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Carbon Sequestration in Agricultural Soils

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Although it is common to alternate between till and no-till practices, past research has considered farmers' tillage options to be limited to the dichotomous choice of whether or not to switch to a long-term no-till regime. This paper expands farmers' options and models their choices of tillage frequency. Less frequent tilling sequesters more carbon but permits a greater accumulation of weeds, whereas more frequent tilling eliminates weeds but releases carbon (tillage emissions). The timing of tillage balances its marginal benefits and costs. Higher payments from industry or government for atmospheric greenhouse gas reductions will increase marginal cost and reduce tillage frequency. Other key parameters, such as higher rates of tillage emissions or reduced weed impact, also influence tillage frequency. However, for the discount rate and the natural decay rate of carbon, the net change depends on the magnitude of other parameters.

Key Words: carbon contract, no-till, offsets, soil carbon sequestration, tillage frequency

Introduction

Carbon sequestration in agricultural soils can reduce atmospheric greenhouse gases; sequestration rates are impacted by farmers' tillage practices. Changing from conventional tillage to no tillage increases sequestration of soil carbon. In addition, longer no-tillage periods lead to larger amounts of sequestered carbon. However, because tillage is beneficial for weed control and other reasons, no-till practices are occasionally or frequently interrupted, resulting in a pattern of tillage within a no-till regime. While earlier studies have been conducted on whether or not to adopt a particular long-term sequestration program such as no-till, and some have considered switching out of the program, none have examined multiple tillage options. This paper presents a model that allows for different tillage frequencies, and shows that payments for reducing atmospheric greenhouse gases can increase sequestration by decreasing tillage frequency, even if such payments do not provide a sufficiently strong incentive for complete no-till adoption.

Costs and Benefits of Less Frequent Tillage

Farmers' tillage decisions are driven by the costs and benefits of postponing tillage. Agricultural tillage controls weeds, relieves soil compaction, incorporates surface residue and fertilizers, and prepares the soil surface for seeding (Phillips et al., 1980). One of the most important reasons for tilling is to eliminate weeds, which allows for higher commercial yields (Hobbs, Sayre, and Gupta, 2008). Hill (2001) finds that U.S. Corn Belt growers tilled at least every two and a half years to avoid decreased yields. No-till practices promote both the build-up of soil carbon and weed growth. While annual tilling may not be necessary for controlling

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weeds, periodic tilling often is. In the Northern Great Plains, weeds such as foxtail barley can be difficult pests in a no-till system, and keeping them in check may require periodic tilling (Derksen et al., 2002).

Less frequent tilling promotes the sequestration of carbon in agricultural soils, leading to improved soil organic carbon (SOC) and subsequently promoting soil fertility and enhancing yields. Farmers will therefore pursue some carbon sequestration practices for the private benefits they receive. Tillage affects SOC accumulation through its influence on soil carbon dynamics, which can be characterized at the most basic level by two carbon pools—a surface labile pool, and a deeper stable pool. Carbon residue inputs initially enter the surface pool where the tillage regime influences their capture and retention. With no-tillage practices, the residue is protected and forms SOC, which is transferred to the deeper pool and sequestered over an extended period of time. Tillage interrupts the stabilization of SOC in the surface pool and releases carbon to the atmosphere (Six, Elliot, and Paustian, 2000). As reported by Conant et al. (2007), frequent tilling of agricultural soils results in less retained carbon. Campbell et al. (1995) show that in the brown chernozem soils of the Canadian prairies, a reversion back to conventional tillage after no-till led to a statistically significant decline in SOC in the surface pool and a resulting smaller input into the slowly decaying deeper soil pool.

The same practices that improve SOC also reduce atmospheric CO_2, but it appears the potential soil carbon sink provided by agricultural soils is not large relative to emissions. Sperow, Eve, and Paustian (2003) estimate the potential sink in the United States to be about 15% of the reduction required by the Kyoto Protocol from 2008 emission levels. Desjardins et al. (2001) estimate Canada's potential sink to be 7% of the required reduction from 2010 emissions. However, there is evidence that increasing the duration of no-till practices also increases the potential amount of stored carbon (Manley et al., 2005). Focusing specifically on periodic tillage, Conant et al. (2007) find that the duration of no-tillage practices has a substantial impact on carbon sequestration. Although the potential sink is currently estimated to be small, tillage frequency can have considerable influence on its size, and agricultural sinks could therefore play an important role in mitigating greenhouse gases.

Despite being a social benefit, reducing atmospheric CO_2 is not a private benefit for farmers. However, this externality could be internalized though the creation of offsets that might be rented or purchased by the government or by private-sector emitters whose emissions are regulated. Since less frequent tillage sequesters more carbon, payments for offsets could create incentives for less frequent tillage.

Carbon Contracts

Recent studies on carbon sequestration contract design focus on long-term contracts for one specific, though vaguely defined, sequestration technology. Feng, Zhao, and Kling (2002) present a model in which agricultural land can be brought into, or removed from, a carbon sequestration program. Because the least expensive land is brought into the program first, the marginal cost of bringing new land into the program (or the marginal benefit from removing it) increases as more land is introduced into the program.

Gulati and Vercammen (2005) model the optimal long-term contract length, which is determined by a rising marginal opportunity cost through time and a declining marginal benefit from sequestration. Marginal costs rise over time because the build-up of soil carbon raises productivity and increases farmers' profits, regardless of whether the sequestration program is continued. Declining marginal benefits result from saturation, which limits further carbon

accumulation. Using a similar model, Gulati and Vercammen (2006) consider problems of time inconsistency when a contract to use carbon sequestering technology is of limited duration, and payments within the contract are discounted to reflect the fact that carbon can be released without liability at the contract's end. Antle et al. (2003) compare the costs of sequestering a precise amount of carbon under contracts that pay farmers for a specific technology versus tons of sequestered carbon. They conclude that the inefficiency associated with requiring a specific technology far outweighs measurement costs associated with per ton payments.

This study extends the earlier work of Gulati and Vercammen (2005) in that the growth of a stock of weeds is a by-product of the build-up of soil carbon, but the timing of a tillage event does not determine optimal contract length. Additionally, we provide for a wider menu of tillage patterns. Also, similar to Antle et al. (2003), we allow farmers to choose tillage patterns. In our model, however, offset payments provide an incentive for less frequent tilling, but do not necessarily lead to long-term no-till practices. We also allow for investments in seeding and weed control technologies to reduce the cost of delaying tillage, making our policy recommendations richer than those from earlier models.

Options for Soil Carbon Sequestration

The Model

We assume farmers operate in a perfectly competitive market for their agricultural product and their tillage choices are based on private costs and benefits. These include the benefit of increasing crop yields resulting from increased SOC, the cost of decreasing crop yields through the build-up of weed infestations, and the offset revenue generated or lost by sequestering atmospheric CO_2 or releasing it into the atmosphere through tillage. The model is nested in that offset payments vary from zero to the full marginal social value of reducing atmospheric CO_2. The optimal control model incorporates delayed weed and carbon stock responses, and is similar to delayed recruitment models in fisheries economics (Clark, 1976) and lagged entry models of limit pricing (De Bondt, 1976; Kamien and Schwartz, 1981).

Farmers face a dynamic optimization problem. The objective is to maximize the present value of a flow of net benefits into the infinite future, subject to the limits imposed by the dynamics of soil carbon sequestration and weed accumulation, and on the amount of land available. The maximization objective is converted to a Hamiltonian composed of two parts. The first is instantaneous net benefits and the second is the marginal value of investments or disinvestments in the stocks of soil carbon and weeds. Instantaneous net benefits include the crop yield benefits in equation (1) and the benefits from the rental of offsets in equation (2):

(1) $$PAY_t = PA(cC_t - dW_t)$$

and

(2) $$\pi(C_t - C_o),$$

where P is the fixed net price per unit of commercial crop yield; $Y_t = cC_t - dW_t$ is the crop yield per hectare, which is a linear function of soil carbon and weed stocks and does not vary across time; c is the constant marginal product of the soil carbon stock; C_t is the soil carbon stock at time t; C_o is the baseline carbon stock with continuous tillage; $-d$ is the constant marginal product of the weed stock; W_t is the weed stock at time t; A is the hectares of land

under crop at time t; and π is the offset rental rate for a unit of carbon stock above the baseline kept out of the atmosphere. No-till practices promote increases in both weed and soil carbon stocks. Incremental additions to soil carbon stock have positive and constant marginal effects on crop yields, while incremental additions to weed stock have negative and constant marginal effects on crop yields.

The second part of the Hamiltonian is the marginal value of investments or disinvestments in the two stocks—weeds and soil carbon. Both stocks must be nonnegative; the shadow prices of these stocks are α_t and λ_t. There are four tillage control variables: v_t, w_t, x_t, and y_t. The variable v_t represents hectares tilled in time t after being last tilled in $t-1$, implying continuous tillage. The variables w_t, x_t, and y_t represent periodic tillage—hectares tilled in t after being last tilled in $t-2$, $t-3$, and $t-4$. After each tillage event, land must be recommitted to no-till. The variable u_t represents hectares of land recommitted in time t. The recommitted land may be tilled again at time $t+1$, $t+2$, $t+3$, or $t+4$, or left in permanent no-till.

Weed Stock

Weed stock at time t, W_t, has a negative influence on yield. Smith et al. (1996) show that no-till practices can lead to a build-up of weed stock, particularly for perennial weeds. Suppose that in year zero, a farmer chooses u_o, the number of hectares to be permanently committed to a tillage rotation, and weeds do not accumulate during year zero. In the next year, s_1 units of weeds per hectare accumulate, as do additional units in the subsequent three years, s_2, s_3, and s_4. The marginal accumulation is positive but decreasing through time: $s_1 > s_2 > s_3 > s_4 > 0$. The total amounts accumulated per hectare after one, two, three, and four years are $S_1 = s_1$, $S_2 = s_1 + s_2$, $S_3 = s_1 + s_2 + s_3$, and $S_4 = s_1 + s_2 + s_3 + s_4$. To simplify, we assume there is no additional weed accumulation after year four.

When there are tillage options, the net increment to the weed stock between years t and $t+1$ will be influenced by the hectares committed to a tillage rotation in the past and not tilled before $t+1$. Land committed to no-till in $t-4$, and not tilled in $t-3$, $t-2$, $t-1$, or t, will contribute an increment of $s_4 n_{t-4} = s_4(u_{t-4} - v_{t-3} - w_{t-2} - x_{t-1} - y_t)$. Land committed in $t-3$ and not tilled in $t-2$, $t-1$, or t will contribute an increment of $s_3 n_{t-3} = s_3(u_{t-3} - v_{t-2} - w_{t-1} - x_t)$. Similar contributions will come from land committed in years $t-2$ and $t-1$. If tillage occurs in time t, not only will the marginal increment to the weed stock be avoided, but any previously accumulated weed stock will be eliminated. The change in the stock W_t is given by:

$$W_{t+1} - W_t = \sum_{i=1}^{4} s_i n_{t-i} - S_1 w_t - S_2 x_t - S_3 y_t \,. \tag{3}$$

The Soil Carbon Stock

Soil carbon stocks are affected by crop residue inputs and decay processes. Even the simplest soil science models have complicated carbon dynamics, involving many different carbon pools that decay at different rates. Our approach is to simplify these models and consider two pools: a labile pool and a more slowly decaying pool. To keep our model tractable, we combine the two pools into one carbon stock, which receives inputs and exhibits two types of carbon releases from short-term decay processes induced by tillage and long-term natural decay processes.

Gross crop residue inputs are treated as identical regardless of tillage system. Short-term decay processes are computed by subtracting tillage releases from residue inputs. Tillage emissions depend on organic matter stocks near the surface (roughly the top 15 centimeters) and previous management practices (Campbell et al., 1995). A tillage event following a period of no-till practices would release a somewhat greater amount of carbon than a regularly occurring tillage event. However, Conant et al. (2007) find that steady-state soil carbon content is greater in less frequently tilled soils. Over a number of years, total tillage emissions will be smaller for fewer tillage events, because carbon inputs in the near-surface labile pool gradually move to pools that are less prone to release (Six, Elliot, and Paustian, 2000).[1] Given carbon inputs in every time period, this gradual movement means that longer periods of no-till practices increase emissions from a subsequent tillage event, but incremental emissions from an increase in tillage rotation will decrease with longer tillage rotations.

All cropland is assumed to provide g tons of carbon input per hectare to the residue pool per year and is assumed to have been under continuous tillage prior to conversion to no-till. We also assume land tilled in time $t-1$ can be committed to no-till at the end of that time period. However, soil carbon inputs are not increased until time t. For a unit of crop residue input deposited in time t, a portion (f) of the input will remain in the labile pool until the end of the time period. While it is in the labile pool, it can be released by tillage. The remaining portion $(1-f)$ enters the more stable pool and will be released slowly at a decay rate of k. Annual net carbon input on continuously tilled land is $g(1-f)$ per hectare.

If a hectare of land is committed in time $t-1$ and never again tilled, none of the carbon added to the labile pool is ever released by tillage, and the net carbon input will include both the labile and stable pool inputs. The net input at time t will be g and will remain at that level indefinitely; the soil carbon stock decays slowly at a rate of k.

If land was converted to no-till at the end of time $t-2$, not tilled in $t-1$, and then tilled in time t, tillage emissions will be generated from time t (gf) and time $t-1$. Although natural decay processes occur, most carbon, $(1-k)gf$, remains in the soil and is carried forward to time t. Like new input, the carry-forward input is divided into two portions: $(1-k)gf(1-f)$ enters the more stable pool and $(1-k)gf^2$ remains in the labile pool and is released by tillage in time t. The total tillage release per hectare in time t is $gf + (1-k)gf^2$. Letting $F = (1-k)f$, the total tillage loss in t is $gf + gfF$ and the net carbon input is $g(1-f) - gfF$. With tilling every second year, tillage emissions in a tillage year exceed the annual continuous tillage losses, $gf + gfF > gf$; but over two years, tillage losses are greater under continuous tillage, $2gf > gf + gfF$.

If land is committed at the end of time $t-3$ and tilling does not occur until time t, tillage emissions per hectare in t will be $gf + gf(F + F^2)$, and the net carbon input will be $g(1-f) - gf(F + F^2)$. These emissions exceed those from tilling every second year and the annual emissions from continuous tillage, $gf + gf(F + F^2) > gf + gfF > gf$. Over six years, however, its emissions are less than those from tilling every second year, and both are less than those from continuous tillage.[2] For land converted at the end of time $t-4$, the net input in time t is written as:

$$g(1-f) - gf \sum_{t=1}^{3} F^i.$$

[1] Soil science models typically recognize multiple pools. For a relatively simple (two-pool) model, see the ICBM Model (Andrén and Kätterer, 1997). For a more complex example, see the Century Model (Parton et al., 1987).

[2] Over six years, the total emissions from tillage every third year versus every second year versus every year are: $2gf + 2gfF + 2gfF^2 < 3gf + 3gfF < 6gf$.

For land converted at $t-5$ or earlier, the net input is assumed to remain the same as for land converted at $t-4$. Incorporating net carbon inputs and natural emissions, the soil carbon dynamics are given by:

$$(4) \qquad C_{t+1} - C_t = g(1-f)A + gf\sum_{i=1}^{\infty} n_{t-i} - gfFw_t - gf(F+F^2)x_t - gf(F+F^2+F^3)y_t - kC_t.$$

This description of soil carbon dynamics is simpler than most soil science models. However, it can be parameterized to give results very similar to models such as those presented in Campbell et al. (1995); McConkey, Liang, and Campbell (1999); and Conant et al. (2007).[3]

Figure 1 shows the time paths for soil carbon stocks under various tillage rotations, with parameters that mimic results in the literature for Great Plains soil carbon dynamics.[4] The initial stock (C_o) is the unique steady state for continuous tillage. With tilling occurring every second year, the soil carbon stock increases prior to a decline resulting from a tillage event. The decline offsets a portion of the increase. With longer tillage rotations, the stock increases by a greater amount prior to a somewhat greater decline. The decline offsets a smaller portion of the increase. Thus, longer tillage rotations build up the stock more quickly and to a higher steady-state level. With periodic tilling, the steady state is cyclical. The stock increases and then slips back by an equal amount upon tillage. The amplitude of the cycle is greater for longer tillage rotations, but the average stock level is also higher. Permanent no-till generates a continuous increase in the soil carbon stock until reaching a unique steady state.

The Farmer's Hamiltonian

The full Hamiltonian contains the instantaneous net benefits, plus the marginal value of investments/disinvestments in the stocks. The constraint $v_t + w_t + x_t + y_t - u_t = 0$ indicates that the amount of land recommitted to no-till in time t must be equal to the amount of land tilled in that time period; β is the shadow price on this constraint. With r as the discount rate and $\rho = 1/(1+r)$ as the discount factor, the Hamiltonian is specified as:

$$(5) \quad H = PA(cC_t - dW_t) + \pi(C_t - C_o) + \rho\alpha_{t+1}\left(\sum_{i=1}^{4} s_i n_{t-i} - S_1 w_t - S_2 x_t - S_3 y_t\right) + \rho g\lambda_{t+1}\left((1-f)A + f\sum_{i=1}^{\infty} n_{t-i}\right) - \rho g\lambda_{t+1} f\left(Fw_t + (F+F^2)x_t + (F+F^2+F^3)y_t\right) - \rho\lambda_{t+1}kC_t + \beta(v_t + w_t + x_t + y_t - u_t).$$

[3] An appendix with a schematic of the soil carbon dynamics is available from the author upon request.

[4] The parameter value for the simulation in figure 1 was chosen to produce very rough equivalence to the results of a tillage experiment on brown chernozem soils in Saskatchewan (Campbell et al., 1995). In this experiment, the initial state was land continuously tilled for 70–80 years, with a starting stock of soil carbon at 30 metric tons per hectare. With this starting stock, a residue input $g = 2$ metric tons, $f = 0.24$, and $k = 0.05$, continuous tillage maintains the steady state of 30 metric tons. Permanent no-tillage increases the carbon stock by about 4 metric tons over 10 years. The steady state with continuous no-till is 40 metric tons. Although this is approached only asymptotically, 39 metric tons are achieved in 46 years. Campbell et al. have no steady-state estimate. However, they suggest estimates of 30 to 50 years to reach steady state. They also introduce tillage in one experiment after a 10–12 year period of no-till. Our rotational tillage simulations yield similar reductions in soil carbon to their experiment. By way of comparison, Conant et al. (2007) use the Century model to simulate soil carbon content for a site near Manhattan, Kansas. The conventional tillage steady state was 31.6 metric tons, and the no-till steady state was 40.2. Tillage every two years produced a steady state of 34.6 metric tons, and every four years of 36.9 metric tons.

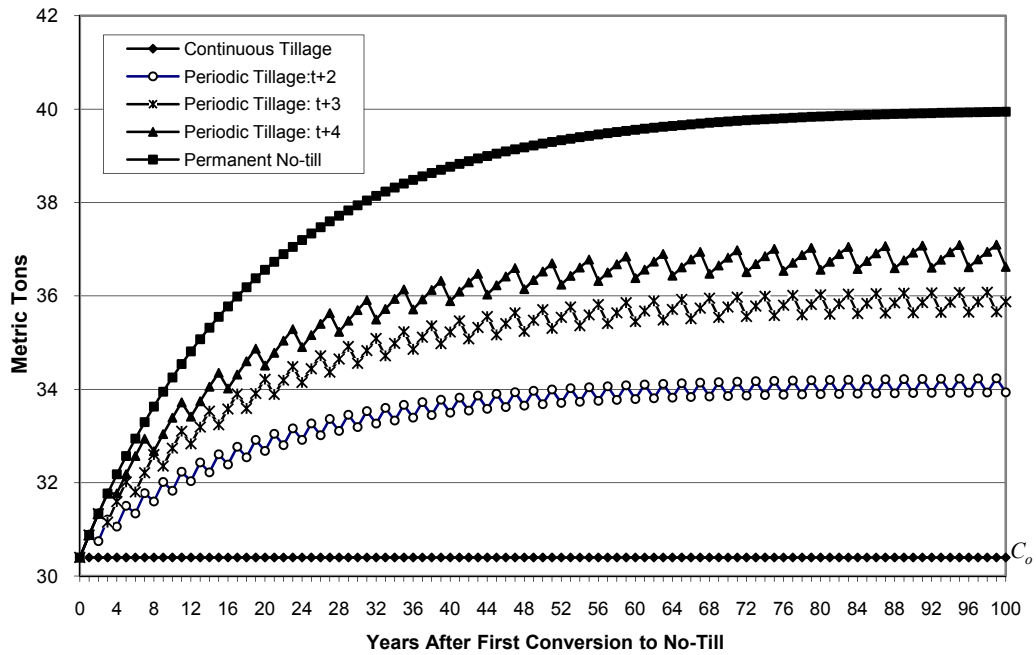

Figure 1. Soil carbon to 100 years after a continuous tillage steady state

The first-order conditions include four tillage rotation conditions, the no-till commitment condition, and the adjoint equations for stocks C_t and W_t, which are represented as:

(6) $$-PcA - \pi + k\rho\lambda_{t+1} = \rho\lambda_{t+1} - \lambda_t$$

and

(7) $$PdA = \rho\alpha_{t+1} - \alpha_t .$$

Unless continuous tilling or permanent no-till is chosen as an option, both the soil carbon and weed stock will exhibit cyclical steady-state behavior. However, their shadow prices,

$$\alpha = \frac{-PdA}{1-\rho} \text{ and } \lambda = \frac{PcA + \pi}{1-\rho(1-k)},$$

are unique steady-state values because of the linearity of the crop yield function. The value of α is the marginal cost of an extra unit of weed stock, while λ is the marginal value of an extra unit of soil carbon sequestration considering both its soil enhancement value and its value for reducing atmospheric carbon. $PcA + \pi$ is the annual marginal benefit per unit of carbon sequestered, and

$$\rho\lambda = \frac{\rho(PcA + \pi)}{1-\rho(1-k)}$$

is the marginal present value of benefits from that unit. The latter discounts future sequestration benefits from a unit of sequestered carbon that decays at rate k.

We now use the steady-state levels for λ and α in the first-order conditions for tillage decisions and for the decision to commit (or recommit) to no-till:

$$(8)\qquad -\alpha\sum_{i=1}^{4}\rho^{i}s_{i}-\frac{\lambda\rho g f}{1-\rho}+\beta\geq<0,\quad (v_t=A,\ v_t=0);$$

$$(9)\qquad -\alpha\sum_{i=1}^{3}\rho^{i}s_{i+1}-\alpha\rho S_1-\frac{\lambda\rho g f}{1-\rho}-\lambda\rho g f F+\beta\geq<0,\quad (w_t=A,\ w_t=0);$$

$$(10)\qquad -\alpha\sum_{i=1}^{2}\rho^{i}s_{i+2}-\alpha\rho S_2-\frac{\lambda\rho g f}{1-\rho}-\lambda\rho g f(F+F^2)+\beta\geq<0,\quad (x_t=A,\ x_t=0);$$

$$(11)\qquad -\alpha\sum_{i=1}^{1}\rho^{i}s_{i+3}-\alpha\rho S_3-\frac{\lambda\rho g f}{1-\rho}-\lambda\rho g f(F+F^2+F^3)+\beta\geq<0,\quad (y_t=A,\ y_t=0);$$

$$(12)\qquad \alpha\sum_{i=1}^{4}\rho^{i+1}s_{i}+\frac{\lambda\rho^2 g f}{1-\rho}-\beta\geq<0,\quad (u_t=A,\ u_t=0).$$

Because the Hamiltonian is linear in the control variables and α and λ are constants, u_t, v_t, w_t, x_t, and y_t are either zero or A; β is the marginal value of the constraint which requires that the amount of land recommitted to no-till following tillage equals the amount of land just tilled. When land is tilled, it becomes available for recommitment to no-till. Here, β can be viewed as the price per hectare that would be offered for the newly tilled land to be used in no-till. For the tillage decision, β is the marginal benefit of making the land available. For the recommitment decision, it is the offer price to be paid for land.[5] Condition (8) gives a necessary condition for the tilling of A hectares one year after commitment to no-till. The sum of the marginal weed reduction benefits from tilling and the marginal benefit of making the land available must be at least as great as the value of marginal soil carbon losses. Conditions (9)–(11) have similar interpretations, except that they refer to tilling two, three, and four years after land has been committed to no-till.

Because the choice of tillage rotation allows at most one of v_t, w_t, x_t, or y_t to equal A, the necessary condition for tillage is not sufficient. The determination of the optimal tillage rotation is similar to the determination of the optimal harvest age in forestry (Heaps and Neher, 1979). Soil carbon and weed stocks are both forms of capital in which a farmer must jointly invest or disinvest. Whenever there is an option to till, a farmer must decide whether to continue investing in the soil carbon and weed stocks through delaying tillage, or to disinvest by tilling. Tilling occurs when marginal net benefits have increased to zero. Although λ and α are constant, both weed stocks and emissions per tillage event increase at a declining rate. As a result, both the marginal benefits and costs of tilling increase at a decreasing rate.

With the additional assumption that weed stocks grow faster than tillage emissions prior to $t-4$, there are three possible outcomes for optimal tillage rotations. First, the sum of marginal

[5] The offer price is the same as the concept of bare land value used in forestry—the value of the land for growing future forests (see Heaps and Neher, 1979).

weed reduction benefits and the marginal benefit of making the land available is always at least as great as the marginal soil carbon losses; this implies continuous tillage, or $v_t = A$. Second, marginal soil carbon losses always exceed the sum of the two marginal benefits, implying $v_t = w_t = x_t = y_t = 0$ and permanent no-till. Third, marginal soil carbon losses initially exceed marginal benefits, but the latter grows more quickly and equals or exceeds the former by $t + 4$. Periodic tilling will result with either w_t, x_t, or y_t equal to A.

If one of conditions (8)–(11) equals zero, then an optimal tillage rotation exists conditional on a previous no-till commitment. Overall optimality requires that both the tillage rotation choice and the no-till commitment choice which preceded it be optimal. Condition (12) is the no-till commitment condition. It compares the marginal benefits of carbon sequestration with the marginal cost of increasing weed stocks plus the offer price for the land. If condition (12) holds with equality, it can be combined with the condition for the optimal tillage rotation to generate an offer price for land committed to no-till with optimal future tillage. If, for example, tilling every second year is conditionally optimal, condition (9) will hold with equality and $w_t = A$. Commitment to no-till in time t requires condition (9) to hold with equality in $t + 2$ and condition (12) to hold with equality in t. Adjusting condition (9) to $t + 2$ and substituting the result into condition (12) yields:

$$\beta(1-\rho^2) = \alpha(1-\rho)\rho^2 S_1 + \lambda(gf\rho^2 - gf\rho^3 F), \tag{13}$$

where the left-hand side is the offer price for keeping the land in no-till from t to $t + 2$. The first term on the right-hand side is the marginal cost associated with weed stock build-up. The second term is the marginal benefit from the net accumulation of carbon stock. The $t + 2$ rotation is optimal if marginal net benefit equals the two-period offer price.

With the $t + 2$ tillage rotations continuing indefinitely, the land is permanently committed and equation (11) can be rewritten as:

$$\beta = \frac{\alpha(1-\rho)\rho^2 S_1 + \lambda(gf\rho^2 - gf\rho^3 F)}{1-\rho^2}. \tag{14}$$

Equation (14) defines a function which can be used to determine the offer price for land to be committed to a tillage rotation of $t + 2$ indefinitely. Using β (the offer price) as the dependent variable, λ (the shadow price for carbon) as the independent variable, and the remaining terms as parameters, we have a linear function with a negative intercept,

$$\frac{\alpha(1-\rho)\rho^2 S_1}{1-\rho^2},$$

and a positive slope,

$$\frac{gf\rho^2 - gf\rho^3 F}{1-\rho^2}.$$

Using the same approach, β functions like equation (14) can be derived for the periodic tillage rotations $t + 3$ and $t + 4$. For continuous tillage, conditions (8) and (12) result in $\beta = 0$. For the permanent no-till condition, (12) alone is used. Table 1 presents the β functions for continuous tillage, the three periodic tillage options, and permanent no-till.

Each β function defines the offer price for a given tillage rotation as a function of the shadow price for carbon (λ). However, because the tillage rotation which maximizes β (the

Table 1. Tillage β Functions

Tillage Description	β Function
Continuous Tillage (t + 1)	$\beta = 0$
Periodic Tillage (t + 2)	$\beta = \frac{\alpha(1-\rho)\rho^2 S_1}{1-\rho^2} + \lambda\left[\frac{gf\rho^2}{1-\rho^2} - \frac{gf\rho^3 F}{1-\rho^2}\right]$
Periodic Tillage (t + 3)	$\beta = \frac{\alpha(1-\rho)\rho^2 S_1}{1-\rho^3} + \frac{\alpha(1-\rho)\rho^3 S_2}{1-\rho^3} + \lambda\left[\frac{gf(\rho^2+\rho^3)}{1-\rho^3} - \frac{gf\rho^4(F+F^2)}{1-\rho^3}\right]$
Periodic Tillage (t + 4)	$\beta = \frac{\alpha(1-\rho)\rho^2 S_1}{1-\rho^4} + \frac{\alpha(1-\rho)\rho^3 S_2}{1-\rho^4} + \frac{\alpha(1-\rho)\rho^4 S_3}{1-\rho^4} + \lambda\left[\frac{gf(\rho^2+\rho^3+\rho^4)}{1-\rho^4} - \frac{gf\rho^5(F+F^2+F^3)}{1-\rho^4}\right]$
Permanent No-Till	$\beta = \alpha(1-\rho)\left(\rho^2 S_1 + \rho^3 S_2 + \rho^4 S_3 + \frac{\rho^5 S_4}{1-\rho}\right) + \frac{\lambda\rho^2 gf}{1-\rho}$

optimal rotation) varies as the value of λ varies, there is also an envelope β function. The envelope function coincides with an individual β function when its tillage rotation is optimal. To illustrate, we simulate β functions for individual rotations using the parameters from figure 1, supplementing them with parameters relating to weed growth and weed losses. The parameters $S_1 = 5$, $S_2 = 8$, $S_3 = 10$, and $S_4 = 11$ describe weed growth. The annual loss from a marginal increase in weed stock is set at $PAd = 1$. With a discount factor of $\rho = 0.97$, the shadow price of a unit of weed stock is $\alpha = 34.33$. While there is no strong empirical documentation for these parameter values, they do serve to illustrate how tillage rotations are chosen. Using these values, figure 2 shows the offer price (β) as a function of the shadow price of carbon (λ) for each of the five β functions in table 1. The optimal tillage rotation is found by choosing the maximum β for a given level of λ. Continuous tillage (t + 1) maximizes β for λ values of 13.4 or less. A tillage rotation of $t + 2$ maximizes β for values of λ between 13.4 and 20.1, a tillage rotation of $t + 3$ for λ values between 20.1 and 28, and a tillage rotation of $t + 4$ for λ values between 28.1 and 33.2. If the value of λ is above 33.2, permanent no-till is the optimal choice. The greater the shadow price of sequestered carbon, the larger the maximum offer price and the longer the optimal tillage rotation.

Our simplifying assumption of no growth in the weed stock or tillage emission losses beyond $t + 4$ ensures that either $t + 4$ or permanent no-till always has at least as great a β value as any intermediate tillage rotation (the proof is given in the appendix). Relaxing the assumption would allow tillage rotations between $t + 4$ and permanent no-till, but the nature of the results would not change.

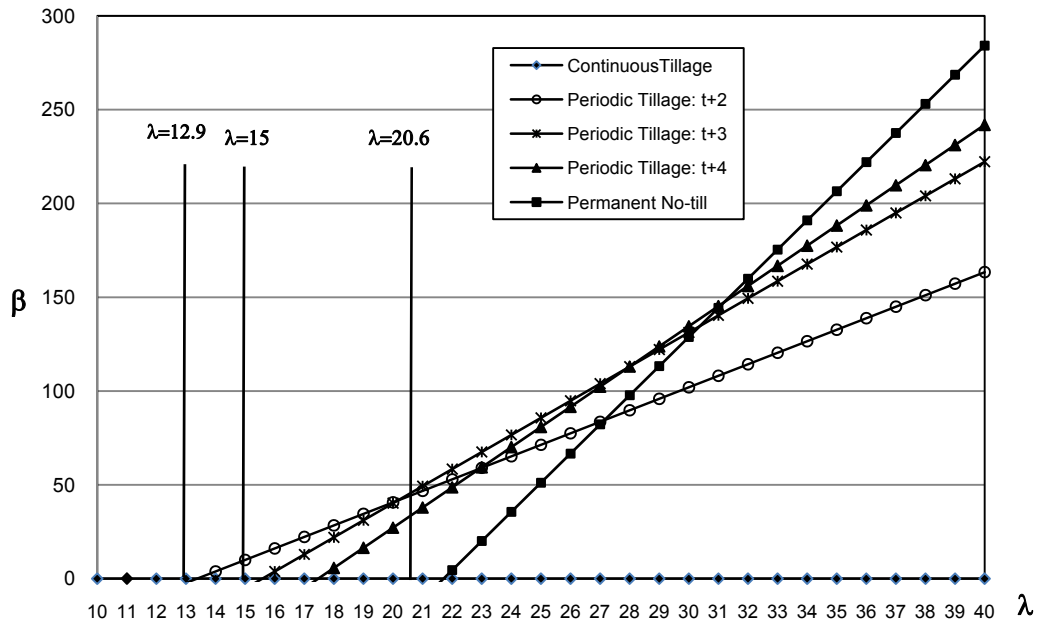

Figure 2. Offer price (β) and tillage rotations for different values of λ

Internalizing the Benefits from Reducing Atmospheric CO_2

Payments for Atmospheric Carbon Reductions

The socially optimal tillage rotation is one that fully internalizes the benefits of reducing atmospheric CO_2 in the offset rental value, π. The shadow price of carbon will incorporate this rental value along with the crop yield benefits, giving

$$\lambda = \frac{PAc + \pi}{1 - \rho(1-k)}.$$

In the absence of a regulatory mechanism to internalize these benefits, $\pi = 0$ and the shadow price of the soil carbon stock will be lower. To illustrate, assume $PcA = 1$. Without internalization, $\pi = 0$, $\lambda = 12.9$, and continuous tillage will be chosen (see figure 2). With internalization, there must be a positive value for π. Assuming a relatively small value of $\pi = 0.165$, $\lambda = 15$, and the optimal tillage rotation will be $t + 2$, or $w_t = A$. With higher values for π, internalization means higher values for λ and longer optimal tillage rotations. A value of $\pi = 0.797$ yields $\lambda = 22$ and an optimal tillage rotation of $t + 3$, or $x_t = A$. A value of $\pi = 1.669$ yields $\lambda = 34$ and permanent no-till. In general, internalizing the value of reducing atmospheric CO_2 leads to longer tillage rotations.

Potential offset payments could be characterized in at least three ways. So far, we have used the annual rental payment of π per ton of stock exceeding the steady-state continuous tillage level. The payment per ton of stock is constant, but since the carbon stock changes over time, the time path of total rental payments follows the same pattern as the carbon stock in figure 1. With permanent no-till, the total rental payment will be constant once the steady-

state carbon stock has been reached. With periodic tillage, annual payments would be cyclic in the steady state, but the total rental payment over the cycle would be constant.

Rather than renting the greenhouse gas-reducing services of the accumulated carbon stock, a farmer could sell increments of carbon stock and be liable for decrements. For an increment, the price is a one-time payment for the stream of services from that increment. Since the increment will degrade naturally at a rate k and future benefits are discounted, the present value of the services from a one-ton increment in time t will be $\rho\pi/(1-\rho(1-k))$. A payment would be made to the farmer at the time of the initial sequestration, and a liability would be created when tillage emissions occur. Table 2 shows a series of payments and liabilities based on the $\rho\pi/(1-\rho(1-k))$ per ton values for different tillage rotations. Continuous tillage generates a zero net payment because the payment generated by committing land to no-till is exactly offset by the liability resulting from tillage. With a tillage rotation of $t+2$,

$$\frac{gfA\rho\pi}{1-\rho(1-k)}$$

is paid to a farmer for gfA tons added in each of $t+1$ and $t+2$. But in $t+2$, tillage occurs and the farmer is liable for

$$\frac{gfA\rho\pi(1+F)}{1-\rho(1-k)}$$

to cover the tillage emissions. This leaves a net liability in $t+2$ of

$$\frac{gfA\rho\pi F}{1-\rho(1-k)}.$$

The other two periodic tillage rotations, $t+3$ and $t+4$, show a similar pattern of payments followed by a liability, while permanent no-till requires the same payment every time period.

For intermediate cycles where increments vary, we can collapse the payments and liability to one payment per cycle by calculating a discount factor weighted sum of the increments and decrements to the carbon stock. For a given tillage rotation, the weighted sum and payment would be the same in every cycle. The longer the tillage rotation, the greater the weighted sum and payment.[6] Although there is no increase in soil carbon stock once the permanent no-till steady state has been reached, continued payments are necessary for soil carbon inputs to balance natural degradation.

We can also aggregate the payments for increments and liabilities for decrements into one up-front payment for a tillage plan. This up-front payment is the capitalized value of all of the payments and liabilities that occur throughout the plan. It is also the capitalized value of the total annual rental payments for the greenhouse gas-reducing services of the accumulated carbon stock. Up-front payments for the five tillage plans are shown in the last row of table 2.

All forms of payments are efficient under certainty with respect to a farmer's actions and future tillage releases. Without certainty, incentive compatibility considerations favor rental payments or sale payments with liability. An up-front payment with no liability allowances would encourage moral hazard, but a combination of random tillage releases and risk aversion by farmers provides a trade-off between moral hazard avoidance and risk reduction. The industry or government purchaser of the offset, who has more opportunity to spread risk, can

[6] For this case and for the rental case, it is possible to create a levelized payment for each cycle. This would give equal annual payments in steady state.

Table 2. Payments for Offsets

Time	Permanent No-Till	t to $t+4$	t to $t+3$	t to $t+2$	t to $t+1$
Payments for Increments to the Carbon Stock:					
$t+1$	$\frac{gfA\rho\pi}{1-\rho(1-k)}$	$\frac{gfA\rho\pi}{1-\rho(1-k)}$	$\frac{gfA\rho\pi}{1-\rho(1-k)}$	$\frac{gfA\rho\pi}{1-\rho(1-k)}$	0
$t+2$	$\frac{gfA\rho\pi}{1-\rho(1-k)}$	$\frac{gfA\rho\pi}{1-\rho(1-k)}$	$\frac{gfA\rho\pi}{1-\rho(1-k)}$	$\frac{-gfA\rho\pi F}{1-\rho(1-k)}$	
$t+3$	$\frac{gfA\rho\pi}{1-\rho(1-k)}$	$\frac{gfA\rho\pi}{1-\rho(1-k)}$	$\frac{-gfA\rho\pi(F+F^2)}{1-\rho(1-k)}$		Repeating
$t+4$	$\frac{gfA\rho\pi}{1-\rho(1-k)}$	$\frac{-gfA\rho\pi(F+F^2+F^3)}{1-\rho(1-k)}$	Repeating	Repeating	
	Repeating	Repeating			
Up-front Payment for a Sequestration Plan:					
	$\frac{gfA\rho\pi}{(1-\rho)(1-\rho(1-k))}$	$\frac{gfA\rho\pi \times (1+\rho+\rho^2-\rho^3(F+F^2+F^3))}{(1-\rho^4)(1-\rho(1-k))}$	$\frac{gfA\rho\pi \times (1+\rho-\rho^2(F+F^2))}{(1-\rho^3)(1-\rho(1-k))}$	$\frac{gfA\rho\pi(1-\rho F)}{(1-\rho^2)(1-\rho(1-k))}$	0

provide insurance for a farmer through an up-front payment for expected sequestration (Eswaran and Kotwal, 1985). In addition, political constraints regarding payments may render liability payments unacceptable (Gulati and Vercammen, 2006).

Sensitivity of the Tillage Rotation to Parameter Changes and Other Policy Options

Paying farmers an offset price that reflects the true social value of sequestered carbon may be the best policy to internalize an externality in an otherwise competitive market. However, if this is not possible, or if there are other market failures (e.g., incomplete information, too high a degree of impatience), other policy recommendations can be extracted from a sensitivity analysis using the β functions.

As a base case, assume no offset value for sequestered carbon. Also assume all of the parameters of the β functions have the same values as in the simulations, giving

$$\lambda = \frac{PAc}{1-r(1-k)} = 12.9.$$

Varying parameters such as k, d, f, or ρ can change the position of λ, and/or the intercepts or slopes of the functions in table 1. In turn, the optimal tillage rotations are changed. As observed from table1, parameters that influence losses from weeds affect the vertical intercept of the β functions, parameters that influence the carbon stock affect their slope, and parameters that influence the shadow price of carbon cause a movement along the horizontal axis. Some parameters, such as the discount rate, exert multiple influences.

A lower k value represents a slower SOC decay rate. Carbon inputs will stay in the soil longer and λ will increase. This will mean a movement to the right along the horizontal axis in figure 2, possibly into the $t + 3$ range, but the higher λ is tempered by the fact that F increases when k decreases. Tillage emissions increase because slowed natural decay leaves more carbon available for release by tillage (Andrén and Kätterer, 1997). The slopes of all the β functions will be decreased by this change. However, the slope change will be small if the terms containing F are small. With $k = 0.02$ (less than the $k = 0.05$ for the base case), the slopes of the β functions change only slightly, but the value of λ is increased to 20.6. The result is an increase in the optimal tillage rotation, in this case to $t + 3$ (figure 2).[7]

Technologies or practices that reduce the negative impact of weed build-up and make d smaller will result in longer tillage rotations. A decrease in d brings the vertical intercepts of all β functions closer to zero, but does not change their slopes or the value of λ. This will tend to increase the tillage rotation. If the absolute value of d is reduced to zero, permanent no-till will generate the highest β value for all values of λ.

An increase in f means that a larger amount of carbon is released through continuous tillage. The amounts released when tillage is delayed will also increase, but if the terms fF, $f(F + F^2)$, and $f(F + F^2 + F^3)$ are all small relative to f, the increase in slope will be greatest for the β functions representing longer tillage rotations. Thus, a longer tillage rotation will be optimal for a given λ value. Figure 3 shows this case when f is increased to 0.3 with the other parameters remaining at their original values. With λ remaining at $\lambda = 12.9$, the optimal tillage rotation increases to $t + 2$.

A decrease in the discount rate (an increase in ρ) will have three effects. First, the vertical intercepts of the β functions are pushed farther apart and tend to reduce the tillage rotation as weed costs increase. Second, the slope of the β functions will increase as future tillage losses are more heavily weighed. Third,

$$\lambda = \frac{PAc}{1 - \rho(1 - k)}$$

increases. The latter two effects tend to increase tillage rotations. Although it is conceivable the first of the three effects could dominate and lead to a shorter tillage rotation, the parameter values that lead to the first effect dominating (small PcA, small f, and large PdA) would result in a short tillage rotation at the initial discount rate. Because the tillage rotation cannot be shorter than continuous tillage, the lower discount rate will simply mean no change from continuous tillage. Figure 4 shows the effect of a decrease in the discount rate to $r = 0.01$. Because the increase in shadow price of carbon dominates, λ increases to 16.8 and the optimal tillage rotation to $t + 2$.

What policy recommendations can be extracted from this sensitivity analysis? Investments in improved seeding technologies and better methods of weed control are possible ways to make d smaller and lengthen farmers' tillage rotation.[8] A similar effect can be achieved by encouraging crop rotation. With continuous production of a single crop, weed populations adapt to become highly competitive with that crop. Crop rotations inhibit such adaptations (Murphy and Lemerle, 2006). None of these changes are costless, so policy incentives to undertake them may be necessary (Kurkalova, Kling, and Zhao, 2006).

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[7] Because the slope changes are so small, figure 2 is used to show the change that results from varying k.

[8] Genetically modified crops are complementary to no-till practices for this reason (see Trayler, 2006).

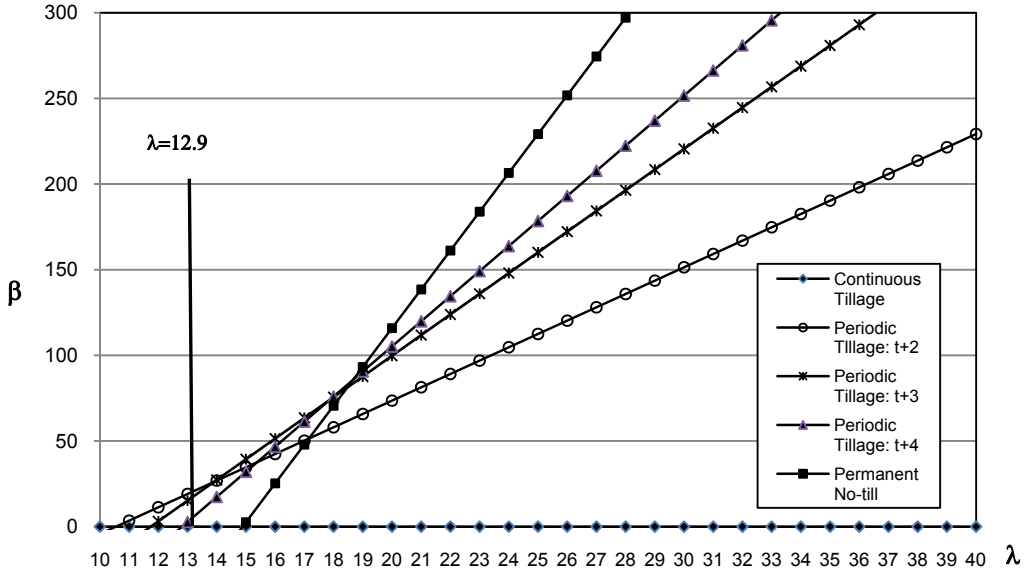

Figure 3. Offer price (β) and tillage rotations: Higher *f*

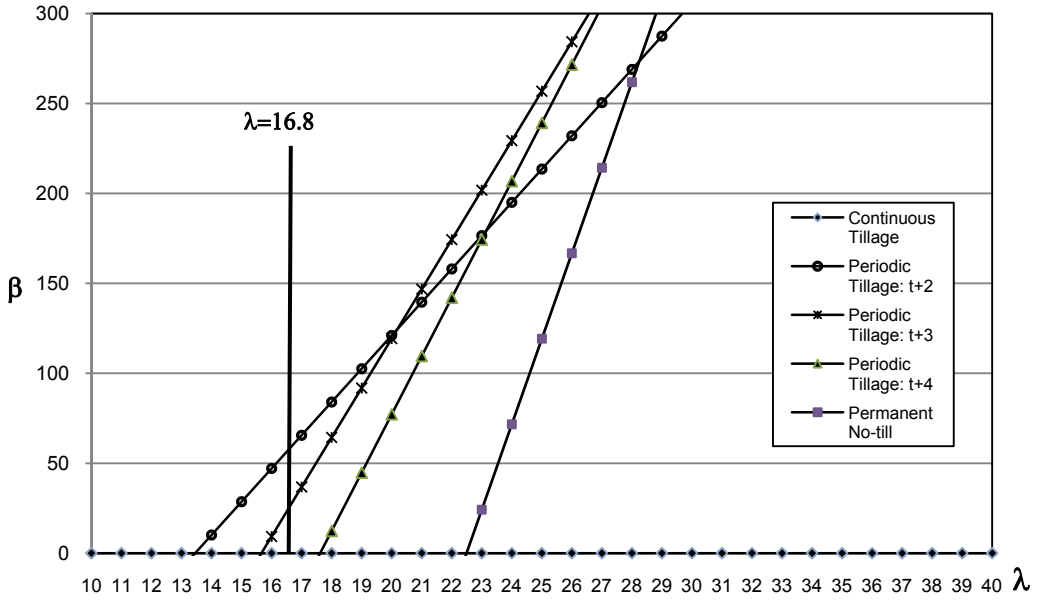

Figure 4. Offer price (β) and tillage rotations: Smaller *r*

Tillage rotations may be short for reasons other than weed control. If k is high and f low, there will be less to be gained by delaying tillage. In contrast, a low k and a high f indicate tillage is an important contributor to soil carbon loss, and there is more soil fertility gain from delaying tillage. Better knowledge of the magnitude of soil fertility gains resulting from delayed tillage could encourage farmers to till less frequently.

The discount rate weights future gains and losses relative to the present. The lower the discount rate, the greater is the weight given to the future. Given that the change in λ dominates, lower borrowing costs could influence a farmer's decision to undertake the slow process of building up soil fertility through longer tillage rotations.

Conclusions

This paper addresses incentives for permanence in soil carbon sequestration by modeling a farmer's choice as one of tillage frequency rather than simply no-till adoption. Less frequent tillage builds up soil fertility, reduces atmospheric CO_2, and allows the build-up of weeds. By allowing for a range of tillage choices, we show that higher offset payments for sequestered carbon reduce tillage frequency. It is also reduced by a higher rate of tillage emissions or reduced weed impact. When the discount rate, or the natural decay rate for carbon, varies, the net change depends on the magnitude of other parameters such as the rate of tillage emissions. These factors provide the basis for a range of supplementary policy mechanisms to influence tillage frequency.

The model is an initial investigation of farmers' tillage choices that allows for occasional or regular tillage within a no-till regime. It incorporates concepts from forestry economics to model the tillage frequency choice. Because the instantaneous benefit function is linear, the benefit from a unit of weed stock reduction is fixed, as is the per unit cost of soil carbon reduction. As in many forestry models, nonlinearity is introduced through the functions that describe the growth of the weed stock and tillage emissions. Unlike tree growth, however, there is little documentation of the growth of weed stocks, nor is there strong evidence on how tillage emissions increase with postponed tillage. Future research would improve the realism and policy relevance of the model though more accurate estimates of these parameters and by allowing for nonlinearities in the benefit function.

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Appendix:
Dominance of $t+4$ or Permanent No-Till

The assumption of no growth in the weed stock or tillage emissions losses beyond $t+4$ results in a β function for a longer tillage rotation that is a clockwise movement of the $t+4$ function around its intersection with the permanent no-till function. Thus, it approaches the permanent no-till function as the rotation length approaches infinity. Hence, either $t+4$ or permanent no-till always has at least as great a β value as any intermediate tillage rotations.

The β functions for $t+4$ and permanent no-till are given by (A1) and (A2), respectively:

$$\text{(A1)} \qquad \beta = \frac{\alpha(1-\rho)}{(1-\rho^4)}\left(\rho^2 S_1 + \rho^3 S_2 + \rho^4 S_3\right) + \lambda\left(\frac{gf(\rho^2+\rho^3+\rho^4)}{1-\rho^4} - \frac{gf\rho^5(F+F^2+F^3)}{1-\rho^4}\right)$$

and

$$\text{(A2)} \qquad \beta = \alpha(1-\rho)\left(\rho^2 S_1 + \rho^3 S_2 + \rho^4 S_3 + \frac{\rho^5 S_4}{1-\rho}\right) + \frac{\lambda\rho^2 gf}{1-\rho}.$$

Setting the two β's equal yields:

$$\text{(A3)} \qquad \frac{\alpha(1-\rho)}{(1-\rho^4)}\left(\rho^2 S_1 + \rho^3 S_2 + \rho^4 S_3\right) - \alpha(1-\rho)\left(\rho^2 S_1 + \rho^3 S_2 + \rho^4 S_3 + \frac{\rho^5 S_4}{1-\rho}\right)$$
$$= \lambda gf\left(\frac{\rho^2}{1-\rho} - \frac{(\rho^2+\rho^3+\rho^4)}{1-\rho^4} + \frac{\rho^5(F+F^2+F^3)}{1-\rho^4}\right).$$

Solving for λ and simplifying yields:

$$\text{(A4)} \qquad \lambda = \frac{\alpha(1-\rho)\left(\rho S_1 + \rho^2 S_2 + \rho^3 S_3\right) - \alpha S_4(1-\rho^4)}{fg(1+F+F^2+F^3)}.$$

Consider a longer rotation of $t+5$. The first-order condition for tillage in $t+5$ is the same as that for $t+4$ [see text equation (11)]. However, it has to be adjusted forward to $t+5$ rather than $t+4$. Substituting into text equation (12) yields the following β function:

$$\text{(A5)} \qquad \beta = \frac{\alpha(1-\rho)}{(1-\rho^5)}\left(\rho^2 S_1 + \rho^3 S_2 + \rho^4 S_3 + \rho^5 S_4\right)$$
$$+ \lambda\left(\frac{gf(\rho^2+\rho^3+\rho^4+\rho^5)}{1-\rho^5} - \frac{gf\rho^6(F+F^2+F^3)}{1-\rho^5}\right).$$

Setting β from (A5) equal to β from (A2) yields:

$$\text{(A6)} \qquad \frac{\alpha(1-\rho)}{(1-\rho^5)}\left(\rho^2 S_1 + \rho^3 S_2 + \rho^4 S_3 + \rho^5 S_4\right) - \alpha(1-\rho)\left(\rho^2 S_1 + \rho^3 S_2 + \rho^4 S_3 + \frac{\rho^5 S_4}{1-\rho}\right)$$
$$= \lambda gf\left(\frac{\rho^2}{1-\rho} - \frac{(\rho^2+\rho^3+\rho^4+\rho^5)}{1-\rho^4} + \frac{\rho^6(F+F^2+F^3)}{1-\rho^4}\right).$$

Solving for λ and simplifying again yields (A4), and the β functions for $t+4$ and $t+5$ intersect the permanent no-till β function at the same level of λ. □