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MORE ON ESTIMATING ELASTICITIES USING

LAGGED ENDOGENOUS VARIABLES

by

S. E. Miller, O. Capps, and G. J. Wells

clemson university
south carolina agricultural experiment station

department of agricultural economics
and rural sociology
clemson university
clemson, south carolina 29631

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S. E. Miller, O. Capps, and G. J. Wells

* Capps is Associate Professor of Agricultural Economics at VPI&SU, Blacksburg, VA, and Miller and Wells are Associate Professors at Clemson University.

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Introduction

In an article appearing in this journal, Bopp and Pendley[1], hereafter abbreviated B-P, have considered the problem of constructing confidence intervals for long-term elasticities estimated from logarithmic equations containing lagged endogenous variables as explanatory variables. The estimated long-term elasticity for explanatory variable i is

$$\hat{\eta}_i = \frac{\hat{B}_i}{(1-\hat{\lambda})} , \tag{1}$$

where $\hat{B}_i$ is the estimated regression coefficient for explanatory variable i, or equivalently, the short-term elasticity for explanatory variable i, and $\hat{\lambda}$ is the estimated coefficient for the lagged endogenous variable. Bopp and Pendley use an estimate of the asymptotic variance of $\hat{B}_i/(1-\hat{\lambda})$ to construct approximate confidence intervals for η_i. The purpose of this note is to present a simple technique for constructing exact confidence intervals for η_i. This technique is also illustrated by application to the data used by B-P.

Exact Confidence Intervals for Long-Term Elasticities

The ratio $\hat{B}_i/(1-\hat{\lambda})$ is, under the usual assumptions, a ratio of two normally distributed random variables with the numerator having an expected value of B_i and variance $\sigma^2_{\hat{B}_i}$, and the denominator having expected value of $1-\lambda$ and variance $\sigma^2_{\hat{\lambda}}$. The covariance of the numerator and denominator is $-\sigma_{\hat{B},\hat{\lambda}}$. Although Hinkley[2] has derived an exact

expression for the exact cumulative distribution function (c.d.f.) for such a ratio, evaluation of that function involves use of tabulations of the bivariate normal distribution function. Since use of these tabulations usually requires trivariate interpolation, construction of exact confidence intervals for η_i using this approach would be at best inconvenient.

However, a result due to Fieller[3] allows construction of exact confidence intervals for η_i and requires no more than solutions to quadratic equations. This result makes use of the fact that the ratio $R = \hat{B}_i/(1-\hat{\lambda})$ is equivalent to

$$\hat{B}_i - R(1-\hat{\lambda}) = 0 \ . \tag{2}$$

A $1-\alpha$ percent confidence interval for R is given by those values of R for which

$$\text{Prob}\left\{\frac{(\hat{B}_i - R(1-\hat{\lambda}))^2}{s^2_{(1-\hat{\lambda})} - 2R\left(\hat{B}_i(1-\hat{\lambda}) - t^2_{\alpha/2} s_{\hat{B}_i,(1-\hat{\lambda})}\right) + \hat{B}_i^2 - t^2_{\alpha/2} s^2_{\hat{B}_i}} \leq t^2_{\alpha/2}\right\} = 1-\alpha \tag{3}$$

where $s^2_{\hat{B}_i}$ is the sample variance of $\hat{B}_i$; $s^2_{(1-\hat{\lambda})}$ is the sample variance of $(1-\hat{\lambda})$; $s_{\hat{B}_i,(1-\hat{\lambda})}$ is the sample covariance of $\hat{B}_i$ and $(1-\hat{\lambda})$; and $t^2_{\alpha/2}$ is the squared value of the upper $\alpha/2$ percentage point of the t-distribution with appropriate degrees of freedom, or equivalently the upper α percentage point of the F-distribution with one degree of freedom in the numerator and the appropriate degrees of freedom in the denominator. For the problem at hand, the appropriate degrees of freedom for the t-distribution or the denominator of the F-distribution are the error degrees of freedom of the regression used to estimate $\hat{B}_i$ and $\hat{\lambda}$. For the confidence interval to be closed, $(1-\hat{\lambda})$ must be significantly different

from zero. The exact $1-\alpha$ confidence limits for η_i are those values of R for which

$$R^2\left((1-\hat{\lambda})^2-t^2_{\alpha/2}s^2_{(1-\hat{\lambda})}\right)-2R\left(\hat{B}_i(1-\hat{\lambda})-t^2_{\alpha/2}s_{\hat{B}_i(1-\hat{\lambda})}\right)+B_i^2-t^2_{\alpha/2}s^2_{\hat{B}_i} = 0. \qquad (4)$$

Note that the solution to this quadratic equation in R need not, and generally will not, be symmetric about $\hat{\eta}_i$. Thus, the use of the asymptotic variance of $\hat{\eta}_i$ to construct approximate confidence intervals for η_i centered upon $\hat{\eta}_i$ can be misleading.

An Application

In order to illustrate the construction of exact confidence intervals for long-term elasticities, we make use of the data analyzed by B-P. They estimated a logarithmic equation expressing residential electrical consumption in Connecticut (Q_t) as a function of real disposable income (Y_t), own marginal price (P_t), and lagged consumption (Q_{t-1}) using annual observations from 1960 to 1975. The estimated short- and long-term elasticities and their respective variances are summarized in Table 1. The estimated coefficient of adjustment $(1-\hat{\lambda})$ was 0.152, and the relevant covariance terms were estimated as follows: $s_{\hat{Y},(1-\hat{\lambda})} = -s_{\hat{Y},\hat{\lambda}} = 0.005$; and $s_{\hat{P},(1-\hat{\lambda})} = -s_{\hat{P},\hat{\lambda}} = 0.0008$. The error degree of freedom is 12. With this information, exact confidence intervals can be constructed for both the short- and long-term elasticities. These are also displayed in Table 1.

Note that the exact confidence intervals for the long-term elasticities are much wider than their respective short-term counterparts. Also, the exact confidence intervals for the long-term elasticities are not symmetric about their respective point estimates, a result not

Table 1. Exact Confidence Intervals for Short- and Long-Term Elasticities

	Estimated Elasticity[a]		Exact 95% Confidence Intervals	
Variable	Short-term	Long-term	Short-term	Long-term
Income	0.17 (0.1221)	1.13 (0.474)	-0.10 to 0.44	-1.51 to 1.78
Price	-0.21 (0.0480)	-1.4 (0.587)	-0.31 to -0.11	-4.08 to -0.52

a. Standard errors of estimated elasticities are shown in parentheses. The standard errors for long-term elasticities are asymptotic.

detected by the use of the asymptotic variance of $\hat{\eta}_i$ in constructing appropriate confidence intervals for η_i. Although the exact confidence interval for the short-term income elasticity is wider than the interval for short-term own price elasticity, the relative widths of the long-term elasticity intervals are reversed, a result agreeing with the findings of B-P.

Summary and Conclusions

The purpose of this note is to demonstrate a technique for constructing exact confidence intervals for long-term elasticities from logarithmic equations containing lagged endogenous variables as regressors. This technique involves only simple computations based upon results generated in the course of parameter estimation.

Notes

[1]Bopp, Anthony E., and Robert E. Pendley. "A Note on Estimating Economic Elasticities Using Lagged Endogenous Variables," Intermountain Economic Review (Fall 1978), pp. 111-7.

[2]Hinkley, D. V. "On the Ratio of Two Correlated Normal Random Variables," Biometrika 56(1969), pp. 635-9.

[3]Fieller, E. C. "The Distribution of the Index in a Normal Bivariate Population," Biometrika 24(1932), pp. 428-40.

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CLEMSON UNIVERSITY, CLEMSON, SOUTH CAROLINA
W. C. GODLEY, ASSOCIATE DEAN AND DIRECTOR
S. C. EXPERIMENT STATION
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