Efficiency and Returns to Scale Measurements with Shared Inputs in Multi-activity Data Envelopment Analysis: An Application to Farmers' Organizations in Taiwan

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Abstract

This paper addresses the question how team production promote efficiency of a firm when some inputs can be rewarded on the basis of outputs but some cannot because they are shared among outputs and non-separable. A multi-activity DEA model with variable returns to scale is proposed to provide information on the efficiency performance for organizations with inputs shared among several closely related activities. The model is applied to study the case of 279 farmers' associations in Taiwan. The result suggests that it is important to improve the efficiency of the non-profit oriented activities to improve their overall performances. Three out of four departments of TFAs can gain from economies of scale through expansion, while the remaining one gains through contraction. Thus, policies promoting structural adjustment and consolidations of TFAs would not be inconsistent with public interests.

Keywords: multi-activity DEA, shared inputs, efficiency measure, directional distance function

1. INTRODUCTION

In this article, a micro-level study of the role of team work in firm-level performance will be presented. Based on Alchian and Demsetz (1972)'s analysis of team production, a firm is an entity which brings together a team which is more productive working together than at arm’s length through the market, because of informational problems associated with monitoring of effort. We are, however, not interested in the knowledge transfer or information sharing, but in how this embeds in firms. The question we wish to address in this study is that how team production systems work in practice when some inputs can be rewarded on the basis of outputs but some cannot because they are shared among outputs and non-separable.

Like many developing countries around the world, Taiwan’s farmers' organizations (TFAs) have played an important role in assisting the government throughout the process of her agricultural development. Subsidize credit programs were offered by the government to promote certain policy goals such as assisting farmers to enlarge their operation or to adopt a new technology. The TFAs serve as a venue to assist farmers or rural poor to acquire the low-interest credits to whom regular lenders would not serve. In recent decades, as the favorable conditions for agricultural production declined, the TFAs have also begun to take on a greater role in promoting village construction and enhancing farmers' welfare, thereby helping to bring about wider development. After Taiwan became a member of the WTO in 2002, the Agricultural Development Act was revised in 2003 and the TFAs were given a new role to minimize the impact of WTO entry through upgrading and promoting local products into the global markets.

Initially, the TFAs were designed to provide credit, extension, insurance, and marketing services to their members, most of which are the rural households. Each association consists of four departments to carry out these services. Profits from the credit departments are used for improving cooperative marketing, insurance and extension services whereas the activities of the extension, insurance, and marketing services attract savings to the TFAs which can later serve as loanable funds available to the eligible members.

The close linkages among the services and close ties between the organization and government have made TFAs the most important institutions in financing rural Taiwan. However, the performance varied greatly among the TFAs. By the mid-1990s, some of credit departments of the successful TFAs have rivaled the commercial banks while the others reply heavily on government subsidized credits. On September 2001, the insolvency problem led the government take over 35 poorly-performing credit departments of TFAs by 10 commercial banks. It is widely believed that these grassroots institutions' financial crises are owing to the cost inefficient operations, which falls short of maximizing profits and maintaining healthy levels of capital asset ratios. Some of the causes are inherent in the TFAs' non-profit maximizing orientations, while others are found to be a direct consequence of inefficient operations. Some argue that the subsidized credits create detrimental effects on the TFA's competitiveness because it impairs their incentive to minimize costs. Others focus on the political involvement of the managers of TFAs with the local politicians and related corruption issues. However, the multi-service nature of the organizations and the intra-firm networking in the creation of business are often ignored.

In this study, we propose a Multi-activity Data Envelopment Analysis (MDEA) method to examine the role of internally shared inputs in the efficiency performance of the TFAs. The MDEA model was first introduced by Beasley (1995) and subsequently revised by Mar Molinero (1996), Cook et al. (2000), Jahanshahloo et al. (2004), and Tsai and Mar Molinero (1998, 2002).[1]

[1] The model of Beasley (1995) was constructed in ratio form. Mar Molinero (1996) subsequently revised the model using Shephard's distance function. Cook et al. (2000) proposed a model similar to Beasley (1995) to evaluate a sample of Canadian banks' multi-component efficiency and discussed the relaxations of Beasley's nonlinear model to

It allows us not only to estimate the performance and returns to scale properties of a set of decision making units (DMUs), but also to deal with the problem of determining how much of shared inputs are associated with each activity simultaneously. The efficiency measure derived from the traditional DEA model implicitly assumed that each DMU is equally efficient in all activities, and that the DMU is free to apply any of its inputs to any of its outputs in the most desirable way (Mar Molinero, 1996). In comparison, the MDEA treats each activity heterogeneously and determines how much of shared inputs are associated with each activity simultaneously. Thus, the MDEA can identify the particular areas of strength and weakness of the DMUs by distinguishing which activity operates under its most productive scale.

Furthermore, due to the consideration of bad loans as an undesirable output for the credit departments into our efficiency measure, the directional graph distance function of Yu and Fan (2006) is used in our study. We will also measure the status of RTS. It is noted that the concept of returns to scale for MDEA model has been explored in Tsai and Mar Molinero (1998, 2002). However, our model adopts the directional graph distance function rather than the Shephard distance function. Therefore, we will discuss how to obtain the status of RTS in the directional graph distance function based MDEA model before conducting the empirical analysis.

The remainder of this paper is organized as follows. The next section describes the methodology of MDEA followed by a description of the empirical model. Section three discusses the data and section four presents the empirical results and the final section concludes.

2. METHODOGY

For the purpose of comparison, the traditional DEA model will be introduced prior to the

a linear one. Jahanshahloo et al. (2004) then extended to consider the situation that all components are involved in producing some outputs and non-discretionary factors. All the above MDEA models are evaluated under the technology of CRS. Tsai and Mar Molinero (1998, 2002) extended Mar Molinero's MDEA model into variable returns to scale, and applied it in the National Health Services in the UK.

MDEA model with variable returns to scale technology. In order to allow the joint production of desirable (good) and undesirable (bad) outputs of a DMU, our DEA model is modified by using the directional distance function approach introduced by Luenberger (1992). The directional distance function generalizes Shephard's input and output distance functions by simultaneously scaling inputs and outputs[2], but not necessarily along the rays from the input and output origin.[3] As a consequence, it encompasses Shephard's input and output distance functions (Chambers et al., 1996; Fukuyama, 2003). Therefore, the directional-distance-function-based DEA model would not as restrictive as the Shephard-distance-function-based model.

2.1 Traditional DEA with Directional Distance Function

Let $x = (x_1, x_2, \ldots, x_N) \in R_+^N$ denotes a input vector and $u = (u_1, u_2, \ldots, u_G) \in R_+^G$ a output vector, where *u* composes of the desirable outputs (*y*) and undesirable outputs (*u*), ie $u = (y, b) = (y_1, y_2, \ldots, y_M;\ b_1, b_2, \ldots b_R) \in R_+^{M+R}$. The directional distance function seeking to increase the desirable outputs and decrease the undesirable outputs and inputs directionally can be defined by the following formulation:

$$\vec{D}(x, y, b;\ g) = \sup\{\beta : (x - \beta g_x, y + \beta g_y, b - \beta g_b) \in T\}, \qquad (1)$$

where the nonzero vector $g = (g_x, g_y, g_b)$ determines the "directions" in which inputs, desirable outputs and undesirable outputs are scaled, and the technology reference set $T = \{(x, u) : x \text{ can produce } u\}$ satisfies the assumptions of variable returns to scale, strong

[2] Details of the relationship between directional distance functions and Shephard distance functions can be found in Chung et al. (1997) and Färe and Grosskopf (2000).

[3] The efficiencies associated with Shephard's distance functions are radial efficiency measures assume equi-proportionate adjustments in the variables to be adjusted, whereas the efficiencies associated with directional distance functions are non-radial efficiency measures permit non-proportional adjustments in these variables. One of the criticisms of radial measures is that it does not permit its input and/or output mixes to change. Non-radial measures avoid this criticism as they do not require the observed input and/or output mix to be preserved in obtaining relative efficiency scores. (Chambers and Mitchell, 2001; Glass et al., 2006)

disposability of desirable outputs and inputs, and weak disposability of undesirable outputs.

Suppose there are $k = 1, \cdots, K$ DMUs in the data set. Each DMU uses input $x^k = (x_1^k, x_2^k, \cdots, x_N^k) \in R_+^N$ to jointly produce desirable outputs $y^k = (y_1^k, y_2^k, \cdots, y_M^k) \in R_+^M$ and undesirable outputs $b^k = (b_1^k, b_2^k, \cdots, b_R^k) \in R_+^R$. The piecewise reference technology allowing for variable returns to scale can be constructed as follows:

$$
\begin{aligned}
T = \{(x, y, b):\ & \sum_{k=1}^{K} z^k y_m^k \geq y_m, && m = 1, \ldots\ldots, M. \\
& \sum_{k=1}^{K} z^k b_r^k = b_r, && r = 1, \ldots\ldots, R. \\
& \sum_{k=1}^{K} z^k x_n^k \leq x_n, && n = 1, \ldots\ldots, N. \qquad (2) \\
& z^k \geq 0, && k = 1, \ldots\ldots, K. \\
& \sum_{k=1}^{K} z^k = 1 \},
\end{aligned}
$$

where z_k are the intensity variables to shrink or expand the individual observed activities of DMU k for the purpose of constructing convex combinations of the observed inputs and outputs.

Relative to the reference technology T constructed in (2), traditionally, for each DMU $k' = 1, \cdots, K$, the directional distance function can be obtained by solving the following linear programming problem with $g = (g_x, g_y, g_b) = (x^{k'}, y^{k'}, b^{k'})$, i.e., when the direction chosen is based on observed inputs and outputs:

$$
\vec{D}(x^{k'}, y^{k'}, b^{k'}; -x^{k'}, y^{k'}, -b^{k'}) = \max\ \beta^{k'}
$$

$$
\begin{aligned}
\text{s.t.}\ ! & \sum_{k=1}^{K} z^k y_m^k \geq (1 + \beta^{k'}) y_m^{k'} && m = 1, \cdots, M \\
& \sum_{k=1}^{K} z^k b_r^k = (1 - \beta^{k'}) b_r^{k'} && r = 1, \ldots\ldots, R. \\
& \sum_{k=1}^{K} z^k x_n^k \leq (1 - \beta^{k'}) x_n^{k'} && n = 1, \cdots, N \qquad (3)
\end{aligned}
$$

$$z^k \geq 0, \qquad k = 1, \cdots, K$$

$$\sum_{k=1}^{K} z^k = 1$$

where $\beta^{k'}$ measures the maximum inflation of all desirable outputs and deflation of all inputs and undesirable outputs that remain technically feasible and can be served as a measure of technical inefficiency. If $\beta^{k'} = 0$, then DMU k' operates on the frontier of T with technical efficiency. If $\beta^{k'} > 0$, then DMU k' operates inside the frontier of T. Therefore, the non-radial technical efficiency can be measured as $1 - \beta$.

The efficiency measurement constructed in (3) expands all desirable outputs and contracts all inputs and undesirable outputs at the same rate β. It can be further generalized to accommodate different expansion and contraction ratios as follows:

$$\vec{D}(x^{k'}, y^{k'}, b^{k'}; -x^{k'}, y^{k'}, -b^{k'}) = \max\ \omega_1\beta_1^{k'} + \omega_2\beta_1^{k'} + \omega_3\beta_1^{k'} = \beta^{k'}$$

$$\begin{aligned}
\text{s.t. } & !\sum_{k=1}^{K} z^k y_m^k \geq (1+\beta_1^{k'}) y_m^{k'} && m = 1, \cdots, M\ ; \\
& \sum_{k=1}^{K} z^k b_r^k = (1-\beta_2^{k'}) b_r^{k'} && r = 1, \ldots\ldots, R\ ; \\
& \sum_{k=1}^{K} z^k x_n^k \leq (1-\beta_3^{k'}) x_n^{k'} && n = 1, \cdots, N\ ; \\
& z^k \geq 0, && k = 1, \cdots, K\ ; \\
& \sum_{k=1}^{K} z^k = 1\ .
\end{aligned} \tag{4}$$

The measure $\vec{D}(x^{k'}, y^{k'}, b^{k'}; -x^{k'}, y^{k'}, -b^{k'})$ given in (4) maximizes hyperbolically $\beta^{k'} = \omega_1\beta_1^{k'} + \omega_2\beta_2^{k'} + \omega_3\beta_3^{k'}$ by comparing the observed $(y^{k'}, b^{k'}, x^{k'})$ with the frontier ($(1+\beta_1^{k'})y_m^{k'}$, $(1-\beta_2^{k'})b_r^{k'}$, $(1-\beta_3^{k'})x_n^{k'}$), where $(1+\beta_1^{k'})y_m^{k'}$, $(1-\beta_2^{k'})b_r^{k'}$, and $(1+\beta_3^{k'})x_n^{k'}$ maximize the value of $\omega_1\beta_1^{k'} + \omega_2\beta_2^{k'} + \omega_3\beta_3^{k'}$. The coefficients ω_1, ω_2 and ω_3 are associated with the priorities given to the inputs and outputs and their sum are normalized to unity. The improvement expressed in terms of the percentage desirable outputs, undesirable outputs, and inputs can be measured by $\beta_1^{k'}$, $\beta_2^{k'}$, and $\beta_3^{k'}$ respectively and then used to calculate the weighted efficiency score $\beta^{k'}$ (Yu and Fan, 2006). Note that if we set $\beta_1^{k'} = \beta_2^{k'} = \beta_3^{k'}$, then model (4) degenerates to model (3).

The dual of (4) is shown to be:

$$
\begin{aligned}
\min \quad & -\sum_{m=1}^{M} u_m^{k'} y_m^{k'} + \sum_{n=1}^{N} v_n^{k'} x_n^{k'} + \sum_{r=1}^{R} \rho_r^{k'} b_r^{k'} + \delta^{k'} \\
\text{s.t} \quad & -\sum_{m=1}^{M} u_m^{k'} y_m^{k} + \sum_{n=1}^{N} v_n^{k'} x_n^{k} + \sum_{r=1}^{R} \rho_r^{k'} b_r^{k} + \delta^{k'} \geq 0 \qquad k = 1,\ldots\ldots,K. \\
& \sum_{m=1}^{M} u_m^{k'} y_m^{k'} = \omega_1 \qquad\qquad (5) \\
& \sum_{r=1}^{R} \rho_r^{k'} b_r^{k'} = \omega_2 \\
& \sum_{n=1}^{N} v_n^{k'} x_n^{k'} = \omega_3 \\
& u_m^{k'}, v_n^{k'} \geq 0, \;\; \rho_r^{k'}, \delta^{k'} \;\; free,
\end{aligned}
$$

where $u_m^{k'}, v_n^{k'}$, and $\rho_r^{k'}$ are multipliers for desirable outputs, inputs, and undesirable outputs respectively. Model (5) shows that a measure of technical inefficiency may be defined as follows:

$$
TIE^{k'} = \frac{-\sum_{m=1}^{M} u_m^{k'} y_m^{k'} + \sum_{n=1}^{N} v_n^{k'} x_n^{k'} + \sum_{r=1}^{R} \rho_r^{k'} b_r^{k'} + \delta^{k'}}{\sum_{m=1}^{M} u_m^{k'} y_m^{k'} + \sum_{n=1}^{N} v_n^{k'} x_n^{k'} + \sum_{r=1}^{R} \rho_r^{k'} b_r^{k'}}.
$$

The first constraint in (5) is used to ensure that the cross efficiencies do not exceed unity.

As pioneered in Banker er al. (1984), the shadow price $\delta^{k'}$ on the convex constraints can be used to characterize the scale properties. Fukuyama (2003) indicated that the criteria to determine the status RTS associated with directional distance function by $\delta^{k'}$ are as follows: (i) if $\delta^{k'} > 0$, then DRS prevail; (ii) if $\delta^{k'} = 0$, then CRS prevail; and (iii) if $\delta^{k'} < 0$, then IRS prevail.

In the traditional DEA model, there are other methods to determine the RTS. For example, Charnes et al.(1978) uses the sum of the optimal intensity variable values as a measure for RTS classifications. The scale efficiency index method proposed by Färe et al. (1985) can also be used to test the nature of RTS. This method states that the scale inefficiency of a DMU is due to DRS if the DMU scores the same value under NIRS technology, otherwise it is due to IRS.

2.2 Multi-Activity DEA with Directional Distance Function

Following Tsai and Mar Molinero (1998, 2002) and Yu and Fan (2006), the traditional DEA model is extended to a multi-activity fashion by allowing each activity to grade its performance and RTS property with its own technology frontier. This multi-activity efficiency measure provides a performance measure with activity-based information as part of the aggregate score.

Consider again there are $k = 1, \cdots, K$ DMUs and each engages in I activities. Let $X_k^1, X_k^2, \cdots, X_k^I$ and $X_k^s = (x_{k,1}^s, x_{k,2}^s, \cdots, x_{k,L}^s)$ denotes the dedicated input vector and shared inputs of DMU k respectively, where X_k^i, $i = 1, ..., I$ be the input vector associated solely with the ith activity in which $X_k^i = (x_{k,1}^i, x_{k,2}^i, \cdots, x_{k,N_i}^i)$ while $x_{k,l}^s$, $l = 1, ..., L$ be the input shared by the I activities. Because $x_{k,l}^s$ is a shared input, it is assumed that some portion

$\mu_{k,l}^{i}$ ($0 < \mu_{k,l}^{i} < 1$, $\sum_{i=1}^{I} \mu_{k,l}^{i} = 1$) of this shared input is allocated to the ith activity. In the MDEA model, $\mu_{k,l}^{i}$ is a decision variable to be determined by the DMU. Thus, the ith activity employs X_k^i and $\mu_k^i X_k^s$ to jointly produce desirable output Y_k^i and undesirable output B_k^i.

Following Tsai and Mar Molinero (1998, 2002) and Yu and Fan (2006), the production technology with variable returns to scale and shared inputs for the ith activity can be defined as follows:

$$
\begin{aligned}
T^i = \{ (x^i, y^i, b^i) : \ & \sum_{k=1}^{K} z_k^i y_{k,m_i}^i \ge y_{m_i}^i, && m_i = 1,\ldots\ldots, M_i. \\
& \sum_{k=1}^{K} z_k^i b_{k,r_i}^i = b_{r_i}^i, && r_i = 1,\ldots\ldots, R_i. \\
& \sum_{k=1}^{K} z_k^i x_{k,n_i}^i \le x_{n_i}^i, && n_i = 1,\ldots\ldots, N_i. \\
& \sum_{i=1}^{I} \sum_{k=1}^{K} z_k^i \mu_{k,l}^i x_{k,l}^s \le \sum_{i}^{I} \mu_{k,l}^i x_l^s, && l = 1,\ldots\ldots, L. \qquad (6) \\
& \sum_{i=1}^{I} \mu_{k,l}^i = 1 && l = 1,\ldots\ldots, L. \\
& z_k^i \ge 0, && k = 1,\ldots\ldots, K. \\
& \sum_{k=1}^{K} z_k^i = 1 \}
\end{aligned}
$$

It is known that the uncontrollable variables (i.e. environmental variables) such as location characteristics, labour union power, and government regulations, etc. (Fried et al., 1999) are not traditional inputs, but could influence the efficiency of a DMU. Therefore, if the DMUs operate in different environments, the following constraint can be added into (6) to incorporate the effects.

$$\sum_{k=1}^{K} z_k^i e_{k,h_i}^i \le e_{h_i}^i, \qquad h_i = 1,......, H_i. \tag{7}$$

where e_{k,h_i}^i, $h_i = 1,......, H_i$, are the environmental variables with positive effect on efficiency faced by the *i*th activity of DMU *k*.

Combining (6) and (7), we construct the production possibility set in which the risky outputs, shared inputs and environmental factors are simultaneously considered. Then the aggregate inefficiency of DMU k' ($\beta^{k'}$) weighted by *I* activities' individual inefficiency ($\beta_{k'}^i$) based on the directional distance function and variable returns to scale technology can be measured by the following MDEA model[4].

$$Max \qquad \beta^{k'} = \sum_{i=1}^{I} w^i \beta_{k'}^i \tag{8}$$

$$\text{s. t.} \quad \sum_{k=1}^{K} z_k^i y_{k,m_i}^i \ge (1+\beta_{k'}^i) y_{k',m_i}^i, \qquad m_i = 1,......, M_i. \quad i = 1,......, I \tag{9}$$

$$\sum_{k=1}^{K} z_k^i b_{k,r_i}^i = (1-\beta_{k'}^i) b_{k',r_i}^i, \qquad r_i = 1,......, R_i. \quad i = 1,......, I \tag{10}$$

$$\sum_{k=1}^{K} z_k^i x_{k,n_i}^i \le (1-\beta_{k'}^i) x_{k',n_i}^i, \qquad n_i = 1,......, N_i. \quad i = 1,......, I \tag{11}$$

$$\sum_{i=1}^{I} \sum_{k=1}^{K} z_k^i \mu_{k',l}^i x_{k,l}^s \le \sum_{i}^{I} (1-\beta_{k'}^i) \mu_{k',l}^i x_{k',l}^s, \qquad l = 1,......, L. \tag{12}$$

$$\sum_{k=1}^{K} z_k^i e_{k,h_i}^i \le e_{k',h_i}^i, \qquad h_i = 1,......, H_i. \quad i = 1,......, I \tag{13}$$

$$\sum_{i=1}^{I} \mu_{k',l}^i = 1 \qquad l = 1,......, L. \tag{14}$$

$$\sum_{k=1}^{K} z_k^i = 1 \qquad i = 1,......, I \tag{15}$$

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[4] For simplicity, we assume that the rates for desirable outputs to expand and for inputs and undesirable outputs to contract are the same.

$$z_k^i \geq 0, \qquad k = 1,......,K. \quad i = 1,......,I \tag{16}$$

$$0 < \mu_{k',l}^i < 1, \;\; \beta_{k'}^i \geq 0, \tag{17}$$

where w^i is a positive number which represents the relative importance given to the various activities and their sum are standardized to be equal to 1. This MDEA model is essentially designed to minimize the inputs and undesirable outputs and at the same time maximize the desirable output for each activity.

As mentioned above, we would also like to examine the returns to scale of a DMU. Therefore, the dual form of the above model was developed as follows:

$$\begin{aligned} Min \quad & -\sum_{i=1}^{I}\sum_{m_i=1}^{M_i} u_{m_i}^i y_{k',m_i}^i + \sum_{i=1}^{I}\sum_{n_i=1}^{N_i} v_{n_i}^i x_{k',n_i}^i + \sum_{i=1}^{I}\sum_{r_i=1}^{R_i} \rho_{r_i}^i b_{k',r_i}^i \\ & + \sum_{i=1}^{I}\sum_{h_i=1}^{H_i} \gamma_{h_i}^i e_{k',h_i}^i + \sum_{l=1}^{L} v_l^s x_{k',l}^s + \sum_{i=1}^{I} \delta_{k'}^i \end{aligned} \tag{18}$$

$$\begin{aligned} \text{s.t} \quad & -\sum_{m_i=1}^{M_i} u_{m_i}^i y_{k,m_i}^i + \sum_{n_i=1}^{N_i} v_{n_i}^i x_{k,n_i}^i + \sum_{r_i=1}^{R_i} \rho_{r_i}^i b_{k,r_i}^i + \sum_{h_i=1}^{H_i} \gamma_{h_i}^i e_{k,h_i}^i \\ & + \sum_{l=1}^{L} v_l^s \mu_{k',l}^i x_{k,l}^s + \delta_{k'}^i \geq 0, \end{aligned}$$

$$k = 1,......,K. \quad i = 1,......,I, \tag{19}$$

$$\sum_{m_i=1}^{M_i} u_{m_i}^i y_{k',m_i}^i + \sum_{n_i=1}^{N_i} v_{n_i}^i x_{k',n_i}^i + \sum_{r_i=1}^{R_i} \rho_{r_i}^i b_{k',r_i}^i + \sum_{l=1}^{L} v_l^s \mu_{k',l}^i x_{k',l}^s \geq w^i,$$

$$i = 1,......,I, \tag{20}$$

$$u_{m_i}^i, v_{n_i}^i, \gamma_{h_i}^i, v_l^s \geq 0, \;\; \rho_{r_i}^i, \delta^i \;\; free, \tag{21}$$

where $u_{m_i}^i, v_{n_i}^i, \rho_{r_i}^i, v_l^s$, and $\gamma_{h_i}^i$ are multipliers for desirable outputs, inputs, undesirable outputs, shared inputs and environmental variables, respectively. When the equality holds in equation (20), the dual model ((18) ~ (21)) shows that an aggregate measure of technical inefficiency may be defined as follows:

$$TIE_{k^{'}}=\frac{-\sum_{i=1}^{I}\sum_{m_i=1}^{M_i}u_{m_i}^{i}y_{k^{'},m_i}^{i}+\sum_{i=1}^{I}\sum_{n_i=1}^{N_i}v_{n_i}^{i}x_{k^{'},n_i}^{i}+\sum_{i=1}^{I}\sum_{r_i=1}^{R_i}\rho_{r_i}^{i}b_{k^{'},r_i}^{i}+\sum_{i=1}^{I}\sum_{h_i=1}^{H_i}\gamma_{h_i}^{i}e_{k^{'},h_i}^{i}+\sum_{l=1}^{L}v_{l}^{s}x_{k^{'},l}^{s}+\sum_{i=1}^{I}\delta_{k^{'}}^{i}}{\sum_{i=1}^{I}\sum_{m_i=1}^{M_i}u_{m_i}^{i}y_{k^{'},m_i}^{i}+\sum_{i=1}^{I}\sum_{n_i=1}^{N_i}v_{n_i}^{i}x_{k^{'},n_i}^{i}+\sum_{i=1}^{I}\sum_{r_i=1}^{R_i}\rho_{r_i}^{i}b_{k^{'},r_i}^{i}+\sum_{l=1}^{L}v_{l}^{s}x_{k^{'},l}^{s}}.$$

This measure is the weighted result of *I* activities' individual inefficiency (See Appendix A for proof). Moreover, the constraint (19) enforces that the efficiencies do not exceed unity. (See Appendix B)

Following the similar criteria stated above, the shadow price δ^i can be used to determine the RTS status for each activity. As Tsai and Mar Molinero (1998, 2002) indicated, there are two interesting consequences about the RTS properties in the MDEA model. First, different activity is allowed to operate under different RTS in each DMU as in the real situation, since every activity has its own production technology. Second, the overall status of RTS of a DMU depends on the sum of all its activity's δ^i (i.e., $\sum_{i=1}^{I}\delta^i$). Thus a DMU may appear to be operating under CRS and scale efficient when it is actually operating under IRS in some activities and DRS in the others and is scale inefficient. Thus, the CRS efficiency is more complex than the traditional DEA model would suggest.

3. DATA AND VARIABLE SPECIFICATION

The empirical application is implemented using the data from the *Farmers' Association Yearbook* of 2003 published by the Taiwan Provincial Farmers' Association. The total number of TFAs is 279, 78 of which are deleted either because their credit departments were taken over by the commercial banks or because of missing data problem.

Regarding the specification of variables, for the marketing activity specific input of operating expenditures (x_1^1) is used to produce two outputs, namely the income from marketing (operation income, y_1^1) and other income (y_2^1). Similarly, the insurance department employs specific input of operating expenditures (x_1^2) to produce total insurance income (y_1^2). The extension department uses operating expenditures (x_1^3) to carry out extension services (y_1^3), farmers' education (y_2^3), and rural welfare programs (y_3^3). For the credit departments, they employed two inputs, loanable funds (x_1^4) and capital expense (x_2^4) to produce two desirable outputs, total loans (y_1^4), non-loan receipts (y_2^4), and one undesirable output, non-performing loans (b_1^4).

Among the four departments, there are two shared inputs: labor (x_1^s) which is defined as the number of employees and managers[5], and fixed assets (x_2^s) which include the net present values of land, buildings, machines, equipments and other fixed capitals.

At last, the rural-urban effect is controlled by introducing the ratio of associate members to total members (e_1) as a proxy for environment variable. The TFA located in more urban areas tends to have more associate members than the regular members so that e_1 reflects the degree of urbanization[6]. Table 1 provides the sample means and standard deviations for all variables and the relationship for them is given in Figure 1.

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[5] There are two reasons to specify total number of employees as a shared input in this study. First, some staffs may officially belong to one department, but are actually responsible for the jobs of more than one department. The manager is responsible for all four departments and thus is a *bona fide* shared input. Second, many TFAs alternate their employees among different departments on a routine basis as part of their human resource training program.

[6] The members of FCUs consist of regular members (or voting members) and associate members (or non-voting members). Only full-time farmers are eligible to become regular members. The associated members are mostly part-time farmers or local residents (Wang and Chang, 2003).

4. EMPIRICAL RESULTS

Three modifications are made before applying it to the TFAs. First, the environmental variables normally cause un-determined directions of impact on the performance of DMUs. Since the TFAs with higher ratios of associated members are more likely to be located in the urban areas with tougher competition from the commercial banks, their credit departments are expected to perform better than those with lower ratios. Therefore, the sign of e_1 is expected to be positive for the credit department. However, for the other three departments, its impacts are undetermined. Therefore, the inequality signs in constraint (13) for the marketing, extension and insurance activities are changed into equalities.

Second, the weights in the objective function of MDEA model (i.e. w^i in equation (8)) are viewed as pre-specified parameters. Tsai and Mar Molinero (1998, 2002) believed that activities may not be considered to be equally important, so they adopted the proportions of individual activities' current operating expenditures in relation to the total expenditures as the initial weights. Diez-Ticio and Mancebon (2002) and Yu and Fan (2006), on the other hand, chose to weight various activities equally, with the aim of not introducing into the analysis any subjective element that is difficult to justify. Here, we adopt both specifications and compare their differences. In Tsai and Mar Molinero's specification, the w^i s are given by a survey results from the Council of Agriculture , the supervising institution of TFAs. They are 0.28, 0.11, 0.27, and 0.34 for the marketing, insurance, extension and credit departments, respectively.

Third, for the unknown allocation of shared inputs, i.e., $\mu^i_{k,l}$, proper bounds should be specified to obtain feasible solutions on these fractions (Cook et al., 2000). For the labor share, the number of employees associated with each activity is available in the published yearbook of

TFA. Therefore, the ratios can be computed for the entire sample period for each TFA, from which the largest and smallest ones are chosen as the upper and lower bounds for the shares of labor input. These bounds are also used as the bounds for the other shared input, i.e., the fixed assets.

Table 2 reports the summary statistics of efficiencies where unequal weights are specified. Note that the efficiency scores should be less than or equal to unity and that a higher score indicates a more efficient status. The results diverge from 0.602 to 1.000 with a sample mean of 0.778. This suggests that there are on average rooms for TFAs to expand 22.2% of their outputs and decrease inputs and their undesirable output by the same proportion to become a fully efficient unit. The second column also shows that, out of the 201 TFAs, only 13 (6.47%) can be considered as globally efficient.

As for individual activities, the performances of marketing and credit departments are in general much better than those of insurance and extension departments. The mean values of insurance and extension departments' efficiencies are 0.588 and 0.441, respectively, with high standard deviations, while the means of the other two departments are 0.958 and 0.959 with much smaller standard deviations. The priority given by the managers of TFAs to the marketing and credit departments, as a consequence of more profit earning, could be the major reason which explains this phenomenon. Nevertheless, the lower average and wider divergent performance of the extension and insurance departments suggest that the challenge to improve the overall efficiency lies in these two departments.

We also compute the efficiency scores using the equal weights following Diez-Ticio and Mancebon (2002). The results in Table 3 show that the mean value of overall efficiency is 0.737 with 0.957, 0.959, 0.580, and 0.450 for marketing, credit, insurance, and extension departments, respectively. Comparing to the results presented in Table 2, it can be found that the overall efficiency deteriorates significantly because the weights assigned to the activities with high efficiency scores are lower than the weights assigned to the activities with low efficiency scores. However, the mean values for the four activities do not alter in a significant fashion. In addition, Table 3 offers Kendall rank correlation coefficients between the two measurement and they all strongly reject the null hypothesis of independency in ranking. This implies that changing the priority about individual activities will neither influence the mean values nor their relative rankings.

For comparison purposes, the traditional DEA efficiency scores are computed and listed in the last column of Table 2[7]. It can be found that the mean value of the traditional DEA is very close to one with 87.56% of TFAs located on technology frontier. The high efficiency scores may be explained by two aspects. First, as Diez-Ticio and Mancebon (2002) indicated, the achievement of maximum efficiency in the MDEA model requires that good productive behavior be demonstrated on the part of every activity, whilst in the traditional DEA model it is possible for them to compensate with each other. Thus, a DMU will reach the production frontier in the traditional DEA model if only one of the activities it carries out outperforms the other DMUs. Second, it is known that for any fixed sample size, the greater the number of input and output variables in a DEA, the higher the dimensionality of the programming solution space, and thus the higher the scores for the DMUs. (Jenkins and Anderson, 2003; Huhhes and Yaisawarng, 2004) In other words, the traditional DEA model which incorporates all activities' input and output variables into an integrated model has less discriminating power than the MDEA model. Although the MDEA model is much more technically demanding, it is more discriminating than

[7] Here we use the model (3) with the environmental constraints for consistency with (8)~(17) where the rates for desirable outputs to expand and for inputs and undesirable outputs to contract are assumed to be equal.

the traditional DEA model.

Next, the nature of RTS of TFAs is explored in Table 4 where the numbers and percentages of TFAs operating under decreasing, constant and increasing RTS by activity are summarized. It can be found that the status of RTS differs considerably among the four activities. Table 3 also indicates that more than 50 percent TFAs experience diseconomies of scale by IRS in their credit, insurance, and extension departments, suggesting that their efficiency performance in three out of four departments can be improved through expansion. However, for the marketing department, the DRS prevails suggesting that this department is either over-capitalized or over-staffed, and should be contracted in most TFAs. Beside the implications on the need for intra-TFA realignment, this result suggests that the marketing service of agricultural products at the local level has reached a limit. It is necessary for the marketing services to operate over broader geographic areas through strategic alliances or consolidations into a regional or even national operation.

At last, the overall status of RTS can be obtained by aggregating the RTS results of all four activities. Table 4 also demonstrates that only 1.5 percent of the TFAs operate under the optimal scale. The number of TFAs considered to be too large (i.e., DRS) is almost identical to the number of those to be too small (i.e., IRS). Therefore, although the recent legislation have increase the pressures for TFAs to consolidate, it is very important to take into account the discrepancies in RTS to ensure that the TFAs are operating under the most productive scale.

5. CONCLUSIONS

This study proposes a modified MDEA model to decompose the efficiency measures into components that are reflective of the multi-purpose characteristics of the TFAs. The directional distance functions are used to construct a non-radial measures with risk and environmental factors adjusted and certain inputs shared among four departments. It is superior to the radial efficiency measures given that it can avoid the criticism of not permitting its input and output mix to change (Glass et al., 2006). To offer policy suggestions in how TFAs can effectively compete in a competitive environment, the primal and dual relationships of the MDEA model are used to estimate the status of returns to scale for the TFAs as a whole and four departments individually.

The empirical results of 201 TFAs in 2003 suggest that there exist significant divergences on the performance among the four departments. The MDEA overcomes the inflexibility of alternative approaches by allowing the allocation of shared inputs to be optimally determined. It ensures that multi-activity efficiencies are fully realized by first generating efficiency scores based on the comparison of individual activities among the peers and then embedding them into a maximization of the overall achievement with constraints on shared inputs. In doing so, individual department benefits from additional efficiency gain which can be difficult to achieve without reallocating the shared inputs.

In the policy aspect, this study strongly suggests that the TFAs should pay more attentions to improve the efficiency of insurance and extension departments despite the fact that they are by nature non-profit oriented operations. As for the returns to scale status, it is found that most TFAs and its four departments experience diseconomies of scale. Thus, policies that promote structural adjustment and consolidations of TFAs would not be inconsistent with public interests. Furthermore, the wide divergences in the RTS status among the TFAs and their four departments warrant continuing deregulations of the TFAs by easing restrictions on their ability to acquire or consolidate with other TFAs to operate over broader geographical areas.

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Table1. Summary Statistics of All Variables

Category	Variable name	unit	Mean	Std. Dev.
1. Marketing department				
Specific inputs	Operating expenditure (x_1^1)	NT$ millions	83.27	113.06
Outputs	Operating income (y_1^1)	NT$ millions	85.11	114.94
	Other income (y_2^1)	NT$ millions	4.76	8.99
2. Insurance department				
Specific inputs	Operating expenditure (x_1^2)	NT$ millions	1.36	3.88
Outputs	Operating income (y_1^2)	NT$ millions	2.26	3.75
3. Extension department				
Specific inputs	Operating expenditure (x_1^3)	NT$ millions	17.33	30.67
Outputs	No.of extension duties (y_1^3)	Thousands	0.33	0.37
	Farmers' education (y_2^3)	NT$ millions	2.11	3.22
	Welfare activity(y_3^3)	Thousands of persons	5.13	10.69
4. Credit department				
Specific inputs	Loanable funds (x_1^4)	NT$ millions	4,931.87	4,551.49
	Capital expense (x_2^4)	NT$ millions	23.72	18.13
Desirable outputs	Total loans (y_1^4)	NT$ millions	1,857.38	1,973.20
	Non-loan receipts (y_2^4)	NT$ millions	2,885.12	2,798.16
Undesirable outputs	Non-performing loans(b_1^4)	NT$ millions	365.82	442.08
5. Shared input				
	Labor (x_1^s)	No. of persons	67.91	37.20
	Fixed assets (x_2^s)	NT$ millions	236.59	258.79
6. Environmental variable				
	Membership ratio(e_1)	%	36.50	23.96

Table 2. Summary Statistics of Efficiency Measures of TFAs

	Multi-activity DEA					Traditional DEA
	Overall	Marketing	Insurance	Extension	Credit	
Mean	0.778	0.959	0.588	0.441	0.958	0.997
SD	0.112	0.036	0.272	0.331	0.051	0.008
Max	1.000	1.000	1.000	1.000	1.000	1.000
Min	0.602	0.793	0.019	0.013	0.746	0.957
No. of fully efficient units	13	52	29	31	84	176
% of fully efficient units	6.47	25.87	14.43	15.42	41.79	87.56

Table 3. Comparison for Different Specifications on Efficiency Weights

	Overall	Marketing	Insurance	Extension	Credit
Using COA weights	0.778	0.959	0.588	0.441	0.958
Using equal weights	0.737	0.957	0.580	0.450	0.959
t statistics[a]	3.459*	0.455	0.291	-0.274	-0.224
Kendall's rank test	0.796*	0.930*	0.971*	0.978*	0.967*

a. the difference in means of these two groups of efficiencies scores are compared.
* Significant at the 1%

Table 4. Numbers and Percentages in Total of TFAs experiencing DRS, CRS or IRS

	Overall	Marketing	Insurance	Extension	Credit
IRS	92(45.8%)	65(32.3%)	105(52.2%)	106(52.7%)	122(60.7%)
CRS	3 (1.5%)	5 (2.5%)	33(16.4%)	5 (2.5%)	6 (3.0%)
DRS	106(52.7%)	131(65.2%)	63(31.3%)	90(44.8%)	73(36.3%)

a Percentages may not add to 1 because of rounding.

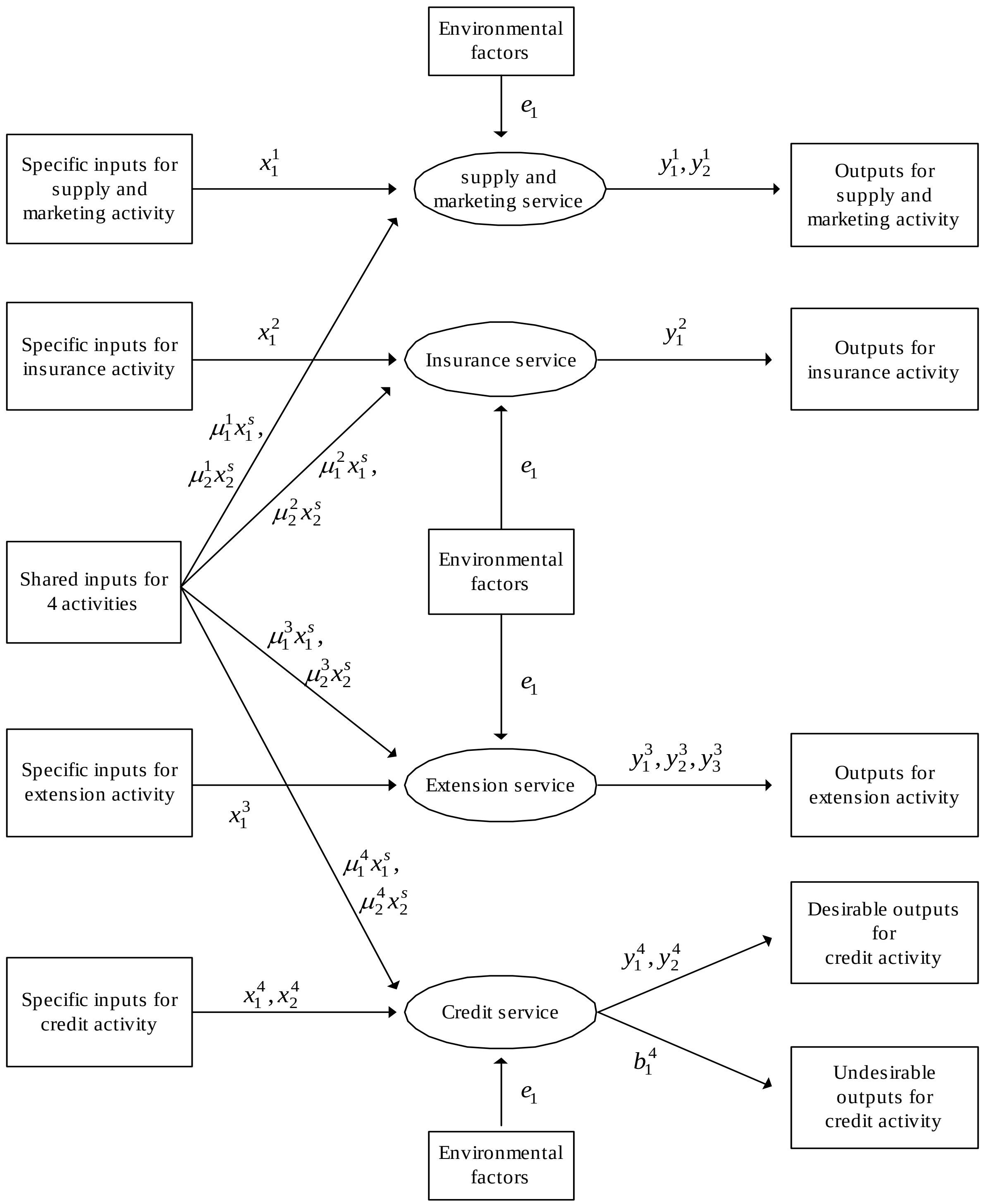

Figure 1. The production process for a TFA

Appendix A.

For notational ease, the proof is shown in the matrix form. In addition to the notation defined above, we also denote $u^i = (u_1^i, u_2^i, \cdots, u_{M_i}^i)$, $v^i = (v_1^i, v_2^i, \cdots, v_{N_i}^i)$, $\rho^i = (\rho_1^i, \rho_2^i, \cdots, \rho_{R_i}^i)$, $\gamma^i = (\gamma_1^i, \gamma_2^i, \cdots, \gamma_{H_i}^i)$, and $v^s = (v_1^s, v_2^s, \cdots, v_L^s)$. The technical inefficiency measure is defines as follows:

$$TIE_{k'} = \frac{-\sum_{i=1}^{I} u^i y_{k'}^i + \sum_{i=1}^{I} v^i x_{k'}^i + \sum_{i=1}^{I} \rho^i b_{k'}^i + \sum_{i=1}^{I} \gamma^i e_{k'}^i + v^s x_{k'}^s + \sum_{i=1}^{I} \delta_{k'}^i}{\sum_{i=1}^{I} u^i y_{k'}^i + \sum_{i=1}^{I} v^i x_{k'}^i + \sum_{i=1}^{I} \rho^i b_{k'}^i + v^s x_{k'}^s}$$

$$= \frac{(-u^1 y_{k'}^1 + v^1 x_{k'}^1 + \rho^1 b_{k'}^1 + \gamma^1 e_{k'}^1 + v^s \mu_{k'}^1 x_{k'}^s + \delta_{k'}^1) + \cdots + (-u^I y_{k'}^I + v^I x_{k'}^I + \rho^I b_{k'}^I + \gamma^I e_{k'}^I + v^s \mu_{k'}^I x_{k'}^s + \delta_{k'}^I)}{\sum_{i=1}^{I} u^i y_{k'}^i + \sum_{i=1}^{I} v^i x_{k'}^i + \sum_{i=1}^{I} \rho^i b_{k'}^i + v^s x_{k'}^s}$$

$$= \frac{(-u^1 y_{k'}^1 + v^1 x_{k'}^1 + \rho^1 b_{k'}^1 + \gamma^1 e_{k'}^1 + v^s \mu_{k'}^1 x_{k'}^s + \delta_{k'}^1)}{\sum_{i=1}^{I} u^i y_{k'}^i + \sum_{i=1}^{I} v^i x_{k'}^i + \sum_{i=1}^{I} \rho^i b_{k'}^i + v^s x_{k'}^s} \times \frac{u^1 y_{k'}^1 + v^1 x_{k'}^1 + \rho^1 b_{k'}^1 + v^s \mu_{k'}^1 x_{k'}^s}{u^1 y_{k'}^1 + v^1 x_{k'}^1 + \rho^1 b_{k'}^1 + v^s \mu_{k'}^1 x_{k'}^s}$$

$$+ \cdots + \frac{(-u^I y_{k'}^I + v^I x_{k'}^I + \rho^I b_{k'}^I + \gamma^I e_{k'}^I + v^s \mu_{k'}^I x_{k'}^s + \delta_{k'}^I)}{\sum_{i=1}^{I} u^i y_{k'}^i + \sum_{i=1}^{I} v^i x_{k'}^i + \sum_{i=1}^{I} \rho^i b_{k'}^i + v^s x_{k'}^s} \times \frac{u^I y_{k'}^I + v^I x_{k'}^I + \rho^I b_{k'}^I + v^s \mu_{k'}^I x_{k'}^s}{u^I y_{k'}^I + v^I x_{k'}^I + \rho^I b_{k'}^I + v^s \mu_{k'}^I x_{k'}^s}$$

$$= \frac{(-u^1 y_{k'}^1 + v^1 x_{k'}^1 + \rho^1 b_{k'}^1 + \gamma^1 e_{k'}^1 + v^s \mu_{k'}^1 x_{k'}^s + \delta_{k'}^1)}{u^1 y_{k'}^1 + v^1 x_{k'}^1 + \rho^1 b_{k'}^1 + v^s \mu_{k'}^1 x_{k'}^s} \times \frac{u^1 y_{k'}^1 + v^1 x_{k'}^1 + \rho^1 b_{k'}^1 + v^s \mu_{k'}^1 x_{k'}^s}{\sum_{i=1}^{I} u^i y_{k'}^i + \sum_{i=1}^{I} v^i x_{k'}^i + \sum_{i=1}^{I} \rho^i b_{k'}^i + v^s x_{k'}^s}$$

$$+ \cdots + \frac{(-u^I y_{k'}^I + v^I x_{k'}^I + \rho^I b_{k'}^I + \gamma^I e_{k'}^I + v^s \mu_{k'}^I x_{k'}^s + \delta_{k'}^I)}{u^I y_{k'}^I + v^I x_{k'}^I + \rho^I b_{k'}^I + v^s \mu_{k'}^I x_{k'}^s} \times \frac{u^I y_{k'}^I + v^I x_{k'}^I + \rho^I b_{k'}^I + v^s \mu_{k'}^I x_{k'}^s}{\sum_{i=1}^{I} u^i y_{k'}^i + \sum_{i=1}^{I} v^i x_{k'}^i + \sum_{i=1}^{I} \rho^i b_{k'}^i + v^s x_{k'}^s}$$

$$= TIE_{k'}^1 \times w^1 + \cdots + TIE_{k'}^I \times w^I$$

$$= \sum_{i}^{I} w^i TIE_{k'}^i$$

Appendix B.

Here, we use the activity 1 of DMU k as an example to present this proof. The technical inefficiency of activity 1 is defined as follows:

$$TIE_k^1 = \frac{-u^1 y_k^1 + v^1 x_k^1 + \rho^1 b_k^1 + \gamma^1 e_k^1 + v^s \mu_k^1 x_k^s + \delta_k^1}{u^1 y_k^1 + v^1 x_k^1 + \rho^1 b_k^1 + v^s \mu_k^1 x_k^s}.$$

So the technical efficiency can be calculated by the following formulation and should not exceed 1.

$$TE_k^1 = 1 - \frac{-u^1 y_k^1 + v^1 x_k^1 + \rho^1 b_k^1 + \gamma^1 e_k^1 + v^s \mu_k^1 x_k^s + \delta_k^1}{u^1 y_k^1 + v^1 x_k^1 + \rho^1 b_k^1 + v^s \mu_k^1 x_k^s} \leq 1.$$

Then, we have

$$\frac{(u^1 y_k^1 + v^1 x_k^1 + \rho^1 b_k^1 + v^s \mu_k^1 x_k^s) - (-u^1 y_k^1 + v^1 x_k^1 + \rho^1 b_k^1 + \gamma^1 e_k^1 + v^s \mu_k^1 x_k^s + \delta_k^1)}{u^1 y_k^1 + v^1 x_k^1 + \rho^1 b_k^1 + v^s \mu_k^1 x_k^s} \leq 1$$

$$\Rightarrow 2u^1 y_k^1 - \gamma^1 e_k^1 - \delta_k^1 \leq u^1 y_k^1 + v^1 x_k^1 + \rho^1 b_k^1 + v^s \mu_k^1 x_k^s$$

$$\Rightarrow -u^1 y_k^1 + v^1 x_k^1 + \rho^1 b_k^1 + v^s \mu_k^1 x_k^s + \gamma^1 e_k^1 + \delta_k^1 \geq 0$$

Thus, we obtain the constraint (19) as i =1. Note that we can use the similar method to show that the combination of all the constraints in equation (19) ensures that the aggregate efficiency for DMU k should not exceed 1.