Read This Paper Later: Procrastination with Time-Consistent Preferences

Carolyn Fischer

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RESOURCES FOR THE FUTURE

1616 P Street, NW
Washington, DC 20036
Telephone 202-328-5000
Fax 202-939-3460

Read This Paper Later: Procrastination with Time-Consistent Preferences[1]

CAROLYN FISCHER[2]
Resources for the Future

Abstract

A model of time-consistent procrastination is developed to assess the extent to which the observed behavior is compatible with rational behavior. When a finite work requirement must be completed by a deadline, the remaining time for leisure is an exhaustible resource. With a positive rate of time preference, the optimal allocation of this resource results in more hours spent working (and fewer in leisure) the closer the deadline. Key qualitative findings of psychological studies of academic procrastination are consistent with the standard natural resource management principles implied by the model, when suitably adapted to task aversiveness, uncertainty, and multiple deadlines. However, quantitatively, the fully rational model requires an extremely high rate of time preference or elasticity of intertemporal substitution to generate serious procrastination; furthermore, it cannot explain undesired procrastination. A companion paper, "Read This Paper Even Later: Procrastination with Time-Inconsistent Preferences" analyzes the extent to which alternative time discounting preferences can better explain such impatience and address the issue of self-control failures.

Keywords. procrastination, natural resource economics

JEL Classification No(s).: Q3, D9, J22, D81

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[2]Resources for the Future, 1616 P St. NW, Washington, DC 20036. E-mail: fischer@rff.org

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In alphabetical order:

δ	daily discount factor
λ	multiplier for cumulative stock of work constraint
σ	rate of time preference
$\psi(\omega)$	$= u(1-\omega) + u(1-\omega)\omega$
ω	work rate
$\phi(\cdot)$	probability distribution function of $\tilde{N}$
a	marginal aversiveness of work
I	number of days taken to complete M
l_t	leisure at time t
M	initial amount of work to be completed before uncertainty resolved
$\tilde{N}$	uncertain remaining work requirement
N_I^*	cutoff remaining work requirement for last stage beginning at I
P	penalty
R	required hours of work
t	time index
T	number of days until deadline (including day 0)
$u(\cdot)$	utility of leisure
$U_i(\cdot)$	discounted utility of leisure in stage i
$V_i(\cdot)$	discounted value of stage i
w_t	work at time t

1 Introduction

It's a familiar scene: the last day of classes and the day the term paper is due. Students straggle in, some depositing papers only to turn around and head back to bed; some take a seat but appear to mistake it for that other piece of furniture; and the rest struggle to pay attention to the lecture, most offering up little more than a blank, red-eyed gaze to the board. At these times, both students and professors must wonder, why do they put themselves through it? (Of course, professors may not really ponder this question until the morning the grade sheets are due.)

Procrastination, particularly in academics, has been the source of several psychological studies, but few economic ones. Presumably, this dichotomy exists because psychologists love to explore irrational behavior and economists usually restrict themselves to rational conduct — and procrastination is perceived to be irrational. Freudians Blatt and Quinlan (1967) attribute procrastination to an attempt to avoid "unconscious death anxiety": "By being continually late, the procrastinator is expressing rebellion at the finality of his or her existence."[3] Missildine, another psychoanalyst of the 1960s, blames poor childrearing as the cause: "an overindulgent parent encouraging underachievement, or an overdemanding parent encouraging rebellious lassitude."[4] However, if not rational, the behavior is certainly normal: Ellis and Knaus (1977) estimated that 95% of college students procrastinate. More recently, empirical studies (such as Aitken (1982), Solomon and Rothblum (1984), and McCown, Petzel and Rupert (1987)) have concentrated on the correlation between procrastination and certain psychological or personality traits, such as compulsiveness, anxiety, and extraversion.

Some economists have also taken the perspective of irrational procrastination. Akerlof (1991) studied procrastination when a task requires action at a single point in time. Opportunity costs of time today are more salient than those tomorrow; that is, today's op-

[3] McCown, Petzel, and Rupert (1987), p. 781.
[4] ibid.

portunities are clear while tomorrow's are vague, making the former seem more pressing. With a "salience cost" to acting today, and none attributed to tomorrow, one always wants to postpone action, even though the stream of benefits is maximized with immediate action. But the dynamically inconsistent preferences generated by salience costs in his model are not sufficient to produce indefinite procrastination: one must have irrational expectations of the future. If a person realizes she will want to postpone again every day, the rational expectations strategy is to perform the task at once. O'Donoghue and Rabin (1999) also examine the decision to procrastinate a one-time task with Akerlof-style salience costs, allowing for rewards or costs to be salient and expectations to be sophisticated (rational) or naïve.[5]

However, not all tasks require a one-time action. This paper considers a particular type of task, distinct from Akerlof's: work that can be divided into many (if not an infinite number of) small actions to be completed over time, such as writing a term paper. Nor is procrastination always characterized by missing deadlines or abandoning tasks; procrastination can exhibit itself in an increasing workload, as more of the task is performed the closer the deadline.

Academic procrastination is a familiar example: assignments may be handed in on time yet procrastination is deemed to have occurred since much of the work was accomplished at the eleventh hour. But can delaying enough of the effort to such a point as the discomfort of all-nighters really be rational?

Modeling time as an exhaustible resource, this paper shows that simple impatience offers a reasonable theoretical explanation of dynamically consistent procrastination. The next section overviews some of the observations psychologists have made regarding procrastinating behavior. Section 3 develops the time allocation decision as an exhaustible

[5]They show that rational expectations can still allow for some procrastination. For example, in a four-period model, if the task is worth performing in period 4, and knowing this the person in period 3 will procrastinate, the person in period 2 may find waiting two more periods for the reward too costly and want to perform it. Consequently, the person in the first period will procrastinate, confident the task will be performed in the next period.

resource utilization problem. Subsequent subsections draw on standard results from natural resource economics to explain key findings in the psychological studies: task aversion, procrastination under uncertainty, and the effectiveness of multiple deadlines. A brief discussion of how to reconcile the high discount rates implicit in substantial procrastination with standard concepts of discounting follows. Section 4 concludes.

2 Stylized Facts

In psychology as well as in economics, definitions of procrastination differ widely; perhaps the one most closely corresponding to mine is presented by Solomon and Rothblum (1984): "the act of needlessly delaying tasks to the point of experiencing subjective discomfort." I interpret "needless" to be in terms of feasibility and "subjective discomfort" to be significant disutility of work near the deadline. A student could work at a steady pace, but a procrastinator builds up her workload, putting off more work until near the deadline, even at the cost of having little or no leisure left at that point.

Some stylized facts about procrastination observed by psychologists are summarized in the box below.

The Psychology of Procrastination: Some Stylized Facts
• Procrastination is pervasive. 95% of college students procrastinate. [5] 46% nearly always or always procrastinate n writing a term paper. [23] • Major reasons cited for procrastination: [23] 1. Too many other things to do (61%) 2. Task is aversive (47+%) 3. Felt overwhelmed by the task (40%) 4. Fear of failure (14+%) • Easier tasks are performed first. [17] • Procrastinators are more likely to miss deadlines. [2] • Extraverts procrastinate more. [17] • Procrastination is perceived to be a problem. [23]

This paper will address each observation in turn, showing that almost every one can be explained by a rational model of time allocation. The notable exception is the last observation, prompting discussion of modifications to the rational model to deal with problem procrastination.

3 Model

Posing the problem succinctly, when the work requirement demands many units of effort over a finite amount of time, when will that effort take place? The answer lies in the simple idea that leisure time is an exhaustible resource. When a fixed amount of work must be done by a distant deadline, leisure time in the interim is an exhaustible resource to be consumed over time until the deadline, subject to a cumulative stock constraint. People like leisure and prefer it sooner rather than later; time can be allocated between work and leisure, but a certain amount of cumulative hours of work is required by some deadline. The person thus weighs the gains from taking leisure now against the utility costs of having to do more work later.

Suppose a person gets utility $u(l_t)$ for l hours of leisure on any day t, and assume $u(\cdot)$ is strictly increasing and concave. Let δ represent the daily discount factor. R hours of work are required to write a paper, the deadline for which is T days from now. A maximum of 24 hours per day can be spent working on the paper;[6] leisure thus equals the excess of 24 over w_t hours of work: $l_t = 24 - w_t$. What is the optimal allocation of time into leisure and work from now until the deadline?

We can think of leisure time as an exhaustible resource with a stock size of $24T - R$ (assumed to be positive; i.e., the task is feasible). In addition, we have a maximum daily extraction rate—a capacity constraint—of 24 hours. The student maximizes her utility from leisure subject to the work requirement:

$$U(R,T) = \max_{w_t \in [0,24]} \sum_{0}^{T-1} u(24 - w_t)\delta^t - \lambda \left(R - \sum_{0}^{T-1} w_t \right). \tag{1}$$

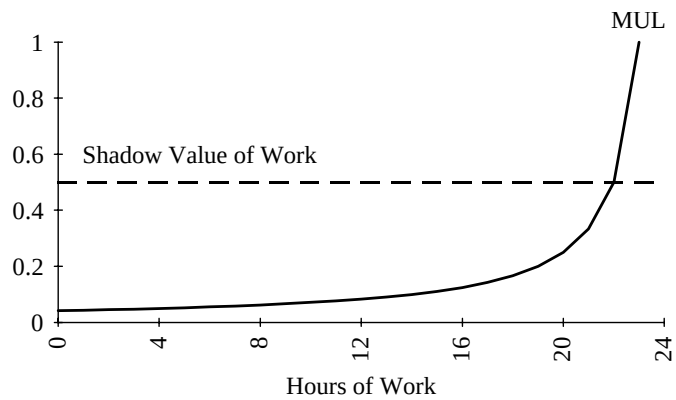
The multiplier on the work constraint, λ represents the shadow value of another hour of leisure. The resulting first-order conditions are really those found by Hotelling. One of the following pairs must hold for all t:

$$\begin{aligned}
&(a) \quad 0 < w_t < 24, \quad u'(24 - w_t)\delta^t = \lambda; \\
&(b) \qquad w_t = 0, \qquad u'(24)\delta^t \geq \lambda; \\
&(c) \qquad w_t = 24, \qquad u'(0)\delta^t \leq \lambda;
\end{aligned} \tag{2}$$

These conditions imply that for any $0 < w_t < 24$, the marginal utility of leisure (MUL) will be growing at the rate of time preference, which in turn implies that w_t will rise monotonically over time. Figure 1 shows the "daily labor supply" curve for a log utility function. The student works until her discounted marginal utility of leisure equals the shadow value of work.

[6] Those of less hardy stock who need a minimum of sleep may feel free to pick a smaller number.

Figure 1: Daily Labor Supply



In addition, one of these pairs must hold:

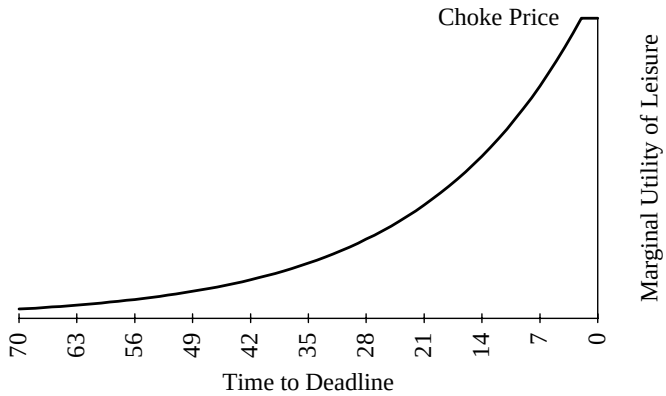
$$(a)\ \lambda > 0,\ \sum_{0}^{T-1} w_t = R; \qquad (b)\ \lambda = 0,\ \sum_{0}^{T-1} w_t \geq R. \tag{3}$$

That is, if λ is strictly positive, the work constraint is binding and cumulative hours of work just equal the requirement; if the constraint does not bind (cumulative work hours exceed the requirement), then λ will be zero. But psychologists and economists alike should recognize that doing more work than necessary is never rational, for if $(3b)$ holds and $u(\cdot)$ is strictly increasing, then $(2b)$ implies that $w_t = 0$ for all $t \in [0, T-1]$. In other words, extra work requires leisure to be foregone and, in the context of this model, has no offsetting benefit; thus, cumulative work hours must exactly equal the requirement.

If marginal utility at no leisure is finite, this value is essentially the "choke price." Once the MUL reaches this level, no leisure is consumed and the student works nonstop 24 hours per day. Suppose the choke price were reached at the last day of work. The student works a little bit more each day until she spends the entire day before the deadline writing. The cumulative hours of work implied by this path are unlikely to equal the requirement except by pure chance. If the work requirement were less, the entire path would shift down (lowering the MUL and w_t for all t and thus lowering the cumulative hours of work until they equalled the requirement), and the choke price would never be reached; in other words,

even at the end she would not spend the entire day working. On the other hand, if the work requirement were greater, the choke price would be reached before the deadline, and the student would spend the last days working nonstop on the paper. This situation is depicted in Figure 2.[7]

Figure 2: Finite Marginal Utility at No Leisure

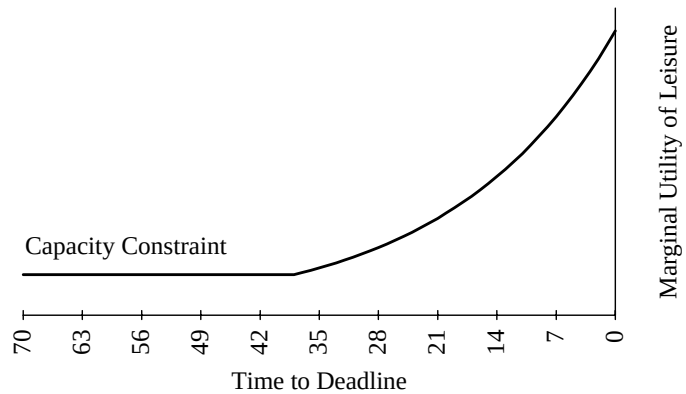


If marginal utility at all leisure is strictly positive, the student is "capacity constrained" at a maximum leisure extraction rate of 24 hours. For small enough work requirements the student does no work at all initially, waiting until the discounted MUL equals the shadow value of work. In a sense, the student would like to be taking more than 24 hours of leisure in the day, but is bound by the technological constraint of the earth's rotation. Thus, she does no work until that constraint no longer binds. Figure 3 shows this path.[8]

In the presence of a binding capacity constraint and a choke price, a situation can be generated where the student plays all day at the beginning, then builds up her workload and works nonstop at the end. A special case is that of Nordhaus (1973), where marginal utility is constant at c. In this case, the student's day is never split between work and leisure; she plays all day, as long as $\delta^{-t}\lambda < c$, and then starts working full-steam once $\delta^{-t}\lambda > c$. Accordingly, $R/24$ days are spent working and the preceding $T - (R/24)$ days are spent playing.

[7] Figure 2 uses quadratic utility and a discount factor of .95.
[8] Figures 3 and 4 use log utility and a discount factor of .95.

Figure 3: Positive Marginal Utility at Leisure All Day



Thus, the shape of the utility function helps determine how quickly the student builds up her workload (and builds down her leisure) over the time period. Substituting in leisure and manipulating the first-order condition (2a), we can see how leisure hours change over time:[9]

$$\frac{\Delta l_t}{l_t} \approx -\frac{(1-\delta)}{\delta}\left(\frac{u'(l_t)}{-u''(l_t)l_t}\right). \tag{4}$$

The ratio $(1-\delta)/\delta$ equals the rate of time preference. The bracketed term is the elasticity of intertemporal substitution (also the inverse of the Pratt-Arrow measure of relative risk aversion). Thus, leisure hours decline faster over time the greater the discount rate and the greater the elasticity of intertemporal substitution. A larger change in leisure hours implies a faster buildup of work, meaning that for the same cumulative number of work hours, the student starts out doing less work and does more later. In other words, the student will procrastinate more the more heavily she weights current consumption of leisure and the less she minds large swings in leisure consumption.

[9]The preceding steps start with the first-order condition (2a) and approximate the change in marginal utility to find the change in l_t:

$$u'(l_t) = \delta u'(l_{t+1}) = \delta(u'(l_{t+1}) - u'(l_t)) + \delta u'(l_t) \approx \delta u''(l_t)\delta l_t + \delta u'(l_t)$$

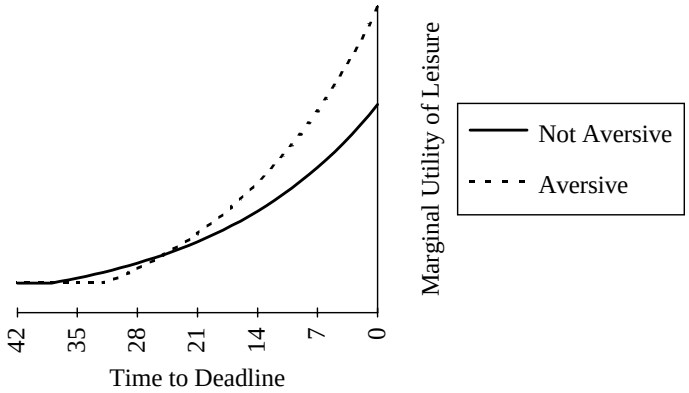
.

3.1 Task Aversiveness

Rather than contradict the psychological literature, this rational model offers a new perspective on some of those studies. Solomon and Rothblum (1984) found that "Aversiveness of the Task" was the major factor cited by procrastinators to explain their behavior.[10] Disutility of work can be easily added to the model, functioning like a variable cost of extraction in the natural resource analog — a negative cost.

Let marginal disutility of work be a constant a. Define "net utility" as utility from leisure minus disutility from work: $u(l_t) - (24 - l_t)a$. The new first-order conditions require that marginal net utility rise by the discount rate: $u'(l_t) + a = \delta^{-t}\lambda$. As a result, marginal utility of leisure rises more steeply; Figure 4 illustrates this change in the time path of marginal utility.

Figure 4: Task Aversion



The cost to working acts like a subsidy to taking leisure. Constant marginal disutility of work has the effect of raising the elasticity of intertemporal substitution, causing the rate of change in leisure extraction to increase:

$$\frac{\Delta l_t}{l_t} \approx -\frac{(1-\delta)}{\delta}\left(\frac{u'(l_t)+a}{-u''(l_t)l_t}\right). \tag{5}$$

[10] In their study of reasons for procrastination, task aversion had several question responses associated with it, leading the 47% estimate to be a lower bound. [23]

McCown, Petzel, and Rupert (1987, henceforth MPR) also observed that procrastinators tend to perform the easier tasks first, putting off the more difficult ones. This behavior conforms with the predictions of standard natural resource theory that, if an extractor is free to draw down deposits of differing costs, the least costly deposit will be extracted first. Consider the simple case of a two-task project where one task is simpler (less aversive) than the other, and the tasks both need to be completed by a single deadline, although in no particular order. Suppose the student performed some of the more aversive task before she finished the easier task; if she switched one of those earlier hours of hard work for one of those later hours of easy work, both tasks would still be completed by the deadline, but her welfare would increase: her utility from leisure would remain unchanged in every period, but she would gain discounted net utility from putting off the harder work. Thus, the optimizing student will work on only one task at a time, beginning with the less aversive one and moving on to the more difficult one after the first is completed.[11]

3.2 Procrastination under Uncertainty

While procrastination may be characterized by a rapid buildup in the workload, procrastinators are often identified by their failure to meet deadlines. Aitken (1982) proposed that the main problem of procrastinators is an inability to estimate adequately the amount of time needed to perform the task. However, prediction abilities held equal, high procrastinators may be more likely to miss deadlines just because they have the aforementioned characteristics: higher rates of time preference, elasticity of intertemporal substitution, or task aversion.

Suppose the actual work requirement is not initially known. How does this uncertainty

[11]The literature in this area of natural resource economics is broad. Herfindahl (1967) considers nondecreasing marginal costs, finding that one should always extract the next unit from the resource pool with the lowest marginal cost. Weitzman (1976) develops a more general method, allowing for marginal costs to decrease with cumulative extraction, which dictates extracting from the resource pool with the lowest implicit cost, the minimum of the average costs over different ranges of consecutive extraction. For task aversiveness, this rule means the student may start first on a task for which the initial units of effort are difficult, as long as it gets easier and over some range is easier on average than the other task, which requires less effort to start.

affect the optimal allocation of work and leisure?[12]

Performance of an uncertain task is somewhat analogous to the problem of extracting from a resource pool of uncertain size. Resource use with reserve uncertainty has been treated in different ways in the natural resource literature, mostly falling into the "cake-eating" genre, since the exhaustible resource problem is analogous to consuming a cake of uncertain or changing size. Gilbert (1979) and Loury (1978) model problems in which the size of the reserves is fixed but unknown, and more information becomes revealed as the resource is used. Among their results is that uncertainty implies a more conservative, i.e., slower, extraction policy.[13] This subsection discusses briefly the impact of uncertainty itself on procrastination, which is quite different from the standard resource model. However, the main question at hand is whether high procrastinators are more likely than low procrastinators to miss a deadline, given the same information about the distribution of an uncertain work requirement.

The model of procrastination under uncertainty is more easily presented in continuous time; to this end, let σ be the rate of time preference and $\omega \in [0, 1]$ represent the instantaneous rate of work.[14] Utility is now a function of the leisure rate, $u(1 - \omega)$, and the units of time can generalize to hours, days, or whatever is desired. This formulation does not change the essential results.

Let us begin with a simple, two-stage model. (This model can easily be extended to incorporate multiple stages, but the two-stage model suffices for demonstrating the intu-

[12]Of course, there are many potential sources of uncertainty. Rewards may be uncertain; other factors may render future marginal utility (i.e., the costs) uncertain. The evolution (and resolution) of uncertainty over time will also affect work/leisure tradeoffs. Work requirement uncertainty, however, is certainly common, and this example illustrates how the presense of uncertainty itself can exacerbate the behavior of high procrastinators.

[13]On the other hand, Pindyck (1980) models the effects of uncertainty when the reserves fluctuate continuously and stochastically over time, finding that the resource may be exploited more quickly than under certainty. This model could be used in an example where the professor's expectations for the paper change randomly, and the student gets information over the course of the semester rather than with the amount of work being done.

[14]While continuous time may pose some conceptual difficulties, such as how one divides an instant of time between work and leisure, the mathematical presentation is more elegant and the basic results are the same as in discrete time.

ition.) Suppose an initial M units of work (say, the literature search) must be completed before the remaining work requirement, $\tilde{N}$, (how long it takes to write the paper) becomes known. As in Gilbert's model, utility from resource (leisure) extraction in the first stage is maximized given expectations about utility in the second stage, which depends upon how much work is left to do.

The traditional resource model cannot be applied directly to procrastination, however. Here, the resource in question is leisure time, but the uncertain requirement is work; this indirect link between the resource and the constraint causes complications. The first problem is that the work requirement M is not associated with a particular amount of consumption of leisure time in the first stage. The second difference grows out of the first: because of the deadline, the optimal path in the last stage depends not only on the remaining work, but also how much time is left to complete it. The longer it takes to get to M, the less time is left to complete $\tilde{N}$.

An additional complication is that states may exist where the student prefers to drop the course rather than complete the term paper. Implicit in the original assumption of complying with the certain assignment is some penalty or reward making it worthwhile. A fixed penalty creates a knife edge in the incentives to work: if at any time the student realizes the task is not feasible, the utility-maximizing strategy would be to give up then and not waste any leisure trying. Furthermore, in some cases where completion is feasible, for some combinations of penalties and remaining work, the student may prefer to give up. If she deems the task worthwhile to complete, she optimizes as under the assumption of making the deadline.

Therefore, the first-stage problem solves for the path of work and leisure hours and for the time I when M is reached, knowing the last-stage response to any given N and I and the distribution of $\tilde{N}$. This response must also depend on the penalty. If, arriving at I, the student realizes completing the paper is infeasible, she will stop work (drop the course) and accept the penalty at the end (perhaps summer school). If the N she realizes at I is feasible,

she still must weigh her alternatives: continue working or give up now.

Let $U_0(N, I)$ be the maximized discounted utility of completing work requirement N by time T starting at time I:

$$U_0(N, I) = \max_{\omega_t \in [0,1]} \int_I^T u(1-\omega_t)e^{-\sigma t}dt - \lambda_0 \left(\int_I^T \omega_t dt - N \right) \tag{6}$$

The first-order conditions with respect to ω_t are essentially the same as those in (2):

$$\begin{aligned} &(a) \quad 0 < \omega_t < 1, \quad u'(1-w_t)e^{-\sigma t} = \lambda_0; \\ &(b) \qquad \omega_t = 0, \qquad u'(1)e^{-\sigma t} \geq \lambda_0; \\ &(c) \qquad \omega_t = 1, \qquad u'(0)e^{-\sigma t} \leq \lambda_0. \end{aligned} \tag{7}$$

In other words, within the stage, for $\omega_t \in (0, 1)$, the marginal utility of leisure will grow at the discount rate.

The value of the last stage is the larger of discounted utility from either finishing the project or stopping work and accepting the penalty P. Let $V_0 = V(N, I)$ be the value of the maximized last-stage utility stream net of the penalty in the states where the deadline is not met, discounted to the beginning of the first stage:

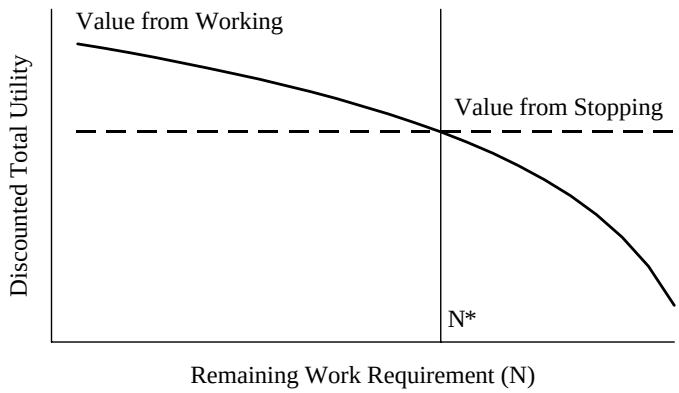
$$V_0(N, I) = \max \left\{ U_0(N, I), \quad U_0(0, I) - e^{-\sigma T} P \right\}. \tag{8}$$

Since a higher workload and less time until the deadline both lower $U_0(\cdot)$, $V_0(\cdot)$ is also decreasing (though not necessarily strictly) in both variables.

Once the last stage is reached, the remaining work requirement is revealed and optimization proceeds under certainty. For any starting time I, the value of working declines as the workload becomes more burdensome, while the value from quitting remains constant. At the workload where these values cross, the student is just indifferent between finishing the course and dropping. Figure 5 depicts her alternatives as a function of the

remaining workload.

Figure 5: Value of the Last Stage Given I



In the first stage, however, $\tilde{N}$ is not known and the optimal work path depends on the expected value of utility in the last period. The expected value of the last stage, and thereby the incentives for timing the completion of the first stage, depends on the distribution of possible workloads and on the penalty for noncompletion:

$$E\{V_0(\tilde{N}, I)\} = \int_{N_L}^{N_I^*} U_0(\tilde{N}, I)\phi(\tilde{N})d\tilde{N} + \int_{N_I^*}^{N^H} \left(U_0(0, I) - e^{-\sigma t}P\right)\phi(\tilde{N})d\tilde{N}, \qquad (9)$$

where $\phi(\tilde{N})$ is the probability distribution function. N_I^* is the cutoff remaining workload: for lesser workloads the student will finish, and for greater workloads she will quit.

Let $U_1(M, I)$ be the maximized discounted utility of completing work requirement M by time I starting at 0:

$$U_1(M, I) = \max_{\omega_t \in [0,1]} \int_0^I u(1 - \omega_t)e^{-\sigma t}dt - \lambda_1 \left(\int_0^I \omega_t dt - M\right) \qquad (10)$$

In this case, $U_1(M, I)$ is strictly increasing in I: Adding another unit of time spent partly in leisure unambiguously raises discounted utility, which may then be further improved by readjusting work rates to correspond to the first-order conditions for ω_t.

In the first stage, discounted utility from performing M by I and the expected value of

utility thereafter are maximized with respect to I:

$$V_1(M, I) = \max_I \left\{ U_1(M, I) + E\{V_0(\tilde{N}, I)\} \right\}. \tag{11}$$

In choosing when to complete the literature review and start writing the paper, the student will want to extend I as long as the marginal benefits in the first stage outweigh the expected marginal costs in the last stage. At the optimal I^*, the student would not want to extend the literature review, nor would she want to shorten it:

$$\frac{\partial U_1(M, I)}{\partial I} = -\frac{\partial E\{V_0(\tilde{N}, I)\}}{\partial I}. \tag{12}$$

Let a negative superscript on I indicate the point immediately preceding the switch into the last stage. The increase in first-stage utility from stretching out the literature review a bit more equals the leisure enjoyed at that point (ω_{I^-}) plus the discounted marginal addition to leisure on preceding days from smoothing:

$$\begin{aligned} \frac{\partial U_1(M, I)}{\partial I} &= u(1 - \omega_{I^-})e^{-\sigma I} - \int_0^I u'(1 - \omega_t)e^{-\sigma t}\frac{\partial \omega_t}{\partial I}dt \\ &= u(1 - \omega_{I^-})e^{-\sigma I} + u'(1 - \omega_{I^-})e^{-\sigma I}\int_0^I \frac{\partial \omega_t}{\partial I}dt \\ &= \left(u(1 - \omega_{I^-}) + u'(1 - \omega_{I^-})\omega_{I^-}\right)e^{-\sigma I}. \end{aligned} \tag{13}$$

This equation is reduced using two facts: (i) according to the first-order conditions, discounted marginal utility is constant, and (ii) the total reduction in work performed before I must equal that performed in the additional unit of time from extending I (i.e., ω_{I^-}).[15]

Correspondingly, the decrease in expected last-stage utility from extending the literature review equals the leisure enjoyed at that point (ω_{I^+}) plus the discounted marginal reduction in leisure at all points in time from smoothing (in states where the work is com-

[15]More precisely, discounted marginal utility is constant when the subject is unconstrained; however, if the subject is constrained, the change in ω_t is zero. This reduced form holds as long as the subject is not constrained at I itself.

pleted):

$$\begin{aligned}\frac{\partial E\{V_0(\tilde{N},I)\}}{\partial I} &= \int_{N_L}^{N_I^*} u(1-\omega_{I^+})e^{-\sigma T}\phi(\tilde{N})d\tilde{N} \\ &+ \int_{N_L}^{N_I^*} u'(1-\omega_{I^+})\omega_t e^{-\sigma T}\phi(\tilde{N})d\tilde{N} \\ &+ \int_{N_I^*}^{N^H} u(1)e^{-\sigma T}\phi(\tilde{N})d\tilde{N}. \end{aligned} \tag{14}$$

While lengthening I also lowers the cutoff work requirement, since at N^* the two alternatives are equal in value, small changes in N^* do not affect the expected value of the last stage.

Let $\psi(\omega) = u(1-\omega) + u'(1-\omega)\omega$. Note that for a non-negative, concave utility function, $\psi(\omega) \geq 0$, and $\psi'(\omega) = -u''(1-\omega)\omega \geq 0$. The first-order condition for I (Equation (12)) can thus be more simply written:

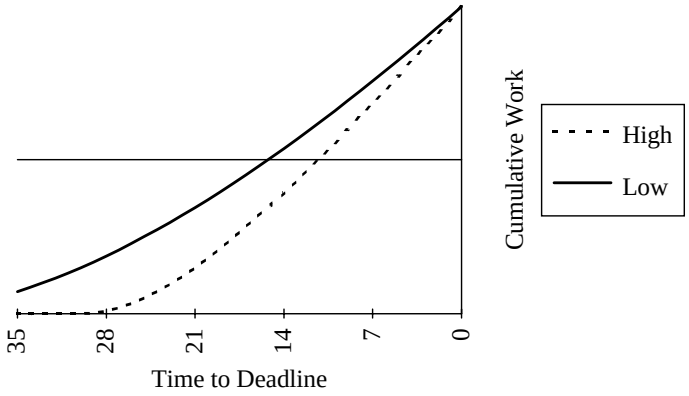
$$\psi(\omega_{I^-}) = E\{\psi(\omega_{I^+})\}. \tag{15}$$

In general, the more work that must be done relative to the time remaining, the more hours of work are performed each day (including day I). Lengthening the first stage lowers M/I and lowers ω_{I^-}. However, lengthening I raises $N/(T-I)$ and raises ω_{I^+}. Thus, extending I lowers the left-hand side of Equation (15) and raises the right-hand side; with the optimal I^* the two sides will equal and the expected path of marginal utility will be smooth. Thus, Equation (15) tells us that the more work expected to be done at the beginning of the last stage, the more work will be done at the end of stage 1 (and each day in stage 1, according to the FOCs). In other words, the greater the expected workload in the last stage, the faster the first-stage work will be performed.

Though not written explicitly, the equilibrium work path is also a function of the discount factor: a lower discount factor implies a steeper path. Consider, for a moment, the case where the last-stage work requirement is certain, illustrated in Figure 6. A student

with a higher rate of time preference will be performing less work at the beginning of the last stage (given any N and I). She will thus also be working less at the end of the first stage, which in turn implies the first stage will take longer to complete.

Figure 6: Cumulative Work and Rates of Time Preference under Certainty



Under uncertainty, these incentives remain: given the same probability distribution of the work requirement, a student with a high rate of time preference will take longer to do the literature review than one with a low subjective discount rate, leaving less time for the last stage. Should the true requirement realized in the last stage prove to be much greater than estimated, the high procrastinator has a greater chance of lacking enough time to perform the task completely.

Furthermore, the high procrastinator, given any I and P, is more likely to want to quit. Since she discounts the penalty at a higher rate, she is induced to complete the task in fewer states.[16] A corollary of this result is that any incentive uncertainty creates to complete the first stage earlier is reduced. "Precautionary work" will be performed if uncertainty raises the expected cost of postponement relative to that cost at the expected remaining work rate (i.e., if $E\{\psi(\omega_{I^+}(\tilde{N}))\} \geq \psi(\omega_{I^+}(E\{\tilde{N}\}))$, or $\psi(\cdot)$ is a convex function of $\tilde{N}$).[17] With

[16]Proof of these statements is in the Appendix.

[17]I am borrowing liberally from the language of the precautionary savings literature of Kimball and others. I will not discuss the application of prudence here, since precautionary work here is a function not only of the relationship between the third and second derivatives of the utility function, but also the relationship between the work rate, the remaining work requirement, and the deadline. I will concentrate on the effect of the quitting alternative on expectations.

the option to quit, $\psi(\cdot)$ is not a strictly increasing, convex function of $\tilde{N}$, since above N^*, $\psi = 0$. For a high procrastinator, with a low N^* given any I, the expected value of $\psi(\cdot)$ can actually be less than the value of $\psi(\cdot)$ at the expected $\tilde{N}$, even if $\psi(\cdot)$ is a convex function of ω.

Figure 7: ψ and its Expected Value

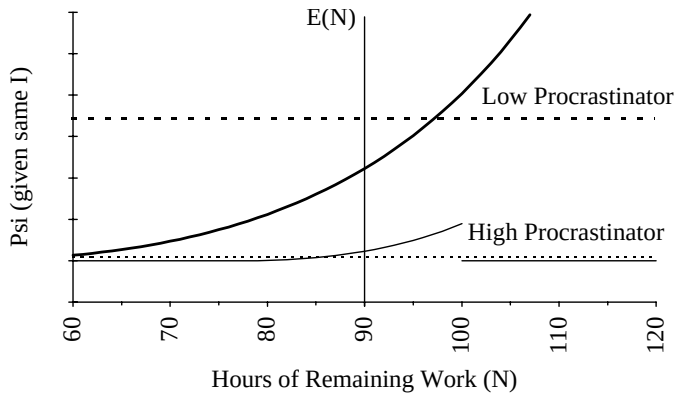


Figure 7 illustrates such a case, presenting $\psi(\cdot)$ as a function of N for both low and high procrastinators (low and high rates of time preference). The solid lines show these functions, and the dotted lines represent their corresponding expected values. Given the same I, for any N, $\psi(\omega_I^L) > \psi(\omega_I^H)$. For a penalty that just induces the low procrastinator to complete the project in any state, the high procrastinator will quit for some remaining work requirements. As a result, while $E\{\psi(\omega_L(\tilde{N}))\} > \psi(\omega_L(E\{\tilde{N}\}))$ and the low procrastinator performs precautionary work, $E\{\psi(\omega_H(\tilde{N}))\} < \psi(\omega_H(E\{\tilde{N}\}))$ and the high procrastinator takes even longer to complete the first stage than under certainty.[18] In a sense, this reasoning is the "fear of failure" explanation for procrastination (a reason affirmed by at least 14% in the Solomon and Rothblum (1984) study): [19] a high procrastinator knows she will give up in more states, lowering the expected change in utility of the second stage and causing her procrastinate more the completion of the first part.

[18]This simple example uses log utilities, making $\psi(\cdot)$ convex in ω and ω linear in N, which is assumed to be uniformly distributed.

[19]As with task averesion, several question responses were associated with fear of failure. [23]

Therefore, one need not conclude, as Aitken did, that high procrastinators miss deadlines because they are less able to estimate work requirements. By their nature of having high rates of time preference, they take longer to perform the first stage, possibly even longer in the presence of uncertainty. Thus, should their estimates hit significantly below the mark, they are less likely to be able or willing to meet the deadline.[20]

3.3 Penalties and Multiple Deadlines

This result is important if someone (such as an instructor, university, or employer) wants to reduce the incidence of failure to meet a deadline or submission of a product that fails to meet expectations. Improving estimation abilities will not necessarily help. Either the students (or employees) must be made more conservative in their preferences for risk or for time discounting, or some method is needed to make them do more work ahead of time.

Psychologists have conducted many studies of student procrastination in self- or instructor-paced courses with various reward and penalty schemes. Reiser (1984) found that in a class presented with a pacing schedule, students facing penalties proceeded at a faster pace than the control group. Wesp (1986) found that students in a section with daily quizzes completed course work faster and earned better grades than those with self-paced quizzing. Lamwers and Jazwinski (1989) followed students enrolled in a self-paced course, each facing one of four course contingencies for failing to meet the deadline: "doomsday" (withdrawal from the course or failing grade), doomsday with tokens for early completion, "contracting" (essentially close instructor supervision and pacing), and no contingencies (baseline). Of the four plans, contracting was found to be the most effective method for reducing procrastination problems, but also the most costly administratively.

The fact that some students make the deadline in the no-contingencies case reveals that some reward is gained from completing the course. This baseline benefit applies to

[20] McCown, Petzel and Rupert (1987) did find in a lab experiment that procrastinators reported on average lower estimates of the time requirement to complete a reading assignment, although actual requirements were not compared to those estimates.

all contingencies, and thus is an unnecessary complication when comparing the effects of different penalty regimes on procrastination. I will address the two main alternatives, doomsday and contracting.

Doomsday, in this paper's model, is represented by the type of fixed penalty just discussed. Increasing the penalty makes completing a feasible task more desirable; consequently, the student will be doing the task in more states of the remaining work requirement. The expected workload at the beginning of the last stage will thus be higher, causing her to perform the first-stage work faster.

The contracting method forces students to do more work earlier by creating multiple deadlines. Suppose the task is divided into two parts, which require R_1 and R_2 hours of work by deadlines T_1 and T_2, respectively.

Work on the second part may be performed before the first deadline. The student's objective function becomes

$$\begin{aligned} U(R_1, R_2, T_1, T_2) = \max_{w_t^1 + w_t^2 \in [0,24]} \sum_{0}^{T_1-1} u(24 - w_t^1 - w_t^2)\delta^t \quad &+ \quad \sum_{T_1}^{T_2-1} u(24 - w_t^2)\delta^t \\ -\lambda_1 \left(R_1 - \sum_{0}^{T_1-1} w_t^1 \right) \quad &- \quad \lambda_2 \left(R_2 - \sum_{0}^{T_2-1} w_t^2 \right) \end{aligned} \tag{16}$$

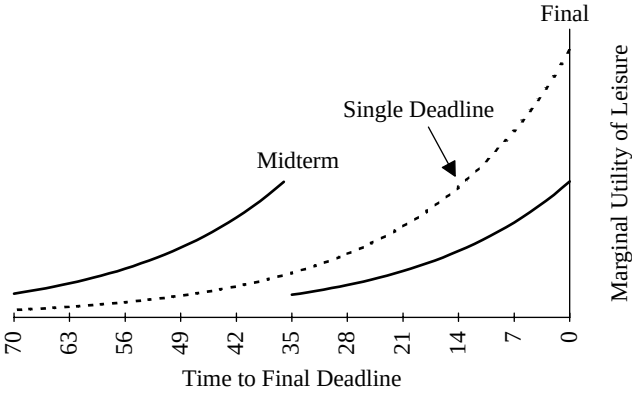
Maximizing with respect to work on both projects produces the following first-order conditions:

$$\begin{aligned} u'(24 - w_t^1 - w_t^2)\delta^t &= \lambda_1, \quad w_t^1 > 0, \quad t < T_1; \\ u'(24 - w_t^1 - w_t^2)\delta^t &= \lambda_2, \quad w_t^2 > 0, \quad t < T_1; \\ u'(24 - w_t^2)\delta^t &= \lambda_2, \quad w_t^2 > 0, \quad t \geq T_1. \end{aligned} \tag{17}$$

If work on the second part is performed before the deadline of the first, $\lambda_1 = \lambda_2$, and the MUL rises smoothly to the second deadline as in a single-part exercise (where $R_1 + R_2$ would have to be completed by T_2); the first deadline does not bind, and the student's pace

of progress is unchanged. If, however, the deadline for the completion of part 1 falls earlier than she would choose on her own, she works on part 1 alone until T_1 and then begins part 2. In this case, $\lambda_1 \neq \lambda_2$, and the MUL rises steadily to the first deadline, drops, and rises again until the second deadline, as pictured in Figure 8.[21] It is never optimal for marginal utility to jump up at T_1, since smoothing could be done by performing some of the second task before the first deadline.

Figure 8: Multiple Deadlines



Under uncertainty, having multiple deadlines with smaller uncertain work requirements (and also smaller variances), high procrastinators are less able to postpone work and risk missing the deadline. Theoretically, there should be some penalty plan which would result in an equal expected failure rate as the multiple deadline scheme. However, for practical or humane reasons, the corresponding penalty may not be implementable (except, perhaps, in Singapore). Similarly, an equivalent reward scheme could be prohibitively costly. Therefore, closer monitoring may be the best method for reducing deadline failure rates.

Multiple deadlines may also be as much a cause of problematic procrastination as the cure. In the real world, people usually have more than one task at hand, and the model shows that the most pressing task with the nearest deadline gets priority while the less pressing one gets postponed ("procrastinated"). "You had too many other things to do"

[21] Of course, the same variations seen in the single-deadline section can also be seen here.

was the most frequently endorsed reason for procrastinating in the study by Solomon and Rothblum (1984) (60.8% of the subjects). MPR found a correlation between extraversion and procrastination; in one layperson's interpretation, this correlation may be explained in part by the fact that extraverts tend to be involved in more activities. Therefore, the best way for a supervisor to give her assignment priority and reduce the risk of missing the ultimate deadline may be to break it into smaller tasks with more deadlines to better compete with the other demands on the student's time.

3.4 Actual and Effective Discount Rates

The exhaustible resource model does a good job explaining the qualitative aspects of procrastination — the effects of task aversion, multiple tasks, uncertainty and, in some circumstances, even the fear of failure. However, quantitatively, the implicit discount rates required to generate such results under exponential discounting may appear unreasonably large. In the basic model, the rate of decrease in daily leisure is the product of the discount rate and the elasticity of intertemporal substitution (EIS). Suppose this "procrastination rate" is just 1 percent, which still seems low for the behavior we observe. Most empirical analysis of consumption behavior has found an EIS less than or equal to one (or a risk aversion coefficient greater than or equal to one). Taking an EIS of one, the corresponding daily discount rate of 1 percent then implies an annual subjective rate of time preference of nearly 3800 percent! While some studies of hypothetical income and delay tradeoffs have produced discount rates of that order of magnitude, of the studies of actual behavior, such as the purchase of consumer durables, the higher estimates of discount rates are on the order of 300 percent.[22] Of course, parameters values obtained from intertemporal studies of long-run consumption and savings may not apply to a model of daily time allocation.

Still, there are several ways to reconcile procrastination with generally lower discount

[22]These studies are cited in Loewenstein and Elster (1992) as evidence for time-inconsistent discounting: pp. 61-2 and 137-8.

rates. Some solutions work within the framework of exponential discounting. As just presented, having multiple tasks can cause the student to postpone work on a particular assignment. Also shown was the case in which task aversiveness causes the marginal utility of leisure to rise faster than the discount rate. Although productivity has not been discussed, it functions similarly to task aversiveness and variable production costs; if returns are increasing to daily work (e.g., getting on a roll), the work path will steepen. Similarly, setup costs to beginning a task can raise the average rate of increase in the marginal utility of leisure.

Setup costs are like Akerlof's one-time task, a fixed loss that must be incurred at some point in time. They could present themselves as an indivisible task which must be performed before the student can proceed with the rest of the project, or a psychic or other cost to starting a phase of the task.[23] Operating in conjunction with divisible work, setup costs can change the average rate of increase in the marginal utility of leisure. The problem is similar to a negative setup cost in the standard extraction model laid out by Hartwick, Kemp and Long (1986), since here one must pay to start consuming less leisure. Suppose the student faces a fixed setup cost to beginning the paper. In choosing the time to start work, she will want to wait as long as the gains from postponement outweigh the losses from less utility smoothing.[24] Marginal utility will jump up when work commences, leading to an equilibrium path which rises faster on average than the discount rate. Figure 9[25] shows how the path of work changes when a setup cost must be incurred at the start of the task.

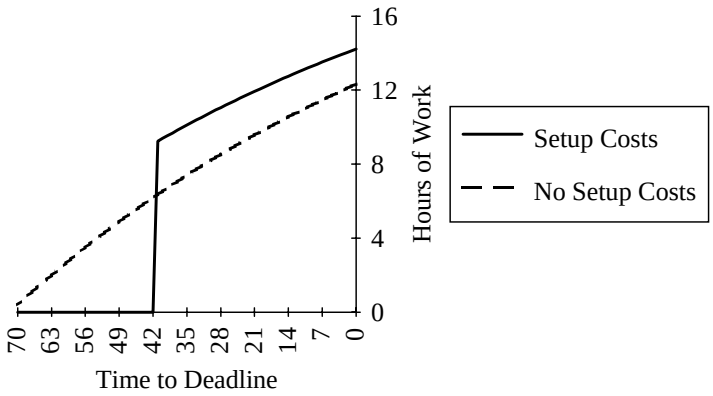
Of course, the value of postponing the setup cost still depends on the discount rate and the EIS. The smaller they are, the larger the setup cost must be to generate a significant

[23]Solomon and Rothblum (1984) reported that 39.6% of the students endorsed "You felt overwhelmed by the task" as a reason for procrastinating. Having to overcome this sensation could be viewed as a setup cost, though not necessarily a rational one.

[24]The problem is similar to that in the uncertainty section, but the gains to extending the first stage include the decrease in the present value of the setup cost for the last stage. This raises the marginal value of postponement, which raises the optimal value of I.

[25]Figure 9 uses log utility and a discount factor of .99

Figure 9: Setup Costs and the Path of Work



postponement effect.

But perhaps the true limits of the rational model are evidenced by the general perception that procrastination is problematic rather than utility maximizing. Solomon and Rothblum (1984) found that about half of American university students surveyed reported that procrastination was a personal problem of "moderate" or more serious proportions. Furthermore, 65% said they wanted to reduce procrastination when writing a term paper. The prospective nature of the problem is revealing. Naturally, at the end of the semester, the student wishes she had done more work (or procrastinated less); one always wants less work and more time to complete it. However, if looking forward to next term she wishes she could change the way she knows she will behave, then we have indications of a self-control problem.

The rational model does not capture issues of self-control failures and undesired procrastination. To this end, other explanations abandon the assumption of time-consistent discounting preferences. For example, people may have different discount rates for different things (say, work versus the reward) or different discount rates for the short and long term.[26] Fischer (1999) picks up where this rational model leaves off to explore the effects of time-inconsistent preferences and finds that, even with rational expectations, they

[26]These alternate forms of discounting can be motivated by the types of salience costs modeled by Akerlof.

exacerbate procrastination of a divisible task by raising the effective discount rate.

4 Conclusion

Of course, some psychologists and even some economists[27] believe any type of time discounting is inherently irrational. Abstracting from this debate, the simple model of impatience presented in this paper offers examples of situations where procrastination can be not only dynamically consistent, but also utility maximizing. Rather than contradict the psychological literature, the model of time as an exhaustible resource offers theoretical underpinnings explaining the results of several empirical psychology studies. It can easily incorporate other adaptations for different types of tasks and incentive schemes and also serves as a baseline for investigating alternate theories of time preference. The psychology studies can offer further insights into how personality traits shape the individual's net utility function and method of time preference, which in turn determine procrastinating behavior.

In conclusion, the resource model provides a useful framework for analyzing procrastination of a task performed over time and for examining policies to reform it. The question is, when will we get around to it?

[27] Including the father of much of environmental economics, Pigou. For an interesting discourse, see Ainslie (1992), p. 56.

Appendix

Proposition 1 *For any N, the minimum penalty required to induce work increases with the rate of time preference.*

The penalty that just induces work equals the discounted difference between all leisure and the optimal work path, evaluated at time T:

$$P = \int_0^T \left(u(1) - u(1 - \omega(R))\right) e^{-\sigma(t-T)} dt \tag{18}$$

Totally differentiating the previous equation, and using the definition of P we find that $dP/d\sigma > 0$:

$$\begin{aligned} \frac{dP}{d\sigma} &= TP - \int_0^T t\left(u(1) - u(1 - \omega(R))\right) e^{-\sigma(t-T)} dt \\ &= \int_0^T (T - t)\left(u(1) - u(1 - \omega(R))\right) e^{-\sigma(t-T)} dt > 0. \end{aligned} \tag{19}$$

Note that since discounted utility is maximized with respect to ω_t, it is unaffected by small changes in σ.

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