RISK, UNCERTAINTY AND FARM MANAGEMENT DECISIONS

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The objective of this paper is to give a brief exposition of the decision criteria commonly propounded for decisions under risk and uncertainty. The review of these criteria in the context of farm management decisions reiterates the inappropriateness of all except the expected utility hypothesis, and it is concluded that utility analysis has considerable potential as an operational tool in farm management.

1 INTRODUCTION

This paper considers decision criteria and utility analysis in the field of farm management decision making. Although no new ground is broken, the aim is to synthesise ideas which previously have either appeared in isolation or not been made explicit.

1.1 THE DECISION PROBLEM

Any situation in which a decision maker is confronted with a choice between alternative actions constitutes a decision problem. Most economic theory has been developed for analysis of decisions under conditions of certainty wherein the precise outcomes of all actions are assumed known. However, most "real world" decisions are taken in the face of risk or uncertainty[1]. That is, precisely what outcome will occur as a result of taking a particular action is not known to the decision maker. It is this latter class of decisions which is our present concern.

It is most convenient to formulate and to analyse any decision problem in a simple framework which consists of: a set of actions, $A = (a_1, a_2, \ldots a_n)$, a set of states of Nature, $S = (s_1, s_2, \ldots s_m)$, and an outcome function which defines for each action a_j and each state s_i, an outcome u_{ij}. While, for the sake of exposition, we will be most concerned with finite sets A and S, these may, of course, be infinite. The outcomes may be specified in terms of gains or losses.

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[1] Initially, we adopt the definitions and terminology formalized in F. H. Knight, *Risk, Uncertainty and Profit* (Boston: Houghton Mifflin, 1921)—reprinted in paperback (New York: Harper and Row, 1965)—pp. 19–20. Risk is "a quantity susceptible of empirical measurement" and uncertainty is "of the non-quantitative type".

In decision problems under risk, a probability distribution can be attached to the states of Nature. Brief account is taken later of the various types of probabilities that can be used. Decision problems under uncertainty exist when no probability distribution can be attached to the states of Nature—that is, there is no information on the likelihoods of these states. Under conditions of certainty, the precise state of Nature is known (i.e., has a probability of unity), and the decision problem reduces to the trivial one of choosing an action using a single outcome vector. Risk and uncertainty thus provide more interesting classes of decision problems.

1.2 RATIONAL BEHAVIOUR

Before it can be determined which decision criterion is "best", a basis for selection is required. Here we judge the worth of a criterion according to a concept of rational behaviour. It is assumed that decision makers attempt to maximize satisfaction in making their decisions. Thus, rational behaviour can be defined as that behaviour which, on the basis of the decision maker's information at the time of decision[2], does most to further his aims and bring him satisfaction. Any criterion consistent with this definition can thus be used to aid in his decision making. Such a criterion is prescriptive (normative[3]) as opposed to descriptive (positive), although naturally there will be situations when it fulfils both roles[4].

2 SOME DECISION CRITERIA FOR UNCERTAINTY

Several criteria which have received considerable attention in the literature on decision problems under uncertainty (DPUU) are the game-theoretic algorithms[5]. These criteria have been proposed for a wide variety of decision problems in economics and agricultural economics[6].

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[2] Of course, in assisting a decision maker in planning his best course of action, any pertinent information additional to that available to him should be used in the analysis.

[3] Without entering any argument, we take "normative" to describe what a decision maker should do, given his goals and aspirations. In G. L. Johnson, "Value Problems in Farm Management", *Journal of Agricultural Economics*, Vol. 14, No. 1 (June, 1960), pp. 13–25, some of the controversies over the use of "normative" and "conditionally normative" are discussed.

[4] Any prescriptive criterion, if it is to be capable of empirical verification, must on occasions be descriptive—see A. N. Halter and H. H. Jack, "Toward a Philosophy of Science for Agricultural Economic Research", *Journal of Farm Economics*, Vol. 43, No. 1 (February, 1961), pp. 83–95.

[5] See R. D. Luce and H. Raiffa, *Games and Decisions* (New York: Wiley, 1957).

[6] Reviewed in J. L. Dillon, "Applications of Game Theory in Agricultural Economics: Review and Requiem", *Australian Journal of Agricultural Economics*, Vol. 6, No. 2 (December, 1963), pp. 20–35. More recently, the use of the maximin criterion in a linear programming model has been illustrated in J. P. McInerney, "'Maximin Programming'—An Approach to Farm Planning Under Uncertainty", *Journal of Agricultural Economics*, Vol. 18, No. 2 (May, 1967), pp. 279–89.

The types of problems for which they can be used can be grouped into two-person zero-sum games and other games, including n-person games. The two-person games can be classified into games in which both players are conscious adversaries and games against Nature in which Nature plays a passive role.

In this paper we will be specifically concerned with game-theoretic criteria in the context of games against Nature, where "Nature" can represent a variety of phenomena including competitive behaviour in a free market. Further, we will consider initially that the states of Nature are uncertain. Knight[7] and other authors have defined uncertainty as existing when there is no objective or empirical evidence on the states of Nature.

2.1 THE LAPLACE, MAXIMIN, MINIMAX REGRET AND HURWICZ CRITERIA

The most common of the game-theoretic criteria are briefly outlined below with definitions for selecting actions.

Laplace Criterion

When the state of Nature is one of uncertainty all the possible events (states of Nature) are given equal weighting. The act with the maximum expected gain is selected; i.e., the act for which

$$\max_j (\Sigma_i u_{ij}/m),$$

with notations as above.

Maximin Criterion

The maximin criterion selects that act with the maximum minimum gain, i.e.

$$\max_j (\min_i u_{ij}).$$

Conventionally, statisticians have framed decision problems in terms of opportunity costs (losses) hence the criterion is better known as the minimax or Wald criterion.

Minimax Regret (Savage) Criterion

The act with the smallest maximum regret, R_{ij}, is selected. The regret is determined by subtracting each payoff u_{ij} from the maximum possible payoff for that event ($\max_i u_{ij}$). That is

$$R_{ij} = (\max_i u_{ij}) - u_{ij},$$

where R_{ij} is the regret for the j-th act and i-th event. Thus the algorithm becomes

$$\min_j (\max R_{ij}).$$

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[7] Knight, *op. cit.*, pp. 19–20.

Hurwicz Optimism-Pessimism Criterion

The Hurwicz criterion takes the best and worst outcomes of each act and weights these in an index according to the pessimism (or optimism) of the decision maker. The act with the largest index is selected. The coefficient of pessimism α is estimated from a single-person game with actions a_1 and a_2, and two states of Nature s_1 and s_2 whose probabilities of occurrence are unknown. For example from the game

	a_1	a_2
s_1	1	x
s_2	0	x

x is determined so that the decision maker is indifferent between acts a_1 and a_2. Then α is obtained by making use of the fact that indifference implies

$$\alpha 0 + (1 - \alpha)\, 1 = \alpha x + (1 - \alpha) x,$$

so that

$$\alpha = 1 - x.$$

Using the value of α determined, the algorithm is

$$\max_j \, [\alpha \min u_{ij} + (1 - \alpha) \max u_{ij}].$$

If $\alpha = 1$ the decision maker is a pessimist and conversely, if $\alpha = 0$ he is an optimist. Intermediate values, of course, give varying degrees of optimism or pessimism.

2.2 HYPOTHETICAL ILLUSTRATION OF THE CRITERIA

Decision situations in which an individual has to choose his optimal strategy against the "strategies" of the neutral opponent, Nature, are the most frequent type of decision at the micro-economic level in agriculture. The selection of strategies by the game-theoretic criteria and the implications of these selections can be usefully examined by considering a hypothetical game against Nature.

A payoff matrix (of utilities[8]) is constructed for the acts a_1, a_2, a_3, and a_4, and states of Nature s_1 and s_2. Money payoffs can be used in testing the criteria[9] but it is then difficult to discover the implications of a decision maker's choice of a criterion because his expectations about the occurrence of events will be confounded with his utilities of the outcomes. For this reason it is customary to use utility units[10].

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[8] Utilities and the concept of utility are discussed later in Section 3.2.

[9] As was done in J. L. Dillon, "Theoretical and Empirical Approaches to Program Selection within the Feeder Cattle Enterprise", *Journal of Farm Economics*, Vol. 40, No. 5 (December, 1958), pp. 1921–31.

[10] Luce and Raiffa, *op. cit.*, p. 279.

Payoffs of a Hypothetical Game against Nature

		Acts			
		a_1	a_2	a_3	a_4
States of	s_1 ..	500	630	700	750
Nature	s_2 ..	800	700	670	650

Table 1 shows the decisions implied by the criteria for this hypothetical game.

TABLE 1

Decisions Implied by the Game-Theoretic Criteria [†]

Act	Laplace (expected gains)	Maximin (minimum gains)	Minimax Regret (maximum regrets[‡])	Hurwicz (α = 0·2[§], minimum gains)
a_1	650	500	250	740*
a_2	665	630	120*	686
a_3	685	670*	130	676
a_4	700*	650	150	670

[†] The table entries are explained in parentheses above each column. The asterisks indicate the act (row) chosen by each criterion.

[‡] There is a difficulty in expressing regrets in terms of utilities. To do so requires a separate utility function to be derived for each state of Nature using the maximum possible gain for each state as the origin of each utility function. It would be infeasible to derive utility functions under these conditions.

[§] α = 0·2 implies the decision maker is fairly optimistic.

2.3 THE CRITERIA AS PRESCRIPTIVE DECISION RULES

Possible Gains From Decisions

Using the previous example we can construct a graph of gains for different acts given s_1 or s_2. The utility of the gain incurred by adopting act a_j when state of Nature s_i occurs is denoted by $u(a_j/s_i)$. Figure 1 describes the convex set of gains for all actions or mixtures of actions.

Admissible Acts

An act a_j is said to be admissible if there is no act a' such that for all i,

$$u(a_j/s_i) \leqslant u(a'/s_i),$$

that is, act a_j is not dominated by any act a'.

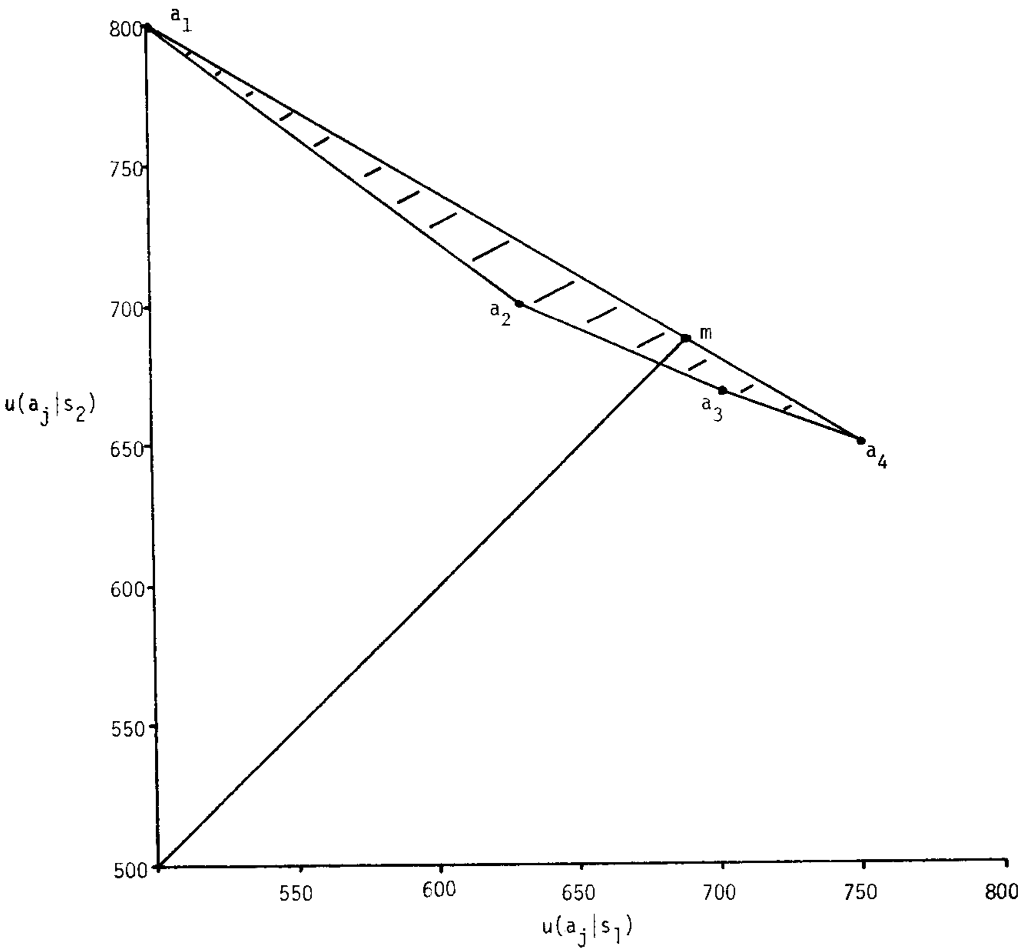

FIGURE 1: *The Convex Set of Possible Gains for Pure and Mixed Acts.*

Pure and Mixed Acts

In the example, acts a_1, a_2, a_3, and a_4 are pure acts. If no mixture of acts was permissible, then only the points corresponding to acts a_1, a_2, a_3, and a_4 would be admissible. When mixed acts are permissible, the cross-hatched area of Figure 1 shows the gains of all possible strategies. But the admissible acts are then only those on the line joining a_1 and a_4, i.e. any combination or random mix of these two acts is an admissible act (strategy), all points below and to the left being dominated by one or more points on the line.

Decision Implications of the Criteria[11]

(i) *Maximin Criterion.* When randomized or mixed strategies are permissible, the maximin strategy is indicated by a point[12] m on the line a_1a_4 of Figure 1. Using the probabilities implied by the slope of a_1a_4 a decision maker should be indifferent between all acts on this line, but in selecting the maximin strategy a decision maker is implying that this is better than all others [13]. Thus, assuming that utilities are correctly specified, a second-order utility resulting from interaction between outcomes and events must explain the choice of the maximin strategy. In fact, in adopting the maximin strategy the decision maker is implying that Nature is a conscious opponent who will change her strategy to minimize his gain, i.e. the conditional gain of an act will affect the probability of the event. This is, of course, inherently wrong. In games against Nature the gain is conditional on the event and not the reverse. It is axiomatic that Nature is neutral, and there is—for the type of situation under consideration here—certainly no justification to assume otherwise. As a prescriptive tool the maximin criterion is therefore unsatisfactory for games against Nature.

(ii) *Minimax Regret (Savage) Criterion.* The Savage criterion is not as conservative as the maximin; that is, it does not assume Nature is motivated by the same implied degree of ill-intent. Nonetheless, in adopting a minimax regret criterion a degree of malevolence is implied. Such a decision maker is more concerned with the *ex post* opportunity losses than the *ex ante* outcomes and this means there is a breakdown in one of the generally accepted axioms of rational behaviour, namely the consistent ordering of preferences[14]. For example, the ordering of preferences for the acts is $a_2 > a_3 > a_4 > a_1$. By dropping the least preferred act, a_1, the preference ordering of the acts becomes $a_4 \geqslant a_3 > a_2$. The ordering of the three acts has thus been reversed by dropping an irrelevant alternative from the set of possible acts. Such indicated behaviour is judged as irrational.

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[11] In A. N. Halter and G. W. Dean, *Decisions Under Uncertainty: Applications to Agriculture* (unpublished manuscript, Oregon State University, 1967), the implications of these criteria are discussed more comprehensively than here although in similar context.

[12] The point m was determined by constructing a line from the origin with slope = 1 (since scales of both axes are the same) to intersect an admissible strategy. See H. Chernoff and L. E. Moses, *Elementary Decision Theory* (New York: Wiley, 1959), p. 148.

[13] There is an alternative explanation that he is indifferent to all points on a_1a_4 and uses the maximin solely to take action. But this is trivial because it could apply equally well for any decision criterion.

[14] This axiom need not be accepted for games against conscious opponents where collusion and trade in payoffs are permissible. Situations of rational action in which the axiom is not necessary have been noted in J. S. Coleman, "The Possibility of a Social Welfare Function", *American Economic Review*, Vol. 56, No. 5 (December, 1966), pp. 1105–22.

(iii) *Hurwicz Criterion.* This criterion is a variation of the maximin (corresponding exactly when $\alpha = 1{\cdot}0$) in which the decision maker becomes less pessimistic about Nature's role in the decision as α approaches 0·5. A decision maker with α less than 0·5 is optimistic. Nature, rather than being an opponent, is then regarded as working for him.

The values of α and $1 - \alpha$ for those games with only two states of Nature may be regarded as prior probabilities of these states occurring. Therefore, by consistently holding to a value of α the decision maker implies Nature is consciously altering her strategies depending on the outcomes. The decision maker who employs the Hurwicz criterion (i.e. uses some α universally) thus demonstrates the same type of irrationality as one who employs the maximin.

(iv) *Laplace Criterion.* Here, equal weights are assigned to all states of Nature—a procedure equivalent to assuming implicitly a rectangular (diffuse) prior probability distribution on the states. However, because our definition of uncertainty precludes use of probabilities, equal weighting cannot be more justified than any other weighting. This then is the logical fallacy[15] of using the Laplace for decision problems under uncertainty (rather than for those under risk). The further criticism which can be directed is the irrelevant one that if there is incorrect specification of the number of states of Nature[16], or if there is no clear and definite separation of the possible states, then the weights given to states are incorrect or cannot be assigned.

Criticisms of the game-theoretic algorithms based on their compatibility with sets of axioms for decision making have been given by Milnor and by Luce and Raiffa[17]. These criticisms illustrate the failure of the criteria to meet under all circumstances with the norms of rational behaviour. Although most of these criticisms are valid, for our purposes it is really unnecessary to go beyond proving the inconsistency of the criteria with the axiom of Nature's neutrality, and the illogicality of implying prior probabilities, to demonstrate their unsuitability as normative decision criteria for uncertainty.

2.4 AN ALTERNATIVE APPROACH TO UNCERTAINTY PROBLEMS

If the view that only objective probabilities are legitimate is strictly adhered to, then the only probabilities of use in decision making are those derived from experiment or observation. This implies that most of the

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[15] The decision maker who uses the Hurwicz criterion with varying α for different problems is illogical on the same grounds.

[16] Irrelevant because our definition of the decision problem precludes the possibility of not specifying the states correctly.

[17] J. Milnor, "Games Against Nature", pp. 49–60 in R. M. Thrall, C. H. Coombs and R. L. Davis (*eds.*), *Decision Processes* (New York: Wiley, 1954) and Luce and Raiffa, *op, cit.*, pp. 286–98. These axioms have been based on characteristics considered desirable in all decision criteria.

decisions of businessmen (including farmers) must be made under uncertainty because for the majority of these operational decisions, even if some form of experimentation was feasible, the cost added to the cost of procrastination would exceed the benefit of experimenting.

By assigning subjective probabilities[18] to the states of Nature, uncertainty reduces to risk—often referred to as subjective risk. The important question is "How often can subjective probabilities be formulated to reduce situations of uncertainty to risk?" It is difficult to envisage a decision problem confronting a decision maker in which he has insufficient subjective knowledge about the problem to formulate subjective probabilities. Without some such knowledge the decision maker is unlikely to recognize the existence of a decision problem. Even if the knowledge he has is hazy or intuitive, he will usually be able at least to specify bounds for the probability of occurrence of an event. If the decision maker does not consciously use probabilities in making his decision he will at least imply them by his decision.

If prior probabilities are attached to the states of Nature, then any act which maximizes expected outcome is a Bayes' strategy. For the two-state case if the prior probabilities are p_1 and p_2 (where $p_2 = 1 - p_1$), the Bayes' strategy is

$$\max E[u(a_j/s_i)] = p_1 u(a_j/s_1) + p_2 u(a_j/s_2).$$

It can be proved that every admissible act is a Bayes' strategy[19]. Therefore, if the utilities are correctly specified for a decision maker, the admissible act he selects will imply a set of prior probabilities. The proof can be seen by considering the convex set in Figure 2.

By drawing the iso-expected utility line

$$p_1 u_1 + p_2 u_2 = k,$$

which corresponds to prior probabilities being attached to the gains u_1 and u_2, and adjusting the level of expected utility k so that the line just touches the convex set, the point at which it touches is a Bayes' strategy. As p_1 goes from 1 to 0, the slope of the line changes from 0 to $-\infty$. It is a property of the convex set that any tangent line on the boundary supports that set. Therefore, for any point on the boundary of the convex set there is a corresponding support line. This means that each admissible act has a support line which implies a set of prior probabilities corresponding to the choice of that act.

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[18] The case for using subjective probabilities in decision problems has been argued most strongly in L. J. Savage, *The Foundations of Statistics* (New York: Wiley, 1954). This issue has been a controversial one involving statisticians, psychologists, and economists. W. Fellner, *Probability and Profit* (Homewood: Irwin, 1965) and, more recently, R. R. Officer, *Decision Making Under Risk* (unpublished M.Ag.Ec. thesis, U.N.E., 1967) have reviewed the controversy. As far as we are concerned, all probabilities used in decision making are necessarily subjective by virtue of their use, even if they are, in fact, based on clearly defined objective probabilities. That is, use of frequency-based "objective" probabilities explicitly assumes that the future will be just like the past.

[19] See, for example, A. M. Mood and F. A. Grabill, *Introduction to the Theory of Statistics* (New York: McGraw-Hill, 1963), p. 285.

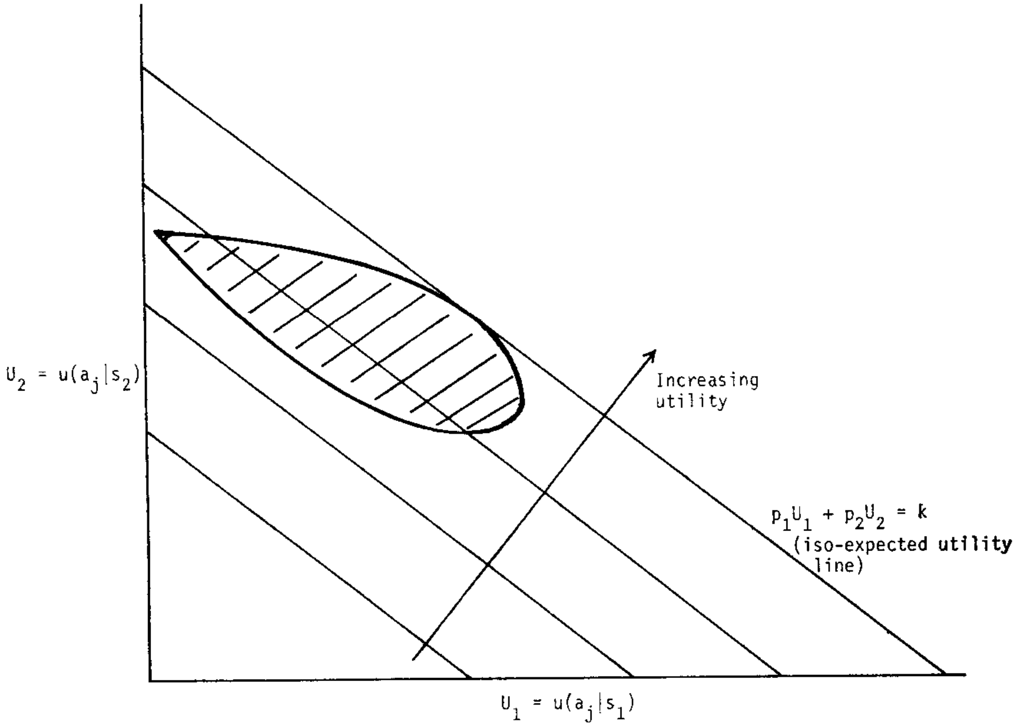

FIGURE 2: *An Investment Opportunity Set.*

Returning to the example of Figure 1, it can be seen that all strategies on a_1a_4 (including a_1 and a_4) are Bayes' strategies. The slope corresponds to a particular weighting of u_1 and u_2, implying for strategies on this line the prior probabilities p'_1 and p'_2. Using these implied probabilities, all the strategies on the line a_1a_4 have the same expected gain, i.e. a decision maker with probabilities p'_1 and p'_2 would be indifferent between all strategies on a_1a_4. At this point it is worth noting that if the prior probabilities are subjective probabilities, then a decision maker adopting a Bayes' strategy is also using the criterion of maximizing expected utility.

The decision maker does not always have to be very precise in his formulation of subjective probabilities. Even the specification of upper and lower bounds for the subjective probabilities will often be sufficient. This can be illustrated in our previous numerical example where $p_1 < 0{\cdot}375$ ($p_2 > 0{\cdot}625$) implies selection of a_1. If $p_1 > 0{\cdot}375$, a_4 would be selected. Each act has a range cf probabilities of the states of Nature for it to be selected and thus allows room for error. When the admissible acts have closely related conditional losses, the chosen act will be sensitive to small changes in probabilities. However, even though some acts may change with a slight change in probability, the difference in terms of losses between adjacent acts will be small. Any error in formulating probabilities, if it is not large, will not result in a large loss if the "best" act is not chosen. This further implies[20] that there may be little value in expending effort to gain more precise information on the problem, such as by improving probability estimates.

If subjective probabilities are implied in real behaviour, then it is far better to make these explicit in a systematic manner—even if they may not be readily extractable. This is a natural extension of the pervading philosophy of farm management that formal analysis and record keeping are superior to informal reasoning and reliance on memory. Accordingly, we suggest that there is virtually no decision problem, recognized by a decision maker, for which he cannot formulate worthwhile subjective probabilities, i.e. the decision maker is never in complete ignorance about state likelihoods in a problem. If he has little confidence in formulating probabilities, the (practical) approach of Bayesian statistics is to use a diffuse prior distribution on the states of Nature, such as is implied in the Laplace criterion. This means he has sufficient knowledge to list the possible states and their likely consequences, but insufficient knowledge about the relative weighting these states should receive. We generalize that in practice decisions which do not have certain outcomes can be formulated as decisions under risk. Although at the extreme, risk may not be clearly differentiated from uncertainty, we suggest the use of a diffuse prior distribution for the "uncertainty" case so that analysis can proceed in an unambiguous systematic manner.

3 SOME DECISION CRITERIA FOR RISK

Two important criteria have found widest acceptance for solution of risky decision problems, namely maximization of either expected profit or expected utility.

3.1 EXPECTED PROFIT MAXIMIZATION

The criterion most commonly used by economists and others for decision problems under risk is to choose the act with the maximum expected profit. The popularity of the criterion is due to the widely held view that business decisions are made to maximize profits, and because the criterion readily fits conventional economic theory. Profit maximization may result in maximizing satisfaction for decisions under certainty but is unlikely to do so for decisions under risk, when the decision problem takes on another dimension. Both income and security become dimensions of satisfaction, although they may not be the only dimensions.

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[20] A similar set of implications in a production economics setting has been discussed by J. Havlicek Jr. and J. A. Seagraves, "The 'Cost of the Wrong Decision' as a Guide in Production Research", *Journal of Farm Economics*, Vol. 44, No. 2 (May, 1962), pp. 157–68.

Profit maximization extended to risky situations implies that consideration of risk plays no part in deciding between alternative courses of action, i.e. the course of action with the greatest expected return is adopted irrespective of the risk or variability associated with that return. It is likely that firms try to maximize returns when risk is negligible relative to their assets, but when risk becomes more substantial there is considerable evidence[21] to suggest they are very conscious of its presence. Therefore we conclude that because the criterion of maximizing expected profits takes no account of the risk associated with a decision, and because security aspects play an important role in many decisions made under risk, the criterion is not consistent with our definition of rational behaviour.

3.2 EXPECTED UTILITY MAXIMIZATION

The only worthwhile criterion[22] which has been strongly proposed as an alternative to maximizing expected profit in risky decisions is the maximization of expected utility. The proponents of this criterion claim that it overcomes all the shortcomings of profit maximization. Units of utility[23] (utiles) are assigned to amounts of money as a result of preferences shown by a decision maker for the monetary outcomes of simple gambles. It is postulated that these preferences indicate the value of the outcomes with respect to his goals. For example, if $2,000 will

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[21] Obvious examples are given by diversification in the face of risk, e.g., many mixed-farming operations, the holding of chains of properties by pastoral companies, and the diversification of stocks held by unit trust companies.

[22] This statement means that we reject, for farm management decisions, criteria such as Baumol's theory of sales maximization (W. J. Baumol, *Business Behavior, Value and Growth* (New York: Macmillan, 1959)) and Williamson's theory of discretionary behaviour (O. E. Williamson, *The Economics of Discretionary Behavior: Marginal Objectives in the Theory of the Firm* (Englewood Cliffs: Prentice Hall, 1964)). Also we consider that theories such as Shackle's theory of potential surprise and focus outcomes (e.g., G. L. S. Shackle, *Expectations in Economics* (Cambridge: Cambridge Univ. Press, 1949)), Simon's theory of the satisficer (H. A. Simon, *Models of Man*, (New York: Wiley, 1957)), Roy's safety-first theory (A. D. Roy, "Safety-first and the Holding of Assets", *Econometrica*, Vol. 20, No. 3 (July, 1952), pp. 431–49), and Fellner's semi-probabilistic theory (W. Fellner, *op. cit.*) are essentially descriptive: that is their role is to describe actual behaviour rather than rational behaviour. Even in this role the theories are only useful as expository aids to understanding possible reasons for people taking particular courses of action. The inconsistency and the large number of variables affecting a decision maker's choice of action render the theories ineffectual as universal descriptive decision models.

[23] The utility used in the criterion of maximizing expected utility should not be confused with the utility underlying the demand curve. Utility in risky decision analysis has no direct welfare implications and is operationally distinct from the utility of welfare economics (see E. Kauder, *A History of Marginal Utility Theory* (New Jersey: Princeton University Press, 1965), pp. 200–217). For an excellent review of utility theory see E. W. Adams, "A Survey of Bernoullian Utility Theory", in H. Solomon (*ed.*), *Mathematical Thinking in the Measurement of Behaviour* (Glencoe: The Free Press, 1960), pp. 151–268.

enable him to educate his children to the level he aspires for them, but \$1,000 is totally inadequate, then he will value \$2,000 more than twice as much as \$1,000. The utilities assigned as a result of such preferences incorporate the ambitions of the decision maker, and therefore reflect the relative satisfaction different monetary outcomes would give him. The action with the greatest expected utility will be chosen for a risky decision situation. Based on the reasonable consistency requirements of utility theory and the assumptions involved in assigning utilities, this action guarantees the decision maker greatest satisfaction, judged *ex ante*, of the alternative courses of action available to him. The criterion, by its definition, is consistent with our concept of rational behaviour.

The criterion of maximizing expected utility by way of a utility function reduces the important dimensions of satisfaction of a risky situation to a single dimension. This lessens any ambiguity and simplifies the analysis of decision problems under risk. The risk element in a decision is accounted for by the weights or utilities given to possible outcomes. Consider an illustration based on one given by Friedman and Savage[24].

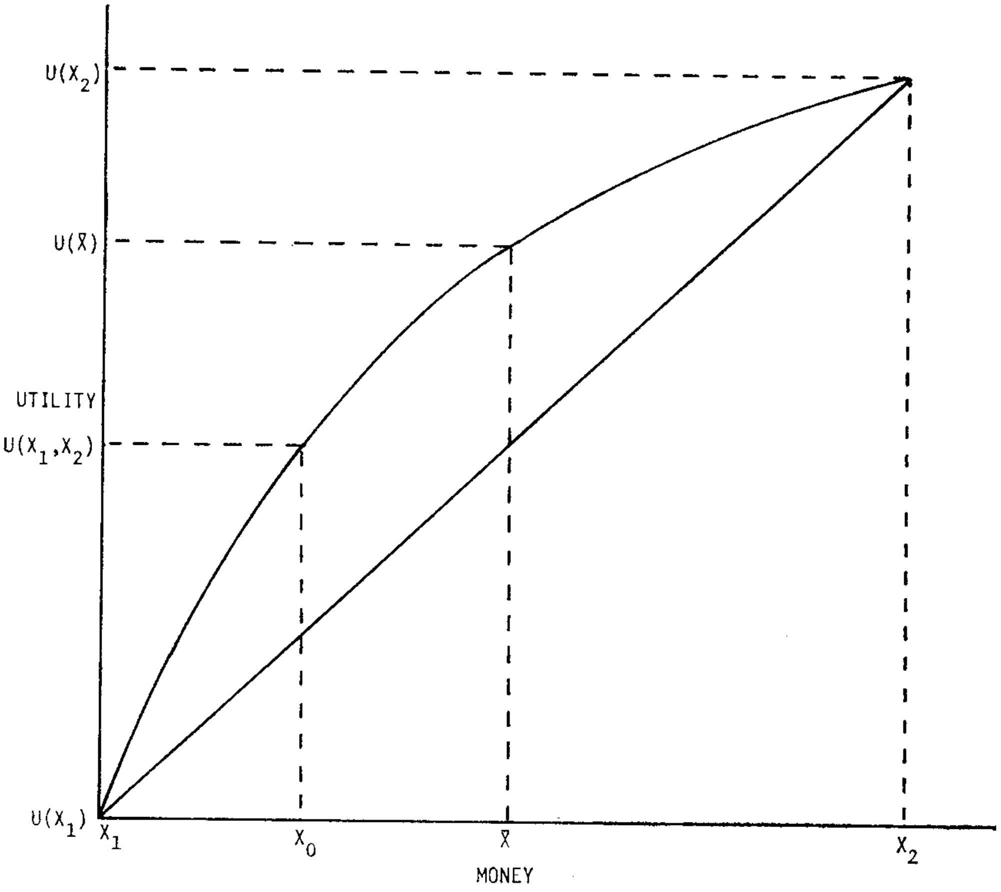

FIGURE 3: *A Utility Function with Diminishing Marginal Utility.*

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[24] M. Friedman and L. J. Savage, "The Utility Analysis of Choices Involving Risk", *Journal of Political Economy*, Vol. 56, No. 4 (August, 1948), pp. 279–304.

A decision maker is faced with a decision between the gamble with equally likely risky outcomes X_1 and X_2 and the certain outcome $\bar{X}$, where $\bar{X}$ is the expected value of the gamble (i.e. $\bar{X} = \frac{1}{2}X_1 + \frac{1}{2}X_2$). He has a utility function with diminishing marginal utility as shown in Figure 3.

The utilities of outcomes X_1 and X_2 are shown on the utility axis as $u(X_1)$ and $u(X_2)$ respectively. The utility of the risky choice (X_1, X_2) is $u(X_1, X_2)$ where

$$u(X_1, X_2) = \tfrac{1}{2}u(X_1) + \tfrac{1}{2}u(X_2).$$

The utility of the certain choice $(\bar{X})$ is $u(\bar{X})$. Because $u(\bar{X}) > u(X_1, X_2)$, the decision maker chooses the certain outcome $\bar{X}$. The risk aversion shown by the decision maker is $\bar{X} - X_0$, in money value, where X_0 is a certain amount of money whose utility value is equal to $u(X_1, X_2)$. This means he would be prepared to pay up to $\bar{X} - X_0$ not to participate in the gamble (X_1,X_2) but instead to receive $\bar{X}$ with certainty. The risk aversion of the decision maker is wholly explained by the utilities of the outcomes of the gamble. The reverse would be true in a similar problem for a person whose utility function shows increasing marginal utility over this range. Where payoffs are small and the risk faced by a decision maker is thus insignificant, the relevant sector of the utility function will be approximately linear, and maximizing expected utility will be equivalent to maximizing expected profit.

4 UTILITY ANALYSIS AND RISK

Utility analysis is the procedure through which the expected outcomes of alternative acts (which may be investment opportunities of various complexity) are transformed into expected utilities, and the criterion of maximizing expected utility is applied to the selection of the best action. There are two methods by which the analysis can be carried out—the most frequently used method being the "direct" method[25]. Using this method, each outcome is transformed into a utility value, weighted by its probability of occurrence and summed to give an expected utility for each act. The method is straightforward and appropriate to analysis of problems with few outcomes.

An alternative procedure is the "moment" method, which is best suited to more complex problems because it reduces the computational burden of analysis. This is done by summarizing the probability distribution of outcomes for an act by its mean and variance (thus ignoring higher moments in the simplest case), which can then be used to determine the

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[25] The direct method has been well demonstrated in C. J. Grayson, Jr., *Decisions Under Uncertainty: Drilling Decisions by Oil and Gas Operators* (Boston: Harvard Business School, 1960).

expected utility of the act[26]. The way in which a person is prepared to sacrifice expected gain to reduce the variance of expected outcomes can also be described by a two-dimensional expected outcome—variance ($E-V$) indifference system, which implies a utility function. For example, the utility function of Figure 3 is equivalent to the $E-V$ indifference system of Figure 4.

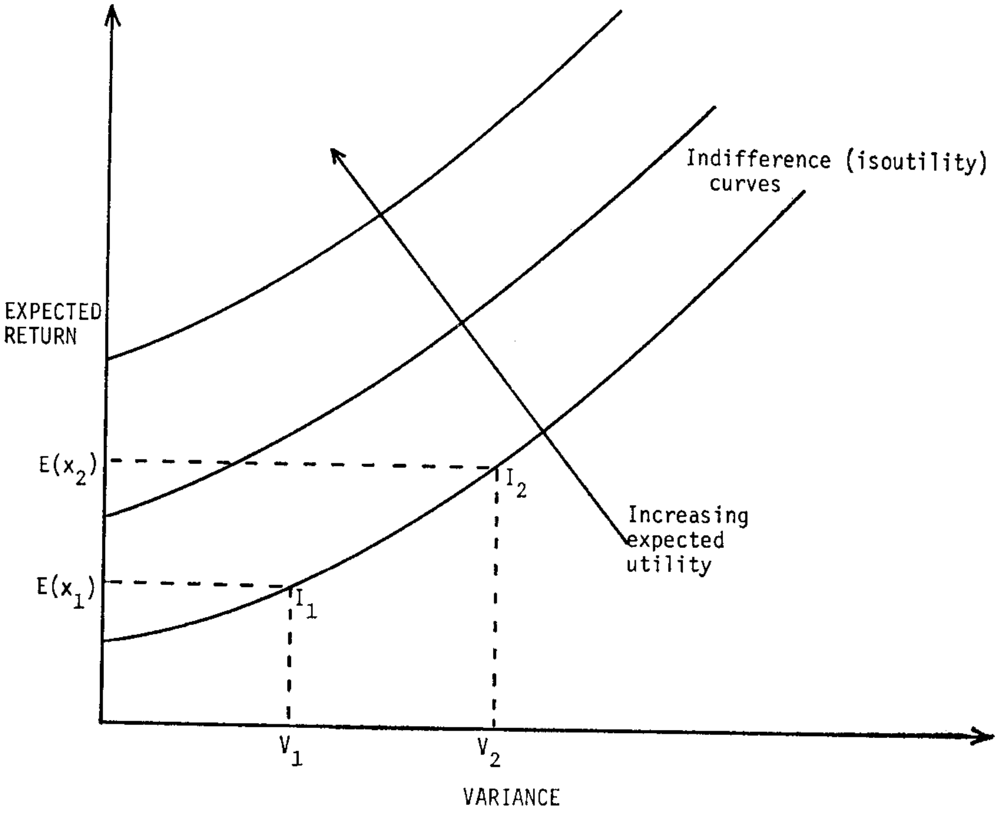

FIGURE 4: *An E-V Indifference System.*

All points on an $E-V$ indifference curve have the same expected utility. For example, in Figure 4 an act I_1 with an expected return $E(x_1)$ with a variance V_1 has the same expected utility as the act I_2 which has an expected return $E(x_2)$ with a variance V_2. The increased risk (variance) of I_2 compared with I_1 has been offset by the increased expected return of $E(x_2)$ over $E(x_1)$. The extent to which the individual discounts

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[26] How this is done has been shown by D. E. Farror, *The Investment Decision under Uncertainty* (Englewood Cliffs: Prentice-Hall, 1962); K. Borch, "A Note on Utility and Attitudes to Risk", *Management Science*, Vol. 9, No. 4 (July, 1963), pp. 697–700; W. T. Morris, *The Analysis of Management Decisions* (Homewood: Irwin, 1964); and H. F. Gale, "Cardinal Utility and Decision Making in the Soybean Processing Industry", *Journal of Farm Economics*, Vol. 49, No. 4 (November, 1967), pp. 942–47.

expected return for risk is indicated by the slope of the indifference curves. Of course, ranking efficient acts along a frontier on the $E-V$ preference map to find a solution where the frontier touches the highest indifference curve, is directly equivalent to ranking acts according to their expected utilities.

5 CONCLUSION: UTILITY ANALYSIS IN PERSPECTIVE

Agricultural economists are usually responsive to new developments in economics (and other disciplines) when some operational usefulness is apparent. Since utility analysis has been widely discussed for over a decade, some explanation of the small impact it has had so far in agriculture is necessary.

It has been shown that utility analysis is the most satisfactory method available for handling risky decision problems. The difficulty of using utility analysis in practical situations has impaired its use. However, in our opinion, utility analysis has so far not been given a fair trial. There has apparently been only one study explicitly set up to explore the use of the expected utility hypothesis as a prescriptive criterion for farm management decisions[27]. The results from this study were encouraging in that the decisions indicated by utility analysis were better, judged by farmers' response, than those indicated by the conventional approach of profit maximization (cost minimization).

One difficulty in using utility analysis is that much of the prescriptive advice given in agriculture tends to be directed at groups or communities rather than individuals. Utility analysis is, strictly speaking, relevant only to the individual's behaviour. There are many difficulties in accounting for risk in broad-scale recommendations—these problems ~~being noted~~ elsewhere[28].

Finally, formal treatment of risk in farm management work is not very old. Study in this area gained impetus following the 1952 Bozeman Conference on Risk and Uncertainty[29], and has developed slowly since then. The demand for algorithms to handle risk in decision making has also been slow to develop. However, judging by recent publications, the situation is rapidly changing to more widespread explicit incorporation of risk in economic analyses of many types including farm management analysis. How useful this empirical work will prove has yet to be

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[27] Officer, *op. cit.* and reported in R. R. Officer and A. N. Halter, "Utility Analysis in a Practical Setting", *American Journal of Agricultural Economics*, Vol. 50, No. 2 (May, 1968), in press.

[28] R. R. Officer, A. N. Halter and J. L. Dillon, "Risk, Utility and the Palatability of Extension Advice to Farmer Groups", *Australian Journal of Agricultural Economics*, Vol. 11, No. 2 (December, 1967), in press.

[29] Great Plains Agricultural Council, *Proceedings of Research Conference on Risk and Uncertainty in Agriculture* (North Dakota Agric. Exp. Sta. Bul. 400, 1955).

evaluated. Such work as that on stochastic production functions[30], stochastic farm programming[31], inventory problems[32] and simulation[33] only goes part of the way if decision problems are formulated in terms of risk (perhaps as an efficiency frontier) but are not solved to the extent of indicating the decision maker's best action. Since we have shown that a decision maker can make such decisions consistently and conveniently by way of his utility function (which, in turn, merely formalizes his preferences), we suggest and anticipate a much wider use of utility analysis in research and practical decision making in farm management.

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[30] e.g., W. A. Fuller, "Stochastic Production Functions for Continuous Corn", *Journal of Farm Economics*, Vol. 47, No. 1 (February, 1965), pp. 105–19.

[31] e.g., R. Freund, "The Introduction of Risk into a Programming Model", *Econometrica*, Vol. 24, No. 2 (July, 1956), pp. 253–63; E. O. Heady and W. Candler, *Linear Programming Methods* (Ames: Iowa State University Press, 1958), Chap. 17; A. M. M. McFarquhar, "Rational Decision Making and Risk in Farm Planning—An Empirical Application of Quadratic Programming in British Arable Farming", *Journal of Agricultural Economics*, Vol. 14, No. 4 (December, 1961), pp. 552–63; B. M. Camm, "Risk in Vegetable Production on a Fen Farm", *Farm Economist*, Vol. 10, No. 2 (1962), pp. 89–98; P. van Moseke, "Stochastic Linear Programming", *Yale Economic Essays*, Vol. 5, No. 1 (Spring, 1965), pp. 196–253; W. C. Merrill, "Alternative Programming Models Involving Uncertainty", *Journal of Farm Economics*, Vol. 47, No. 3 (August, 1965), pp. 595–610; J. G. Stovall, "Income Variation and Selection of Farm Enterprises", *Journal of Farm Economics*, Vol. 48, No. 5 (December, 1966), pp. 1575–79; S. R. Johnson, "A Re-examination of the Farm Diversification Problem", *Journal of Farm Economics*, Vol. 49, No. 3 (August, 1967), pp. 610–21; and S. R. Johnson, K. R. Tefertiller and D. S. Moore, "Stochastic Linear Programming and Feasibility Problems in Farm Growth Analysis", *Journal of Farm Economics*, Vol. 49, No. 4 (November, 1967), pp. 908–19.

[32] e.g., J. L. Dillon and A. G. Lloyd, "Inventory Analysis of Drought Reserves for Queensland Graziers: Some Empirical Analytics", *Australian Journal of Agricultural Economics*, Vol. 6, No. 1 (September, 1962), pp. 50–67, and R. G. Heifner, "Determining Efficient Seasonal Grain Inventories: An Application of Quadratic Programming", *Journal of Farm Economics*, Vol. 48, No. 3, Part I (August, 1966), pp. 648–60.

[33] e.g., A. N. Halter, "Simulation Models in Decision Making", *Conference Proceedings of the Committee on Economics of Range Use and Development, Western Agricultural Economics Research Council* Report No. 7 (July, 1965), pp. 131–67, and P. Zusman and A. Amiad, "Simulation: a Tool for Farm Planning Under Conditions of Weather Uncertainty", *Journal of Farm Economics*, Vol. 47, No. 3 (August, 1965), pp. 574–94.