Environmental Labeling and Technology Adoption in the Presence of Strategic Interactions

Yoshifumi Konishi
Department of Applied Economics, University of Minnesota, USA
E-mail: konis005@umn.edu

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ABSTRACT

This manuscript analyzes the effect of binary ecolabeling on the strategic competition of Cournot duopolists in environmental technology and the output market. Under binary labeling, firms’ abatement technologies are not directly observable by consumers but are certified if they satisfy preset ecological standards. Given this asymmetry, I set up the regulator’s problem as one of choosing a technology standard, or "cutoff," in emissions per unit of output, below which all abatement efficiency levels are certified. The regulatory authority faces a trade-off in choosing the socially optimal cutoff: The regulator would like to raise the standard to reduce emissions but needs to lower it in order to induce technology adoption. There are three important findings: (1) ecolabeling is the second-best instrument in that choosing the optimal cutoff per se can never achieve the first-best outcome; (2) efficiency loss in terms of the difference between the first-best and the second-best total surpluses may or may not be large, depending on the extent of the certification barriers; and (3) setting too high or too low a standard is not only inefficient, but can also lead to an increase in total emissions relative to the status quo. Thus, setting the technology cutoff optimally is of crucial importance.

Keywords: ecolabeling, emissions, product differentiation, technology adoption

JEL codes: D43, L13, Q53, Q58
I. Introduction

This paper has a dual goal. It is to investigate the effects of binary environmental labeling (or "ecolabeling") on the interaction of firms and consumers and to examine optimal rules for setting a technology standard for ecolabeling given this interaction.

A binary label simply indicates that the labeled brand is produced in an environmentally friendly manner. Thus, under binary labeling, the firms’ abatement technologies are not directly observable by consumers, but can be certified if they satisfy preset ecological standards. In fact, many ecolabeling initiatives worldwide are binary. Canada’s Eco-logo, Germany’s Blue Angel, Japan’s Eco-Mark, the Nordic Council’s Nordic Swan, and the US’s Green Seal are well-known examples of binary ecolabeling. Because information on environmental attributes is often complex and is hard to communicate, binary labeling has been promoted as a preferred labeling mode.

Under binary labeling, firms’ supply-side responses are often sensitive to technology standards set force by certification programs. For example, a series of surveys by Japan’s Eco-Mark Office (JEO) indicate that there is a large variation in the ecolabeled brands’ market shares even within a similar product category (e.g. toilet paper and tissue paper) (Table 1). The JEO’s labeling standard for tissue papers and toilet papers are essentially the same — the percentage content of secondary or recycled papers. However, tissue papers require

2 "The ISO defines three types of ecolabels. Type I labels compare products with others in the same category, awarding labels to those that are environmentally preferable throughout their whole life cycle. The criteria are set by an independent body and monitored through a certification or auditing process... Type II labels are environmental claims made about goods by their manufacturers, importers or distributors. They are not independently verified, do not use predetermined and accepted criteria for reference, and are arguably the least informative of the three types of environmental labels. A label claiming that a product is ‘biodegradable’ without defining this term is a Type II label. Type III labels provide a menu of a product’s environmental impacts throughout its life cycle. These labels are similar to nutrition labels on food products that detail fat, sugar or vitamin content. Unlike Type I labels, these labels do not judge products. That task is left to consumers. Critics question whether the average consumer has the time and knowledge to determine whether, for example, emissions of sulphur are more hazardous than those of cadmium” (UNEP, 2006, p.3). Up to date, many ecolabeling programs worldwide use either Type I or Type II. Germany’s Blue Angel, Canada’s Eco-logo, the US’s Green Seal and Japan’s Ecomark are well-known examples of Type I ecolabeling programs. This manuscript’s analyses apply to these Type I programs. Type II ecolabels are also ubiquitous. Type III ecolabels exist, but are rare. For example, the U.S. Scientific Certification Systems has prepared an eco-profile that can be applied to any product category.
more subtle and softer textures than toilet papers do. Thus, it is more costly to produce tissue papers than toilet papers that satisfy the same standard while maintaining the other quality attributes discernible to consumers. Thus, if consumer preferences for environmental attributes are the same with tissue papers and toilet papers, then the difference in production and investment costs to meet the standard can explain the observed difference in market shares.³

Table 1.
Market Shares of Eco-Marked Products in Sales to Consumers
(Japan Eco-Mark Office Surveys, 2001-2004)

<table>
<thead>
<tr>
<th>Product category</th>
<th>(%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ball-point pens</td>
<td>1.8</td>
</tr>
<tr>
<td>Mechanical pencils</td>
<td>2.7</td>
</tr>
<tr>
<td>Markers/highlighters</td>
<td>3.1</td>
</tr>
<tr>
<td>White-outs</td>
<td>19.8</td>
</tr>
<tr>
<td>Laser printers</td>
<td>26.6</td>
</tr>
<tr>
<td>Inkjet printers</td>
<td>25.3</td>
</tr>
<tr>
<td>Multi-copying papers</td>
<td>10.2</td>
</tr>
<tr>
<td>Inkjet papers</td>
<td>4.0</td>
</tr>
<tr>
<td>Toilet papers</td>
<td>39.4</td>
</tr>
<tr>
<td>Tissue papers</td>
<td>1.5</td>
</tr>
</tbody>
</table>

With these factors in mind, I construct a model of Cournot duopolists in which consumer demand is generated by binary labeling and heterogenous altruistic preferences. The duopolists play a two-stage simultaneous-move game: In stage 1, firms simultaneously choose abatement technology levels and whether to certify in the first stage; and in stage 2, they play a Cournot game of quantity competition with the corresponding demand system. Given this

³It is certainly plausible that consumers have slightly different motivations for purchasing durables and non-durables, which may in turn affect their preferences for ecolabeled durables and non-durables. For example, O’Brien and Teisl (2001) find that consumers are more inclined to purchase environmentally labeled brands if the product was a frequently purchased item, because this would allow them to make a greater environmental impact. However, there seems to be no apparent reason why consumers would have different preferences for ecolabeled toilet papers and ecolabeled tissue papers.
game structure, I set up the regulator's problem (in stage 0) as one of choosing a voluntary
technology standard, or "cutoff," in emissions per unit of output, below which all abatement
efficiency levels are certified for environmental labeling. Set up this way, it is immediate to
observe a trade-off in setting the socially optimal cutoff: The regulator would like to raise
the standard to reduce total emissions from the industry but need to lower it in order to
induce firms' technology adoption. Moreover, because the duopolists' supply responses are
determined in part by their cost functions as well as consumer demand, total emissions from
the industry is not necessarily decreasing in the equilibrium number of certifying firms.

This regulator's instrument is imperfect in a number of important regards, however.
First, because labeling is only binary, it can only offer an incomplete signal to consumers.
Second, because consumers are heterogeneous in altruistic interests, consumers can interact
with one another in an important way in the product market. Labeling can cause altruistic
consumers to purchase ecolabeled goods, which increases (decreases) the price of ecolabeled
(non-labeled) goods. However, selfish or environmentally unaware consumers do not mind
buying non-labeled goods and benefit from the low price of non-labeled goods. The equilib-
rium prices are, therefore, likely to depend on the distribution of altruism among consumers.
Third, firms compete strategically both in the output market and in environmental tech-
nology adoption given the consumer demand and the cutoff chosen by the regulator. Thus,
imperfect competition among the firms erodes away the effectiveness of environmental label-
ing. Because of these market distortions, the regulator cannot, in choosing the optimal level
of ecolabeling standard, maximize social net benefit in the first-best manner. Thus, binary
ecolabeling is second-best in nature.

The magnitude of efficiency loss in terms of the difference between the first-best and the
second-best total surpluses depends crucially on the primitive parameters of the economy
such as the dispersion of altruism and the fixed cost of technology adoption. Since no
clear-cut analytical results are feasible, I construct a numerical example to demonstrate how
changing these parameters affects efficiency loss. For example, the regulator can attain the
total surplus sufficiently close to the first-best level when the fixed-cost parameter is small whereas when it is high, the regulator can set the standard only at the level that is too loose compared to the first-best level. The result is robust to perturbations of the other model parameters.

Although my analyses primarily deal with the case of identical technology endowments, I also briefly analyze the heterogeneous case. When firms are endowed with different abatement efficiency levels, the regulatory authority may opt for a discriminatory standard in favor of firms with more environmentally friendly technology. This case is important, because proponents of ecolabeling initiatives often appear to presume that such a discriminatory labeling can automatically reduce emissions from the industry. In contrast to the popular view, I show formally that choosing a discriminatory ecolabeling standard at an inappropriate level can lead to an increase in total emissions relative to the status quo. Intuitively, this happens because the consumers may respond to ecolabeling in such a way that the demand for the labeled good increases too much to the extent that outweighs the benefit of the demand shift (from the non-labeled good to the labeled good). Thus, the paper calls upon a question into the current ecolabeling practice, which often relies on an engineering approach to determine the technology standards with no or little economic consideration.

This paper complements two large strands of literature. First, my model is related to, but is significantly different from, previous theoretical studies on ecolabeling, many of which typically use a vertical product differentiation model (Cremer & Thisse, 1994; Arora and Gangopadhyay, 1995; Bansal & Gangopadhyay, 2003; Amacher et al., 2004; Engel, 2004; Conrad, 2005; Lombardini-Piipinen, 2005). Vertical differentiation offers an obvious advantage because it captures the underlying trade-off firms face: competition among firms may be less intense if they offer products that are less substitutable, but they may reap more profits by selecting an undifferentiated product for which demand is strong (Hotelling, 1929; Mazzeo, 2002). However, a vertical differentiation model assumes perfect information, and therefore, its applicability may be limited in the context of binary labeling and on the ground
that detailed information on environmental attributes is much harder to communicate than other product qualities that are readily observable by consumers.\footnote{In recent years, information on environmental performance has been publicized via various public disclosure programs such as the U.S. Toxics Release Inventory and EPA's 33/50 program. Firms are increasingly cautious about their reputation as environmentally friendly entities. However, prior empirical studies indicate that it is questionable to claim that consumers can and do relate such detailed information to the exact environmental impact of each product and take it into purchasing considerations. A field-experiment study by Hiscox and Smyth (2005), for example, seems to suggest that it is the power of labeling rather than the detailed information that derives consumers' purchasing decisions.} My analysis is appropriate for many ecolabeled products of interest, such as office papers, construction woods, sanitary goods, stationary goods, staple agricultural commodities, and electricity, which have a thin margin for quality competition. Thus, the paper complements a growing literature on firms' environmental quality competition.

Second, earlier research efforts have focused on showing whether or not ecolabeling has a positive influence on consumer behavior. Along this line, a number of empirical studies based on stated preferences or experiments have concluded that many consumers would select ecolabeled products over standard ones both at equal and at different prices (Blend and Ravenswaay, 1999; Teisl \textit{et al.}, 1999; Wessells \textit{et al.}, 1999; Loureiro \textit{et al.}, 2001; Moon \textit{et al.}, 2002; Conner, 2002; Sergienko and Nemudrova, 2002; Wechel and Wachenheim, 2002; Johnson \textit{et al.}, 2002). Studies on actual in-store demand followed and found the positive demand as well as the large willingness to pay (WTP) for ecolabeled products (Teisl \textit{et al.}, 2002; Bjorner \textit{et al.}, 2004; Hiscox and Smyth, 2005). However, this paper suggests that the positive consumer demand for ecolabeled goods does not necessarily translate into the positive effects of ecolabeling and the failure to account for the (potential) interactions between firms and consumers could result in biased policy implications.

Though this paper focused on emissions, its main contention can readily generalize to environmental externalities from unsustainable use of renewable resources such as forest and marine resources. For example, sustainability concept adopted by the Forest Stewardship Council or the Marine Stewardship Council may be used as a measure of negative externality per unit of output. The trade-offs the labeling authority faces are essentially the same as...
those in the case of emissions. If the standard is set too loose, the increased consumption of
the labeled products may outweigh the benefit of the sustainable harvesting practice. If the
standard is too high, however, producers may withdraw from certification and no significant
impacts may result. Thus, an optimal standard must strike an appropriate balance.

II. The Model

A. Firms

Consider a market for a homogenous (i.e. physically identical) good with two identical
producers. The good can be produced using different production processes or inputs. The
abatement efficiency $\omega$ of each production technology is measured in terms of emissions per
unit of output, so that the total emissions are given by $E = \omega_1 X_1 + \omega_2 X_2$, where $X_j$ denotes
the quantity produced by firm $j$. Two identical firms, endowed with initial abatement effi-
ciency $\bar{\omega}$, play a two-stage game. In the first stage ("Stage 1"), firms simultaneously decide
to certify or not certify their products for environmental labeling, and choose a new abate-
ment efficiency $\omega$. Because $\omega$ is measured in emissions per unit of output, a higher $\omega$ implies
lower abatement efficiency. If they decide to certify, they must pay the investment cost $k$
and face the new marginal cost $c$, both of which depend on the new abatement efficiency $\omega$.

In the second stage ("Stage 2"), firms play a Cournot game of quantity competition with the
corresponding demand system depending on the number of certifying firms. The investment
cost depends on the distance between the initial and the new abatement efficiency levels. I
assume throughout that the capital and other costs required to attain $\omega > \bar{\omega}$ (i.e. the costs

\footnote{Investment cost here includes all tangible and intangible fixed costs of entry, such as certification fees, legal licensing fees, documentation costs as well as investments in physical or human capitals.}

\footnote{We could employ Bertrand competition instead. However, with Bertrand competition, when both firms certify or do not certify, $x_1$ and $x_2$ become literally identical under binary labeling. Thus, equilibrium prices will be $p = c(\omega_1) = c(\omega_2)$. When only one firm certifies, two products become incomplete substitutes, so that we can solve for demand functions: $x_j = D_j(p_1, p_2)$. In this case, it is well known that Cournot competition is dual to Bertrand competition in that the Cournot reaction functions, equilibrium strategies, and profits can be derived from the Bertrand ones, though Cournot competition is typically more monopolistic than Bertrand competition in that Cournot quantities (prices) are lower (higher) than in Bertrand competition (Singh and Vives, 1984).}
of ‘divesting’) is sufficiently high relative to the cost-saving gains from it, so that no firm would ever find it optimal to choose \( \omega > \bar{\omega} \). I take \( \bar{\omega} \) to be the fixed primitive parameters of the model. Therefore, under these conditions, we only need to deal with the compact set of possible abatement technologies, \( \Omega = [0, \bar{\omega}] \), so we write \( k(\omega, \bar{\omega}) = k(\omega) \). Furthermore, I impose the following regularity assumptions:

**A1.** \( c(\omega) \) is twice-continuously differentiable with \( c' < 0, c'' \geq 0 \) and \( c(0) = \alpha \).

**A2.** \( k(\omega) \) is twice-continuously differentiable with \( k' < 0, k'' \geq 0 \), \( k(\bar{\omega}) = 0 \) and \( k(0) = K > 0 \).

As will be explained below, \( \alpha > 0 \) is a preference (and demand) parameter and, therefore, **A1** implies that it is economically not viable for firms to produce the good with zero emissions.

### B. Consumers

The economy consists of a continuum of identical consumers indexed with \( i \). Under binary labeling, consumers do not observe firms’ abatement efficiencies \( \omega \)’s directly, but their products can be certified by a third-party program if \( \omega \) satisfies a predetermined technology standard. Each consumer has quasi-linear preferences \( U_i(x_i, E) = \alpha x_i - x_i^2/2 - E^2/2 \), which are separable in the *numeraire* good. If \( \omega \)’s and everybody’s actions were perfectly observable, consumers could calculate environmental damages from their own consumption \( E = \int \omega_1 x_1 + \omega_2 x_2, di \). However, in a large economy like ours, consumers know their contribution to environmental damages is non-measurable, and therefore, will completely free ride on others and will not buy ecolabeled products.

To allow for a purchasing incentive for the ecolabeled good, I assume that consumers have genuine altruistic interests (Kennett, 1988; Johansson, 1997).\(^7\) By genuine altruism, we mean

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\(^7\)Johansson (1997) classifies altruism into four broad categories: (a) pure altruism, (b) paternalistic altruism, (c) impure altruism, and (d) genuine or semi-Kantian altruism.
that individuals "care for other individuals, through their behavior, without deriving any utility from it" (Johansson, 1997). Moreover, in our context, consumers do not know the precise impacts of their behavior on others. Thus, I assume that each consumer acts as if collective action, given everyone acts the same as hers, will affect everyone’s utility. Therefore, the consumer acts as if she maximizes the adjusted net benefit

\[ U(x_1, x_2, E) = v(\beta_i) - p_1 x_{1i} - p_2 x_{2i}, \]

where \( v(\beta_i) = \beta_i (\sum_j \chi_j x_{ji})^2 / 2 \) and \( \beta_i \in (0, 1) \) is an altruistic parameter. \( \chi_j \) is an indicator function, which equals one if \( j \) is non-labeled and zero otherwise. Thus, a higher \( \beta_i \) means that \( i \) is more altruistic. Furthermore, I assume there is a known distribution of altruism with its cumulative distribution function \( \lambda \), so that \( \int \beta_i \, di = \int \beta \, d\lambda(\beta) = E\beta_i \).

The assumption of genuine altruism is partly made for analytical tractability as well as for consistency with binary labeling. As shown in Johansson (1997), in a large economy, pure altruism in the sense of Becker (e.g. 1974) imposes restrictive assumptions on the (relative) size of an indirect utility effect through others’ utilities: As the population size grows, the indirect effect must decrease proportionally. On the other hand, impure altruism in the sense of Andreoni (e.g. 1989, 1990) could be used in our context. However, a demerit of impure altruism is that the total surplus depends not only on the consumption and environmental damages, but also on the utility gains by each consumer through impure altruism. This makes calculation of the total surplus intractable.

Given this setup and assuming an interior economy, we obtain an individual demand given by the following system: If both firms certify, \( x_1^*(\beta) + x_2^*(\beta) = \alpha - p \); If only firm 1 certifies, \( x_1^*(\beta) = \alpha - p_1 - (p_1 - p_2) / \beta \) and \( x_2^*(\beta) = (p_1 - p_2) / \beta \); If none certifies,

8Except the altruism term \( v \), this is a standard treatment in the literature (see, for example, Qiu (1997), Singh and Vives (1984), Spence (1976) and Vives (1985)). Note that the usual utility maximization problem, \( \max\{u(x_1, x_2, z) \mid p_1 x_1 + p_2 x_2 + z \leq m\} \), is equivalent to maximizing the net benefit function, \( U(x_1, x_2) - \beta x_1 - p_2 x_2 \), if \( u \) is quasilinear in \( z \), i.e., \( u(x_1, x_2, z) = U(x_1, x_2) + z \).

9In this manuscript, I use the terms "selfish" and "altruistic" to describe an individual, respectively with a low and a high \( \beta \). These words, however, need not be interpreted literally. Readers may interpret, for example, "selfish" to mean individuals who are not environmentally aware. The wording choice was made so it is consistent with the notion of genuine altruism. All that is required for the results in this manuscript is that when we calculate total surplus, we do not need to deal with the intractable integration over the distribution of allocations and preferences.

10If \( \beta \) is sufficiently close to zero, \( x_2^* \) will be bounded by the budget constraint: \( m/p_2 \). By interiority assumption, however, I will ignore this case, as the relative price \( p_1/p_2 \) will adjust and become sufficiently
\[ x_1^*(\beta) + x_2^*(\beta) = (\alpha - p) / (1 + \beta). \] This system yields sensible comparative statics. When both firms certify, \( \beta \) plays no role and two goods are identical. When only one firm certifies (i.e. both labeled and non-labeled goods are available), \( \partial x_1^*/\partial \beta > 0, \partial x_1^*/\partial p_1 < 0, \) and \( \partial x_1^*/\partial p_2 > 0 \) and \( \partial x_2^*/\partial \beta < 0, \partial x_2^*/\partial p_1 > 0, \) and \( \partial x_2^*/\partial p_2 < 0 \) as long as \( p_1 > p_2 \). If \( p_1 = p_2 = p \), then \( x_1^* = \alpha - p \) and \( x_2^* = 0 \) for all \( \beta \in (0, 1) \). When no labeled good is available, consumers know that the goods available are not produced in an environmentally friendly manner, so that an increase in \( \beta \) decreases the demand for these goods.

The average (inverse) demand, corresponding to each case, is given by the following: if both firms certify,

\[ P_1 = P_2 = \alpha - (X_1 + X_2); \] (1)

if only firm 1 certifies,

\[ P_1 = \alpha - (X_1 + X_2), \quad P_2 = \alpha - X_1 - (1 + 1/\sigma) X_2, \] (2)

where \( \sigma = E[1/\beta] \in (1, \infty) \); if none certifies,

\[ P_1 = P_2 = \alpha - (1/\tau) (X_1 + X_2), \] (3)

where \( \tau = E[1/(1 + \beta)] \in (1/2, 1) \).

The parameters \( \sigma \) and \( \tau \) have an intuitive appeal. \( \sigma \) may be considered a measure of ‘dispersion’ of altruism in that a mean-preserving spread (MPS) of the distribution increases the value of \( \sigma \) (Rothschild and Stiglitz, 1970), and therefore, it increases, via (2), the average demand for non-certified goods. When both labeled and non-labeled goods are produced, each firm needs to consider not only the effects of its production decisions on its own price but also on its competitor’s, because the demand for the labeled good is determined in part by the price of the non-labeled good and vice versa. Furthermore, this interaction between close to zero if a sufficiently large number of consumers have such \( \beta \).
two market segments is intermediated by the distribution of altruism in such a way that an increase in dispersion increases (decreases) the non-certifying (certifying) firm’s ability to raise its own price. Intuitively, this happens because MPS moves the population mass of both the selfish and the altruistic in such a way that preserves the expected value of the distribution. However, selfish consumers with \( \beta \) close to zero take advantage of the lower price of \( x_2 \) and consume more of \( x_2 \). Thus, its overall impacts favor the non-labeled product. On the other hand, \( \tau \) is a measure of the ‘market size’. Because \( 1/(1 + \beta) \) is a convex function, MPS again raises the value of \( \tau \). The increase in \( \tau \) increases the demand at a given price \( p \). Thus, the difference \( (1 - \tau) \) in the average demand between (1) and (3) may be viewed as the size of the positive labeling impact on consumer demand: It becomes large when consumers are largely altruistic while it becomes smaller as the mass of the selfish population increases.

C. Regulator

At the beginning of this two-stage game, a third-party ecolabeling program sets a technology cutoff point, denoted \( \theta \), on \([0, \bar{\omega}]\). I assume away measurement errors occurring during the certification process or the stochastic nature of investment outcomes. Hence, all \( \omega \leq \theta \) will be labeled as environmentally friendly. Because the labeling is only binary, consumers cannot identify firms’ abatement technologies \( \omega \). Under these assumptions, it is immediate that profit-maximizing firms will choose \( \omega = \theta \), if they decide to certify given the cutoff \( \theta \). In this paper, I only solve for pure strategies. As will be made clear in the later analysis, the Subgame Perfect Nash Equilibria of the two-stage game above depend crucially on \( \theta \). Thus, it is natural to think of the regulator’s problem in setting a standard for \( \theta \). However, in setting the standard, the regulator faces a natural tension between its own and the firms’ objectives. Ideally, the regulator would like to lower the cutoff, so that the certified firms would attain the higher level of environmental technology. However, if the cutoff is too tight, the firms may refrain from investing in environmental technology. Moreover, because
the duopolists’ supply responses are determined in part by their cost functions as well as consumer demand, it is not necessarily true that the total emissions from the industry is lower when two firms certify than when one firm or none certifies. The regulator, therefore, chooses an optimal cutoff $\theta^*$, which maximizes the (average) net surplus:\footnote{We are concerned only with the ‘average’ net surplus, because in the model, all quantities are expressed in averages only.}

$$NS = U(x^*(\theta), E(x^*(\theta))) - \sum_{j=1,2} C_j(x^*_j(\theta)) - TS(\bar{\omega}) - F,$$

(4)

where $x^*(\theta) = (x^*_1(\theta), x^*_2(\theta))$ is a vector of equilibrium quantities given $\theta$, $C_j$ is $j$’s total production cost, $TS(\bar{\omega})$ the status-quo total surplus when no firm certifies, and $F$ the fixed cost of establishing a coherent labeling program. A few things must be clarified. First, the altruistic term $v$ is ignored, because by definition of genuine altruism it does not affect consumer surplus. Second, I ignore the fixed cost $k$ in the calculation of net surplus, because I treat $k$ to be a variable part of the model primitives and examine the impacts of the changes in $k$ on the maximal surplus and because the inclusion of $k$ in (4) does not meaningfully affect our main results. Lastly, I assume the following regularity condition to hold throughout the paper:

**A3.** Model primitives $\delta = (\alpha, \bar{\omega}, c, k, \lambda)$ are such that the maximum possible level of net total surplus is increasing in the number of certifying firms.

The condition essentially says that the underlying structure of the economy favors ecolabeling: Higher welfare levels can be potentially achieved if more firms certify. This condition effectively precludes the possibility of a trivial boundary solution $\theta^* = \bar{\omega}$. I emphasize, however, that the condition does not necessarily imply the social optimum always attains when both firms certify. In fact, as will be discussed later, it is sometimes optimal to induce only one certifying firm even under **A3**.
III. The First-Best Outcome

As a benchmark, let us first examine the first-best outcome of the economy. Because firms have identical technologies and consumers have identical preferences, we can impose a symmetric solution to the social planner’s problem. Imposing $X_1 = X_2 = X$, $\omega_1 = \omega_2 = \omega$ and $x_{ji} = x_j = x$ for all $i \in (0, 1)$, we obtain $X_j = \int x\,di = x$, $E = \int \omega_1 x_{1i} + \omega_2 x_{2i}\,di = 2\omega x$, and $\int U(x_{1i} + x_{2i}, E)\,di = \int U(2x, 2\omega x)\,di = U(2x, 2\omega x)$. Given these relationships, we can derive the following:

**Lemma 1.** The first-best interior emissions standard $\omega^{FB}$ (in per unit of output) maximizes the reduced-form net benefit function $W$, defined as

$$W(\omega) \equiv \frac{1}{2} \frac{(\alpha - c(\omega))^2}{1 + \omega^2}. \quad (5)$$

*Proof:* The first-best outcome of the economy must maximize the total surplus given by

$$TS = \int U(x_{1i} + x_{2i}, E)\,di - \sum_{j=1,2} c(\omega_j) x_j$$

$$= U(2x, 2\omega x) - 2c(\omega) x$$

$$= 2(\alpha - c(\omega)) x - 2x^2 - 2(\omega x)^2,$$

where the second line follows from the symmetry and the last line follows by substituting the explicit expression for $U$. The social planner would choose $x$ and $\omega$ that maximize this expression. Note that this function is concave in each of the arguments $\omega$ and $x$, but is not globally concave in $(\omega, x)$. Thus, I will require the first-best output $x$ to trace out an interior optimal path given $\omega$. Given this assumption, the first-order condition with respect to $x$ gives

$$x^* = \frac{\alpha - c(\omega)}{2(1 + \omega^2)}. \quad (6)$$

Substituting this back to the original total surplus function above, we obtain the desired
Equation (6) says that the competitive output level, $(\alpha - c(\omega))/2$, must be decreased to reflect the negative externality by the factor of $1/(1 + \omega^2)$. This first-best outcome can be achieved, for example, by directly controlling $x$ and $\omega$ or by combining an ad valorem subsidy on output price and an emissions tax equal to the marginal environmental damage. Note that the regulator (or ecolabeling authority) in our setup does not have direct access to this net benefit function, because he can only choose the ecolabeling standard to affect consumers’ and firms’ behaviors. As will be shown later, the regulator can set the socially optimal technology standard $\theta^*$ only at the second-best level. In this paper, therefore, I reserve the term "first-best" for the fully efficient outcome that maximizes (5).

III. Characterization of Two-Stage Game

A. The Second Stage Quantity Competition

Given the technology choice $\omega_j$, each firm chooses production quantity $x_j$, which solves:

$$\max P_j(x_1, x_2) x_j - c(\omega_j) x_j - k(\omega_j).$$

As discussed earlier, the inverse demand $P_j$ changes according to the number of eco-certified firms. In each of the three generic cases, there exists a unique Cournot-Nash equilibrium. Substituting (1), (2) or (3) for each case, taking the first-order condition, and manipulating, we obtain the equilibrium quantities ($\tilde{x}_1, \tilde{x}_2$) and profits $(\Pi_1, \Pi_2)$ (Table 2). Note that in the table, I use the fact that certifying firms must choose $\omega = \theta$ and non-certifying firms $\omega = \bar{\omega}$ with $k(\bar{\omega}) = 0$. Furthermore, note that $A(\theta) = \alpha - c(\theta) \geq 0$ and $B(\theta) = c(\theta) - c(\bar{\omega}) \geq 0$ for all $\theta \in [0, \bar{\omega}]$ by A1. However, there is no a priori reason why the profits net of investment

---

12Note that $W$ becomes a downward-sloping convex function of $\omega$ if $c$ is not sufficiently convex, in which case the solution becomes trivial — $\omega^{FB} = 0$. That is, it is optimal to require firms to produce output with zero emissions. If, on the other hand, $c$ is sufficiently convex, $W$ will be concave in $\omega$ on a relevant section, so that at least one of the roots satisfying the first-order condition will be the solution.
costs are nonnegative in Case (1) or (2). Thus, I assume that the firm ceases to produce at \( \theta \), at which the equilibrium net profit becomes nonpositive. The firm’s equilibrium quantity and profit are constant only when both firm 1 and firm 2 do not eco-certify. Even when firm 1 does not certify, its equilibrium quantity and profit are functions of \( \theta \) as long as its opponent certifies, because the equilibrium quantities depend on the marginal production costs of both firms.

Table 2.
Equilibrium Levels of Quantities and Profits for Firm 1

<table>
<thead>
<tr>
<th>I. Both certify</th>
<th>Quantities</th>
<th>Profits</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \frac{A(\theta)}{3} )</td>
<td>( \max \left{ 0, \left( \frac{A(\theta)}{3} \right)^2 - k(\theta) \right} )</td>
<td></td>
</tr>
</tbody>
</table>

| II. Firm 2 certifies | \( \frac{A(\theta) + 2B(\theta)}{3+4/\sigma} \) | \( (1+\frac{1}{\sigma}) \left[ \frac{A(\theta) + 2B(\theta)}{3+4/\sigma} \right]^2 \) |

| III. Firm 1 certifies | \( \frac{(1+2/\sigma)A(\theta) - B(\theta)}{3+4/\sigma} \) | \( \max \left\{ 0, \left[ \frac{(1+2/\sigma)A(\theta) - B(\theta)}{3+4/\sigma} \right]^2 - k(\theta) \right\} \) |

| IV. None certifies | \( \tau \frac{D}{3} \) | \( \tau \left( \frac{D}{3} \right)^2 \) |

Note: \( A(\theta) = \alpha - c(\theta), B(\theta) = c(\theta) - c(\bar{\omega}), \) and \( D = \alpha - c(\bar{\omega}) \).

B. The First Stage Certification Decisions

Because any entering firm chooses \( \omega_j = \theta \), the duopolists simply engage in a two-by-two simultaneous-move game, with \{certify (ec), not certify (nc)\} being the set of possible actions. Let us define \( \Pi_{1}^{I}(\theta) \equiv \Pi_{1}(ec, ec), \Pi_{1}^{II}(\theta) \equiv \Pi_{1}(nc, ec), \Pi_{1}^{III}(\theta) \equiv \Pi_{1}(ec, nc), \) and \( \Pi_{1}^{IV}(\theta) \equiv \Pi_{1}(nc, nc) \). With \( \bar{\omega}_1 = \bar{\omega}_1 = \bar{\omega} \), the game reduces to a symmetric one with the following payoff matrix for firm 1 at the first stage:

\(^{13}\)To be more precise, firm 1’s equilibrium quantity in III may not be positive under some region. We can ignore this region, because on such a region, no firm would ever find it profitable to certify.
Fact 1. Let $n^*$ be the equilibrium number of firms in the ecolabeled segment. Then we have the following relationships: $n^* = 0$ if $\Pi^I_1 < \Pi^{II}_1$ and $\Pi^{III}_1 < \Pi^{IV}_1$, $n^* = 1$ if $\Pi^I_1 < \Pi^{III}_1$ and $\Pi^{II}_1 > \Pi^{IV}_1$, $n^* = 2$ if $\Pi^I_1 > \Pi^{II}_1$ and $\Pi^{III}_1 > \Pi^{IV}_1$, and $n^* = 0$ or $2$ if $\Pi^I_1 > \Pi^{III}_1$ and $\Pi^{II}_1 < \Pi^{IV}_1$.

Multiple equilibria arise when the last case occurs. Moreover, if one of the inequality in each case holds with equality, then multiple equilibria occur. For example, if $\Pi^I_1 = \Pi^{II}_1$ and $\Pi^{III}_1 > \Pi^{IV}_1$, the corresponding equilibria will be $n^* = 1$ and $n^* = 2$. The structure of the game depends on the cutoff $\theta$, because the set of SPNE outcomes depends only on combinations of ordered pairs: $\Pi^I_1 (\theta) \leq \Pi^{II}_1 (\theta)$ and $\Pi^{III}_1 (\theta) \leq \Pi^{IV}_1 (\bar{\omega})$. Therefore, the characterization of the two-stage game is equivalent to characterizing how these ordered pairs change as a function of $\theta$.

Let us define

$$L^{nm} (\theta) = \Pi^n_1 (\theta) - \Pi^m_1 (\theta),$$

$$\theta^{nm} \in \{\theta | L^{nm} (\theta) = 0\}.$$ 

for $n, m = 1, .., 4$, which correspond to cases I, ..., IV. As will become apparent, though only the signs of $L^{12}$ and $L^{34}$ determine the SPNE of this game, we need information on the signs of $L^{13}$, $L^{14}$, and $L^{24}$ to describe how $L^{12}$ and $L^{34}$ change signs as a function of $\theta$.

From Table 2, it is immediate that $\Pi^I_1$ is increasing in $\theta$ with its first derivative $d\Pi^I_1 / d\theta =$
\[-(2/\theta) A(\theta) c' - k' > 0 \text{ for } \theta \text{ with } \Pi_1^I(\theta) > 0; \] \[\Pi_1^{II} \text{ strictly decreasing with } d\Pi_1^{II}/d\theta = 2a(\sigma) [A(\theta) + 2B(\theta)] c' < 0; \] \[\Pi_1^{III} \text{ increasing with } d\Pi_1^{III}/d\theta = -4a(\sigma) [(1 + 2/\sigma) A(\theta) - B(\theta)] c' - k' \text{ for } \theta \text{ with } \Pi_1^{III}(\theta) > 0; \] \[\Pi_1^{IV} \text{ decreasing with } d\Pi_1^{IV}/d\theta = 0, \text{ where } a(\sigma) = (1 + 1/\sigma) / (3 + 4/\sigma)^2.\]

This is intuitively trivial: The advantage of certification decreases as its technology standard becomes more stringent, which requires higher investment and production costs; The tightening of the standard is advantageous to the non-certifying firm because it increases the firm’s ability to exploit its lower production costs. In general, these profit functions need not be concave in $\theta$.\(^{14}\) By the monotonicity of the second-stage profit functions, we also see that $\Pi_1^I(\theta) \geq \Pi_1^{II}(\theta)$ for all $\theta \geq \theta^{12}$ and $\Pi_1^{III}(\theta) \geq \Pi_1^{IV}(\theta)$ for all $\theta \geq \theta^{34}$. In Lemma 2 in Appendix, I show that these two important ‘thresholds’ $\theta^{34}$ and $\theta^{12}$ exist and are in deed unique. These relationships, combined with Fact 1, establish the following:

**Fact 2.** The equilibrium number $n^*$ of certifying firms is monotonically increasing in $\theta$ in the following sense:

(i) If $\theta^{34} < \theta^{12}$, $n^* = 0$ for $\theta \leq \theta^{34}$, $n^* = 1$ for $\theta^{34} \leq \theta \leq \theta^{12}$, and $n^* = 2$ for $\theta \geq \theta^{12}$.

(ii) If $\theta^{12} \leq \theta^{34}$, $n^* = 0$ or $2$ for $\theta \leq \theta^{12}$ and $n^* = 2$ for $\theta \geq \theta^{34}$.

A corollary to this fact is that a unique Subgame Perfect Nash Equilibrium wherein only one firm certifies attains if and only if $\theta^{34} < \theta^{12}$ and $\theta \in (\theta^{34}, \theta^{12})$. Figure 1 illustrates graphically how the structure of the first-stage game is determined as a function of $\theta$ for the case in which $\theta^{34} < \theta^{12}$.

\[\text{[Figure 1]}\]

From a regulator’s point of view, the case with $\theta^{34} < \theta^{12}$ is more important. As will be discussed in detail, neither total output nor total emissions from an equilibrium with $n^* = 1$ is necessarily higher than those from an equilibrium with $n^* = 2$. As a result, net total surplus, calculated as in (4), will not necessarily be lower when $n^* = 1$ than when $n^* = 2$.

\(^{14}\)Under A1 and A2, $\Pi_1^{II}$ is surely convex whereas $\Pi_1^I$ and $\Pi_1^{III}$ are concave if $c$ is sufficiently convex.
$n^* = 2$. If $\theta^{12} < \theta^{34}$ and, therefore, $n^*$ is either 0 or 2, then the regulator’s problem simplifies significantly. Because the net total surplus is negative (i.e. $-F$) when $n^* = 0$, assuming that the total surplus is sufficiently large when $n^* = 2$ at least in some non-empty segment on $\Omega$, it is sufficient to pick $\theta^s$ that maximizes the total surplus given $n^* = 2$. Moreover, my numerical simulations appear to show that the set of primitives that support the case with $\theta^{12} \leq \theta^{34}$ is limited. For these reasons, I focus on the case with $\theta^{34} < \theta^{12}$ in the subsequent analyses.\footnote{A sufficient condition for $\theta^{34} < \theta^{12}$ and its proof are available from the author upon request.}

One interesting characteristic of our model setup is that it can generate a continuum of cutoff values that support the Prisoner’s Dilemma (PD) outcome. In previous theoretical studies, it is often presumed, implicitly or explicitly, that firms invest in environmental technologies and enter green markets if such markets offer profitable investment opportunities (e.g. Cremer & Thisse, 1994; Arora and Gangopadhyay, 1995; Swallow & Sedjo, 2000; Sedjo & Swallow, 2002; Bansal & Gangopadhyay, 2003; Amacher et al., 2004; Engel, 2004; Conrad, 2005; Lombardini-Piipinen, 2005). However, it appears equally plausible that cases exist where firms must invest and enter the green markets because such strategy is a strictly dominant strategy even though they know that such a strategy will strictly decrease their profits. Figure 2 exhibits two numerical examples, one in which the PD outcome attains and the other in which it does not. Using Lemmas 3 and 4 in Appendix, we can establish sufficient conditions for the existence of the PD outcome.

**Lemma 5.** For a given set of primitives $\delta = (\alpha, \bar{\omega}, \epsilon, \lambda)$ with $L^{24}(\omega; \delta) < 0$, there exists an investment cost function $k$ that yields a range $(\theta^{12}, \theta^{14})$, each $\theta$ on which supports a SPNE wherein both firms certify even though such an outcome is strictly Pareto-dominated by the status-quo outcome.

\[\text{[Figure 2]}\]
Of course, the net welfare effects may be still positive even when the PD outcome arises. Thus, the regulator need not to avoid such an outcome necessarily. Nonetheless, there are at least three reasons why the regulator may need to care about this outcome. First, ecolabeling has been promoted as a voluntary mechanism. However, the existence of the PD outcome implies that firms may be forced to adopt environmental technologies that strictly reduce their profits. This may interfere with the spirit of the voluntary labeling scheme.\footnote{In the mechanism design literature, some authors define "voluntary participation" constraints as no decrease in participants’ private profits relative to their status-quo or no-participation profits (e.g. Smith and Shogren, 2002).} Second, the PD outcome may not be sustainable if the duopolists have an opportunity to collude (say, play an infinitely repeated version of this game). Finally, the net social surplus as calculated in (4) does not include investment costs, and thus, may overestimate the welfare effects when this outcome attains.

IV. Socially Optimal Technology Standard

The regulatory authority’s objective is to set a technology standard \( \theta^* \) that maximizes the net surplus (4) given the primitives \( \delta \). However, in so doing, the authority faces two dilemmas. First, as shown in the previous section, \( n^* \) is an increasing function of \( \theta \): i.e. if the technology standard is tight, less firms would find it profitable to enter the ecolabeled segment. Second, because \( \theta \) affects marginal production costs, total output and total emissions are (non-linear) functions of \( \theta \) for each fixed \( n^* \). Because the net total surplus depends on the equilibrium output \( X^*(\theta) \) as well as \( n^*(\theta) \), it will be a discontinuous function of \( \theta \):

Assuming \( \theta_{34} < \theta_{12} \),

\[
NS(\theta) = NS(X^*(\theta), n^*(\theta)) \equiv \begin{cases} 
-F & \text{if} \; \theta \leq \theta_{34} \\
TS(\theta, 1) - TS(\bar{\omega}, 0) - F & \text{if} \; \theta_{34} \leq \theta \leq \theta_{12} \\
TS(\theta, 2) - TS(\bar{\omega}, 0) - F & \text{if} \; \theta_{12} \leq \theta
\end{cases} \quad (7)
\]

where \( TS(\theta, n) \) is the total surplus net of environmental damages as a function of \( \theta \) when...
the number of certifying firms is \( n \). Moreover, as mentioned earlier, multiple equilibria can arise at \( \theta = \theta^{12} \) or \( \theta^{34} \). For analytical tractability, I simply assume that the regulator can choose the equilibrium that attains the largest net surplus.\(^{17}\)

Let us first show the main result of this section that binary ecolabeling per se can never achieve the first-best efficient outcome. The result obtains because binary labeling is an imperfect policy instrument on a number of important accounts. First, binary labeling gives an incomplete signal to consumers. Second, ecolabeling gives rise to a number of economic interactions between and across firms. The interaction among consumers via equilibrium pricing distorts (average) consumer demand for the labeled and non-labeled goods. Furthermore, firms interact strategically both in the output market and in technology adoption. These factors jointly limit the regulator’s ability to achieve the first-best outcome. As a result, the "socially optimal" standard is second-best from the outset.

**Proposition 1.** Under binary ecolabeling, the first-best efficient outcome can never be achieved, with the following inequality:

\[
W(\omega^{FB}) > \max_{\theta} TS(X^*(\theta), n^*(\theta)).
\]

Therefore, the socially optimal standard is second-best in nature.

*Proof:* First, the maximal \( \theta^s \) of (7) maximizes the RHS of the above inequality. Second, \( \theta^s \) must maximize \( TS(\theta, n) \) for some \( n \in \{0, 1, 2\} \). Furthermore, by A3, we have \( \max_\theta TS(\theta, 2) \geq \max_\theta TS(\theta, 1) \geq TS(\bar{\omega}, 0) \). Thus, I only need to show \( W(\omega^{FB}) = \max_\theta W(\theta) > \max_\theta TS(\theta, 2) \). This holds trivially if \( W(\theta) > TS(\theta, 2) \) for all \( \theta \).

By (5), we have

\[
W(\theta) = \frac{1}{2} \frac{(\alpha - c(\theta))^2}{1 + \theta^2},
\]

where \( \omega \) is simply replaced by \( \theta \) because both \( \omega \) and \( \theta \) are in the same unit. To compare

\(^{17}\)Of course, we can eliminate multiple equilibria by imposing additional equilibrium-selection criteria such as the elimination of weakly dominated equilibria. However, I decide not to do so because it will only complicate our discussion without providing real policy implications.
and $TS(\theta, 2)$, we can simply compare the two multiplicative factors on $(\alpha - c(\theta))^2$.

Suppose by contradiction that

$$\frac{1}{2(1 + \theta^2)} \leq \frac{2}{3} - \frac{2}{9} (1 + \theta^2).$$

Manipulating both sides yields

$$1 \leq \theta^2 (1 - \theta^2).$$

But $\theta^2 (1 - \theta^2) \in [0, 1)$ for $\theta^2 \in [0, 1)$ and $\theta^2 (1 - \theta^2) < 0$ for all $\theta^2 \geq 1$, a contradiction.

$Q.E.D.$

As the proof of the proposition suggests, the first-best outcome cannot be achieved even without the fixed-cost barriers to certification. In practice, the magnitude of efficiency loss depends crucially on the structure of the economy, particularly the distribution of altruism $\lambda$ and the fixed-cost function $k$. Since no clear-cut analytical results are feasible as to the magnitude of inefficiency, I will use a concrete numerical example to "demonstrate" the following facts:

**Fact 3.** Non-trivial cases exist in which the regulator can only set the optimal technology standard for ecolabeling at a level much looser than the first-best level and, therefore, efficiency loss, defined as the difference between the first-best total surplus and the maximum attainable level of total surplus via ecolabeling, is quite large.

In the numerical example, I assume identical consumers with $\beta_i = \beta$, $c(\omega) = \alpha \exp(-d\omega)$, and $k(\omega) = K(\omega - \bar{\omega})^2$, where $\alpha = 10$, $\beta = 0.5$, $d = 3.5$, $\bar{\omega} = 1$ and $K = 10$ or 70. We can verify that these parametric assumptions satisfy $A1$-$A3$, and the relative size $k/c$ is in a reasonable range. Figure 3-(a) and 3-(b) illustrate how the first-best and the second-best (ecolabeling) outcomes differ, respectively, with $K = 10$ and $K = 70$. First, note that there are efficiency losses in both cases. More importantly, efficiency loss is small in Figure 3-(a) whereas it is quite large in Figure 3-(b). This contrast pins down the dilemma that the
The increase in investment costs decreases the firms’ profit margins and undermines the regulator’s ability to set a tight technology standard. As a result, the economically viable region for certification (i.e. $[\theta^{34}, \theta^{12}] \cup [\theta^{12}, \bar{\omega}]$) is smaller in Figure 3-(b) than in Figure 3-(a). When $K = 70$, the authority would wish to choose $\hat{\theta}$, which attains the maximum of $TS(\cdot; 2)$ on $\Omega$. This is the maximum level of (net) total surplus that could have been attained if both firms had participated in certification. However, at such a cutoff level, no firm would actually certify because firms act strategically. If the authority indeed chooses $\hat{\theta}$, it obtains a negative social cost $-F$ (because $TS(\bar{\omega}, 0)$ could have been attained without the labeling effort). This point must be taken seriously, as many ecolabeling initiatives worldwide often set technology standards based on engineering or scientific criteria, with no or little consideration of economic incentives. Similar results hold for changes in the distribution of altruism. An increase in the mass of the selfish would decrease an average demand for the labeled good, and therefore, it will prevent the regulator from raising the standard.

More formally, we can establish the following, the proof of which can be found in Appendix.

**Proposition 2.** (i) For a given set of primitives $\delta = (\alpha, \bar{\omega}, c, \lambda)$, let $k_0$ and $k_1$ be investment cost functions, which satisfy $A2$ and $k_0(\omega) < k_1(\omega)$ for all $\omega \in \Omega$. Then we have $\theta^{12}(\delta, k_0) \leq \theta^{12}(\delta, k_1)$ and $\theta^{34}(\delta, k_0) \leq \theta^{34}(\delta, k_1)$. (ii) For a given set of primitives $\delta = (\alpha, \bar{\omega}, c, k)$, let $\lambda_0$ and $\lambda_1$ be two cumulative distribution functions on $(0, 1)$ such that $\sigma(\lambda_0) < \sigma(\lambda_1)$ and $\tau(\lambda_0) < \tau(\lambda_1)$. Then we have $\theta^{12}(\delta, \lambda_0) < \theta^{12}(\delta, \lambda_1)$ and $\theta^{34}(\delta, \lambda_0) < \theta^{34}(\delta, \lambda_1)$.

This result has a real policy implication. If the labeling authority wishes to induce high levels of firms’ abatement technology adoption, the authority may opt for policies to lower entry costs associated with certification such as subsidies on certification fees or abatement
investments. Furthermore, if the authority observes either the consumer demand is highly heterogeneous or the fixed costs of meeting the standard are too high, it needs to compromise on the loose standard. For example, Fischer et al. (2005) cites the estimates of the costs of forest certification in the U.S., which vary but can be as low as $0.55 per acre for preparation. For those engaged in illegal or unsustainable logging in developing countries, however, the costs of meeting the certification standard will be prohibitively high. This may explain why developing countries currently account for only 8% of the total certified area (Fischer et al., 2005), although, ironically, many forest certification programs were originally established to tackle deforestation in these countries.

There is a caveat to the analyses presented above. There can be a "jump" in $\theta^*$ in that, as $\lambda$ or $k$ changes, the optimum $\theta^*$ may jump from $[\theta^{12}, \tilde{\omega}]$ to $[\theta^{34}, \theta^{12}]$. This means that it can be optimal to induce only one firm into the ecolabeled market rather than two firms. This case happens even under A3, because the total surplus on the range of $\theta$ that supports $n^* = 1$ can be still higher than that on the range that supports $n^* = 2$, for the following three reasons. First, total output can be higher when $n^* = 1$ than when $n^* = 2$ because the decrease in demand for a non-labeled good could be well offset by the increase in demand for a labeled good. Second, total emissions can be lower when $n^* = 1$ than when $n^* = 2$ because the emission per unit of output (= the cutoff) that supports $n^* = 1$ may be significantly lower than that supports $n^* = 2$. Third, the total production costs can be lower when $n^* = 1$ than when $n^* = 2$.\(^{18}\)

V. Heterogeneous Technology Endowments

All analyses in the preceding sections can extend easily to the case with heterogeneous technology endowments. In general, there will be threshold cutoffs $\theta'$ and $\theta''$ such that $n^* = 0$ on $[0, \theta']$, = 1 on $[\theta', \theta'']$, and = 2 on $[\theta'', \tilde{\omega}]$ where $\tilde{\omega} = \max \{\tilde{\omega}_1, \tilde{\omega}_2\}$. An optimal standard $\theta^*$ needs to be chosen so as to maximize the net total surplus, and thus, the regulatory

\(^{18}\)I have confirmed this "jump" result with numerical examples similar to the one above.
authority must consider how setting $\theta^*$ will affect $n^*$. However, there are subtle differences in the analysis when firms are endowed with different technology endowments.

A key difference is that the regulator can now choose a discriminatory standard such that $\bar{\omega}_1 \leq \theta \leq \bar{\omega}_2$. Such a discriminatory policy may be often popular in political debates, for several reasons. First, large corporations with advanced abatement technologies may lobby for the policy that favors their technologies. Second, firms are generally against the ‘technology-inducing’ standard, which requires technologies that are not currently available or economically viable.\(^\text{19}\) Thus, the regulator may be constrained to choose $\theta \geq \min \{\bar{\omega}_1, \bar{\omega}_2\}$. An important question then is, how different such a discriminatory policy is from the technology-inducing standard. In this section, I focus on the impact of labeling standard $\theta$ on total emissions rather than on (net) total surplus. This is because we know from the outset that a constrained maximum subject to $\theta \in [\bar{\omega}_1, \bar{\omega}_2]$ is no greater than an unconstrained maximum and that if the constraint is binding, such a discriminatory policy will be inefficient. I expect, therefore, that the policy evaluation is done in terms of second-best criteria such as emissions reductions relative to the status quo. Moreover, there appears to be a presumption among ecolabeling practitioners that discriminatory labeling would automatically decrease emissions. On the contrary, the analysis in this section suggests that discriminatory labeling could increase emissions relative to the status quo.

Without loss of generality, I assume $\bar{\omega}_1 < \bar{\omega}_2$. Assume further that the cost of certification except for technology investments is negligible. Under these assumptions, any $\theta \in [\bar{\omega}_1, \bar{\omega}_2]$ would favor firm 1 (advantageous firm) because it can certify its product without additional costs. Note, however, that even when $\bar{\omega}_1 \leq \theta$, firm 1 can also choose not to certify, in which case consumers treat goods produced by firm 1 as non-environmentally friendly because they cannot directly observe $\bar{\omega}_1$. It is easy to show that when $\bar{\omega}_1 \leq \theta$, it is a dominant strategy

\(^{19}\)EPA often locks in technologies at the best available control technologies.
for firm 1 to certify. Furthermore, provided that firm 1 certifies, firm 2 certifies if and only if
\[
\left( \frac{\alpha + c(\tilde{\omega}_1) - 2c(\theta)}{3 + 4/\sigma} \right)^2 - k(\tilde{\omega}_2 - \theta) \geq \left( 1 + \frac{1}{\sigma} \right) \left[ \frac{\alpha + c(\tilde{\omega}_1) - 2c(\tilde{\omega}_2)}{3 + 4/\sigma} \right]^2. \tag{8}
\]

The left-hand side of this inequality monotonically decreases as $\theta$ decreases whereas the right-hand side is constant given the model primitives. Let $\gamma$ be such that (8) holds with equality. If $\gamma > \tilde{\omega}_1$, there will be regions $[\tilde{\omega}_1, \gamma]$ where $n^* = 1$ and $[\gamma, \tilde{\omega}_2]$ where $n^* = 2$. Clearly, the number of firms with effective technology adoption is $n^* - 1$. If firm 2 does not certify, for example, there will be no effective technology adoption because firm 1’s technology will be unchanged. To see that ecolabeling may increase emissions, let us consider two cases: (a) $\gamma > \tilde{\omega}_1$ and $\theta \in [\tilde{\omega}_1, \gamma]$ and (b) $\gamma \leq \tilde{\omega}_1$ and $\theta \in [\tilde{\omega}_1, \tilde{\omega}_2]$.

Case (a): $\gamma > \tilde{\omega}_1$ and $\theta \in [\tilde{\omega}_1, \gamma]

Note that this corresponds to the case of no effective technology adoption. Without a labeling program, consumers treat two goods identical and equally ecologically unfriendly. By the argument above, with labeling firm 1 certifies whereas firm 2 does not. Total equilibrium emissions in these cases are given by

\[
E^*_{\text{no label}} = \tilde{\omega}_1 \left( \frac{\tau A}{3} \right) + \tilde{\omega}_2 \left( \frac{\tau B}{3} \right),
\]

\[
E^*_{\text{label, } n^* = 1} = \tilde{\omega}_1 \left( \frac{A + 2/\sigma(\alpha - c(\tilde{\omega}_1))}{3 + 4/\sigma} \right) + \tilde{\omega}_2 \left( \frac{B}{3 + 4/\sigma} \right),
\]

where $A = \alpha + c(\tilde{\omega}_2) - 2c(\tilde{\omega}_1)$, $B = \alpha + c(\tilde{\omega}_1) - 2c(\tilde{\omega}_2)$. To see the maximal impact of labeling, assume that all consumers are altruistic, so we have $\tau \simeq 1/2$ and $\sigma \simeq 1$. Manipulating these

---

\[1\text{The main result of this section may not hold if I assume that, in the absence of the labeling program, consumers consider both firms’ products are neutral, so that the distributional parameter } \tau = E[1/(1 + \beta)] \text{ does not enter the demand function. Under the alternative assumption, we would have } E^*_{\text{no label}} = \tilde{\omega}_1 \left( A/3 \right) + \tilde{\omega}_2 \left( B/3 \right), \text{ instead. In this case, we can prove that binary environmental labeling always reduces total emission relative to the status quo without labeling.} \]
expressions with $\tau = 1/2$, we obtain $E_{\text{no label}}^* > E_{\text{label}}^*$ if and only if

$$\bar{\omega}_1 \left(3A - \frac{4}{\sigma}C\right) > \bar{\omega}_2 \left(3B - \frac{4}{\sigma}B\right),$$

(9)

where $C = 2\alpha - c(\bar{\omega}_1) - c(\bar{\omega}_2)$. Note that $A < B$ because $\tau A/3$ ($\tau B/3$) is the equilibrium quantity for firm 1 (firm 2), which has a higher (lower) production cost. Moreover, because $\alpha > c(\bar{\omega}_1) > c(\bar{\omega}_2)$, we have $C > B$. Thus, the bracketed term in LHS is always smaller than that in RHS. Moreover, because $\sigma \simeq 1$, the bracketed terms on both sides of the inequality are negative. This can be interpreted to mean that the equilibrium quantity for the certifying firm (i.e. firm 1) is higher with labeling than without labeling. The relationship reverses for the non-certifying firm. Thus, when the majority of consumers are altruistic, ecolabeling yields an expected impact. However, given $0 < \bar{\omega}_1 < \bar{\omega}_2$, we can certainly have a combination of $(\bar{\omega}_1, \bar{\omega}_2)$ such that LHS is less than RHS.

Remark: This result is intuitive. When two products differ by abatement technology and one with low emissions is certified while the other with high emissions is not certified, the consumer demand for the labeled good increases whereas that for the non-labeled good decreases. Duopolists choose quantities in response to this demand change. However, the demand for the certified product may increase too much to the extent it outweighs the positive impact from the shift in demand. In general, we would expect that the total emissions from the industry will be lower with the labeling program than without if and only if this quantity impact (i.e. LHS of (9)) is smaller in magnitude than the relative abatement efficiency (i.e. $\bar{\omega}_2/\bar{\omega}_1$). The above result says that such a condition is not always satisfied.

Remark: Note that this result obtains because in our model, the distribution of consumers can give rise to the asymmetric impacts of labeling on demand for labeled and non-labeled goods. In the vertical differentiation model, in contrast, each consumer buys a fixed number of units from exactly one of the firms. Thus, if the demand shifts from a dirtier firm to a cleaner firm, the overall impact must always result in a net reduction in emissions. To see
this, suppose that the duopolists’ abatement technologies are \( \bar{\omega}_1 < \bar{\omega}_2 \) (prior to the third-party labeling) and that the labeling makes the firms’ technologies perfectly observable. Assuming that ‘divesting’ from the current technologies is costly, the new technology levels after introduction of labeling must be such that \( \omega_j \leq \bar{\omega}_j, j = 1, 2 \). If the demand shift occurs, it happens simply as the changes in the market share that must sum to zero. Let \( x_1 \) and \( x_2 \) be the pre-labeling equilibrium quantities and \( d \geq 0 \) the demand shift. Then, it must follow that

\[
E^*_{\text{no label}} = \bar{\omega}_1 x_1 + \bar{\omega}_2 x_2 > \omega_1 (x_1 \pm d) + \omega_2 (x_2 \mp d) = E^*_{\text{label}} \text{ provided that } \omega_j \leq \bar{\omega}_j, j = 1, 2.
\]

Thus, in the vertical differentiation model, emissions cannot increase with the introduction of ecolabeling.

**Case (b):** \( \gamma \leq \bar{\omega}_1 \) and \( \theta \in [\bar{\omega}_1, \bar{\omega}_2] \)

In this case, effective technology adoption occurs because firm 2 adopts a technology that meets the standard. It is easy to see that total emissions will be minimized at \( \theta = \bar{\omega}_1 \), because \( n^* = 2 \) for all \( \theta \geq \bar{\omega}_1 \) and total emissions when \( n^* = 2 \) will be given by

\[
E^*_{\text{label}, n^* = 2} = \bar{\omega}_1 \left( \frac{\alpha + c(\theta) - 2c(\bar{\omega}_1)}{3} \right) + \theta \left( \frac{\alpha + c(\bar{\omega}_1) - 2c(\theta)}{3} \right),
\]

which has a positive first derivative with respect to \( \theta \) provided that \( c' < 0 \) and \( 0 \leq \bar{\omega}_1 \leq \theta \). Now, let \( \theta = \bar{\omega}_1 \) and compare \( E^*_{\text{no label}} \) with \( E^*_{\text{label}, n^* = 2} \). In this case, we can see that an increase in the mass of the altruistic population (i.e. a decrease in \( \tau \)) negatively impact this comparison. When \( \tau \approx 1/2 \), we obtain \( E^*_{\text{no label}} > E^*_{\text{label}, n^* = 2} \) if and only if

\[
\bar{\omega}_2 (\alpha + c(\bar{\omega}_1) - 2c(\bar{\omega}_2)) > \bar{\omega}_1 (3\alpha - 2c(\bar{\omega}_1) - c(\bar{\omega}_2)).
\]

However, it is immediate to see that a set of primitives \( (\alpha, c, \bar{\omega}_1, \bar{\omega}_2) \) exists such that this inequality does not hold. We thus "proved" by cases (a) and (b) the following:

**Proposition 3.** Suppose abatement technology endowments are heterogenous (i.e. \( \bar{\omega}_1 < \bar{\omega}_2 \)).

With a discriminatory standard, \( \bar{\omega}_1 \leq \theta \leq \bar{\omega}_2 \), binary environmental labeling may not always
reduce total emissions relative to the status quo. This is true even if a Subgame Perfect Nash Equilibrium of the game is for both firms to certify for all \( \theta \in [\bar{\omega}_1, \bar{\omega}_2] \) and the standard is chosen to lock in the best available abatement technology (i.e. to set \( \theta = \bar{\omega}_1 \)).

VI. Discussion

Ecolabeling has the potential to achieve significant emissions reductions and improve the social welfare. This paper investigates the effects of binary ecolabeling on the strategic competition of firms in environmental technology and in the output market. Because consumers can not directly observe the actual abatement efficiency of the products under binary labeling, setting an appropriate technology standard for certification can have significant welfare impacts.

The paper offers a set of negative results on binary ecolabeling along with important policy implications. One should not, however, interpret these negative results to mean that regulatory authorities must disregard the idea of binary labeling to help solve environmental problems. On the contrary, these results are meant to guide ecolabeling policies on a large class of environmental and resource problems. As demonstrated with a numerical example, there are cases in which setting an appropriate standard per se can potentially achieve the welfare level sufficiently close to the first-best level. Binary ecolabeling should be directed to this class of environmental and resource problems, provided that no first-best policies (e.g. taxes or permit markets) are readily available to tackle them due to, say, political inertia.

The paper also shows that arbitrary discriminatory labeling to favor environmentally more advanced firms may not only result in an inefficient outcome but can also increase total emissions relative to the status quo. This calls upon a question into the current ecolabeling practice, which often uses an engineering approach to set technology standards. The paper does not argue either for or against binary labels. Rather, it argues that, provided that the binary scheme continues to be a popular mode for many ecolabeling initiatives worldwide, attending to the economic impacts of setting particular technology standards for certification
is critical.

The model presented in this paper can be extended in a number of important ways. Previous studies have discussed the impact of taxes and subsidies when consumers are environmentally aware and firms’ abatement technologies are perfectly observable. Unlike in the previous studies (e.g. Arora and Gangopadhyay, 1995; Bansal & Gangopadhyay, 2003; Lombardini-Riipinen, 2005), neither a uniform ad valorem tax nor subsidy necessarily affects firms’ technology adoption. Under binary labeling, firms’ abatement technologies are unobservable to consumers and firms have no incentive to invest more in abatement technologies than required by the standard even when they have a larger profit margin in the output market. Because the uniform ad valorem tax (subsidy) decreases (increases) the equilibrium quantities (and thereby the equilibrium profits) almost equally across all three cases in the second stage (i.e. in the quantity competition), the Nash outcomes of the first-stage game are essentially intact.

On the contrary, a discriminatory ad valorem tax/subsidy or an emission tax likely affects firms’ technology adoption because such a policy will change the relative profitability of certification. Via a slightly different mechanism, a subsidy on abatement technology investments also facilitates technology adoption, and thereby, it may help achieve an outcome sufficiently close to the first-best. When adopting an emission tax, however, one needs to account for three disturbing factors: (1) partial internalization of pollution externality due to consumer’s altruistic behavior; (2) the negative externality from oligopolistic competition in the output market; and (3) the negative externality from fixed costs and oligopolistic competition in technology adoption (Spence, 1976). All these factors jointly affect the efficient emission tax rate, and therefore, the tax rate that achieves the first best outcome may be lower or higher than the marginal social cost of emissions. The paper did not consider these issues in order to maintain its focus on the trade-off in setting the standard, leaving them for future research.

This paper analyzed a static three-stage game describing the competing interests be-
etween firms and the labeling authority. Given the results presented, an interesting extension of the model would be to investigate dynamic updating rules for technology standards in the presence of endogenous technology innovation. The authority would like to update technology standards as firms’ technologies advance. On the one hand, the prospect for the future update of the technological standards gives firms incentives to slow down investments in abatement technologies. On the other hand, firms may be better off investing all at once rather than in a chunky manner, in which case they may invest more than required in the current period in expectation of the future update. Furthermore, firms may prefer to invest (and certify) to quickly reap the benefits of higher demand for certified products before they face tighter technology standards in the following periods. In the presence of stochastic investment outcomes, firms are also likely to engage in a ‘patent race.’ Firms’ incentives to influence the standard-making process as well as heterogeneity of firms’ efficiencies in technology innovation are also important in this context. Because these competing interests affect the speed of investments as well as the equilibrium number of certifying firms in each period, the labeling authority must face a more difficult decision to set an appropriate standard in each period as the industry technology advances over time. The analyses of this type are also left for future research.
Lemma 2. Suppose A1 and A2 hold. Then there exist $\theta^{12}$ and $\theta^{34}$ in the interior of $[0, \bar{\omega}]$ such that (i) $\theta^{12} \leq \theta \implies L^{12}(\theta) \geq 0$ and (ii) $\theta^{34} \leq \theta \implies L^{34}(\theta) \geq 0$. These thresholds are unique in $[0, \bar{\omega}]$.

Proof: We first establish that $L^{12}(\bar{\omega}) > 0$ and $L^{34}(\bar{\omega}) > 0$. Substituting $k(\bar{\omega}) = 0$ (by A2) and $B(\bar{\omega}) = 0$ into the expressions in Table 2, we readily see that

$$L^{12}(\bar{\omega}) = \left\{\frac{1}{9} - \frac{1+1/\sigma}{3+4/\sigma} \right\} A(\bar{\omega})^2 > 0,$$

$$L^{34}(\bar{\omega}) = \left\{\left(1 + \frac{2}{\sigma}\right)^2 / \left(3 + 4/\sigma\right)^2 - \frac{\tau}{9}\right\} A(\bar{\omega})^2 > 0,$$

for all combinations of $\sigma \in (1, \infty)$ and $\tau \in (1/2, 1)$. Now, from the expressions in Table 2 and using $c(0) = \alpha$ and $k(0) = K > 0$, we have

$$L^{12}(0) = -\left(1 + \frac{1}{\sigma}\right) \left[\frac{2B(\theta)}{3+4/\sigma}\right]^2 < 0,$$

$$L^{34}(0) = -\tau \left(\frac{A(\bar{\omega})}{3}\right)^2 < 0.$$

Moreover, $L^{12}(\theta)$ is continuous and increasing with the first derivative $dL^{12}/d\theta = d\Pi^I_1/d\theta - d\Pi^I_1/d\theta > 0$, for all $\theta$ at which $\Pi^I_1(\theta) > 0$ and $= 0$ for $\theta$ with $\Pi^I_1(\theta) = 0$. Thus, by the intermediate value theorem, there exists $\theta^{12} \in (0, \bar{\omega})$ such that $L^{12}(\theta^{12}) = 0$. Moreover, we must have $dL^{12}/d\theta > 0$ at $\theta^{12}$. Suppose not. Then it must follow that $\Pi^I_1 = \Pi^{II}_1 = 0$, which contradicts that $\Pi^{II}_1 > 0$ for all $\theta \in [0, \bar{\omega}]$. Therefore, strict monotonicity at $\theta^{12}$ implies that $\theta^{12}$ is unique and that $\theta^{12} \leq \theta \iff L^{12}(\theta) \geq 0$. Similarly, $L^{34}(\theta)$ is continuous, differentiable, and strictly increasing with the first derivative $dL^{34}/d\theta = d\Pi^{II}_1/d\theta > 0$, for all $\theta$ at which $\Pi^{II}_1(\theta) > 0$ and $= 0$ for $\theta$ with $\Pi^{II}_1(\theta) = 0$. Thus, analogous arguments establish the existence and uniqueness of $\theta^{34}$ with $\theta^{34} \leq \theta \iff L^{34}(\theta) \geq 0$.

Q.E.D.
Lemma 3. Suppose A1 holds. There exists unique $\theta^{24}$ in the interior of $[0, \bar{\omega}]$ such that $\theta^{24} \leq \theta \iff L^{24}(\theta) \leq 0$ if and only if $L^{24}(\bar{\omega}) < 0$.

Proof: Again, using the expressions in Table 2 and $c(0) = \alpha$, we have

$$L^{24}(0) = \left(1 + \frac{1}{\sigma}\right) \left[ \frac{2A(\bar{\omega})}{3 + 4/\sigma} \right]^2 - \tau \left(\frac{A(\bar{\omega})}{3}\right)^2 > 0.$$  

We know that $\Pi_{1}^{I}$ is strictly increasing in $\theta$. Thus, if $L^{24}(\bar{\omega}) > 0$, there should be no $\theta^{24}$ with $L^{24}(\theta^{24}) = 0$. If $L^{24}(\bar{\omega}) = 0$, $\theta^{24} = \bar{\omega}$. This establishes necessity. To show sufficiency, suppose $L^{24}(\bar{\omega}) < 0$. Again by the intermediate value theorem, there exists $\theta^{24} \in (0, \bar{\omega})$ such that $L^{24}(\theta^{24}) = 0$. Moreover, strict monotonicity of $\Pi_{1}^{I}$ implies that $\theta^{24}$ is unique and $\theta^{24} \leq \theta \iff L^{24}(\theta) \leq 0$. Q.E.D.

Lemma 4. Let $\delta = (\alpha, \bar{\omega}, c, \lambda)$ be a given set of primitives (except $k$). Suppose that A1 holds and $L^{24}(\bar{\omega}; \delta) < 0$. Then there exists an investment cost function $k$ such that there is a range $(\theta^{12}, \theta^{14}) \subset [0, \bar{\omega}]$.

Proof: The proof proceeds in two steps. In the first step, I show that provided that $(\delta, k)$ satisfies A1 and A2 with $L^{24}(\bar{\omega}; \delta) < 0$, a necessary and sufficient condition for $\theta^{12} < \theta^{14}$ is that $L^{14}(\theta^{24}) < 0$. In the second step, we shall see that given $\delta$, which satisfies A1 and $L^{24}(\bar{\omega}; \delta) < 0$, there is always a function $k$ that satisfies A2 and $L^{14}(\theta^{24}) < 0$.

Because $\Pi_{1}^{I}$ is increasing, $\Pi_{1}^{II}$ decreasing, and $\Pi_{1}^{IV}$ constant under A1 and A2, the uniqueness of $\theta^{12}$, combined with $L^{24}(\bar{\omega}) < 0$, implies that $\theta^{12} < \theta^{14}$ if and only if $\theta^{24} < \theta^{14}$. Thus, we only need to show that under A1 and A2, $\theta^{24} < \theta^{14}$ if and only if $L^{14}(\theta^{24}) < 0$.

To show sufficiency, suppose $L^{14}(\theta^{24}) < 0$. Because $L^{14}(\bar{\omega}) < 0$, by the intermediate value theorem, there exists $\theta^{14} \in (\theta^{24}, \bar{\omega})$, so that we obtain $\theta^{24} < \theta^{14}$. For necessity, suppose $\theta^{24} < \theta^{14}$ and suppose by contradiction $L^{14}(\theta^{24}) \geq 0$. It then follows that by strict monotonicity of $\Pi_{1}^{I}$, $L^{14}(\theta) \geq 0$ for all $\theta \geq \theta^{24}$. We have two cases: if $L^{14}(\theta^{24}) = 0$, $\theta^{24} = \theta^{14}$; and if $L^{14}(\theta^{24}) > 0$, we have $\theta^{14} < \theta^{24}$. Thus, we obtain $\theta^{14} \leq \theta^{24}$, a contradiction.
Now it remains to show the existence of an investment function that satisfies A2 and yields the property $L^{14}(\theta^{24}) < 0$. In Lemma 3, we have seen that $\theta^{24}$ exists and unique given $L^{24}(\bar{\omega}) < 0$. Recall that we did not need A2 for this result. For a given set of primitives $\delta = (\alpha, \bar{\omega}, c, \lambda)$ (without an investment cost function $k$), we can define $\theta^{24}(\delta)$. If $L^{14}(\theta^{24}(\delta)) < 0$, we are done and any $k$ satisfying A2 would yield $L^{14}(\theta^{24}(\delta; k)) < 0$. Suppose $L^{14}(\theta^{24}(\delta)) \geq 0$. Find $\varepsilon > 0$ such that $L^{14}(\theta^{24}(\delta)) - \varepsilon < 0$. Then define $k(\theta) = K - \varepsilon \theta / (\bar{\omega} - \theta^{24}(\delta))$ where $K = \varepsilon \bar{\omega} / (\bar{\omega} - \theta^{24}(\delta)) > 0$. This function trivially satisfies A2 and $L^{14}(\theta^{24}(\delta; k)) < 0$. \hfill Q.E.D.

Proof of Lemma 5.

By Lemma 4, we can always find $k$ such that $\theta^{12} < \theta^{14}$ given $\delta$ with $L^{24}(\bar{\omega}; \delta) < 0$. It remains to show that each $\theta \in (\theta^{12}, \theta^{14})$ supports SPNE in which both enter. There are two generic cases: (i) $\theta^{34} \leq \theta^{12}$ and (ii) $\theta^{12} < \theta^{34}$. If $\theta^{34} \leq \theta^{12}$, for all $\theta \in (\theta^{12}, \theta^{14}) \subset [\theta^{34}, \bar{\omega}]$, an associated SPNE is $(ec, ec)$ with $\Pi^{IV}_I > \Pi^I_I$. If $\theta^{12} < \theta^{34}$, then it may be $\theta^{12} < \theta^{14} \leq \theta^{34}$ or $\theta^{12} < \theta^{34} < \theta^{14}$. In the former case, each $\theta \in (\theta^{12}, \theta^{14})$ supports two distinct SPNEs: one in which both enter and the other in which none enters. Thus, it still supports $(ec, ec)$ with $\Pi^{IV}_I > \Pi^I_I$. In the latter case, by the property of real numbers, there exists a real number $\theta'$ with $\theta^{34} < \theta' < \theta^{14}$. For such $\theta'$, there is a unique SPNE with $(ec, ec)$ and $\Pi^{IV}_I > \Pi^I_I$. \hfill Q.E.D.

Proof of Proposition 2.

(i) Let us first show that $\theta^{12}(\delta, k_0) \leq \theta^{12}(\delta, k_1)$ and $\theta^{34}(\delta, k_0) \leq \theta^{34}(\delta, k_1)$. Suppose by contradiction that $\theta^{12}(\delta, k_0) > \theta^{12}(\delta, k_1)$. Let $\tilde{\Pi}^I_I(\theta) \equiv \Pi^I_I(\theta) + k(\theta)$. We know that $d\tilde{\Pi}^I_I/d\theta - d\Pi^{IV}_I / d\theta > 0$. Thus, $\theta^{12}(\delta, k_0) > \theta^{12}(\delta, k_1)$ implies $\tilde{\Pi}^I_I(\theta^{12}(\delta, k_0)) - \Pi^{IV}_I(\theta^{12}(\delta, k_0)) > \tilde{\Pi}^I_I(\theta^{12}(\delta, k_1)) - \Pi^{IV}_I(\theta^{12}(\delta, k_1))$. By definition, $\Pi^I_I(\theta^{12}) - \Pi^{IV}_I(\theta^{12}) = \tilde{\Pi}^I_I(\theta^{12}) - k_1(\theta^{12}) - \Pi^{IV}_I(\theta^{12}) = 0$ for $i = 0, 1$. We thus have $k_0(\theta^{12}(\delta, k_0)) > k_1(\theta^{12}(\delta, k_1))$. It then follows that $k_0(\theta^{12}(\delta, k_0)) > k_1(\theta^{12}(\delta, k_1)) > k_0(\theta^{12}(\delta, k_1))$. However, because $k'_0 < 0$ by A2
and \( \theta^{12} (\delta, k_0) > \theta^{12} (\delta, k_1) \), we must have \( k_0 (\theta^{12} (\delta, k_0)) < k_0 (\theta^{12} (\delta, k_1)) \), a contradiction.

Analogous arguments establish \( \theta^{34} (\delta, k_0) \leq \theta^{34} (\delta, k_1) \).

(ii) To show \( \theta^{12} (\delta, \lambda_0) < \theta^{12} (\delta, \lambda_1) \), note that

\[
\frac{\partial L^{12}}{\partial \sigma} = -\frac{(\sigma^{-2}) (5 + 4/\sigma) [A(\theta) + 2B(\theta)]}{(3 + 4/\sigma)^3} < 0.
\]

It follows then that for each \( \theta \), we have \( L^{12} (\theta; \lambda_0) > L^{12} (\theta; \lambda_1) \). Therefore, \( L^{12} (\theta^{12} (\delta, \lambda_0); \lambda_0) = L^{12} (\theta^{12} (\delta, \lambda_1); \lambda_1) = 0 \), combined with \( \partial L^{12} / \partial \theta > 0 \), must imply \( \theta^{12} (\delta, \lambda_0) < \theta^{12} (\delta, \lambda_1) \).

Moreover, note that

\[
\frac{\partial L^{34}}{\partial \sigma} = -\frac{(4\sigma^{-2}) X^{III} (\theta) [A(\theta) + 2B(\theta)]}{(3 + 4/\sigma)^3} \leq 0 \quad \text{and} \quad \frac{\partial L^{34}}{\partial \tau} = -\frac{1}{9} < 0.
\]

Hence, we have \( L^{34} (\theta; \lambda_0) > L^{34} (\theta; \lambda_1) \) for each \( \theta \), which implies \( \theta^{34} (\delta, \lambda_0) < \theta^{34} (\delta, \lambda_1) \).

Q.E.D.
References


