When Should Uncertain Nonpoint Emissions be Penalized in a Trading Program?

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Abstract

When nonpoint source pollution is stochastic and the damage function is convex, intuition might suggest it is more important to control a nonpoint pollution source than a point source. Earlier research has provided sufficient conditions such that the permit price for a unit of ex-ante expected emissions should be higher than the permit price for a unit of certain emissions. Herein we provide a set of necessary and sufficient conditions such that this is the case. An approach to testing for the validity of the condition set is available, and has been applied to a related problem.

Keywords: agricultural pollution, multiple inputs, permit trading, social optimality, trading ratio, water quality.

JEL classification: Q1, Q2, D2, D8

The categorization of pollutant emission sources into point and nonpoint has proven to be useful in large part because point sources are generally viewed as being more certain, more readily monitored, and more readily controlled. Nonpoint sources, such as nitrogen and phosphorus entering waterways from cropland, can depend on such random events as rainfall and temperature. A question that has been raised in the literature is how these distinctions should affect the use of instruments to optimally manage the expected damage from pollution. In particular, suppose point source emissions and expected nonpoint source emissions are both subject to permit requirements where free trade in permits is allowed. Then should the permit price for a unit of expected emissions from a nonpoint source exceed the price for a unit of emissions from a point source?

The issue is important because nonpoint emissions can dominate loadings in watersheds.[1] A focus on point sources creates economic distortions and can severely limit the ability to control overall emissions. A variety of point-nonpoint trading schemes have been implemented over the years. These have achieved only limited success, in part because of problems with specifying what is to be traded and the terms of trade.[2] The issue is also important because, in practice, the implemented point-nonpoint trading ratio has tended to place a higher price per unit of pollution on point source emissions permits.[3] By contrast, theoretical models to date lean toward a higher price per unit (expected) pollution on nonpoint source emissions permits (Horan 2001).

Aspects of the impact of the existence of emissions uncertainty on optimal incentives have been addressed in Shortle (1990, p. 794), in Malik, Letson, and Crutchfield (1993, p. 964), in Zhang and Wang (2002, p. 171), in Horan and Shortle (2005, p. 346), and elsewhere. The intent of this article is to provide definitive conditions under which emissions uncertainty should induce a larger price on expected emissions from nonpoint sources. To make our point we study

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[1] See Table 2 in Horan, Shortle, and Abler (2002) on nitrogen loads in the Susquehanna River Basin.

[2] Further discussions on design issues can be found in Horan *et al.* (2001), Horan, Shortle, and Abler (2002) and Farrow *et al.* (2005).

a version of the standard model (Shortle 1990; Horan and Shortle 2005).

Model

Our model is as in Horan and Shortle (2005). In it there is a single point source, labeled as 'firm,' and a single nonpoint source, labeled as 'farm.' The regulator seeks to minimize the expected total cost to society as represented by the sum of the *i*) cost of reducing point source emissions, *ii*) cost of reducing the level of an input associated with nonpoint source emissions, and *iii*) expected damage done by emissions. The regulator controls emissions by imposing a binding maximum total level of point source emissions, imposing a binding maximum total level of expected nonpoint source emissions, allocating permits as property rights and allowing trade in these permits. Permits in point source and nonpoint source markets can be converted to the other at an exchange rate determined by the regulator. As a special case of the Horan-Shortle (H&S) framework, we assume that the total levels of point source and nonpoint source expected emissions permits are set at their socially optimal levels. Equilibrium permit prices can be compared to identify the regulator-determined exchange rate, what is known in the literature as the trading ratio.

The firm produces point source pollution to the amount e. This firm can control the extent of emissions, but at cost $c(e)$. It is costly at the margin to do so but the marginal cost of control decreases as the emission level increases, i.e., $c_e(e) < 0 < c_{ee}(e)$ where subscripts indicate derivatives. The farm's emissions are stochastic, but do depend on a single input choice made by the farm. Actual emissions cannot be observed by the regulator. With input choice level $x \geq 0$, farm emissions amount to $r(x,\theta)$. Here θ is a reference random variable with finite support $[\theta_l, \theta_u]$, distribution $F(\theta)$ and density $f(\theta)$. The consequence of an increase in θ for nonpoint emissions is held to be adverse, or $r_\theta(x,\theta) \geq 0 \; \forall \theta \in [\theta_l, \theta_u], \forall x \geq 0$.

[3] Horan (2001) provides evidence on this and suggests a reason why it is so.

The input-conditioned mean value of farm emissions is $\mu(x) = E[r(x,\theta)] = \int_{\theta_l}^{\theta_u} r(x,\theta) dF(\theta)$. Quantity $\mu(x)$ is the expected load from the nonpoint source and it is assumed that the input increases expected load, $\mu_x(x) > 0$. The nonpoint source can reduce use of the input in order to reduced expected emissions, but at profit loss $g(x)$ where $g_x(x) < 0 < g_{xx}(x)$. The social damage function is of form $D(z)$, $z = e + r(x,\theta)$, where the argument is the additive sum of point and nonpoint emissions. Damage is strictly increasing and weakly convex in emissions, or $D_z(z) > 0$ and $D_{zz}(z) \geq 0$.

The regulator sets point emissions and expected nonpoint emissions permit levels such that the equilibrium permit prices are q and p, respectively. These prices are set to support the socially optimal point and expected nonpoint emissions levels. Price ratio p/q can be viewed as the number of units of point emissions that can be exchanged for one nonpoint emission in permit markets that allow these permit conversions.[4]

Suppose the point source has initial allocations amounting to e^0 units of point source emissions permits and zero units of nonpoint source expected emissions permits. Suppose too that the nonpoint source has initial allocations amounting to r^0 units of nonpoint source expected emissions permits and zero units of point source emissions permits. The private optimality problems under permit market price-taking are:

$$\begin{aligned} &\min_e S^{pt}(e), \qquad && S^{pt}(e) = c(e) + q \times (e - e^0); \\ &\min_x S^{npt}(x), \qquad && S^{npt}(x) = g(x) + p \times (\mu(x) - r^0). \end{aligned} \tag{1}$$

The regulator seeks to align incentives with the social optimality problem of minimizing the sum of private costs and expected social damage.

The social objective function and first-order optimality conditions are:[5]

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[4] For example, were $p/q = 1.1$ then eleven units of point source pollution emissions would trade for ten units of nonpoint source units of pollution emissions.

[5] An alternative objective function that appears to find favor with policy makers is the

(2) $$T^{uncer}(e,x) = c(e) + g(x) + \int_{\theta_l}^{\theta_u} D(z^*)dF(\theta);$$
$$c_e(e^*) + \int_{\theta_l}^{\theta_u} D_z(z^*)dF(\theta) = 0; \qquad g_x(x^*) + \int_{\theta_l}^{\theta_u} D_z(z^*)r_x(x^*,\theta)dF(\theta) = 0;$$

where (e^*, x^*) is the socially optimal choice vector, $z^* = e^* + r(x^*, \theta)$, and $c_e(e^*)$ is understood to mean the obvious derivative evaluated at the point $e = e^*$. Bear in mind that choosing x is equivalent to choosing $\mu(x)$. From (1) and (2) it is clear that the socially optimal permit prices, $q = q^*$ and $p = p^*$, and optimal trading ratio, $\tau^* = p^* / q^*$, are[6]

(3) $$q^* = \int_{\theta_l}^{\theta_u} D_z(z^*)dF(\theta) = -c_e(e^*); \qquad p^* = \frac{\int_{\theta_l}^{\theta_u} D_z(z^*)r_x(x^*,\theta)dF(\theta)}{\mu_x(x^*)} = -\frac{g_x(x^*)}{\mu_x(x^*)};$$
$$\tau^* = \frac{p^*}{q^*} = \frac{g_x(x^*)}{\mu_x(x^*)c_e(e^*)} = \frac{\int_{\theta_l}^{\theta_u} D_z(z^*)r_x(x^*,\theta)dF(\theta)}{\mu_x(x^*)\int_{\theta_l}^{\theta_u} D_z(z^*)dF(\theta)} = 1 + \frac{\mathrm{Cov}\left(D_z(z^*), r_x(x^*,\theta)\right)}{\mu_x(x^*)\int_{\theta_l}^{\theta_u} D_z(z^*)dF(\theta)}.$$

By contrast, the objective function to be minimized and first-order conditions under certain nonpoint pollution at level $\mu(x)$ are

(4) $$T^{cer}(e,x) = c(e) + g(x) + D\left(e + \mu(x)\right);$$
$$c_e(e^*) + D_z\left(e^* + \mu(x^*)\right) = 0; \qquad g_x(x^*) + D_z\left(e^* + \mu(x^*)\right)\mu_x(x^*) = 0.$$

In this case the optimal trading ratio is

(5) $$\hat{\tau} = \frac{g_x(x^*)}{\mu_x(x^*)c_e(e^*)} = 1,$$

so that

(6) $$\tau^* - \hat{\tau} = \tau^* - 1 = \frac{\mathrm{Cov}\left(D_z(z^*), r_x(x^*,\theta)\right)}{\mu_x(x^*)\int_{\theta_l}^{\theta_u} D_z(z^*)dF(\theta)} \overset{sign}{=} \mathrm{Cov}\left(D_z(z^*), r_x(x^*,\theta)\right).$$

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minimization of costs subject to a maximum specified probability of exceeding a threshold damage level. We chose to follow the present specification because the threshold model will only maximize social welfare under certain circumstances.

[6] In other studies the trading ratio is defined as $t^* = q^* / p^*$, but the algebra and intuitive interpretations are more direct under the inverse ratio we consider.

So whenever the covariance is positive then $\tau^* \geq 1$. This means that the optimal ratio of the permit price for an expected unit of uncertain nonpoint emissions to the price for a unit of certain point emissions exceeds that when nonpoint emissions are certain. That is, if $\tau^* > 1$ then comparatively stronger incentives are provided to control nonpoint sources. Whenever the covariance is negative then the reverse is true.

To summarize, our structural assumptions are:

SA: *i*) $c_e(e) < 0 < c_{ee}(e)\ \forall e \geq 0$, *ii*) $g_x(x) < 0 < g_{xx}(x)\ \forall x \geq 0$, *iii*) damage is of form $D(z)$, $z = e + r(x,\theta)$, where $D_z(z) > 0$ and $D_{zz}(z) \geq 0$, *iv*) nonpoint pollution level $r(x,\theta)$ satisfies $r_\theta(x,\theta) \geq 0\ \forall \theta \in [\theta_l, \theta_u], \forall x \geq 0$, and *v*) θ has distribution $F(\theta)$ such that $d\int_{\theta_l}^{\theta_u} r(x,\theta)dF(\theta)/dx > 0\ \forall x \geq 0$.

Results in the Literature

Our eqn. (6) is a variant of eqn. (10) in H&S.[7] Although the model is slightly different, our eqn. (6) is analogous to the expressions given in (6) and (7) of Shortle (1990). The well-known covariance, or Čebyšev, inequality (Mitrinović 1970) asserts the following sufficient conditions to sign covariance; if $m(\theta)$ and $n(\theta)$ are increasing functions of random variable θ then $\mathrm{Cov}[m(\theta), n(\theta)] \geq 0$. We will present findings from the literature in context with the covariance inequality.

With $m(\theta) = D_z[e^* + r(x^*,\theta)]$ then $m_\theta(\theta) = D_{zz}[e^* + r(x^*,\theta)]r_\theta(x^*,\theta) \overset{sign}{=} r_\theta(x^*,\theta)$. With $n(\theta) = r_x(x^*,\theta)$ then $n_\theta(\theta) = r_{x\theta}(x^*,\theta)$. So, given assumptions SA, eqn. (6) implies that $\tau^* \geq 1$ at equilibrium choices whenever R*i*) $r_{x\theta}(x^*,\theta) \geq 0\ \forall \theta \in [\theta_l, \theta_u]$. On the other hand, $\tau^* \leq 1$ at equilibrium choices whenever R*ii*) $r_{x\theta}(x^*,\theta) \leq 0\ \forall \theta \in [\theta_l, \theta_u]$. This captures the essence of

[7] Bear in mind that $\tau^* = 1/t^*$, see footnote 6.

statements made in Shortle (1990), H&S and others on the role of emissions uncertainty in determining whether some penalty should be placed on expected emissions. Appendix A provides further details, as well as some clarifications, on conclusions in the literature.

R*i*) and R*ii*) are sets of sufficient conditions to sign $\tau^* - 1$ for all increasing and convex damage functions. But they are not necessary conditions since, as we shall show, monotonicity on $r_x(x^*, \theta)$ in the sense of (say) $r_{x\theta}(x^*, \theta) \geq 0 \; \forall \theta \in [\theta_l, \theta_u]$ is not required. Neither are they particularly weak conditions since, for example, condition set R*i*) is likely to be quite onerous in practice. The main intent of this article is to provide a complete characterization of conditions such that $\tau^* - 1$ can be signed for all increasing and convex damage functions. We will also point to empirical methods for doing so.

Analysis

With $E[r_x(x^*, \theta) \,|\, \theta \leq \hat{\theta}]$ as the expected value of $r_x(x^*, \theta)$ given that $\theta \leq \hat{\theta}$, the following is demonstrated in Appendix B:

Proposition 1. Under SA, then the optimal trading ratio satisfies $\tau^* \geq (\leq) 1$ for all considered damage functions if any one of the three condition sets *i*)-*iii*) are satisfied: *i*)

(7) $$E[r_x(x^*, \theta) \,|\, \theta \leq \hat{\theta}] \leq (\geq)\, E[r_x(x^*, \theta)] \quad \forall \hat{\theta} \in [\theta_l, \theta_u].$$

ii) $r_x(x^*, \hat{\theta}) \geq (\leq)\, E[r_x(x^*, \theta) \,|\, \theta \leq \hat{\theta}] \; \forall \hat{\theta} \in [\theta_l, \theta_u]$.

iii) $r_{x\theta}(x^*, \theta) \geq (\leq)\, 0 \; \forall \theta \in [\theta_l, \theta_u]$.

Condition set *i*) is both necessary and sufficient. Condition set *iii*) implies condition set *ii*) and *ii*) implies condition set *i*).

For $E[r_x(x^*, \theta) \,|\, \theta \leq \hat{\theta}] \leq E[r_x(x^*, \theta)] \; \forall \hat{\theta} \in [\theta_l, \theta_u]$, the interpretation of (7) is that the expected marginal nonpoint emissions conditional on $\theta \leq \hat{\theta}$ is less than the unconditional expected marginal nonpoint emissions.[8] The condition captures a weak form of conditional dependence between the nonpoint emission and the source of randomness. So, for the $\leq$ direction, at low θ values the marginal contribution of input x to nonpoint emissions is low on average. This is unfortunate, bearing in mind the convex damage function. The marginal contribution of the input to nonpoint emissions tends to be low when marginal damage is low (when θ is low) and high when marginal damage is high. So the input tends, on average, to cause more emissions under states of nature such that additional damage at the margin is more costly. Relation (7) is necessary in the following sense. If the inequality in the $\leq$ direction does not apply for some interval on $[\theta_l, \theta_u]$ then there exists an increasing and convex damage function such that $\tau^* < 1$. Part *iii*) is the standard application of the covariance inequality, as discussed earlier.

From a policy perspective, Proposition 1 is interesting because it is not clear to the authors why (7) in the $\leq$ direction should be considered to be more reasonable than in the $\geq$ direction. If (7) is true in the $\geq$ direction, then $\tau^* \leq 1$ and the socially optimal price of a permit to emit an expected unit from a nonpoint source is lower than the socially optimal permit price to emit a unit from a point source. Suppose, for the sake of concreteness, that θ is rainfall and that x is nitrogen. Using *iii*), the case could be put forward that more nitrogen makes nonpoint emissions more sensitive to rainfall so that $r_{x\theta}(\cdot) \geq 0 \; \forall \theta \in [\theta_l, \theta_u]$ and $\tau^* \geq 1$. But the case is very much dependent upon context. If nitrogen promotes plant survival then more plants survive to absorb soil nitrogen in high rainfall states and less nitrogen enters waterways.

A special damage function warrants attention. Suppose that $D(z) = \alpha_0 + \alpha_1 z + \alpha_2 z^2, \alpha_1 > 0$, $\alpha_2 > 0$, so that $\mathrm{Cov}\left(D_z(z^*), r_x(x^*, \theta)\right) \overset{sign}{=} \mathrm{Cov}\left(r(x^*, \theta), r_x(x^*, \theta)\right)$. The latter is the condition studied in Shortle (1990). Although $r_\theta(x, \theta) \geq 0$ has been assumed, the covariance still cannot be

[8] From a study of Shaked and Shanthikumar (2007) and elsewhere, we cannot identify a

signed unless further assumptions are made on how $r_x(x^*,\theta)$ changes with θ. Turning to when (7) does not apply in either direction over all $\hat{\theta} \in [\theta_l, \theta_u]$, then the only recourse is to attempt to refine one's beliefs about $D(z)$ and $r(x,\theta)$. Stronger conditions can allow for a weaker condition set to replace (7).[9] The authors do not have beliefs about $D(z)$ and $r(x,\theta)$ beyond those outlined in SA.

Plural different inputs are generally applied in nonpoint production activities, so it would be reassuring to establish that condition (7) has multivariate analogs. We will confirm this to be true for just two inputs, where the multi-input extension is then straightforward. Let there be a second nonpoint input labeled as y. The input-conditioned mean value of farm emissions is now $\mu(x,y) = E[r(x,y,\theta)] = \int_{\theta_l}^{\theta_u} r(x,y,\theta)dF(\theta)$, with the obvious extension in notation. The second input also increases expected load, $\mu_y > 0$. The profit decline associated with this second nonpoint source is $g(x,y)$ where $g_y < 0 < g_{yy}$. The social objective function and optimality conditions become:[10]

$$T^{uncer}(e,x,y) = c(e) + g(x,y) + \int_{\theta_l}^{\theta_u} D(z^*)dF(\theta);$$

$$(8) \qquad c_e(e^*) + \int_{\theta_l}^{\theta_u} D_z(z^*)dF(\theta) = 0; \qquad g_x(x^*,y^*) + \int_{\theta_l}^{\theta_u} D_z(z^*)r_x(x^*,y^*,\theta)dF(\theta) = 0;$$

$$g_y(x^*,y^*) + \int_{\theta_l}^{\theta_u} D_z(z^*)r_y(x^*,y^*,\theta)dF(\theta) = 0.$$

The socially optimal permit prices $(q, p^x, p^y) = (q^*, p^{*,x}, p^{*,y})$ and optimal trading ratios $(\tau^{*,x}, \tau^{*,y}) = (p^{*,x}/q^*, p^{*,y}/q^*)$, are

stochastic dominance relation that is equivalent to condition (7).

[9] Perform a further integration-by-parts on (B2) in Appendix B.

[10] Second-order sufficient convexity conditions are assumed to hold.

$$q^* = \int_{\theta_l}^{\theta_u} D_z(z^*)dF(\theta); \qquad p^{*,x} = \frac{\int_{\theta_l}^{\theta_u} D_z(z^*) r_x(x^*, y^*, \theta)dF(\theta)}{\mu_x(x^*, y^*)};$$

$$(9) \qquad p^{*,y} = \frac{\int_{\theta_l}^{\theta_u} D_z(z^*) r_y(x^*, y^*, \theta)dF(\theta)}{\mu_y(x^*, y^*)};$$

$$\tau^{*,x} = 1 + \frac{\mathrm{Cov}\left(D_z(z^*), r_x(x^*, y^*, \theta)\right)}{\mu_x(x^*, y^*)\int_{\theta_l}^{\theta_u} D_z(z^*)dF(\theta)}; \qquad \tau^{*,y} = 1 + \frac{\mathrm{Cov}\left(D_z(z^*), r_y(x^*, y^*, \theta)\right)}{\mu_y(x^*, y^*)\int_{\theta_l}^{\theta_u} D_z(z^*)dF(\theta)}.$$

A faithful adaptation of the Proposition 1 proof shows that $\tau^{*,i} \geq (\leq) 1$ for $i \in \{x, y\}$ and all convex damage functions if and only if $E[r_i(x^*, y^*, \theta) \,|\, \theta \leq \hat{\theta}] \leq (\geq)\, E[r_i(x^*, y^*, \theta)]\; \forall \hat{\theta} \in [\theta_l, \theta_u]$ for the chosen $i \in \{x, y\}$. It is interesting that, beyond ensuring convexity, interactions between x and y in $\mu(x, y)$ are not relevant for these qualitative results.

Discussion

Can condition set (7) be tested to provide statistical evidence in either direction? Empirical estimates of $E[r_x(x^*, \theta)]$ and $E[r_x(x^*, \theta) \,|\, \theta \leq \hat{\theta}]$ could be arrived at using observations from experimental plots. Or they could be estimated by simulation using agronomic models that seek to account for rainfall and nitrogen, such as the Soil & Water Assessment Tool (SWAT). There are, however, likely to be violations of (7) in one direction or the other due to sampling error. Some formal structure on errors will be necessary if statistical tests of the hypotheses generated by (7) are to be conducted. Concerning a different problem, signing the marginal risk premium of an input under uncertainty, Roosen and Hennessy (2003) have used nitrogen application and corn yield data when testing a condition that is almost identical to (7).[11] They applied methods from the literature on testing stochastic orderings. These methods have seen significant advances in recent years, in particular to allow for dependence across observations.[12] Dependence is likely

[11] See especially their equations (5) and (7).
[12] Early work includes Tolley and Pope (1988), while Davidson and Duclos (2000), Zheng

to exist in any agronomic application. Thus, tests for (7) are possible if the data are available.

Conclusion

This article has identified a set of necessary and sufficient conditions under which the existence of nonpoint emissions uncertainty motivates a price on a permit to emit a unit of expected pollution that is larger than the price on a permit to emit a certain unit of pollution. The condition set is testable using existing empirical methods. Tests are warranted because in practice trading ratios are not consistent with the preponderant belief in the literature that emissions from nonpoint sources should be penalized.

(2002), and Linton, Maasoumi, and Whang (2005) have provided increasingly robust models.

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Appendix A

Using our model notation, i.e., paraphrasing, Shortle (1990) states (p. 794) "The assumption that the damage cost function is convex ($D_{zz} > 0$) implies that the sign of $\text{Cov}\left(D_z(z^*), \partial z^* / \partial x\right)$ is the same as that of $\text{Cov}\left(z^*, \partial z^* / \partial x\right)$." This is not true because <u>all</u> convex damage functions must be considered. It is true that statement '$\text{Cov}\left(D_z(z^*), \partial z^* / \partial x\right) \le (\ge)\, 0$ for all convex damage functions' implies statement '$\text{Cov}\left(z^*, \partial z^* / \partial x\right) \le (\ge)\, 0$.' This follows because $D(z) = (½)z^2$ is convex. But statement '$\text{Cov}\left(z^*, \partial z^* / \partial x\right) \le (\ge)\, 0$' does not imply '$\text{Cov}\left(D_z(z^*), \partial z^* / \partial x\right) \le (\ge)\, 0$ for all convex damage functions.' This can be demonstrated by counter-example. Let there be three equi-probable states of nature. In ordinate form the first is $(z^*, \partial z^* / \partial x) = (1,1)$. The ordinates for the other two states are $(2,3)$ and $(3,0.8)$. Then $\text{Cov}\left(z^*, \partial z^* / \partial x\right) = -0.06\dot{6} < 0$. But consider also some other function, say $D(z) = (⅓)z^3$. This is convex on $z > 0$. Then $\text{Cov}\left((z^*)^2, \partial z^* / \partial x\right) = 1.13\dot{3} > 0$, i.e., the inequality is not valid for a different convex function.

Appendix B

Proof of Proposition 1. Part *i*): To establish <u>sufficiency</u>, write

(B1) $$\text{Cov}\left(D_z(e^* + r(x^*,\theta)), r_x(x^*,\theta)\right) = \int_{\theta_l}^{\theta_u} D_z(e^* + r(x^*,\theta))\left(r_x(x^*,\theta) - E[r_x(x^*,\theta)]\right) dF(\theta),$$

where $\int_{\theta_l}^{\theta_u} \left(r_x(x^*,\theta) - E[r_x(x^*,\theta)]\right) dF(\theta) \equiv 0$. Using an integration-by-parts, eqn. (B1) may alternatively be written as

(B2) $$\begin{aligned} &\text{Cov}\left(D_z(e^* + r(x^*,\theta)), r_x(x^*,\theta)\right) = D_z(e^* + r(x^*,\hat{\theta}))\int_{\theta_l}^{\hat{\theta}} \left(r_x(x^*,\theta) - E[r_x(x^*,\theta)]\right) dF(\theta)\Big|_{\hat{\theta}=\theta_l}^{\hat{\theta}=\theta_u} \\ &-\int_{\theta_l}^{\theta_u} D_{zz}(e^* + r(x^*,\hat{\theta}))r_\theta(x^*,\hat{\theta})\int_{\theta_l}^{\hat{\theta}} \left(r_x(x^*,\theta) - E[r_x(x^*,\theta)]\right) dF(\theta)\, d\hat{\theta} \\ &= -\int_{\theta_l}^{\theta_u} D_{zz}(e^* + r(x^*,\hat{\theta}))r_\theta(x^*,\hat{\theta})\int_{\theta_l}^{\hat{\theta}} \left(r_x(x^*,\theta) - E[r_x(x^*,\theta)]\right) dF(\theta)\, d\hat{\theta}, \end{aligned}$$

where $\int_{\theta_l}^{\theta_l}\left(r_x(x^*,\theta)-E[r_x(x^*,\theta)]\right)dF(\theta)=\int_{\theta_l}^{\theta_u}\left(r_x(x^*,\theta)-E[r_x(x^*,\theta)]\right)dF(\theta)=0$ has been used to simplify. Thus, from (6) and $D_{zz}(e^*+r(x^*,\hat{\theta}))r_\theta(x^*,\hat{\theta})\geq 0$ (by SA), it follows that $\tau^*\geq(\leq)1$ if $\int_{\theta_l}^{\hat{\theta}}\left(r_x(x^*,\theta)-E[r_x(x^*,\theta)]\right)dF(\theta)\leq(\geq)0\ \forall\hat{\theta}\in[\theta_l,\theta_u]$ or

(B3) $\quad \int_{\theta_l}^{\hat{\theta}} r_x(x^*,\theta)dF(\theta)\leq(\geq)\int_{\theta_l}^{\hat{\theta}} E[r_x(x^*,\theta)]dF(\theta)=F(\hat{\theta})E[r_x(x^*,\theta)] \quad \forall\hat{\theta}\in[\theta_l,\theta_u]$.

Upon rearrangement, i.e., use of $E[r_x(x^*,\theta)\,|\,\theta\leq\hat{\theta}]=\int_{\theta_l}^{\hat{\theta}} E[r_x(x^*,\theta)]dF(\theta)/F(\hat{\theta})$, (B3) is shown to be as in (7) above.

The standard approach to demonstrate <u>necessity</u> in *i*) is taken. For condition set (7), suppose the sign of $\int_{\theta_l}^{\hat{\theta}}\left(r_x(x^*,\theta)-E[r_x(x^*,\theta)]\right)dF(\theta)$ in (B2) is reversed and non-zero on a positive measure set $A\subseteq[\theta_l,\theta_u]$. The value of $D_{zz}(e^*+r(x^*,\hat{\theta}))$ could be arbitrarily large, but still finite, on a positive measure subset of A such that the sign of the covariance in (B2) is reversed.

Part *ii*): Note that $E[r_x(x^*,\theta)\,|\,\theta\leq\theta_u]=E[r_x(x^*,\theta)]$ so that $E[r_x(x^*,\theta)\,|\,\theta\leq\hat{\theta}]$ increasing (decreasing) in $\hat{\theta}$ implies $E[r_x(x^*,\theta)\,|\,\theta\leq\hat{\theta}]\leq(\geq)\,E[r_x(x^*,\theta)]\,\forall\hat{\theta}\in[\theta_l,\theta_u]$, i.e., relations (7) as given in part *i*). Differentiation establishes $r_x(x^*,\hat{\theta})\geq(\leq)\,E[r_x(x^*,\theta)\,|\,\theta\leq\hat{\theta}]\,\forall\hat{\theta}\in[\theta_l,\theta_u]$ whenever $E[r_x(x^*,\theta)\,|\,\theta\leq\hat{\theta}]$ is increasing (decreasing) in $\hat{\theta}$.

Part *iii*): If $r_x(x^*,\theta)$ is increasing (decreasing) in θ then it follows that $r_x(x^*,\hat{\theta})\geq(\leq)$ $E[r_x(x^*,\theta)\,|\,\theta\leq\hat{\theta}]\,\forall\hat{\theta}\in[\theta_l,\theta_u]$, as given in part *ii*).

As for the statements that *iii*) implies *ii*) and *ii*) implies *i*), it is clear that condition $r_{x\theta}(x^*,\theta)$ $\geq 0\,\forall\theta\in[\theta_l,\theta_u]$ implies condition $r_x(x^*,\hat{\theta})\geq E[r_x(x^*,\theta)\,|\,\theta\leq\hat{\theta}]\,\forall\hat{\theta}\in[\theta_l,\theta_u]$ while we have shown above that this latter condition implies $E[r_x(x^*,\theta)\,|\,\theta\leq\hat{\theta}]\leq E[r_x(x^*,\theta)]\,\forall\hat{\theta}\in[\theta_l,\theta_u]$.
Implications in the other direction follow similarly. ■