Optimal Density for Municipal Revenues

Soji Adelaja
Land Policy Institute
305 Manly Miles Building
1405 south Harrison Road
East Lansing, MI 48823, USA.
e-mail: adelaja@msu.edu
Tel: 517-432-8800 X 102
Fax: 517-432-8769

Malika Chaudhuri
Michigan State University
Department of Agricultural Economics
12 Cook Hall
East Lansing, MI 48824
e-mail: mukher21@msu.edu
Phone #: 765-532-2869
Fax: 517-432-1800

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Soji Adelaja is the John A. Hannah Distinguished Professor in Land Policy and Director of the Land Policy Institute (LPI) at Michigan State University (MSU). Malika Chaudhuri is a graduate student in the Department of Agricultural Economics at MSU and former Graduate Assistant at LPI. The data support of Michigan Realtors is greatly appreciated.

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Abstract

Distribution of lot sizes and improvements affect property values, hence, zoning affects property tax revenues. If targeted zoning density diverges from optimal, municipal revenue can be increased through zoning changes. This paper derives optimal lot size that maximizes municipal tax revenues. Hedonic analyses of a Michigan community suggest that optimal lot size is lower than current zoning on existing properties. The possibility that municipal revenue can be enhanced through greater zoning density hints of a cost associated with exclusionary zoning. Local governments should therefore seriously consider the fiscal implications of their zoning decisions as they pursue growth control.

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I. INTRODUCTION

In the United States, property taxes are the primary mechanism through which local communities raise revenues to support the provision of services to their residents (Campbell, 1951). Local units of government are constrained largely by the revenue generating capacity of their community's existing real estate endowment in deciding the level of services to deliver (Florestano, 1981). Zoning is important because it affects the nature, volume and tax-rateability of future real property, all of which ultimately affect future municipal revenue.

The links between zoning and other land use regulations, development of various land use classes, future size and value distribution of various land use classes and future municipal tax revenues are becoming more obvious to communities. Commercial, industrial and agricultural land uses are usually viewed as good tax ratables - their revenues exceed their dependencies on services (American Farmland Trust, 2004). However, whether or not a residential property is a good tax ratable is a function of its attributes (nature and value) of improvement and of lot size. Both of these are a function of zoning. Regulation of the residential property type is of utmost importance as it has perhaps the most potential impact on municipal land use patterns, growth, sprawl and service consumption.

A homestead is a bundle of attributes. Hence, property attributes are expected to be correlated with lot size preferences. A relationship should therefore exist between lot size distribution and aggregate property valuation in a community. Certain lot sizes and their associated property attributes should be of greater demand than others due to consumer preferences, affordability, demographics and past land use regulations (Spangenberg and McCormick). One should therefore expect a lot size range to exist that yields maximum per home or per acre municipal revenues for a community.

Particularly important to communities is the ability to finance municipal infrastructure that contribute to the quality of life (QOL). Well-planned communities that accommodate market forces while balancing compelling government interests regarding density are better able to support infrastructure such as parks, forests, farmland and wetlands as well as high quality public infrastructure and protective services (Hulten and Peterson, 1984) that enhance QOL. An economic analysis performed for the East Bay Regional Park District in California concluded that parks, open space, trails, associated recreational and educational opportunities, environmental and cultural preservation, alternative transit modes, and sprawl-limiting characteristics all contribute positively to quality of life, helped to boost the economy and provide extensive economic benefits for all area residents (EPS, 2000). Such balance is the foundation of the Smart Growth Movement.[1]

The financial stability of a community is an important element of sustainable growth and development and of QOL. As the principal control mechanism for growth in most communities, zoning can also be implemented with a goal of financial stability and QOL in mind. To the extent to which a community is not built out, but understands the relationship between lot size and municipal revenues, it can target that "Optimal Lot Size (OLS)" through zoning to maximize municipal tax revenue. Therefore, a more systematic approach to zoning may be implemented to help to maximize revenue through a better distribution of density.[2]

A number of studies have examined the non-price or non-value effects of zoning restrictions, both from a theoretical and empirical perspective (Mills, 1989; Foley, 2004; Gottlieb and Adelaja, 2005,a). For example, Mills examines the effects of zoning on resource allocation and its net social benefits. Foley examines the impact of zoning on the rate of land consumption and concludes that large lot zoning results in decreased consumption of land, up to a point where successive decreases in density results in greater land consumption. More recently, Gottlieb and Adelaja (2005a) examined the political and economic dynamics that lead to zoning change.

The nature and direction of the effects of zoning on land values are ambiguous. For example, on one hand, one expects the withholding of land from development (restricting supply through zoning or other means) to increase the equilibrium price of land and housing. On the other hand, however, limiting density is expected to make raw land less valuable as an input into new housing production. These effects run counter to each other, making the total impact of density restrictions on land prices relatively difficult to ascertain (Quigley and Rosenthal, 2005). One plausible perspective is that the ultimate effect depends on supply characteristics in the particular land market, the nature of consumer demand, and associated elasticities.

A number of studies have looked at the impact of zoning on property value. The analysis of the effect of urban zoning on the price of single-family residential property in North Carolina confirm primarily that large lot zoning, especially in residential areas, significantly reduces the price of single-family residential property, making housing more affordable (Jud, 1980). Colton and Sheehan, on the other hand, concluded that zoning adversely affects housing affordability. Contradicting the findings, the research work by Gottlieb and Adelaja (2005b), found that down zoning of agricultural land results in enhanced values for residential properties in the same communities. Econometric evidence for fiscal zoning based on sample set drawn from Portland, Washington DC, Seattle and Ramapo conclude that the changes in or variations among suburban zoning restrictions are directly reflected through the property value of the homesteads. On one hand, adoption of more restrictive zoning reduces the value of underdeveloped suburban land subject to the restrictions and on the other hand, increases the value of already-developed homes (Fischel, 1992). Despite the obvious municipal revenue connection, no study has directly examined the impacts of zoning on municipal tax revenues or developed a framework for identifying optimal revenue implications of density.

This paper aims to fill the gap in the literature on the effects of zoning on municipal revenues through lot values and improvement values. It develops a conceptual model for evaluating optimal municipal revenues and utilizes a hedonic pricing framework to investigate the relationship between lot size and municipal revenues. The associated empirical hedonic property valuation models were specified to include power terms on lot size so as to allow the estimation of an optimal lot size. Multiple listing data from Meridian Township in Michigan is used in the hedonic analysis. Meridian Township is a metropolitan town strategically located in the intersection of two major highways. It is also geographically close to Lansing, a major urban center and the capital of the state.

II. REVENUE IMPLICATIONS OF PROPERTY LOT SIZE

Conventional urban location theory by Alonso (1964) posits that lot size is inversely related to population density and monotonically increases with increased distance from the central business district (CBD). This implies that in a perfect land market, at any given distance from the CBD, a mixture of different lot sizes would not be expected. In reality, however, distance from the CBD and population density is not the only determinants of lot size. Differences in consumer preferences, constraints on land, local regulations, income and affordability result in observed differences in lot size even at a given location. Land assembly and subdivision are costly and sometimes even prohibitive, hindering Alonso-type parcel size arbitrage and leading to variations in per acre price of land in a given community (Tabuchi, 1996). Moreover, observed lot size differences may be a result of history and a consequence of a cumulative development processes under visionary or shortsighted decision making by developers and landowners (Harrison and Kain, 1974).

Localities impose restrictions on new development by regulating lot size through down zoning, or thorough other indirect means such as purchase of development rights (PDR) on agricultural or open land, transfer of development rights (TDR), infrastructure concurrency requirements (ICR), development impact fees, clustering requirements, urban growth boundaries (UGBs) etc (ICMA, 2002). However, zoning is the focus of the paper. By impacting on lot size, zoning should affect the number of parcels that could be developed, the product mix, housing choices, the demand mix in the community, per acre lot value, improvement size and attribute, the value of housing and ultimately municipal tax revenues (see Figure 1 for an illustration of the zoning – revenue pathway).

The ultimate direction of the effects of zoning on revenues is not clear. On one hand, large lot zoning should reduce the number of build-able lots in a community and per acres. This can affect tax revenue positively or adversely depending on the elasticity of demand for housing. The municipal revenue impact is the product of two opposite effects: the impacts on (1) property value per acre, and (2) the number of buildable lot per acre. Hence the net impact entails two countervailing effects. The ultimate effect clearly depends on the relative value of the elasticity of price (per acre) and the elasticity of number of houses (per acre) with respect to lot size ($\xi_{P_L,X}$ and $\xi_{L,X}$). Since improvements are related to parcel size and value, the ultimate effect should also depend on the elasticity of improvement size and attributes, and the elasticity of improvement value with respect to lot size ($\xi_{I,X}$ and $\xi_{P_I,X}$).

A review of some of the previous studies related to zoning is appropriate at this point. In order to understand the pressure on local officials, we start with studies that look at the motive for zoning. Economic self-interest seems to be a motive for zoning. According to Quigley and Rosenthal (2005), local homeowners seek to maximize home values and minimize tax burdens by controlling the politics underlying land use enactments. Land use restrictions, whether voluntary, market driven or regulatory, tend to promote amenities that make communities more attractive, which can in turn lead to higher housing prices and reduces housing availability. The effects on total property valuation is not clear in the literature.

Through down zoning, the local units of government tend to favor higher minimum lot sizes to limit growth. By eliminating and restricting high-rise apartments and allowing only low-rise apartments or single-family homes, or prohibiting industrial uses and allowing retail uses only, exclusionary zoning also tend to increase price and decrease availability. [3]

Gottlieb and Adelaja (2005b) reinforce the notion that economic self-interest and growth are central motives affecting zoning choices. They estimated that downzoning of agricultural land has a significant positive impact on the price of the typical homeowner's property in the same community. Another Gottlieb and Adelaja (2005a) study concludes that the likelihood of down zoning increases with the increase in the amount of open space that remains to be protected, decreasing in farm population, declining population growth and land values in the community and the presence of alternative growth management tools. Homeowners seem to be using their political clout to influence the lot size mix in their community to achieve their property value and other goals.

Figure (2) shows the medium and average lot sizes of new single-family houses sold in the US from 1992 and 2005 by region. Figure (3) presents information on housing floor area by region. The lot sizes generally fell between 1992 and 2005 in all regions except the north-east region and outside Metropolitan Statistical Area (MSA). Conversely, median and average square footage of housing increased within the period 1978 to 2005 in the US, with the north-east region lying above the mean and areas outside of Metropolitan Statistical Area (MSA) lying below the mean. The relative growth of livable space vis-à-vis lot size suggests that the built infrastructure is more elastic with respect to income and other drivers. Differential impacts of specific drivers on lot size and square footage can be estimated through hedonic pricing models of housing (Glaeser and Gyourko, 2002).

Conceptual Model

A homestead (H) has two major attributes: (1) land (the size of which is measured by lot-size), and (2) improvements (typically measured in terms of attributes such as square footage). Obviously, the value of land is directly related to lot size while the value of improvements is directly related to the intensity of improvement attributes. Denote the value of a homestead (land and improvements) as follows:

$$V_H = V_L + V_I = P_L L + \sum_{i=1}^{n-1} P_i I_i \ . \qquad [1]$$

V_H is the value of the entire homestead. V_I is the value of improvements. V_L is the value of land. P_L is the per acre value of land (or price). The P_i vector is a vector of per unit prices or values of the ith improvements and I_i is the degree of magnitude or scope of the ith improvement type. Note that in equation (1), n is the number of attributes associated with a homestead, n-1 of which are non lot-size related. The attributes of improvements can include home square footage, number of bedrooms, number of non-bedrooms or number of garages. The attributes of land include such things as width (frontage), depth and shape. Other homestead attributes not tied to improvements or land includes such things as density of housing and school quality in the area.

Information on how the relative values of land and improvements vary with lot size is important in understanding property valuations, particularly when there is a need to determine optimal lot size for municipal finance considerations. In the rest of this section, the relationship between lot-size and other components of housing are conceptualized.

The relative shares of improvements and land in the total valuation of a property can be derived as follows. Assuming that each homestead can be quantified in terms of value, differentiating each side of equation (1) with respect to H, one obtains:

$$\partial V_H / \partial H = P_L \partial L / \partial H + L \partial P_L / \partial H + \sum_{i=1}^{n-1} \left(P_i \partial I_i / \partial H + I_i \partial P_i / \partial H \right). \qquad [2]$$

Equation (2) can further be expressed in terms of elasticities:

$$\begin{aligned}(V_H / H)(H \partial V_H / V_H \partial H) &= (LP_L / H)(H \partial L / L \partial H) + (LP_L / H)(H \partial P_L / P_L \partial H) + \\ &\sum_{i=1}^{n-1} \left((P_i I_i / H)(H \partial I_i / I_i \partial H) + (P_i I_i / H)(H \partial P_i / P_i \partial H) \right).\end{aligned} \qquad [3]$$

Since $V_L = P_L L$ and $V_I = \sum_{i=1}^{n-1} P_i I_i$, the following can be derived from equation (3)

$$(V_H / H) \xi_{V_H, H} = (V_L / H) \xi_{L,H} + (V_L / H) \xi_{P_L, H} + \sum_{i=1}^{n-1} \left((V_i / H) \xi_{I_i, H} + (V_i / H) \xi_{P_i, H} \right) \qquad [4]$$

Where $\xi_{V_H, H} = (H \partial V_H / V_H \partial H)$, $\xi_{L,H} = (H \partial L / L \partial H)$, $\xi_{P_L, H} = (H \partial P_L / P_L \partial H)$, $\xi_{I_i, H} = (H \partial I_i / I_i \partial H)$, and $\xi_{P_i, H} = (H \partial P_i / P_i \partial H)$.

Substituting these elasticities into equation (4), yields:

$$\xi_{V_H, H} = (V_L / V_H) \xi_{L,H} + (V_L / V_H) \xi_{P_L, H} + \sum_{i=1}^{n-1} \left((V_i / V_H) \xi_{I_i, H} + (V_i / V_H) \xi_{P_i, H} \right). \qquad [5]$$

In equation (5), $S_L = V_L / V_H$ and $S_i = V_i / V_H$ are shares of total value attributable to land and each improvement. Equation (5) suggests that the elasticity of homestead value with respect to homestead demand is positive and depends on the elasticities given above, all of which are positive. Now, consider a given lot size. X is of course the variable that is regulated via zoning. Further, define $\xi_{H,X} = X\partial H / H\partial X$, where $\xi_{H,X}$ is the elasticity of homestead demand with respect to lot size choice. The elasticity of homestead value with respect to lot size choice (X) is:

$$\xi_{V_H,H} / \xi_{X,H} = S_L\left(X\partial L / L\partial X + X\partial P_L / P_L\partial X\right) + \sum_{i=1}^{n-1} S_i\left(X\partial I_i / I_i\partial X + X\partial P_i / P_i\partial X\right), \qquad [6]$$

or

$$\xi_{V_H,X} = S_L\left(\xi_{L,X} + \xi_{P_L,X}\right) + \sum_{i=1}^{n-1}\left(S_i\left(\xi_{I_i,X} + \xi_{P_i,X}\right)\right), \qquad [7]$$

where $\xi_{P_L,X} = X\partial P_L / P_L\partial X$. This elasticity, which is the price elasticity of land with respect to the availability of land of a given lot-size, measures the price responsiveness to the availability of land across a range of available lot sizes. With a decrease in the supply of build-able land resulting from a decrease in the availability of land in a given lot size, the price of land will increase, resulting in negative value of the elasticity. Similarly, $\xi_{L,X} = X\partial L / L\partial X$, which measures the responsiveness of total build-able land to availability of land in a given lot size category. It measures the impact of lot size restrictions on developed land in the community. $\xi_{P_i,X} = X\partial P_i / P_i\partial X$, which is the price elasticity of i^{th} improvement with respect to lot size, measures the responsiveness of the price of improvements to lot size. Finally, $\xi_{I_i,X} = X\partial I_i / I_i\partial X$, which measures the responsiveness of homestead improvements to lot size restrictions.

According to equation (7), the impact of lot size availability on property value is a function of the relative value of the lot size, versus improvement, plus elasticities depicting the effect of lot size on price, land consumption, improvement attribute values and improvement attribute demand. To understand these relations, it is important to know how these elasticities vary with lot size. We examine these elasticities below.

Hitherto, these elasticities have been treated as fixed. In reality, they are context sensitive and vary by location. To evaluate how some of these elasticities might vary, consider the land dimension. The value of a lot is obviously correlated with the lot size itself. There is evidence to suggest that price per unit of land on residential properties is inversely related to the size of the parcel (Tabuchi, 1996). Due to economies of scale in infrastructure and land construction, the unit land price may decrease as the size of the lot increases. The value of a lot should also be inversely related to its distance from the Central Business District (CBD). This is consistent with Mills-Muth model of urban spatial structure, which suggests that land tends to be more available and low valued at greater distances from the urban core. Moreover, higher-wage workers tend to live farther from the CBD than do low-wage workers (Fernandez and Su, 2004). This concept is in line with the theory of bid-rent curve. [4]

On the other hand, the value of improvements (homestead less the lot size) should be directly related to lot size, at least over a range of lot sizes. As income rises, both lot size and improvement demand should increase (Euler's Theorem) but probably not proportionately. The demand for improvements should grow at a greater proportion than the demand for lot size due to the fact that the former is more of a necessity than the other. The Engle curve for improvements is expected to become flat earlier than the Engle curve for lot size. While both components are normal goods, lot size is more nearly a luxury good. Mitigating factors which affect income elasticity include affordability. Fewer people can afford large homes. Both lot size and improvements distributions must relate to income distribution.

It is demonstrated above that the price elasticity of land with respect to lot-size, $\xi_{P_L,X}$, elasticity of land with respect to lot-size, $\xi_{L,X}$, price elasticity of improvements with respect to lot size, $\xi_{P_i,X}$, elasticity of improvements with respect to lot size, $\xi_{I_i,X}$ affect the total housing value and therefore municipal revenue.

To explore the relationship between optimal property tax revenue and lot size, consumer preferences and demographics become relevant. To illustrate this point, consider the following equation where aggregate housing demand is expressed as:

$$H = H\left(P_H, P_H^*, Y, T, E, M\right) \qquad [8]$$

where P_H is the price of the housing, P_H^* is the vector of the prices of complements and substitutes for housing, Y is the income, T is a vector of taste and preference variables (such as township, public open space, educational quality and access to highways), E is proxy for expectations about (such things as prices, appreciation, future of neighborhood and relocation), and M is the vector of miscellaneous factors (such as household characteristics, family size, etc). The demand for housing attributes is derived demand for housing. Thus the derived demand for lot size (L) and housing attributes (I) (components of the housing bundle) can be expressed as:

$$L = H_L\left(P_H^L, P_H^{L^*}, Y, T, E, M\right),\text{and} \qquad [9]$$

$$I = H_I\left(P_H^I, P_H^{I^*}, Y, T, E, M\right). \qquad [10]$$

P_H^L is the price of lot-size, $P_H^{L^*}$ is the vector of prices of complements and substitutes for lot-size, P_H^I is the price of improvements, and $P_H^{I^*}$ is the vector of prices of complements and substitutes for improvements.

Consider the total land consumed by lot size category in a community. Lot price per acre can be denoted as $P_L(Y, X)$ and the total land being consumed in a lot size categorized as $L(X)$ where X is the lot size of the category. Hence, a variation of the V_L component of Equation (1) can be expressed as follows:

$$V_L = P_L(Y, X)(L(X)). \quad [11]$$

Assume, for simplicity sake, that $L(X)$ follows a normal distribution with mean μ_L and variance σ_L^2. Hence,

$$L(X) \sim 1/\sigma_L \sqrt{2\pi}\left(\exp\left(-(X-\mu_L)^2 / 2\sigma_L^2\right)\right). \quad [12]$$

Similarly, the value of the improvements is the product of the price for each improvements, $P_I(Y, X)$ and the total magnitude of improvements, $I(X)$.i.e.

$$V_I = P_I(Y, X)(I(X)). \quad [13]$$

where $I(X)$ is the level of improvements. We also assume that $I(X)$ follows a normal distribution with mean μ_I and variance σ_I^2. Hence

$$I(X) \sim 1/\sigma_I \sqrt{2\pi}\left(\exp\left(-(X-\mu_I)^2 / 2\sigma_I^2\right)\right). \quad [14]$$

The total value of the homestead can be expressed as:

$$V_H = P_L(Y, X)(L(X)) + P_I(Y, X)(I(X)). \quad [15]$$

To find the value of lot size X that maximizes the total value of the homestead, differentiate equation (15) with respect to X

$$\partial V_H / \partial X = L(X)(\partial P_L / \partial X) + P_L(Y, X)(\partial L / \partial X) + I(X)(\partial P_I / \partial X) + P_I(Y, X)(\partial I / \partial X). \quad [16]$$

At optimum valuation of the value function $\partial V_T / \partial X = 0$ defines the first order condition for optimization. Therefore,

$$L(X)(\partial P_L / \partial X) + P_L(Y, X)(\partial L / \partial X) + I(X)(\partial P_I / \partial X) + P_I(Y, X)(\partial I / \partial X) = 0. \quad [17]$$

Manipulating equation (22) in order to express it in terms of elasticity, one obtains

$$\begin{aligned} &L(X)(P_L / X)(X\partial P_L / P_L \partial X) + P_L(Y, X)(L / X)(X\partial L / L\partial X) + \\ &I(X)(P_I / X)(X\partial P_I / P_I \partial X) + P_I(Y, X)(I / X)(X\partial I / I\partial X) = 0. \end{aligned} \quad [18]$$

which implies that

$$\begin{aligned}&(V_L / X)\xi_{P_L,X} + (V_L / X)\xi_{L,X} + (V_I / X)\xi_{P_I,X} + (V_I / X)\xi_{I,X} = \\ &(V_L / X)(\xi_{P_L,X} + \xi_{L,X}) + (V_I / X)(\xi_{P_I,X} + \xi_{I,X}) = 0.\end{aligned} \quad [19]$$

Hence,

$$V_I / V_L = -(\xi_{P_L,X} + \xi_{L,X})/(\xi_{P_I,X} + \xi_{I,X}). \quad [20]$$

where V_I / V_L is the relative value of improvements to lot size. Further manipulation yields

$$V_L / V_H = -(\xi_{P_I,X} + \xi_{I,X})/(\xi_{P_L,X} + \xi_{L,X} + \xi_{P_I,X} + \xi_{I,X}) \quad [21]$$

Similarly,

$$V_I / V_H = -(\xi_{P_L,X} + \xi_{L,X})/(\xi_{P_L,X} + \xi_{L,X} + \xi_{P_I,X} + \xi_{I,X}) \quad [22]$$

To derive the expression for $\xi_{L,X}$, one can differentiate equation (12) with respect to X as follows:

$$\partial L(X)/ \partial X = 1/\sigma_L \sqrt{2\pi} \left(\exp\left(-(X - \mu_L)^2 / 2\sigma_L^2\right)\right)\left(-2(X - \mu_L)/ 2\sigma_L^2\right). \quad [23]$$

Therefore,

$$X \partial L(X)/ L(X) \partial X = -X(X - \mu_L)/\sigma_L^2 \quad [24]$$

which implies that

$$\xi_{L,X} = -X(X - \mu_L)/\sigma_L^2 \quad [25]$$

Similarly, considering equation (14), in order to derive the expression for $\xi_{I,X}$,

$$\partial I(X)/ \partial X = 1/\sigma_I \sqrt{2\pi} \left(\exp\left(-(X - \mu_I)^2 / 2\sigma_I^2\right)\right)\left(-2(X - \mu_I)/ 2\sigma_I^2\right). \quad [26]$$

Therefore,

$$\xi_{I,X} = -X(X - \mu_I)/\sigma_I^2. \quad [27]$$

Finally, substituting values from equation (24) and (27) into equation (20), one obtains

$$(V_I / V_L) = -(\xi_{P_L,X} - X(X - \mu_L)/\sigma_L^2)/(\xi_{P_I,X} - X(X - \mu_I)/\sigma_I^2). \quad [28]$$

The expression for V_I / V_L is central to property value determination and therefore tax revenues and can be derived in terms of the above referenced elasticities. It shows that at the optimal level, the relative value of improvement and lot also depends on the elasticities above. The extent to which a lot size restriction impacts revenues depends on the ratio of V_I / V_L which itself is determined by the price elasticity of land with respect to lot-size, $\xi_{P_{L,}X}$, elasticity of land with respect to lot-size, $\xi_{L,X}$, price elasticity of improvements with respect to lot size, $\xi_{P_{i,}X}$, elasticity of improvements with respect to lot size, $\xi_{I_{i,}X}$. Hence property valuation and hence municipal tax revenues are functions of the elasticities mentioned above, along with lot size restrictions, which in turn may vary by jurisdiction due to differences in income, demographics, taste and preferences, property mix, housing stock as determined by the supply and demand of housing. The objective of this study is to understand the relationship between lot size and optimal valuation of land, with implications for optimal municipal revenue. The basic hypothesis is that due to the difference in the community structure, optimality in sale value and hence taxable value are determined by elasticities.

III. EMPIRICAL MODEL SPECIFICATION

To operationalize the conceptual model above, the proposed empirical framework is to estimate the relationship between property value and its determinants, with a special focus on lot size. We propose the hedonic pricing model as the basic empirical framework for determining the effect of lot size on the taxable value of a homestead. In the case of lot size, our target variable, a unique functional specification will be used that allows the identification of an optima or several optima.

Since the introduction of the hedonic pricing model by Griliches (1971), an extensive literature has developed on the application of the model to value location, structural and environmental amenities associated with residential property. The hedonic procedure is frequently used to quantify the effect of various housing and neighborhood characteristics on house prices. Empirically, the technique uses regression analysis to variations in market values to the property's characteristics (lot size, age of the house, number of bedrooms, number of bathroom etc) (Goodman and Thibodeau, 1995).[5]

Based on previous studies by Goodman and Thibodeau, 1995, specific attributes included in our empirical analysis include: (1) Lot characteristics (Z_L) such as lot size, frontage, depth etc; (2) structural characteristics (Z_S) such as number of bedrooms, number of half-bath and full bath, number of garages, basement, number of stories etc; (3) neighborhood variables (Z_N) such as percentage of nonresidential areas, percentage of undeveloped land, employment density etc; (4) proximity variables (Z_P) such as proximity of fire station, police station, schools, parks, libraries, recreational facilities and highways etc. Hence the general specification for the hedonic house price equation is:

$$V(H) = f(Z_{Lj}, Z_{Sj}, Z_{Nj}, Z_{Pj}) \qquad [29]$$

where $V(H)$ is the value of the homestead and Z_{Nj} is the neighborhood characteristic, Z_{Sj} is the structural characteristic of the house etc. A consumer with a vector of socio-economic characteristics ω derive utility from the various characteristics of the house $Z_{Lj}, Z_{Sj}, Z_{Nj}, Z_{Pj}$ and from the numeraire non-housing good, τ.

The utility function of the buyer is specified as follows:

$$U = U(Z_{Lj}, Z_{Sj}, Z_{Nj}, Z_{Pj}, \tau, \varpi). \qquad [30]$$

The homebuyer's problem is to maximize U (.) subject to:

$$Y = \tau + P(Z_{Lj}, Z_{Sj}, Z_{Nj}, Z_{Pj}) \qquad [31]$$

where Y denotes level of income and τ represents non-housing expenditure and this would be a standard consumer optimization problem except that the budget constraint may be non-linear.

In a hedonic prices model, there are two equations to be estimated. The hedonic price function and the individual's marginal willingness to pay function are, respectively

$V(H) = f(Z_{Lj}, Z_{Sj}, Z_{Nj}, Z_{Pj})$ and [32]

$$b_{ij} = b_{ij}(Z_{Lj}, Z^*_{sj}, Z_{Nj}, Z_{Pj}) \quad [33]$$

In equation (33), b_{ij} is the marginal willingness to pay for the ith attribute of the jth household. Z^*_{sj} is the vector of other structural characteristics, and z_{sj} is the particular structural (S) characteristic for which we want to derive the marginal willingness to pay function. Equation (32) can be estimated assuming a basic linear functional relationship. It can, however, be estimated through a non-linear function.

For illustrative purposes, simple plots of the taxable value of a house against the lot size are presented in Figure (4) for Meridian Township, our case study. These indicate some correlation between taxable value and lot size. These illustrations hint at the endogeneity of taxable value of a homestead vis-à-vis lot size. A hedonic pricing analysis will systematically identify the functional relationship between the variables and level of significance.

A generalized Box-Cox transformation (Greene, 1997) is used to identify the appropriate functional form in order to determine the range of lot sizes that maximizes the tax revenue. The flexible form approach aids in the estimation of the amenity values with no prior restrictions on the hedonic relationships and allows for the likelihood ratio tests of more traditional functional forms (Milon, Gressel and Mulkey, 1984). Rosen's (1974) pioneering work motivated semilog, log-log as well as alternative specifications using the Box-Cox. The Box-Cox transformation is given by:

$$V_H^{\lambda} = \left(V_H^{\lambda} - 1\right)/\lambda \quad \text{if } \lambda \neq 0 \text{ and} \qquad [34]$$

$$V_H^{\lambda} = \ln V_H \quad \text{if } \lambda = 0 \qquad [35]$$

where λ is the 'Box-Cox parameter'. The Box-Cox transformation, below, can be applied to a regressor, a combination of regressors, and/or to the dependent variable in a regression. The objective of doing so is usually to make the residuals of the regression more homoskedastic and closer to a normal distribution (Greene, 1997).

The virtue of the Box-Cox form is that it requires no prior restrictions on the attribute relationships. For example, if $\lambda_1 = \lambda_2 = 1$, the specification is linear; if $\lambda_1 = \lambda_2 = 0$, it is a double log and if $\lambda_1 = 0; \lambda_2 = 1$, it is semi log. Other combinations yield quadratic and exponential forms. Nested hypotheses testing with the unrestricted Box-Cox form and the traditional functional forms can be conducted using a likelihood ratio test statistic (Milon, Gressel and Mulkey, 1984).

By estimating equation (34) under the various linear hypotheses, restricted maximum likelihood values can be calculated. Under the null hypothesis, minus two times the log of the ratio of the restricted to the unrestricted likelihood value is distributed asymptotically as chi-squared with two degrees of freedom (since we have 2 parametric restrictions).

IV. DATA

Multiple listing data from Michigan Realtors is utilized in this study. Meridian Township in the Ingham County, Michigan is a fast growing metropolitan area. For the year 2005, the residential parcel count for the township was 12,308, commercial parcel count was 663, industrial parcel count was 47, and agricultural parcel count was 5. The total assessed value of residential property was $1,274,286,150, which is approximately 72.26% of the total real value of properties in the county (Michigan State Tax Commission, 2005). Data on all real estate transactions from October ‘04 through March ‘05 for which actual sales had occurred were used in the analysis. Property tax was not used as the dependent Michigan law provides a cap on property tax increases as long as property ownership remains the same. Hence the sales of property triggers an adjustment of taxable value and therefore tax liability. The sluggishness of assessed value and the fact that it does not reflect market value is the primary reason for using sales price data. The data consisted of 137 observations (homesteads).

Table (1) provides a detailed discussion on the nature of the variables in the estimated model. The dependent variable is ‘SALEPRICE’. The independent variables include the total square footage of the house above the ground (SQFTABOVE), the total lot size of the house (TOTLOTSIZE), the age calculated as the difference between the year the house was built and the sale date (AGE), the dummy variable indicating whether the house has a basement or not (DBSMT), the variable indicating the number of car places or garages (NOGARAGE), the dummy variable representing whether the garage is attached to the house (DATTACHGAR), the dummy variable indicating whether the house has a sewage facility (DSEWER), the total number of bedrooms (BDRMS), the total number of non-bedrooms or rooms other than bedrooms (NON_BDRMS), the number of full bath (FULLBATH), the number of half bath (HALFBATH), the number of stories of the house (DTYPE) and the number of days the house has been in the market (DOM). In the following section, the results of the Box-Cox transformation and the hedonic pricing models estimated through using the method of ordinary least square (OLS) are presented.

V. RESULTS

The linear Box-Cox hedonic result is reported in Table (2). The Box-Cox transformation tests the null hypothesis whether the data fits a linear demand function as opposed to a non-linear demand function. The Box-Cox transformation parameters, θ and λ measure the degree by which the dependent and the independent variables have been transformed. 'L' signifies the value of the likelihood ratio test (LRT) statistic and 'χ^2' denotes the chi-square value. The LRT is a statistical test of the goodness-of-fit between two models. A relatively more complex model is compared to a simpler model to see if it fits a particular dataset significantly better. The LRT begins with a comparison of the likelihood scores of the two models: LR = 2*(lnL1-lnL2); This LRT statistic approximately follows a chi-square distribution.

To determine if the difference in likelihood scores among the linear and non-linear models is statistically significant, we next must consider the degrees of freedom. In the LRT, degrees of freedom is equal to the number of additional parameters in the more complex model. Using this information we can then determine the critical value of the test statistic from standard statistical tables (Greene, 1997). According to the values of the likelihood ratio test statistics, the third model and the forth models, i.e., the inverse and the linear functional forms are strongly rejected due to high values of chi-square in both the data sets whereas the second model, i.e., the log-log model is not rejected. This suggests that the log-log model is a better fit.

With the demand functional form identified, a regression analysis was performed using Ordinary Least Square (OLS) method with sales price as the dependent variable and the other variables in Table (3) as independent variables. Box-Cox transformations, along with the inclusion of squared and cube terms of lot size, have been used to be able to capture the possible curvature in the estimated relationships for distance-related variables.

Table (3) summarizes the regression result for the log-log model and specifies the level of significance for each of the independent variables. Since in this model the explanatory non-dummy attributes are transformed to logarithms, the coefficients of these variables can be interpreted as the respective elasticities. Since log transformation is only applicable when all the observations in the data set are positive, the dummy variables were not transformed. The model specifications allow the examination of the effects in valuation of lot size using lot size, squared lot size and cubed lot size. The 3rd order of the lot size variable allows one to observe the peak level of lot size from a revenue perspective.

The variables ‘TOTLOTSIZESQ’ and ‘TOTLOTSIZECUBE’ are significant at 1% level of significance. The variables ‘FULLBATH’, ‘DAGE10TO20’ and ‘TOTLOTSIZE’ are significant at 5% level of significance. The variables ‘SQFTABOVE’, ‘DAGE20TO30’, ‘DAGEGREATER30’, ‘DBSMT’, ‘DATTACHGAR’ and ‘NOGARAGE’ are significant at 10% level of significance. The variables ‘BDRMS’, ‘HALFBATH’ and ‘DOM’ are not statistically significant at the 10% level of significance. Considering that the data is cross sectional in nature, R-Square of 89% for Meridian Township is surprisingly high. The coefficients are generally consistent with expectations, except for ‘DATTACHGAR’, which is negative implying that a house with attached garage is 55% less valuable than a house with no attached garage. The effects of the lot size variables were of great interest. For example, a positive relationship between square footage above and sale price was estimated. Hence, improvement influences price and therefore tax revenue. This is consistent with the study of Bin and Polasky, 2003.

The variable ‘SQFTABOVE’ is positive and statistically significant, suggesting a square footage price elasticity of 0.44. This inflexible price response is consistent with previous studies (Mahan, Polasky and Adams, 2000). The elasticity of price with respect to bedrooms is 0.07, also suggesting an inflexible price response. Similarly, the elasticity of price with respect to full bath is 0.14. Houses that are 10 to 20 years old are 19% more valuable than the numeraire group, i.e., age lying within 0 to 10. This may reflect the effects of community attributes: mature communities with well-established neighborhood likely to be more valuable than upstart communities. However, as expected, houses within the age of 20 to 30 are 36% less valuable than the numeraire age group. Houses over the age of 30 years are even less valuable. This reflects the impact of housing vintage (age) on property value. The coefficient of ‘NOGARAGE’ is 0.2993703 and it is statistically significant. This implies that for each additional garage, the value of the house increases by 30%.

Impact of Lot Size on Revenue

To determine the lot size that maximizes property value, ceteris paribus, set $\partial LNSALEPRICE_i / \partial TOTLOTSIZE_i = 0.$ Therefore

$$\partial LNSALEPRICE_i / \partial TOTLOTSIZE_i = (.0150 - 2(.0228)TOTLOTSIZE + 3(0.0102)TOTLOTSIZE^2)$$
$$= (.0150 - (.0457)TOTLOTSIZE + (0.0306)TOTLOTSIZE^2). \quad [36]$$

Setting equation (41) equal to zero yields

$$TOTLOTSIZE = (.0457 + / - sqrt((.0457^2 - 4(.015)(0.0306))/2(0.0306)); \quad [37]$$

which can be expressed as

$$TOTLOTSIZE = (.0457 + / - .00158)/0.0613; \quad [38]$$

The equation (38) above suggests that there are two optima for Meridian Township, 0.49 acres and 1.00 acres. This is consistent with the two roots expected from a quadratic function.

VI. CONCLUSIONS

This paper conceptualizes the relationship between lot size and municipal tax revenues by examining the lot size that maximizes property values. Double optimum with respect to the impact of lot size on property values was identified with the two peaks being at 0.49 acres and 1.00 acres. The 0.49 acre peak is the higher peak. The fact that the average zoning density on all property in Meridian township is currently 0.8 acres suggests that greater density than the current standard would yield greater municipal property tax revenue than the current density. The fact that peak property values, and therefore municipal revenues, vary along the range of lot sizes suggests that communities should be mindful about the relative position of their township optima in making zoning decisions. In almost all debates about zoning, this issue hardly ever comes up. This finding is novel and is an important addition to the literature.

A more comprehensive study will consider the cost side of the municipal finance. While revenue can be easily attributable to property, obtaining cost data is difficult since cost analysis would require clear understanding of the allocation of municipal taxes to alter services. That information is not always available. The authors of these studies are currently working on the decomposition of school cost. This would rely on accessor data, augmented by data from the corresponding school district on the number of kids originating from each home. The issue of optimal zoning for school financial optimization is clearly an issue of significant interest and the ongoing analysis would shade some light on this issue.

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Endnotes

[1] Smart Growth America, url: http://www.smartgrowthamerica.org/

[2] Zoning can benefit some individuals and impose cost on others. The extent to which zoning imparts a net social loss depends on how skillfully it is applied (Crecine, Davis and Jackson, 1967).

[3] It has been argued that since it is one of the most popular tools for limiting development and curbing suburban sprawl, the government should fairly reimburse the landowner for any negative valuation that may occur to his property (San Diego Association Of Realtors, 2003).

[4] Bid-Rent is equivalent to the maximum land rent a potential user would be willing to pay for a given site / location The Bid-Rent Curve shows how the individual's bid-rent changes as a function of the distance from some critical central point (CP). Central point is the point at which transport costs are minimized and bid-rent maximized for the given use. Each potential use has its own bid-rent curve and also central point (Chapter 4: Inside the City I: Some Basic Urban Economics URL: web.mit.edu/11.431j/www/Fall91202/431_GMch04.ppt, viewed on 06/13/06). Larger bundles will be found in general at a greater distance from the center of the community where prices should be much higher. In other words, lot sizes will be higher at great distances from the center where property values are higher. This suggests that an inverse relation between lot size and value of vacant land.

[5] Hedonic method have been used extensively in studying housing by regressing the price of a property on its internal characteristics such as size, appearance, features and conditions as well as the external neighborhood characteristics such as the accessibility to schools and shopping, level of water and air pollution, value of other homes, etc.

Table 1: Variables Used in the Hedonic Pricing Analysis of Meridian Township, MI.*

Dependent Variable	**Variable Symbol**	**Description**	**Nature of Variable**
Sale Price	SALEPRICE	Sale price of the house	Continuous
Independent Variables			
Lot characteristics (Z_L)			
Square footage above	SQFTABOVE	Total square footage of the house above the ground	Continuous
Total lot size	TOTLOTSIZE	Total lot size of the house	Continuous
Structural Characteristics (Z_S)			
Age	AGE	Difference between year the house was built and sale date.	Dummy variable with the following classes: 0 to10 years, 10 to 20 years, 20 to 30 years and greater than 30 years.
Presence of basement	DBSMT	Whether or not the house has a basement.	Dummy variable with the following classes: 1 if house has basement, 0 otherwise.
No. of garages	NOGARAGE	No.of garages.	Discrete, 1 or 2 or 3 etc
Presence of garage	DATTACHGAR	Whether or not garage is attached to the house.	Dummy variable with the following classes: 1 if the house has an attached garage, 0 otherwise.
Sewage facility	DSEWER	Whether or not house has a private sewage facility.	Dummy variable with the following classes: 1 if the house has the house has sewage facility, 0 otherwise.
Bedrooms	BDRMS	Total number of bedrooms	Discrete, 1 or 2 or 3 etc
Non-bedrooms	NONBDRMS	Total number of non-bedrooms	Discrete, 1 or 2 or 3 etc
Full bath	FULLBATH	No.of full baths	Discrete, 1 or 2 or 3 etc
Half bath	HALFBATH	No.of half baths	Discrete, 1 or 2 or 3 etc
Type / Stories	DTYPE	No. of stories	Dummy variable with the following classes: 1 if house is a 'Ranch', 0 otherwise.
Days on Market	DOM	No. of days house was in the market	Discrete, 1 or 2 or 3 etc

* The data came from multiple listings information from Meridian Township, Michigan. Due to the slow market, only 137 observations of actually sold property were available.

Table 2: Linear Box-Cox Hedonic Results

	Model 1	Model 2	Model 3	Model 4
	Linear Box-Cox	**Log-Log**	**Inverse**	**Linear**
Meridian Township				
θ	-.203405	0.00	-1.00	1.00
λ	-.203405	0.00	-1.00	1.00
L	-1450.6565	-1451.5871	-1474.847	-1490.7292
χ^2	–	1.86	48.38	80.15

Table 3: Effects of Parcel Attributes on Property Values in Meridian Township

Variable Name	**% Impact on Sold Price (Log dependent Variable)**
constant	8.633211***
LnSQFTABOVE	0.4371927***
lnBDRMS	0.0770392
lnFULLBATH	0.1490905**
DAGE10TO20	0.1890373**
DAGE20TO30	- 0.3628232***
DAGEGREATER30	- 0.3537223***
DBSMT	0.2900491***
DATTACHGAR	- .5549429***
NOGARAGE	0.2993703***
HALFBATH	0.0158526
DOM	- 0.0000596
TOTLOTSIZE	0.01499342**
TOTLOTSIZESQ	-.0228491*
TOTLOTSIZECUBE	0.01021*
R-square	0.8945

*, **, and *** represent significance at the 1, 5 and 10% levels, respectively.

Figure 1: Conceptualized Effects of Zoning on Municipal Revenue

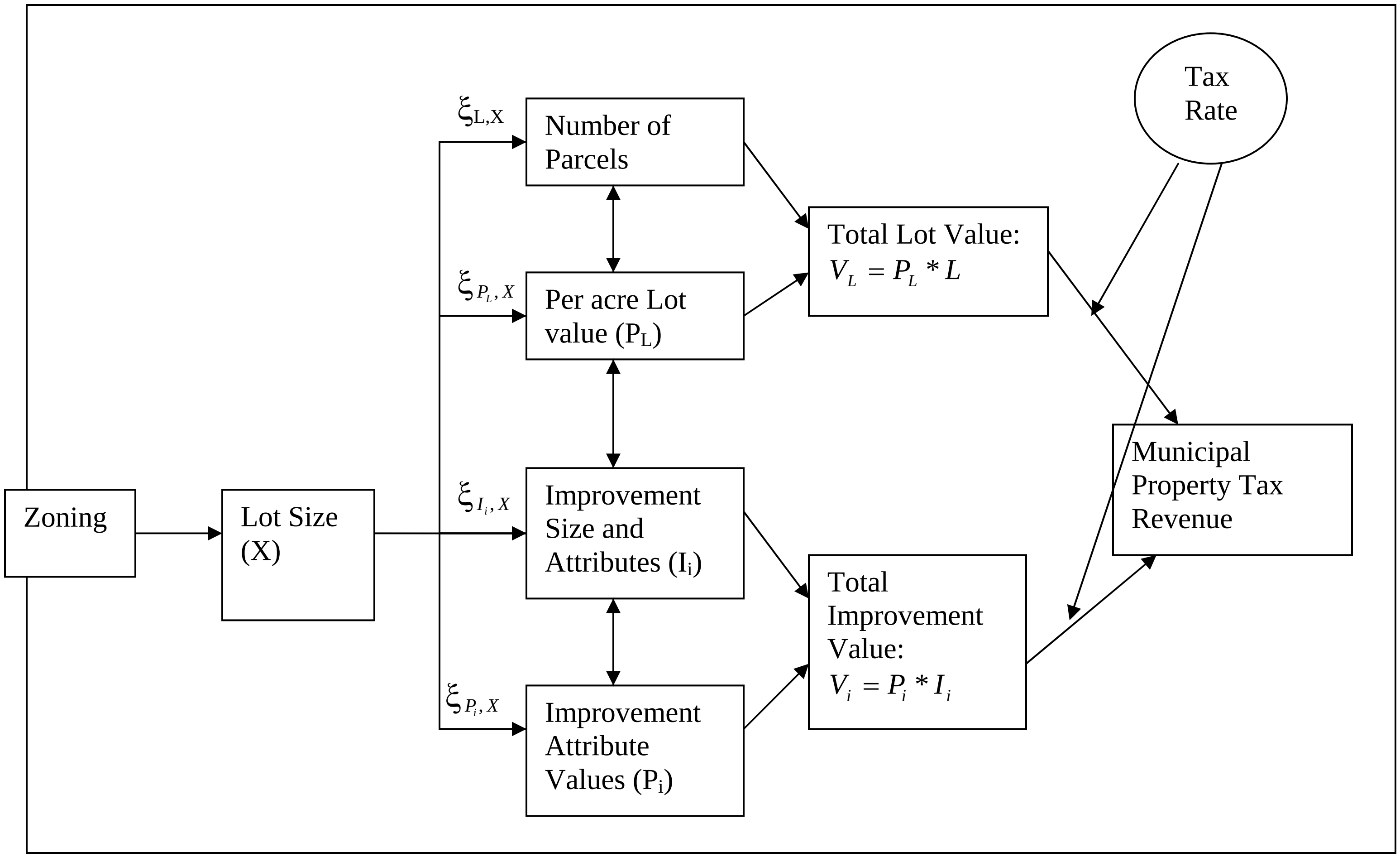
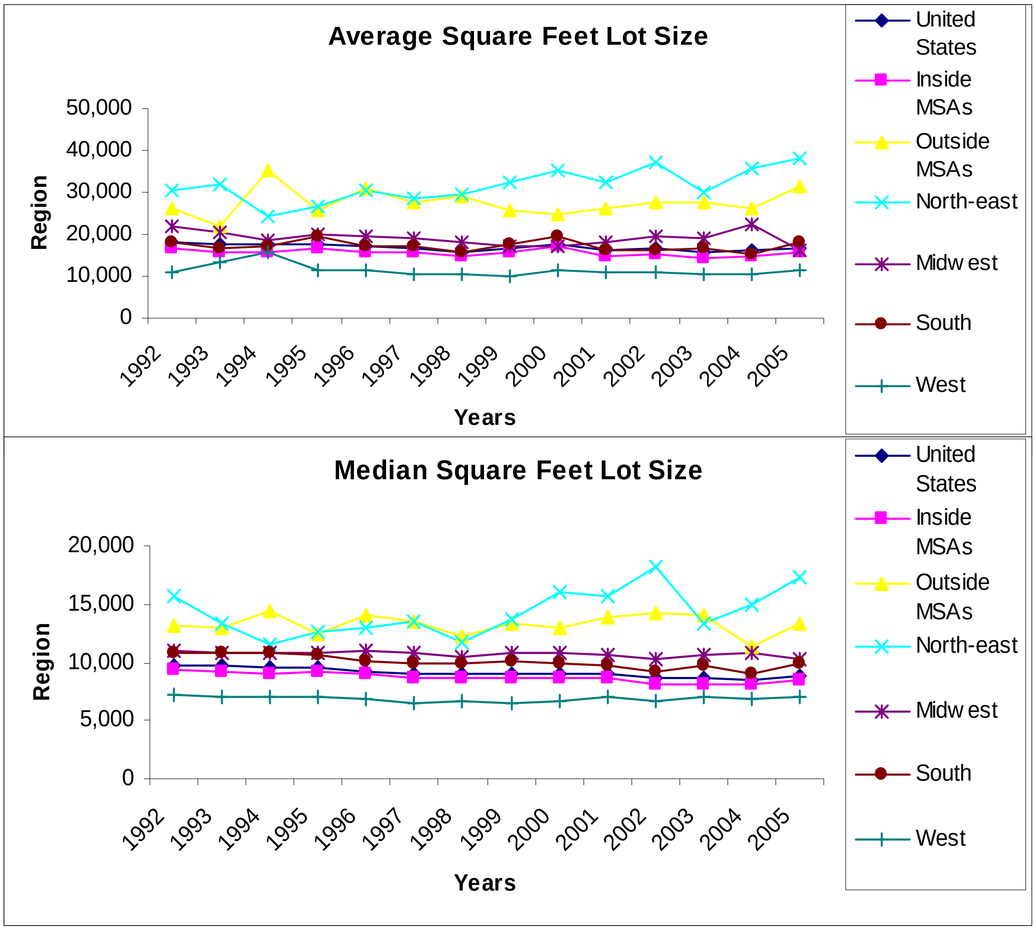
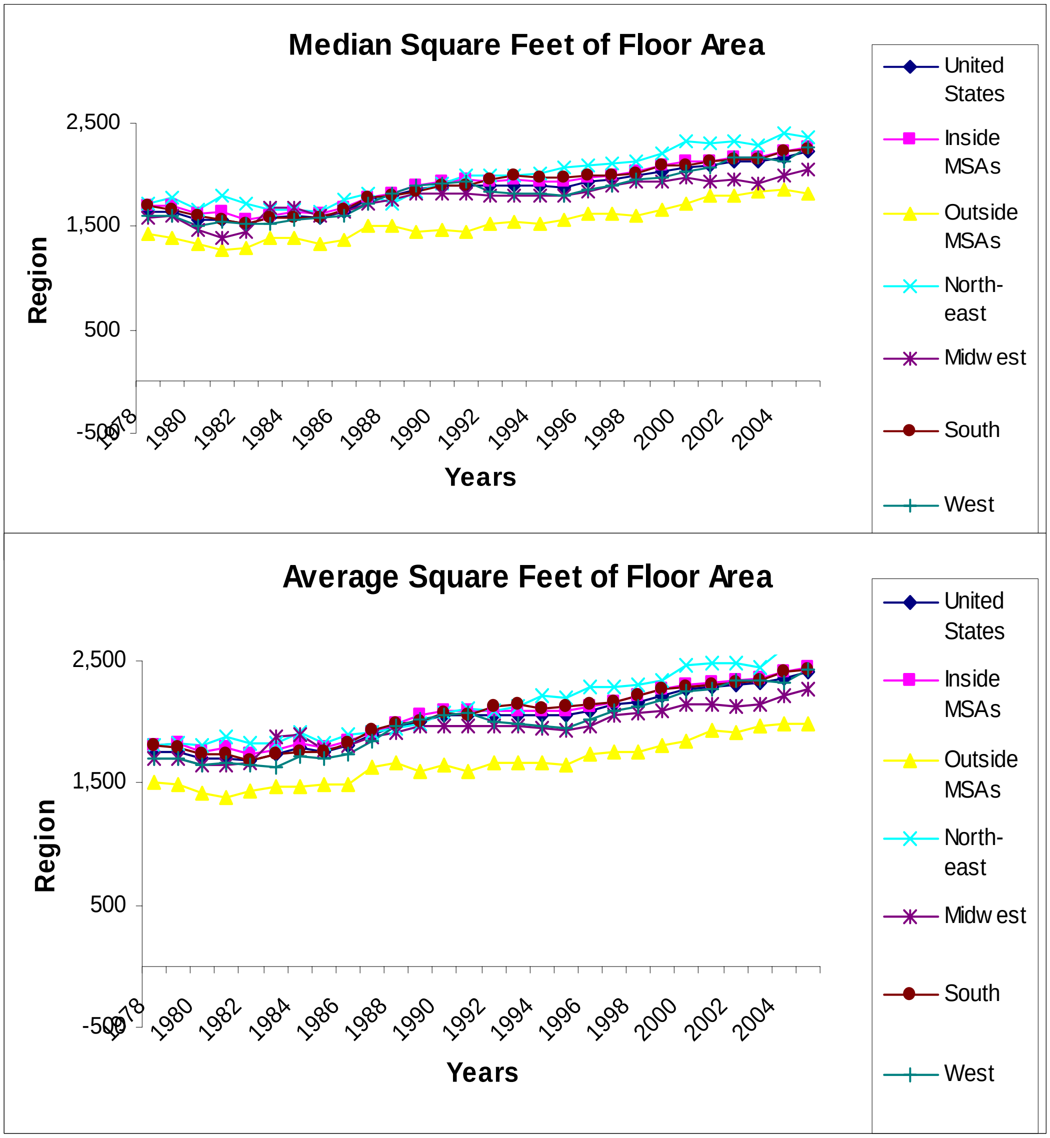


Figure 2: Median and Average Square Footage of Lots Under New Single-Family Houses in the US, 1992-2005



Source: Median and Average Square Feet by Location, Characteristics of New Housing, US Census Bureau, http://www.census.gov/const/C25Ann/malotsizesold.pdf

Figure 3: Median and Average Floor Area in New Single-Family House, USA, 1978 to 2005



Source: Median and Average Square Feet by Location, Characteristics of New Housing, US Census Bureau, http://www.census.gov/const/C25Ann/malotsizesold.pdf

Figure 4: Taxable Value and Lot-size for Recently Sold Houses in Meridian Township, MI (Oct 2004-March 2005), by Lot Size and Range of Lot Size.