Impacts of Weather and Time Horizon Selection on Crop Insurance Ratemaking:
A Conditional Distribution Approach

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Abstract

An important issue in the agricultural actuarial literature is the extent to which sample period selection affects the accuracy of insurance rating. A conditional Weibull distribution approach is developed which explicitly models the interaction of weather, technology, and other variables on probabilistic yield outcomes to address this issue. Results from an application with an extensive producer-level yield dataset representing commercial-scale Illinois firms suggest that the impact of weather heterogeneity on risk estimation across reasonable samples is likely not as great as is often claimed. The results also suggest that yield risk is decreasing significantly through time, and indicate the presence of trend acceleration. A rating analysis indicates that violations in the risk evolution assumptions of the rating approaches used in the Federal Crop Insurance Program—which implicitly assume increasing yield risk through time when yields trend—result in severely biased rates, with typical overstatements of 200% to 400% for Midwest corn.

Keywords: Conditional Weibull Distribution, Conditional Production Function, Catastrophic Risk Modeling, Sample Selection, Yield Risk, Crop Insurance, Ratemaking

Introduction

A longstanding question within the crop insurance and yield risk literatures is to what extent the time horizon of a given sample impacts estimates of production risk and insurance rates. Similarly, many questions remain as to what the appropriate sample period length “should” be for determining crop insurance rates. Simple answers to these questions have remained elusive though. First, the catastrophic nature of adverse weather events can result in high year-to-year variability in crop losses. Second, these questions are confounded by the dynamic nature of production technology through time, which arguably has led to crops that are more resistant to adverse weather and other perils. Addressing these questions is of paramount importance in the current debate regarding the appropriateness of crop insurance rates in the Federal Crop
Insurance Program (FCIP), as sample period selection, treatment of data, and model assumptions (both implicit and explicit) can potentially have large impacts on the assessment of historical loss performance and in the rating of insurance (see e.g., Woodard, Sherrick, and Schnitkey, 2011; Woodard et al., 2011). The adequacy of insurance rates (i.e., unit prices of insurance) is important for government insurance programs such as the FCIP since inaccurate ratings adversely impact the functioning of insurance markets, resource allocation, and can result in excess costs to taxpayers (Priest, 1996; Brown, 2010). Inaccurate rates can also impact planting decisions, which in turn affect land-use and lead to other environmental and economic consequences (Lubowski et al., 2006).\footnote{See Woodard et al. (2011) for a more in depth discussion of these issues in the context of the FCIP.}

Currently, much great disagreement exists regarding the accuracy of FCIP insurance rates and the appropriateness of associated methodologies used to derive them (see e.g., Woodard et al., 2011; Woodard, Sherrick, and Schnitkey, 2010; Coble et al., 2010; Woodard, 2008). Many recent empirical studies cast doubt on the appropriateness of the insurance rating methods employed by the Risk Management Agency—or RMA, a branch of the USDA charged with administering the FCIP (Woodard et al., 2011; Woodard, Sherrick, and Schnitkey, 2011; Yu and Babcock, 2009; Vado and Goodwin, 2010). Meanwhile, other researchers have questioned the validity of existing empirical studies on the grounds that available data periods could be “too short” to facilitate empirical evaluation insurance losses (see e.g., Coble et al., 2010; Smith and Goodwin, 2010).

Despite the obvious importance of the question regarding FCIP insurance rates and the current disagreement over the issue, no relevant work has been conducted to investigate the impacts of sample period selection on yield risk estimation and crop insurance ratemaking.
While some work has investigated rate sampling variability in the context of yield distribution estimation in hypothetical single risk exposure contexts (Ramirez, Carpio, and Rejesus, 2009; Lanoue et al., 2010) and for Group Risk Insurance Products (Woodard and Sherrick, 2010), issues related to the interaction of weather and technology in the context of insurance systems over alternative periods has received much less attention.

The objective of this study is thus to assess the impact that sample period length and sampling variability in weather have on yield risk estimation. The application employs a rich farm-level dataset for Midwest corn in a high premium volume region. A conditional Weibull distribution approach is developed which allows for assessment of yield risk under various sets of weather events by explicitly modeling the impacts of weather and technology change on probabilistic yield outcomes. The conditional distribution approach is advantageous as it allows for straightforward assessment of how sample period selection will likely impact risk estimation under various levels of technology. This allows the analyst, for example, to model the yield distribution under a specific weather event (i.e., conditional on a given weather event) given today’s technology, as well as the distribution over a specific set of weather events (i.e., conditional on a given distribution for weather). This is accomplished by manipulating the conditioning weather distribution once the conditional yield distribution model has been estimated. That is, the conditioning weather distribution used to fit the model can be substituted with a weather distribution representing a wider spectrum of weather outcomes. This allows for assessment of how a yield distribution estimated with, say 30 years of data, would likely differ from a distribution estimated with 100 years of data. This method may be preferred to the simple regression approaches that have been used in the literature until now to investigate similar issues (e.g., Schlenker and Roberts, 2006; Yu and Babcock, 2009; Vado and Goodwin, 2010), as those
studies have primarily focused on assessment of the in-sample *conditional mean* only, but do not carefully consider the impact of weather and technology gains on yield *risk* explicitly, nor the impacts of the chosen weather/time horizon.²

This study provides several contributions. First, the study takes a step toward resolving the current debate regarding the appropriateness of RMA rating assumptions and the associated empirical questions, and also provides a coherent framework within which to approach these key issues. Second, several results of potential interest to production economists are developed for the conditional Weibull model, including conditional elasticity derivations for several risk measures. A bootstrap method to estimate standard errors for conditional distribution elasticities is also developed—extending the work of Nelson and Preckel (1988). Third, the study explicitly models the effect of weather when assessing the impacts of technology change on meaningful measures of yield *risk* and insurance rates for producer-level yields. Last, the results shed light on the trend acceleration issue that has been the focus of some recent debate (Tannura, 2008).

The findings suggest that the impact of weather sample heterogeneity on risk estimates is not likely to be as great as is often suggested in this large premium volume region. Estimates generated under the weather experienced over the 1980-2009 period are found to be—for all practical purposes—very similar to those generated when accounting for weather over the longer period of 1895-2009 in the Midwest. The results also confirm those of Woodard, Sherrick, and Schnitkey (2011) and Woodard (2008) that yield risk for Midwest corn has declined significantly through time, a result in contrast with the results of Schlenker and Roberts (2006), albeit for a

² While it is true that a standard Gaussian regression model with conditional variance is essentially a conditional normal distribution model, previous studies have not carefully modeled and analyzed the conditional and unconditional variance processes in order to generate the rich risk results developed here. Furthermore, the normal distribution is questionable as applied to yield distribution estimation (see e.g., Woodard and Sherrick, 2010, for a thorough discussion). Last, previous studies typically use county yields, which are less relevant for producer rating.
different dataset. The results also suggest the presence of trend acceleration. Finally, the results provide confirmatory evidence that RMA rates in the Midwest are likely severely inflated (see e.g., Woodard, 2008; Woodard et al., 2011; Yu and Babcock, 2009; Vado and Goodwin, 2010), and that the geographic inequities in loss experience identified in past work are most likely not simply due to sample selection issues. Rather, it appears to be due to the fact that the methods employed by RMA are not appropriate given the evolution of crop production technology and resulting risks.

Collectively, the evidence to date suggests that a fundamentally different rating approach or a reweighting of historical loss data to account for the dynamic nature of agricultural production risk —such as that developed in Woodard (2008)—will be needed to rectify geographic rating inequities in the FCIP identified in several high premium volume regions. The geographic rating inequities identified historically are likely to persist if the RMA rating continues to ignore these important risk features when making rates.

The Rate Debate

Woodard et al. (2011) present evidence that, historically, rates in the FCIP have been geographically inequitable, and that these inequities are linked to fundamental and persistent flaws in the approaches employed by the RMA in making rates for the program. Chief among these is the use of unadjusted loss cost approaches by RMA, which have been shown by Woodard (2008) and Woodard, Sherrick, and Schnitkey (2011) to result in biased rating structures unless certain restrictive distributional conditions are met. Specifically, the LCR approach implicitly assumes that yield risk is increasing through time if yields trend upward.
Those studies test these assumptions against a large representative farm-level database and find that they are starkly violated for Illinois, a high premium volume market. They also perform statistical tests that indicate that the impact of weather variability within the sample cannot explain the violations in the risk assumptions. Last, these studies posit that the rating problems identified are caused by improvements in crop technology which have resulted in lower yield risk and trending yields through time.

One criticism of those studies and related work asserts that the sample periods available (approximately 30 years) are “simply too short” to adequately evaluate losses. The potential flaw in the logic that 30 years is “simply too short” is that it ignores the fact that thick panels of producer-level data that are readily available have much higher information content than aggregate indexes, and thus may allow for more accurate risk estimation. However, if the distribution of weather over a longer horizon is much different than the recent 30 year period, differences could arise. The approach in this study allows for assessment of this claim directly by modeling and evaluating the yield distribution which would likely be generated over a longer time horizon.

**Model**

A conditional Weibull model is developed in order to assess the impact of alternative weather/time horizons on yield risk estimation. In addition, the models are used to explore changes in the response of crop yield risk to weather stresses. While similar approaches exist for the normal and Beta distributions, this study employs the Weibull distribution for several reasons. First, the Beta distribution has been shown to have a tendency to overfit in yield
distribution modeling applications (see e.g., Woodard and Sherrick, *forthcoming*) due to the higher number of unconditional distribution parameters in that model (four) versus the Weibull (two). The Beta is also somewhat more difficult to work with computationally. While the normal distribution is less prone to overfitting than the Beta due to its two-parameter nature, it has generally not been found to be a good representation of yield distributions in many regions. For example, Woodard and Sherrick (*forthcoming*) find that the normal distribution is rejected in 47% of Midwest corn counties.

The normal distribution is also restricted to have zero skewness, whereas the Weibull allows for negative skewness. Negative skewness has been identified as a common characteristic of yields distributions in many regions, including the region under investigation here (see e.g., Sherrick et al., 2004). Hennessy (2009a) provides a formalized theoretical motivation for this common finding, arguing that negative yield skewness is likely to occur in tightly controlled environments where the left tails of the resource availability distributions are thin. Furthermore, Hennessy (2009b) develops a theory which implies that negative yield skew is likely to arise in cases where the weather-conditioned mean yield has diminishing marginal product with respect to weather. Last, the support of the normal distribution has a lower bound of negative infinity; thus, it has the potential to imply implausible negative yields, whereas the Weibull distribution’s support has zero as a lower bound.

As a kick-off point, the (unconditional) Weibull distribution can be expressed as,

\[ f(y | a, b) = ba^{-b}y^{b-1}e^{-(y/a)^b}, \]

where \( y \) is yield, and \( a \) and \( b \) are parameters to be estimated. The conditional Weibull is similar except that \( a \) and \( b \) are a function of some other variables, \( a(x_a, \beta_a) = g_a(x_a, \beta_a) \) and
$b(x_b, \beta_b) = g_b(x_b, \beta_b)$, where subscripts $a$ and $b$ are used to denote the respective parameter model, $g(\bullet)$ is some functional form (e.g., linear, Cobb-Douglas, quadratic, etc.) $x$’s are $1$-by-$K$ design matrices of explanatory variables (e.g., weather, soil, acreage), and $\beta$’s are $K$-by-$1$ vectors of parameters to be estimated. If we let $a$ and $b$ have the same set of explanatory variables $x$, and same functional form $g(\bullet)$, the conditional distribution can be expressed as,

$$f(y | x, \beta_a, \beta_b, g(\bullet)) = g(x, \beta_b)g(x, \beta_a)^{-g(x, \beta_b)}y^{g(x, \beta_b)-1} \exp \left( - \frac{y}{g(x, \beta_a)} \right)^{g(x, \beta_b)},$$  \hspace{1cm} (2)

Parameter estimates can be obtained via maximum likelihood as follows. Letting $Y$ and $X$ be an $N$-by-$I$ and $N$-by-$K$ sample of $N$ observations, the conditional model parameters can be estimated as

$$\hat{\beta} = [\hat{\beta}_a, \hat{\beta}_b] = \arg \max_{\beta_a, \beta_b} \left( \prod_{i=1}^{N} f(Y_i | X_i, \beta_a, \beta_b, g(\bullet)) \right),$$  \hspace{1cm} (3)

where in this case $Y_i$ and $X_i$ correspond to data for observation $i$.

Interpretation of the conditional model parameters, $\beta$, resulting from changes in $x$ are somewhat difficult to interpret since the impact of a change in $a$ and $b$ on mean yields and yield risk are—depending on the distribution type—often some nonlinear function of both parameters.\(^3\) Indeed, this is the case with the Weibull. Thus, next we derive mean and variance elasticities for the conditional Weibull. Assessment of these elasticities provides a means to assess the impacts of various conditioning variables on the resulting distribution. Here and

\(^3\) The Normal distribution is an exception.
throughout, it is assumed that \( a \) and \( b \) have common design matrices, \( \mathbf{x} \), and common functional form \( g(\bullet) \). The mean of the Weibull distribution can be expressed as a function of \( a \) and \( b \) as,

\[
\mu(y | a, b, \mathbf{x}, \beta_a, \beta_b) = \frac{a(\mathbf{x}, \beta_a)}{b(\mathbf{x}, \beta_b)} \Gamma\left(\frac{1}{b(\mathbf{x}, \beta_b)}\right),
\]

(4)

where \( \Gamma(\bullet) \) is the gamma function. The conditional mean elasticity with respect to \( x_k \) quantifies the proportional change in the conditional mean yield, \( \mu(y | a, b, \mathbf{x}, \beta_a, \beta_b) \), resulting from a proportional change in \( x_k \), and can be expressed as,

\[
\varepsilon_k^\mu = \frac{\partial \ln(\mu(y | a, b, \mathbf{x}, \beta_a, \beta_b))}{\partial \ln(x_k)} = x_k \frac{\partial \mu(\bullet)}{\partial x_k} \frac{\partial \ln(\mu(\bullet))}{\partial x_k} \mu(\bullet)
\]

\[
= -\left\{ a(\bullet) \Psi'(\bullet) \left( \frac{1}{b(\bullet)} \right) \Gamma\left( \frac{1}{b(\bullet)} \right) \left( \frac{\partial b(\bullet)}{\partial x_k} \right) - a(\bullet) \Gamma\left( \frac{1}{b(\bullet)} \right) \left( \frac{\partial b(\bullet)}{\partial x_k} \right) \right\} \frac{x_k}{\mu(\bullet)}
\]

\[
= -\left\{ x_k a(\bullet) \Psi'(\bullet) \left( \frac{1}{b(\bullet)} \right) a(\bullet) b(\bullet) \left( \frac{\partial b(\bullet)}{\partial x_k} \right) - x_k b(\bullet) \left( \frac{\partial a(\bullet)}{\partial x_k} \right) \right\} \frac{x_k}{a(\bullet) b(\bullet)^2},
\]

(5)

where \( \Psi'(\bullet) \) is the derivative of the log-gamma function (i.e., the di-gamma function). Note, these results are expressed in terms of \( a(\mathbf{x}, \beta_a) \) and \( b(\mathbf{x}, \beta_b) \), but can easily accommodate any functional form simply by substituting in the specific function \( g(\bullet) \) into the equation above.

Thus, all that needs to be known to investigate alternative functional forms is \( g(\bullet) \) itself and its first derivative with respect to \( x_k \). This setup greatly simplifies programming and analysis of
alternative forms in practice. In a similar vein, the variance of the conditional Weibull
distribution is,

\[
\sigma^2(y \mid a, b, x, \beta_a, \beta_b) = a(\bullet)^2 \left\{ \Gamma \left( \frac{b(\bullet) + 2}{b(\bullet)} \right) - \Gamma \left( \frac{b(\bullet) + 1}{b(\bullet)} \right) \right\},
\]

(6)

and thus the conditional variance elasticity can be derived as,

\[
\varepsilon^\sigma_k = \frac{\partial \ln(\sigma^2(\bullet))}{\partial \ln(x_k)} = \\
\frac{-\left\{ 2x_k a(\bullet) \left( \frac{\partial b(\bullet)}{\partial x_k} \right) \right\} \Psi_0 \left( \frac{b(\bullet) + 2}{b(\bullet)} \right) - 2x_k b(\bullet)^2 \left( \frac{\partial a(\bullet)}{\partial x_k} \right) \right\} \Gamma \left( \frac{b(\bullet) + 2}{b(\bullet)} \right)}{

\left\{ 2x_k b(\bullet)^2 \left( \frac{\partial}{\partial x_k} a(\bullet) \right) - 2x_k a(\bullet) \left( \frac{\partial b(\bullet)}{\partial x_k} \right) \right\} \Psi_0 \left( \frac{b(\bullet) + 1}{b(\bullet)} \right) \Gamma \left( \frac{b(\bullet) + 1}{b(\bullet)} \right)^2}

\times \frac{1}{a(\bullet) b(\bullet)^2 \Gamma \left( \frac{b(\bullet) + 2}{b(\bullet)} \right) - a(\bullet) b(\bullet)^2 \Gamma \left( \frac{b(\bullet) + 1}{b(\bullet)} \right)^2}.
\]

(7)

Note, elasticities of scale as well as elasticities of other moments / distributional statistics can be
derived similarly. For example, the median and median elasticity \(w.r.t.\ x_k\) can be expressed as,

\[
Med(y \mid a, b, x, \beta_a, \beta_b) = a(\bullet) \ln(2)^\frac{1}{b(\bullet)},
\]

(8)

and,

\[
\varepsilon^\text{Med}_k = \frac{\partial \ln(Med(\bullet))}{\partial \ln(x_k)} = \frac{\ln(\ln(2))x_k a(\bullet) \left( \frac{\partial b(\bullet)}{\partial x_k} \right) - x_k b(\bullet)^2 \left( \frac{\partial a(\bullet)}{\partial x_k} \right)}{a(\bullet) b(\bullet)^2}.
\]

(9)
Recovery of the Unconditional Distribution

A convenient feature of working with conditional distributions of this form is that the unconditional distribution can be recovered in a straightforward manner by integrating out the explanatory variables, $x$ (provided their joint distribution is known). Additionally, any one of the $x_k$ can be integrated out individually, with the resulting conditional distribution being unconditional on $x_k$, but conditional on all other $x_{-k}$.

It also allows for manipulation of the distributions of $x$. These features are particularly advantageous in answering the question of sample period/weather regime impact on risk estimation. Specifically, the conditional distribution approach allows the analyst to employ a longer span of weather data in order to construct a weather density that embodies longer time-horizons. That is, the models themselves can be fit with available data (say 1980-present) then an augmented distribution for weather can be substituted in the model and integrated out in order to assess the impact of sample length.

This method can also be used to assess changes in various risk measures through time (i.e., under changing technology through time). Last, it also allows for assessment of the impacts of various weather events under current technology.

Formally, with estimates of $\hat{\beta}_a$ and $\hat{\beta}_b$ in hand, the unconditional distribution (i.e., unconditional on $x$) can be recovered from the conditional as,

\[
\hat{f}(y \mid \hat{\beta}_a, \hat{\beta}_b, g(\bullet)) = \int \left[ \hat{f}(y \mid \bar{x}, \hat{\beta}_a, \hat{\beta}_b, g(\bullet)) \cdot g(\bar{x}) \right] d\bar{x},
\]

(10)

where $g(\bar{x})$ is the distribution of interest for $\bar{x}$ (e.g., a particular weather distribution of interest).

Thus, the resulting unconditional distribution in the case of a continuous (discrete) distribution

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4 Technically, this may require that the variables being integrated out should be independent of the variables not integrated out in order for the result to be meaningful.
for $\bar{x}$ is essentially an infinite (finite) mixture of Weibull distributions. Similarly, the analyst can choose to integrate out only some $x_i$’s, while fixing others. One can also obtain unconditional elasticities by integrating the conditional moment elasticities (derived above) over the distribution of $\bar{x}$. Analytical solutions are not feasible for those expressions, and thus numerical methods must be employed, though this is relatively straightforward computationally.

**Data and Methods**

The producer-level corn yield data used in the study are from the Illinois Farm Business Farm Management database (FBFM). The data span the period 1972-2008 and contain for 30,467 corn yield observations from 5 large contiguous production counties in Central Illinois (LaSalle, Livingston, Marshall, McLean, and Woodford). The database also contains records for acreage ($ACRE$) and soil productivity ($SOIL$), both of which have potential impacts not only on mean yields, but also risk more generally. $SOIL$ is derived from on-farm soil tests according to the Circular 1156 Soil Productivity Rating methodology published by the University of Illinois. Yields are measured in bushels per acre. The use of $SOIL$ and $ACRE$ are also needed to control for farm heterogeneity as well as non-linear technology/land interactions through time. The weather data ($WEATHER$) are from the National Climatic Data Center (NCDC) for the Palmer Drought Severity Index (PDSI) for the period 1895-2009. The PDSI is published monthly as a district level index which indicates overall moisture conditions.  

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While other weather measures could have been employed, preliminary analysis did not indicate that using other PDSI index types, months/month combinations, or straight temperature/precipitation measures had any qualitative impact on the results. Clearly, while this choice is of second order, it is still potentially an avenue through which slight improvements in efficiency could be had, and so is left as a potential area for future research.
(June, July, and August) is constructed and employed here to control for the major weather events that affect crop yield growth during the critical growing season.

**Estimation**

The conditional Weibull model is estimated using a common quadratic specification for both \( a(x, \beta_a) \) and \( b(x, \beta_b) \). The quadratic specification is desired as it allows for the modeling of non-linearities and interactions in the parameter responses among the variables. Logged terms are used for \( SOIL \) and \( ACRE \). A time trend (\( TRED \)) is also included to capture changes in technology through time. When fitting the conditional Weibull model parameters, the \( WEATHER \) index is employed for the 1972-2008 period to match the yield dataset. Later in the analysis, we compare various horizons for \( WEATHER \) when recovering the unconditional distribution (namely, the periods 1895-2009 and 1980-2009). Explicitly, we have

\[
x = [\ TRED, \ln(ACRE), \ln(SOIL), WEATHER]\,.
\]

The model parameters are solved using maximum likelihood.\(^7\)

Conditional elasticities are constructed using the equations above. The “unconditional” elasticities are estimated by numerically integrating out the empirical distribution for \( WEATHER \) (either 1895-2009 or 1980-2009). \( ACRE \) and \( SOIL \) are evaluated at their medians throughout. Thus, the elasticities and other results are conditional on \( ACRE \) and \( SOIL \) at their median values, but unconditional on the chosen \( WEATHER \) distribution.\(^8\) Results are also presented for various

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\(^6\) This model was selected as it appeared to have the best fit out of candidate models investigated (including Cobb-Douglass and linear). Overall, the exact choice of model/variables did not appear to have a qualitative impact on the conclusions of the results.

\(^7\) MATLAB code to implement the procedures is available from the author upon request.

\(^8\) This choice did not have a qualitative impact on the results of the analysis.
statistics conditional on different levels of \textit{TREND} in order to assess impacts of changing technology through time, net of any \textit{WEATHER} impacts. Standard errors for the model parameter estimates are calculated directly from the Hessian using the BFGS method.\textsuperscript{9} Deriving standard errors for the \textit{elasticities}, on the other hand, is difficult if not impossible analytically. Thus, in this study bootstrap methods are used to estimate standard errors for elasticities. This is done in a straightforward manner by simply resampling observations with replacement from the main dataset and successively re-estimating the model parameters; the bootstrapped parameter estimates \((N = 1,000)\) are then used to calculate the bootstrapped elasticities and to derive their sampling distributions.

In order to assess impacts on insurance rates over time and under different weather horizons, expected loss cost ratios, \(E(LCR)\), are also calculated and analyzed. \(E(LCR)\) is sometimes referred to as the actuarially fair insurance rate, and is expressed as,

\[
E(LCR) = \sqrt{\int_{0}^{\infty} \text{Max}(0, E(Y) \cdot \text{Cov} - y) \cdot f(y) dy} / \left( E(Y) \cdot \text{Cov} \right), \tag{11}
\]

where \(E(Y)\) is the expected yield, \(\text{Cov}\) is the coverage level (which defines the deductible), \(y\) is yield, and \(f(y)\) is the desired yield distribution (the conditional notation is suppressed here). As outlined in Woodard, Sherrick, and Schnitkey (2011), the RMA uses an empirical loss cost approach as the basis of their rating system, whereby annual average loss costs from historical data are first calculated, and then a simple average of the annual average loss costs is used as a

\textsuperscript{9} Bootstrapped standard errors and significances were also calculated to verify the accuracy of the BFGS method. With the exception of \(LN(ACRE)^2\) in the \(b\) model—which was only significant at the 5\% level under the bootstrap method instead of the 1\% level under the BFGS direct Hessian method—all the other conclusions regarding parameter significance and the level of significance were identical.
proxy for $E(LCR)$. The yield distribution must follow a restrictive and specific process through time in order for RMA’s method to result in unbiased forward-looking rates, $E(LCR)$. In short, if yield risk is decreasing through time, the loss cost ratio will decrease through time, with the implication that a simple average will result in a persistently biased forward looking $E(LCR)$.

Results

Results for the parameter estimates for the conditional Weibull models are presented in Table 1.\(^{10}\) The first column presents parameter estimates for $a(x, \beta_a)$ while the second column presents those for $b(x, \beta_b)$. In general, most of the terms are significant at a high level of significance, indicating a reasonable choice of functional form. Of course, results from the parameter estimates are difficult to interpret directly due to the presence of non-linear and interaction terms. Thus, Table 2 presents production elasticities for the model conditioned on the 1895-2009 WEATHER distribution and 2008 technology (i.e., setting TRENDS to 2008 levels); elasticities for three distribution statistics are reported: expected yield $E(Y)$, standard deviation, $\sigma(Y)$, and coefficient of variation (“relative risk”), $\sigma(Y) / E(Y)$. We present elasticities for standard deviation instead of variance for ease of interpretation.\(^{11}\) Note, since

$$\ln(e^x) - \ln(x) = n \left[ \ln\left((e^x)^{1/n}\right) - \ln\left((x)^{1/n}\right) \right], \forall x \in \mathbb{R},$$

the variance elasticity can be recast as the standard deviation elasticity simply by dividing the variance elasticity by $n$ (in this case, $n=2$).

\(^{10}\) Standard errors for the parameter estimates are calculated directly from the Hessian using the BFGS method approximation. Bootstrapped standard errors and significances were also calculated to verify the accuracy of the BFGS method. With the exception of $LN(ACRE)^2$ variable in the model—which was only significant at the 5% level under the bootstrap method instead of the 1% level under the BFGS direct Hessian method—all the other conclusions regarding levels of significance were identical.

\(^{11}\) Derivation of the coefficient of variation elasticity is available from the author upon request.
since standard deviation is the square root of variance). The production elasticities \textit{w.r.t.} to \textit{TREND} are presented such that the change in time is converted to an equivalent 1-year basis, again for ease of conceptualization.\textsuperscript{12}

As expected, the elasticities of $E(Y) \text{ w.r.t } TRED$ and \textit{SOIL} are positive, large, and significant, reflecting the fact that yields trend through time, and that better soil results in higher yields. For example, the percentage increase in the expected yield over a 1-year horizon is 0.9293\%, and 2.5924\% for every 1\% increase in soil quality.\textsuperscript{13} \textit{ACRE} is also positive and significant, but relatively small (0.0829), indicating small positive scale effects. Referring to the second row of results in Table 2, the elasticity of $\sigma(Y) \text{ w.r.t. } TRED$ is negative and significant, indicating that yields are becoming less risky through time in an absolute sense on the order of 0.3615\% per year. The same is also true for the elasticity of $\sigma(Y) / E(Y) \text{ w.r.t. } TRED$. Note, these elasticity estimates are net of the impacts of weather, indicating that yield risk in the Midwest is decreasing currently, and that this is not simply an appearance due to recent weather patterns. The elasticity of $\sigma(Y) \text{ w.r.t. } ACRE$ was also negative but insignificant. However, the elasticity of $\sigma(Y) / E(Y) \text{ w.r.t. } ACRE$ is negative and significant, reflecting the fact that larger units will tend to have lower yield risk due to aggregation. Improving soil quality also is estimated to reduce risk significantly.

Next, we turn attention to the issue of the impact of weather over alternative sample periods. Figure 1 presents the PDSI data from 1895-2009. While it is often suggested that the

\textsuperscript{12} For \textit{TREND}, this is approximated by multiplying the elasticity by the number associated with the trend year, \textit{TREND}. This result can be interpreted directly as the percentage change in the risk statistic as a result of a 1 year change in technology/time. Since the model was estimated in logs for \textit{SOIL} and \textit{ACRE}, so that the resulting elasticities can be interpreted in terms of their true values, an adjustment is made to the reported elasticities whereby it is the elasticity is divided the logged value of \textit{SOIL} or \textit{ACRE} value. These interpretations and adjustments follow straight from the definition of elasticity.

\textsuperscript{13} Note that “soil quality” is dependent on the index, so the magnitude is somewhat arbitrary. Thus, the magnitude of the elasticity estimates may vary among indexes, but would likely still be significant. The magnitude relative to the index is likely to also be identical among competing soil indexes.
last 30 years is not representative enough, or not adequately representative of longer horizons, this is not apparent from Figure 1. Figure 2 presents kernel density estimates of the WEATHER values from 1895-2009 and 1980-2009. While the recent period appears to have a slightly higher occurrence of wet conditions, the variances of the two appear to be similar, as does the frequency of droughts (as indicated by the secondary mode in the left tail).

In order to shed more light on what impacts the distribution of weather over various horizons will have on risk estimation, we compare conditional Weibull yield distributions which are conditioned on different distributions for WEATHER. Figure 3 presents the conditional Weibull yield distributions (under 2008 Technology), for both the 1895-2009 and 1980-2009 weather conditioning distributions. The Figure illustrates that there is only a small difference between the generated yield distributions, indicating that little is to be gained by taking into account the longer horizon of weather events in this application. This finding is in direct conflict with assertions by other researchers that a 30 year sample period is “too short” of a horizon for evaluating yield risk.

Next, we also explore the impacts of changing technology through time on expected yields (sometimes referred to as “trend yield”) and yield standard deviation, unconditional on weather. Figures 4 and 5 present expected yields and standard deviation conditional on different levels of TREND (i.e., technology). Again, the yield distribution is conditional on the median values of SOIL and ACRE, but are unconditional on weather (i.e., WEATHER is integrated out of the conditional yield distribution; but is still conditional on TREND, SOIL, and ACRE). Results are presented for both the 1895-2009 and 1980-2009 weather conditioning distributions. Again, there is little difference between the 1895-2009 and 1980-2009 distribution results.
The implied yield trend in Figure 4 is non-linear, and also appears to be growing at an increasing rate, a phenomenon referred to as “trend acceleration”. The increase in mean yields over the last 30 years is quite dramatic, with an increase in expected yields of over 50% from 1980 to 2008. Turning attention to Figure 5, after appearing to increase slightly early in the period, since the early 1990’s yield risk has steadily decreased. Note, under some mild distributional assumptions (Woodard, Sherrick, and Schnitkey, 2008), the RMA loss cost method requires that standard deviation grow at a rate proportional to the expected yield in order for it to result in unbiased rates. This requirement clearly does not hold here. While mean yields have increased over 50% for the period, standard deviation has decreased by almost 10%. Figures 6 and 7 present the entire yield distribution under technology for each year from 1980-2008 (presented as the inverse for ease of exposition). Figure 6 conditions the yield distribution on the distribution of WEATHER for the 1980-2009 period, while Figure 7 conditions on the WEATHER distribution for 1895-2009. Again, there is little difference between which conditioning distribution is employed for WEATHER. The Figures illustrate that yields have consistently increased over time at all quantiles of the yield distribution. Consistent with the assertion that yield risk in extreme stress events has decreased significantly, the Figures illustrate that the lower tail area has increased at a faster rate than the upper tail. Of course, in particularly acute events, large yield losses are still possible under 2008 technology, albeit not on the order of magnitude as under 1980 technology obviously.

In order to investigate the impact that a 1988 style drought would have under current technology versus older technology, Figure 8 presents simulated distributions conditional on a

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14 This functional form does not impose trend acceleration, but rather can exhibit constant, decelerating, accelerating, and even negative trends. Simple calculations using the first and second derivatives of the mean with respect to time can be conducted to easily show that all of these cases and combinations thereof are supported.
1988-type drought event occurring under both 1988 technology and 2008 technology. Figure 8 also presents results for both levels of technology (2008 and 1988) conditional on a weather event occurring that is similar to that which occurred in 2008 (a year with quite favorable weather). Focusing on the 2008 technology results, the Figure illustrates that a 1988 style drought event would indeed result in large yield losses. However, yields fair much better under 2008 technology than under the 1988 technology.

The next natural question is, “what would the insurance losses be today if a 1988 style event occurred, versus what occurred under 1988 technology?” Table 3 presents results for the simulated expected loss cost ratio under these scenarios. As suggested by the elasticity results, the rate results indicate that a 1988 event would result in significant losses, but substantially less than those that occurred under 1988 technology. For example, under 2008 technology, a 1988 drought event would only be expected to result in a loss cost ratio of 0.0551, 0.1133, and 0.1902 at the 65%, 75%, and 85% coverage level, versus an expected loss cost ratio of 0.1861, 0.2428, and 0.3004 in the event of a 1988 drought under 1980 technology. Thus, this finding suggests that if one were to estimate 2008 rates by means of a simple average loss cost approach by incorporating historical loss cost information that was observed in 1988, one would expect such loss information to be over-weighted by a factor (0.1861 / 0.0551 = ) 337.5%, 214.3%, and 158.0%, at each respective coverage level. Yet, this is essentially the process that the RMA employs. With this in mind, it is not difficult to see why RMA rates have performed poorly.

In order to generate a clearer picture of the evolution of expected loss costs over time, Figure 9 and 10 present expected loss costs (unconditional on weather) through time. Consistent with earlier results regarding decreasing yield risk, the Figures clearly indicate a strong downward trend in the $E(LCR)$. Again, both time horizons (1895-2009, and 1980-2009) result in
very similar $E(LCR)$ evolution, suggesting that 1980-2009 is indeed adequate in this case. Clearly, if the $E(LCR)$ is declining over time due to changes in technology, then taking a simple average of historical loss costs would necessarily be expected to result in persistently upward biased rates. Table 4 presents the expected bias one would expect under a simple average LCR approach. Overall, the results suggest that premium biases ranging between 218.93% and 420% are to be expected. This level of premium bias is consistent with the levels identified in historical data for the region, and is also consistent in magnitude with the results of Woodard, Sherrick, and Schnitkey (2008).

Results were also investigated for the conditional elasticities (not presented)—that is, conditional on a particular weather event such as a 1988 drought. The results were similar to those discussed above (i.e., “unconditional” on a single weather event) in that the conditional mean elasticities were large, positive and significant; however, in many cases the conditional variance elasticities with respect to time were positive. At first glance, it may then seem that the elasticity of risk in say a drought situation is increasing, meaning that risk in droughts is increasing. This is somewhat misleading though, since the conditional means over all weather events not only offset much of the increase in conditional variance, but are also converging. The net effect is that—conditional on an extreme weather event occurring—the net expected shortfall in the yield relative to the expected yield is still decreasing through time, both in an absolute and a relative sense. That is, relative to the expected yield (which is of importance for insurance), risk in extreme weather situations as well as under “normal” weather is decreasing. This is shown more clearly by analyzing the expected loss cost results (Figures 9 and 10). Again, the period used for conditioning the weather distribution (1895-2009 or 1980-2009) did not appear to have any meaningful impact on the results.
Conclusion

There is currently much disagreement over the impact of sample period selection when estimating yield distributions and making crop insurance rates, as the presence of non-constant weather and occasional catastrophic risk significantly confound their estimation. This consideration severely complicates the assessment of yield risk evolution through time. This study develops a conditional Weibull model for modeling yield risk which explicitly takes into account the number of droughts and other major weather events over the standard 1980-present period typically used when making crop insurance rates, as well as a longer period of 115 years spanning 1895-2009. The results for this dataset of Illinois corn producers do not suggest that there are any important differences between these periods, in terms of the severity of weather events observed, nor that such differences have any important impacts on the rating of insurance or the evaluation of yield risk in this sample. This finding suggests that the use of the recent 30 year sample period in the Midwest is likely adequate for empirically evaluating producer-level crop insurance risk in this region. In some sense, the results generated under the 1895-2009 weather distribution would be expected to be more efficient on theoretical grounds. However, in this application, the efficiency gains appear to be small relative to using the 1980-2009 period. Of course, in other regions this may not always be the case. That is simply an empirical question, which this framework can be applied to investigate.

The results also suggest that even after controlling for weather that yield risk is significantly declining through time due to technology gains in this sample, both in an absolute and relative sense. The implication of this result is that the current RMA procedures will result in biased rates since the simple average annual expected loss cost approach is inappropriate when relative yield risk is declining through time. A rating analysis indicates that these violations are
expected to result in overstatements in RMA rates of 200% to 400% for this region. The fact that this study explicitly takes into account non-constant weather through time and also longer weather horizons adds credibility to this conclusion. The results also lend credence to the trend acceleration argument that not only have yields trended upward through time in this region, but also that the trends themselves are increasing through time due perhaps to dramatic improvements in biotechnology. Last, this work corroborates the findings of earlier related studies (Woodard, 2008; Yu and Babcock, 2010; Woodard, Sherrick, and Schnitkey, 2011).

The models developed here have potentially useful practical applications. For example, such models could be used to derive weighting factors in order to reweight historical loss experience data using the reweighting methodology illustrated in Woodard (2008). In practice, instead of working with yields directly, the methods here could perhaps also be applied to loss cost data as well. The policy implications of this study and related supporting research for RMA rating are also far reaching. Since yield risk appears to be decreasing through time (even after accounting for weather) in this high premium volume region, this suggests that methods should be investigated by RMA to correct the significant rating problems identified in previous work.

Some qualifications are in order. First, the presence of adverse selection and moral hazard in some insurance pools could manifest in other data differently. For example, the data investigated here are production data for enterprise units, which cover the entire crop produced on the farm. While it is doubtful that other data from this region would give starkly different results, the occurrence of switching fraud (Atwood et al., 2006) in smaller optional unit structures and the presence of classical information asymmetries in some markets could lead to different findings in some datasets. I also caution that the results regarding declining yield risk are specific to this region, and should not be generalized to other areas unless supported by
empirical evidence. Thus, future research should focus on investigations for other crops, regions, and datasets, and on the implementation of such models into crop insurance rating systems. The methods developed here also have many potential uses outside the insurance arena. For example, noting that a weather distribution essentially just describes “climate”, the methods used here could be applied to investigate the impacts of climate change under various climatic and technology evolution scenarios by manipulating the distributions used to describe weather (to reflect climate change) and the parameters governing the technology process. Application of these approaches to the evaluation of other risk exposures (e.g., property insurance losses) could also be promising, as results for markets that are exposed to lower frequency catastrophic risks would in many cases differ greatly across various time horizons.
References


Table 1 - Parameter Estimates for Conditional Weibull Model

<table>
<thead>
<tr>
<th>Conditioning Variable</th>
<th>Weibull Model Parameters</th>
<th>( \beta_a )</th>
<th>( \beta_b )</th>
</tr>
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<tbody>
<tr>
<td>INTERCEPT</td>
<td></td>
<td>61.311***</td>
<td>191.683***</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(12.3168)</td>
<td>(35.3533)</td>
</tr>
<tr>
<td>TREND</td>
<td></td>
<td>-3.011***</td>
<td>-1.354***</td>
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<tr>
<td></td>
<td></td>
<td>(0.6631)</td>
<td>(0.1600)</td>
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<tr>
<td>LN(ACRE)</td>
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<td>3.578</td>
<td>-3.987***</td>
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<tr>
<td></td>
<td></td>
<td>(4.1475)</td>
<td>(1.5394)</td>
</tr>
<tr>
<td>LN(SOIL)</td>
<td></td>
<td>-60.798***</td>
<td>-82.997***</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(8.2514)</td>
<td>(16.0117)</td>
</tr>
<tr>
<td>WEATHER</td>
<td></td>
<td>36.648***</td>
<td>-2.742***</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(4.1435)</td>
<td>(0.5096)</td>
</tr>
<tr>
<td>TREND(^2)</td>
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<td>0.064***</td>
<td>0.011***</td>
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<tr>
<td></td>
<td></td>
<td>(0.0015)</td>
<td>(0.0004)</td>
</tr>
<tr>
<td>TREND * LN(ACRE)</td>
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<td>-0.124***</td>
<td>0.006</td>
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<tr>
<td></td>
<td></td>
<td>(0.0200)</td>
<td>(0.0050)</td>
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<tr>
<td>TREND * LN(SOIL)</td>
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<td>0.645***</td>
<td>0.230***</td>
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<tr>
<td></td>
<td></td>
<td>(0.1468)</td>
<td>(0.0355)</td>
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<tr>
<td>TREND * WEATHER</td>
<td></td>
<td>0.084***</td>
<td>0.029***</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.0085)</td>
<td>(0.0021)</td>
</tr>
<tr>
<td>LN(ACRE)(^2)</td>
<td></td>
<td>0.290**</td>
<td>-0.080***</td>
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<tr>
<td></td>
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<td>(0.1247)</td>
<td>(0.0297)</td>
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<tr>
<td>LN(ACRE) * LN(SOIL)</td>
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<td>0.065</td>
<td>1.196***</td>
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<tr>
<td></td>
<td></td>
<td>(0.9251)</td>
<td>(0.3316)</td>
</tr>
<tr>
<td>LN(ACRE) * WEATHER</td>
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<td>-0.036</td>
<td>0.095***</td>
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<td></td>
<td></td>
<td>(0.1007)</td>
<td>(0.0186)</td>
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<td>LN(SOIL)(^2)</td>
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<td>16.011***</td>
<td>9.097***</td>
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<td></td>
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<td>(1.5803)</td>
<td>(1.8339)</td>
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<tr>
<td>LN(SOIL) * WEATHER</td>
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<td>-7.351***</td>
<td>0.493***</td>
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<td></td>
<td></td>
<td>(0.9123)</td>
<td>(0.1134)</td>
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<tr>
<td>WEATHER(^2)</td>
<td></td>
<td>-1.633***</td>
<td>-0.060***</td>
</tr>
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</table>

Table presents parameter estimates from maximum likelihood estimation for conditional Weibull parameter models \( a(\bullet) \) and \( b(\bullet) \), using FBFM yield data from 1972-2008. Significance is denoted as *** = 1\%, ** = 5\%, and * = 10\%. Standard errors estimated are below the parameter estimates in parentheses.
Table 2-Production Elasticities, 2008 Technology, 1895-2009 Weather Conditioning Distribution

<table>
<thead>
<tr>
<th>Risk Measure</th>
<th>TRED</th>
<th>ACRE</th>
<th>SOIL</th>
</tr>
</thead>
<tbody>
<tr>
<td>E(Y)</td>
<td>0.8293*** (0.0168)</td>
<td>0.0829*** (0.0145)</td>
<td>2.5924*** (0.0684)</td>
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<tr>
<td>σ(Y)</td>
<td>-0.3615*** (0.0986)</td>
<td>-0.1375 (0.1041)</td>
<td>-2.7435*** (0.2697)</td>
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<tr>
<td>σ(Y) / E(Y)</td>
<td>-1.1810*** (0.1037)</td>
<td>-0.2202** (0.1104)</td>
<td>-5.2010*** (0.2798)</td>
</tr>
</tbody>
</table>

Table presents weather unconditional elasticity estimates for expected yield, yield standard deviation, and yield coefficient of variation (relative risk). Significance is denoted as *** = 1%, ** = 5%, and * = 10%. Bootstrap standard errors are located below the parameter estimates in parentheses. Elasticities are evaluated at the median of ACRE and SOIL, and at the year 2008 for TRED to reflect current technology.

Table 3-Expected Loss Cost Ratios Conditional on Specific Weather Events (1988 and 2008) and under Different Technology Levels (1988 and 2008)

<table>
<thead>
<tr>
<th></th>
<th></th>
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</thead>
<tbody>
<tr>
<td>E(Y)</td>
<td>125.7747</td>
<td>181.1213</td>
<td>181.1213</td>
<td>125.7747</td>
</tr>
<tr>
<td>65% E(LCR</td>
<td>WEATHER)</td>
<td>0.1861</td>
<td>0.0001</td>
<td>0.0551</td>
</tr>
<tr>
<td>75% E(LCR</td>
<td>WEATHER)</td>
<td>0.2428</td>
<td>0.0005</td>
<td>0.1133</td>
</tr>
<tr>
<td>85% E(LCR</td>
<td>WEATHER)</td>
<td>0.3004</td>
<td>0.0022</td>
<td>0.1902</td>
</tr>
</tbody>
</table>

Table 4-E(LCR) and RMA LCR Method Rate Comparison, 2008 Crop Year

<table>
<thead>
<tr>
<th>Coverage Level</th>
<th>1980-2009 Weather Distribution</th>
<th>65% Cov.</th>
<th>75% Cov.</th>
<th>85% Cov.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Simple Avg. LCR (RMA Method)</td>
<td>1.18%</td>
<td>2.13%</td>
<td>3.70%</td>
<td></td>
</tr>
<tr>
<td>E(LCR)</td>
<td>0.29%</td>
<td>0.73%</td>
<td>1.67%</td>
<td></td>
</tr>
<tr>
<td>RMA Method LCR Rate Bias</td>
<td>403.39%</td>
<td>292.52%</td>
<td>222.07%</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Coverage Level</th>
<th>1895-2009 Weather Distribution</th>
<th>65% Cov.</th>
<th>75% Cov.</th>
<th>85% Cov.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Simple Avg. LCR (RMA Method)</td>
<td>1.31%</td>
<td>2.33%</td>
<td>3.99%</td>
<td></td>
</tr>
<tr>
<td>E(LCR)</td>
<td>0.31%</td>
<td>0.79%</td>
<td>1.82%</td>
<td></td>
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<tr>
<td>RMA Method LCR Rate Bias</td>
<td>418.58%</td>
<td>293.81%</td>
<td>218.93%</td>
<td></td>
</tr>
</tbody>
</table>
Figure 1-Palmer Drought Severity Index, 1895-2009

Figure 2-Palmer Drought Severity Index Kernel Density Distributions, 1895-2009 versus 1980-2009 Sample Periods
Figure 3—Conditional Weibull Yield Distributions under 2008 Technology, 1895-2009 versus 1980-2009 WEATHER Conditioning Distributions (Median SOIL and ACRE)

Figure 4—Expected Yield, 1895-2009 versus 1980-2009 WEATHER Conditioning Distribution, (Median SOIL and ACRE), under Differing Technology Levels through Time
Figure 5—Yield Standard Deviation, 1895-2009 versus 1980-2009 WEATHER Conditioning Distribution, (Median SOIL and ACRE), under Differing Technology Levels through Time

Figure 6—Conditional Weibull under Differing Levels of Technology (Median SOIL and ACRE), 1980-2009 Conditioning WEATHER Distribution
Figure 7-Conditional Weibull under Differing Levels of Technology (Median \textit{SOIL} and \textit{ACRE}), 1980-2009 Conditioning \textit{WEATHER} Distribution

Figure 8-Conditional Weibull Yield Distribution under 1980 versus 2008 Technology/\textit{TREND} (Median \textit{SOIL} and \textit{ACRE}), and under 2008 versus 1988 \textit{WEATHER}
Figure 9-Expected Loss Cost Ratios, E(LCR), 1895-2009 WEATHER Conditioning Distribution, (Median SOIL and ACRE), under Differing Technology Levels through Time at Various Coverage Levels

Figure 10-Expected Loss Cost Ratios, E(LCR), 1980-2009 WEATHER Conditioning Distribution, (Median SOIL and ACRE), under Differing Technology Levels through Time at Various Coverage Levels