspond to the myopic model's simulated levels. Columns eight and nine indicate sector-specific consumer surpluses, that is, annual welfare derived from agricultural and domestic use of groundwater, respectively. Note that in the last year, total quantity demanded has been set so as to exactly exhaust the aquifer. This produces a small producer surplus in year 3 (because price is above pumping cost) equal to £0.75 million (or equal to £0.68 million if discounted at 5 per cent rate of interest). However, we have ignored this in the calculation of overall welfare as this is just an artefact of discrete approximation.

As shown in Table 4, under myopic groundwater extraction the aquifer is exhausted in 3 years. Adding up welfare and discounting at 5 per cent yields net present value (NPV) welfare equal to £27.26 million. This calculation assumes that when the aquifer is completely exhausted, natural recharge disappears and as a result there are zero benefits after irreversible depletion.

However, given the availability of a backstop technology, as soon as the aquifer is exhausted the price jumps to £0.50 per m^3 and desalination is introduced. This approximates the actual experience in the region, where desalination was introduced in 2001. Once the aquifer is exhausted in the myopic case, all demands are satisfied entirely by desalination, rather than partly from groundwater as is true in the optimal case. Welfare in this case will be identical to that found in the optimal steady state shown in Table 3, except for one thing. The producer surplus that exists when groundwater reaches steady state will be absent in the myopic case, as the price of water reflects the supply cost for all units. Hence, annual welfare in the desalination phase will be £7.91 million, capitalised at 5 per cent that is worth £158.20 million, and discounted back to year 1 that is worth £136.66 million. Adding this to the NPV from the mining phase as described in Table 4, gives total NPV of welfare, derived from myopic extraction with desalination, of £163.92 million. Comparing this figure with total welfare under optimal control with desalination (given in Table 3 at £170.36 million), provides a welfare improvement of 3.8 per cent. This suggests that GSE persists in the presence of a backstop substitute. This is an intuitive result, as the availability of a backstop substitute effectively reduces the scarcity of the resource.

The obvious question that arises at this point is how robust is GSE in the absence of backstop substitute, when irreversible depletion of the resource is probable in the near future. To calculate welfare under optimal control in the absence of a backstop substitute, we take a discrete time approximation to our model, which yields Table 5.

Columns 1 to 7 are defined as in Table 4. Columns 8 to 11 indicate sector-specific consumer and producer surpluses. Note that in the last year the total quantity demanded has been set so as to exactly exhaust the aquifer. Hence, under the regime of optimal control the aquifer is exhausted in 4 years. Adding up consumer and producer welfare for both sectors and discounting at 5 per cent, yields NPV welfare equal to £22.83 million.

Table 5 Calculation of welfare under optimal control (no backstop)

t	h_0 m a.s.l.†	$c(\cdot)$ £/m^3	p £/m^3	q_A Mm^3	q_D Mm^3	h_c m a.s.l.†	CS_A £m	CS_D £m	PS_A £m	PS_D £m
1	3.45	0.30	0.50	6.38	3.41	2.47	3.32	4.59	1.29	0.69
2	2.47	0.32	0.52	6.24	3.38	1.51	3.18	1.13	1.28	0.69
3	1.51	0.34	0.55	6.10	3.36	0.58	3.04	1.08	1.27	0.70
4	0.58	0.36	1.16	2.31	2.57	0.00	0.44	0.03	0.49	0.71

† m a.s.l. stands for metres above sea level.

To calculate social welfare for this scenario we use Tables 1 and 2. Natural recharge of the aquifer is equal to 4.00 Mm^3 and the steady-state quantity demanded of groundwater under optimal control is equal to 3.20 Mm^3. To limit demand to this level a price of £1.3920/m^3 is needed, at which agriculture consumption is 0.92 Mm^3 and domestic consumption is 2.28 Mm^3. These correspond to £9.07 million annual consumer surplus for agriculture and £2.05 million annual consumer surplus for domestic. However, there is considerable producer surplus because pumping costs of recharged water are £0.37 per m, whereas the selling price is £1.39 per m^3. Applying this surplus to 3.20 Mm^3 is worth £3.28 million annually. Thus total annual benefits are £5.40 million, and capitalising at 5 per cent makes that worth £107.90 million. Assuming this regime did not start until year 4 after the aquifer is (almost) exhausted, gives NPV welfare of £88.77 million. Adding this amount to £22.83 million, which is the total welfare received during the first 4 years of optimal groundwater extraction before exhaustion of the aquifer, gives an overall NPV welfare of £111.60 million. This is social welfare derived under the scenario of optimal extraction and sustainable aquifer recharge, in the absence of a backstop substitute.

The disappearance of aquifer recharge in the decentralised solution, even though the size of the stock is the same as in the centralised solution, is supported by hydro-geological empirical research (George Socratous, pers. comm., 2000). That is, hydrogeologists argue that irreversible loss of recharge can be avoided at little or no cost with careful (optimal) management, even if the stock of water is exhausted (or nearly exhausted as in our empirical illustration). This is a hydrological possibility for aquifers made of fine materials (e.g., silty sands), whereas it is less relevant for those made of coarse materials (e.g., gravels). Coastal aquifers, such as the one we are modelling, are made of fine materials; hence, our decision to investigate the scenario of sustaining aquifer recharge after water-stock exhaustion. Avoiding hydrogeological jargon, the explanation for such a hydrogeological possibility in coastal aquifers is that uncontrolled excess demand for groundwater under the decentralised solution puts extra pressure on the hydrogeological conditions of the aquifer, which leads to irreversible damage to (and extinction of) the aquifer recharge flow. In contrast, under the centralised solution, there is no excess demand as the social planner rations demand by price to match the available flow. Hence, no extra pressure is put on the hydro-geological conditions of the aquifer, which results in sustaining the recharge flow of the aquifer.

Comparing the above value of social welfare with the one derived under myopic conditions of groundwater extraction (£27.26 million) gives a huge welfare improvement of 409.4 per cent; thus, GSE disappears. Table 6 summarises welfare derived under different extraction regimes with and without backstop substitute. The circumstances of this case, however, make discounting virtually irrelevant; the aquifer's mining phase, as well as its recharge capacity, last for only 4 years. The intuition is that with a positive discount rate, the effect of common-property induced recharge destruction would be much lower in the case of long-lived aquifers.

Table 6 Examining the robustness of the GSE

Regime	Backstop	Welfare	Improvement
Optimal control	Available	£170.36 M	3.8%
Myopic	Available	£162.62 M	GSE persists
Optimal control	Not available	£111.60 M	409.4%
Myopic	Not available	£27.26 M	GSE disappears

Our results suggest that in the presence of a backstop substitute GSE persists, whereas in the absence of a backstop substitute and sustainable natural recharge GSE disappears. This is an intuitive result because it suggests that when the scarcity of the resource is reduced because of the presence of an abundant backstop substitute, welfare gains from controlling resource extraction are not significant for any practical purposes. However, in the absence of a backstop substitute, a considerable welfare improvement can be derived from controlling extraction when myopic behaviour results in irreversible loss of the recharge capacity of the resource.

In effect, this result indicates that for aquifers located in arid and semiarid regions – which are where groundwater management matters most – the avoidance of imminent irreversible depletion gives rise to significant welfare benefits. Irreversible depletion is not an issue for any of the empirical studies that identified GSE, and thus, where the introduction of a backstop was not relevant. Moreover, this result is in agreement with the theoretical result derived by Tsur and Zemel (1995, 2001). Both in their model (with uncertainty concerning the irreversible event occurrence) and ours (with perfect myopia concerning the imminent irreversible event occurrence), a policy that avoids the irreversibility is welfare increasing.

4. Conclusion

Although the notion of a backstop technology as a basis for resource management remains controversial, we submit that the existence per se of a backstop is not the critical issue. Resource substitutes do exist. The critical issue is at which unit costs can they be made effective substitutes for exhaustible resources and what happens in the interim on the way to the steady state, as it is the trajectory to the steady state that captures immediate policy-relevant concerns. The theoretical model of the paper solves for the optimal time for adoption of this technology and identifies an endogenously defined trajectory to the steady state that allows dynamic adaptation to resource scarcity changes. Possible extensions of the model could allow investigation of a number of possible features of this trajectory. If domestic demand for water is driven by population, is urban emigration endogenous to water price? That is, as water price increases with aquifer mining, does emigration increase, thereby causing inward shifts in demand? How would such a scenario be affected by income growth? Moreover, other applications of the approach could describe

the chronological pattern of endogenous adoption of additional backstop substitutes, possibly less expensive than desalination (e.g., water from a dam or a reservoir, artificial recharge, water recycling).

The empirical contribution of the paper is a partial resolution of the Gisser–Sanchez paradox. Results indicate that when a backstop substitute is available, GSE persists, whereas in its absence it is possible that the GSE disappears. Intuitively, benefits from extraction management are significant when myopic behaviour results in imminent exhaustion of the stock and irreversible loss of the recharge capacity of the resource. A comparison of groundwater across other common property resources (e.g., tropical forest in Pearce 2001) indicates that in the absence of a backstop substitute, a similar result to GSE arises (i.e., sustainable forest management is even less profitable than non-sustainable forestry) if imminent irreversibilities are ignored. It is worth noting, however, that it is not purely the presence of a backstop that releases us from the GSE. The fact that the backstop is perfectly elastic in supply rather than subject to increasing marginal costs is a major feature of our model, which contributes to our result. GSE becomes less important the higher the marginal costs of supply and the more steeply they increase. Indeed, the conclusion of this paper is that the optimal control of groundwater is more important when alternatives are economically scarce; that is, they have a high economic cost. In essence, the simulation in the absence of the backstop has looked at the case where the backstop is infinitely expensive.

The generality of our results is somewhat compromised by two of our model assumptions. Firstly, one could argue that a rational extracting agent will eventually learn that its pumping decisions do affect the stock of groundwater and will bring this information to bear in its pumping decision, hence, compromising the relevance of the purely myopic assumption. Dixon (1989), Negri (1989), and Provencher and Burt (1993) construct game theoretic models that can accommodate such strategic behaviour under common-property arrangements. In these models, a firm's strategy is the groundwater extraction plan defining its behaviour in each period of its planning horizon. An equilibrium in Nash strategies is a set of M admissible groundwater extraction plans, the jth element of which maximises the value of groundwater to the jth firm, given the other $M-1$ groundwater extraction plans in the set. The precise nature of the equilibrium depends on whether firms pursue path (open loop) or decision rule strategies (closed loop). Nash equilibria in path strategies reflect the inclination of firms to take the extraction paths of the other firms exploiting the resource as given. Nash equilibria in decision-rule strategies reflect the inclination of firms to take the decision rules of the other firms exploiting the resource as given. The relevant equilibrium concept for decision rule strategies is a type of Markov–Nash equilibria, in which the decision rules of firms at time t are a function of only the current values of the state variables. As shown by Negri, path strategies capture only the pumping cost externality, whereas decision-rule strategies capture both the pumping cost externality and the strategic externality, and exacerbate inefficient aquifer

exploitation compared with path strategies. In general, Provencher and Burt (1993) conclude that the steady-state groundwater reserves attained when firms use decision-rule strategies are bounded from below by the steady state arising when firms are myopic and from above by the steady state arising from optimal exploitation. Hence, if we had adopted the decision rule structure in our model, our simulation results would be expected to produce a smaller welfare difference between optimal management of the aquifer and no control of groundwater extraction (when the behaviour of the extracting agents is strategic). That is, such a structure would make the GSE more robust. However, one might expect the perfectly myopic structure adopted by our model to perform reasonably well when the groundwater resource is exploited by a large number of small firms (which is the case for the aquifer in the Kiti region and most aquifers with overpumping problems), just as the assumption of competitive 'price-taking' behaviour no doubt accurately depicts the situation in many input and output markets. In other words, each firm is too small a part of the whole to give serious consideration to how its pumping decision affects future water supplies. Support for our model comes from a survey of farmers in Kern County, California, conducted by Dixon (1989). Secondly, the assumption of infinite hydraulic conductivity, mentioned in Section 2, exaggerates the degree of common property in some aquifers. This assumption is, for practical purposes, correct for aquifers made of fine materials (e.g., coastal aquifers such as the Kiti aquifer). However, it is less relevant for those made of coarse materials. Hence, the GSE effect is expected to be more persistent in aquifers made of coarse materials, both in the presence and absence of a backstop technology in perfectly elastic supply.

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