Measuring Inverse Demand Systems and Consumer Welfare

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An inverse (price-dependent) demand system, in which prices are functions of quantities demanded and income, is theoretically sound within the framework of classical demand theory. The demand system is important for applied demand analyses and is useful for situations in which policy options are directly related to quantity changes, such as marketing agreements for restricting availability through the use of a quota or the Government acreage control program. These are geared toward controlling supplies and/or stabilizing or raising commodity prices. Therefore, quantities rather than prices are appropriate instrumental or control variables in a demand system for evaluating some policy and program effects.

The major focus of this study is on implementing the conceptual demand relationships into empirical modeling and estimation of inve in table 1rse demand systems and consumer welfare. An application to *U.S. Personal Consumption Expenditures* data provides an example.

Methodology

The modeling and estimation of an inverse demand system and its consumer welfare measure include the following three steps:

Step 1--Apply distance function: $d(u, \boldsymbol{q}) = \min_{\boldsymbol{p}} [(\boldsymbol{p}' \boldsymbol{q}) / c(u, \boldsymbol{p})]$

Let $\boldsymbol{q}$ be a vector of n quantities demanded, $\boldsymbol{p}$ a vector of the corresponding prices, m the per capita income, and $u(\boldsymbol{q})$ the utility function. A distance function $d(u, \boldsymbol{q})$ is dual to a cost function $c(u, \boldsymbol{p})$ and can be used as a means to explore the properties of an inverse demand relationship such as homogeneity, symmetry, and scale aggregation. In addition, the Malmquist-quantity index defined as the ratio of two distance functions representing the constant utility quantity index for quantity changes can be regarded as a measure of efficiency in quantity metric welfare.

Step 2--Specify inverse demand system: $dr_i / r_i = \Sigma_j f_{ij}^* (dq_j^* / q_j^*) + g_i (ds / s)$

Armed with the concept of a distance function, an empirical inverse demand system can be derived from the Hotelling-Wold-identity. Where f_{ij}^* be the compensated price flexibility of the ith commodity with respect to a quantity change in the jth commodity, s is a scale variable defined as $\log s = \Sigma_j w_j \log q_j$. A reference quantity vector $q_j^*$is defined by using the scale variable to deflate a quantity as $q_j^* = q_j / s$.

Step 3--Measure consumer welfare: $CV = \boldsymbol{p}^h (u^0, \boldsymbol{q}^1)' \boldsymbol{q}^1 - \boldsymbol{p}^0{}' \boldsymbol{q}^0$

For measuring consumer welfare, an alternative form of the Malmquist-quantity index is defined as the difference between two distance functions and then converted it into Hicksian compensating variation in expenditure (CV). Where $\boldsymbol{p}^h (u^0, \boldsymbol{q}^1)$ be a vector of estimated compensated inverse demand at given quantity vector $\boldsymbol{q}^1$ and at the same initial utility level u^0. A positive CV implies a requirement of more spending to achieve the same utility level as before

the quantity changes and causing a decrease in consumer welfare. By contrast, a negative *CV* implies a reduction in spending and thus a gain in consumer welfare.

Application

The developed methodology is applied to estimate an inverse demand system consisting of 11 expenditure categories using the data from *U.S. Personal Consumption Expenditures* covering 1960-2006. The estimated demand system is then applied to evaluate the consumer welfare effects of quantity changes in each expenditure category.

Compensated own-price flexibilities. The compensated own-price flexibilities in table 1 show how much a category price must change to induce consumers to purchase more quantity of that category. For example, the compensated own-price flexibilities for food consumed at home and energy are -0.5302 and -0.4311, respectively. These estimates are relatively larger than most of other categories and explain well the recent soaring prices of food and energy, because it takes a large increase in the prices of these basic goods in response to a small reduction of their quantities available in the market.

Compensated cross-price flexibilities. The compensated cross-price flexibility in table 1, for example, between the price of food consumed at home and the quantity of clothing is -0.0478 which implies that the two expenditure categories are substitutes. A marginal 10-percent increase in the quantity of clothing is associated with a 0.478 percent decrease in the price of food to induce consumers to purchase the same quantity of food. In contrast, the compensated cross-price flexibility between the price of food and the quantity of medical care is positive at 0.3017 indicating a complementary relationship between the two categories.

Consumer welfare effects of quantity changes. The estimates of the demand system are then applied to the analysis of consumer welfare effects in response to quantity changes in various expenditure categories. Among the calculated welfare effects in table 2, a 10-percent decrease in the quantities of either food consumed at home or energy would increase per capita annual expenditures or incur consumer welfare losses by $1,491 or $798, respectively.

Conclusion

The concept of a distance function is useful for empirical modeling of an inverse demand system and consumer welfare measurement. The proposed differential-form inverse demand system has linear parameters for easy estimation, and the estimates can be interpreted directly as price flexibilities. The compensating variation in expenditures reflecting the quantity changes in distance functions is a proper measure for representing the efficiency in quantity metric welfare. The developed procedures are used to estimate an inverse demand system consisting of 11 U.S. expenditure categories and show the compensated price flexibilities and the consumer welfare effects of reduced quantity on a specific expenditure category.

Table 1--Compensated price flexibilities for U.S. personal consumption expenditures, 1960-2006

Price						*-- Reference quantity --*						*Scale*	*Constant*	*RMS*
	F.home	F.away	Energy	Clothing	O.nondur.	Cars	Furniture	O.durable	Transport	Medical	O.service			*errors, %*
Food at home	**-0.5302**	-0.0046	-0.0550	-0.0478	0.0915	-0.0114	0.0373	0.0960	-0.0199	0.3017	0.1424	**-1.0166**	-0.0166	1.15
	0.1077	0.0374	0.0939	0.0426	0.0542	0.0334	0.0369	0.0210	0.0265	0.0866	0.1504	0.1972	0.0060	
Food away home	-0.0074	**-0.2104**	0.1392	-0.1030	0.1325	-0.0313	-0.0734	0.0228	0.0191	0.1041	0.0076	**-0.7354**	0.0030	0.70
	0.0607	0.0583	0.0650	0.0432	0.0492	0.0245	0.0326	0.0141	0.0218	0.0572	0.0868	0.1191	0.0035	
Energy	-0.0977	0.1522	**-0.4311**	0.1166	0.0901	-0.0979	0.1356	0.1250	-0.0211	0.4257	-0.3975	**-1.7354**	0.0126	7.31
	0.1669	0.0711	0.2513	0.0860	0.1099	0.0676	0.0755	0.0454	0.0530	0.1844	0.2972	0.5064	0.0170	
Clothing	-0.0910	-0.1207	0.1250	**-0.5977**	-0.0763	-0.0205	0.1105	-0.0048	-0.0153	0.2136	0.4772	**-0.2858**	-0.0459	1.13
	0.0811	0.0506	0.0921	0.0798	0.0712	0.0350	0.0443	0.0201	0.0290	0.0792	0.1290	0.1807	0.0055	
Other nondurable	0.0929	0.0828	0.0515	-0.0407	**-0.4272**	0.0172	-0.0362	0.0499	0.0073	-0.1020	0.3044	**-1.2077**	0.0073	1.02
	0.0550	0.0308	0.0628	0.0379	0.0603	0.0239	0.0306	0.0141	0.0193	0.0549	0.0918	0.1257	0.0038	
Cars and parts	-0.0168	-0.0282	-0.0808	-0.0158	0.0248	**-0.0084**	0.0341	0.0619	0.0014	0.0822	-0.0545	**-1.3631**	0.0045	2.59
	0.0490	0.0221	0.0558	0.0270	0.0345	0.0339	0.0245	0.0189	0.0259	0.0629	0.0706	0.2276	0.0068	
Furniture	0.0675	-0.0818	0.1382	0.1052	-0.0645	0.0422	**-0.1833**	0.0278	0.0268	-0.2982	0.2200	**-0.7136**	-0.0131	1.33
	0.0668	0.0363	0.0770	0.0421	0.0545	0.0302	0.0473	0.0178	0.0240	0.0695	0.1118	0.1665	0.0051	
Other durable	0.3318	0.0486	0.2433	-0.0087	0.1700	0.1461	0.0531	**-0.7368**	0.1102	0.0824	-0.4399	**0.2398**	-0.0200	2.03
	0.0725	0.0300	0.0884	0.0365	0.0480	0.0446	0.0340	0.0330	0.0335	0.0900	0.1398	0.2742	0.0080	
Transportation	-0.0387	0.0229	-0.0231	-0.0157	0.0140	0.0019	0.0288	0.0620	**-0.0406**	0.0409	-0.0525	**-1.3721**	0.0059	2.61
	0.0516	0.0261	0.0580	0.0296	0.0370	0.0344	0.0258	0.0189	0.0293	0.0656	0.0654	0.2244	0.0067	
Medical care	0.1665	0.0354	0.1322	0.0619	-0.0554	0.0309	-0.0909	0.0131	0.0116	**-0.2088**	-0.0967	**-1.1016**	0.0268	1.04
	0.0478	0.0194	0.0573	0.0229	0.0298	0.0237	0.0212	0.0144	0.0186	0.0731	0.0764	0.1573	0.0048	
Other services	0.0327	0.0011	-0.0514	0.0576	0.0689	-0.0085	0.0279	-0.0292	-0.0062	-0.0218	**-0.0710**	**-0.9533**	0.0048	0.95
	0.0345	0.0123	0.0384	0.0156	0.0208	0.0111	0.0142	0.0093	0.0077	0.0272	0.0760	0.0917	0.0028	
Expenditure share	0.0841	0.0518	0.0261	0.0442	0.0828	0.0574	0.0464	0.0243	0.0432	0.1524	0.3874	1.0000		

Note: For each pair of estimates, the upper part is the estimated compensated price flexibility, and the lower part is the standard error.

RMS = Root-mean-square errors.

Table 2--Consumer welfare effects of reduced quantity on a specific expenditure category

Expenditure			Reduced quantity on a specific expenditure category by 1 percent								
category	F.home	F.aw ay	Energy	Clothing	O.nondur.	Cars	Furniture	O.durable	Transport	Medical	O.service
	Compensating variation (CV) -- Increased expenditures (dollars)										
Food at home	5.61	1.39	2.50	2.25	-0.18	1.69	0.24	-1.73	1.55	-3.56	5.58
Food aw ay home	1.06	-7.67	-1.59	2.07	-1.09	1.12	1.64	-0.08	0.19	0.12	3.99
Energy	3.87	-0.99	0.42	-0.63	0.85	3.13	-0.87	-1.31	1.53	-2.56	16.40
Clothing	1.32	1.56	-1.28	2.54	1.15	0.42	-1.12	0.14	0.32	-1.96	-4.29
Other nondurable	0.21	-0.50	0.14	2.30	1.33	1.27	2.25	-0.50	1.10	6.99	3.37
Cars and parts	1.94	1.46	2.15	1.12	1.30	-12.23	0.43	-0.43	0.85	1.86	8.19
Furniture	-0.10	1.52	-1.34	-0.94	1.58	-0.02	-7.25	-0.13	0.05	5.20	0.53
Other durable	-2.38	-0.41	-1.72	-0.01	-1.29	-1.08	-0.43	3.13	-0.82	-0.81	2.38
Transportation	1.68	0.53	0.96	0.83	1.09	0.84	0.38	-0.31	-8.75	1.84	6.07
Medical care	-3.71	1.09	-4.03	-0.67	7.38	1.62	7.14	0.69	1.81	-12.41	25.14
Other services	5.06	5.15	10.29	-1.65	1.08	6.74	1.74	5.59	5.05	17.81	-17.05
Total	14.57	3.11	6.49	7.21	13.20	3.52	4.15	5.04	2.89	12.52	50.32
Expenditure	2428	1525	1588	1150	2444	1479	1278	677	1093	5028	10662
Change (percent)	0.60	0.20	0.41	0.63	0.54	0.24	0.32	0.74	0.26	0.25	0.47