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Estimating a Demand System with Seasonally Differenced Data

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Several recent papers have used annual changes and monthly data to estimate demand systems. Such use of overlapping data introduces a moving average error term. This paper shows how to obtain consistent and asymptotically efficient estimates of a demand system using seasonally differenced data. Monte Carlo simulations and an empirical application to the estimation of the U.S. meat demand are used to compare the proposed estimator with alternative estimators. Once the correct estimator is used, there is no advantage to using overlapping data in estimating a demand system.

Key Words: autocorrelation, demand system, Monte Carlo, overlapping data, seasonal differences

JEL Classifications: C13, Q11, Q13

Some past research has estimated demand systems using seasonally differenced models (Brown and Lee, 2000; Brown, Lee, and Seale, 1995; Duffy, 1990; Eales, Durham, and Wessells, 1997; Lee, 1988; Chiang, Lee, and Brown, 2001; Muhammad, 2007; Muhammad, Jones, and Hahn, 2007; Seale, Marchant, and Basso, 2003). With monthly data, for example, the first observation of annual differences might be the January 2001 to January 2002 change and the next observation would be the February 2001 to February 2002 change. Thus, the January 2001 to January 2002 and the February 2001 to February 2002 changes have 11 monthly changes in common. The 11 common monthly changes are the February 2001 to March 2001, March 2001 to April 2001, and so on, until December 2001 to January 2002 change. Therefore, such data in the form of January 2001 to January 2002 and February 2001 to February 2002 changes are said to overlap since 11 of the 12 monthly changes included in the two annual changes in the example are the same. These models allow the researchers to use the higher frequency data and do not require specifying the form of seasonality present in the high frequency data. For our example, using seasonal annual changes does not require the specification of monthly seasonality. Fraser and Moosa (2002) provide a motivation for avoiding having to specify the form of the seasonality since they show that if seasonality is stochastic and it is assumed to be deterministic, the model will be misspecified

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and estimates will be inconsistent. But, as we show, except in the unlikely case of seasonal unit roots, such models become autocorrelated with the degree of autocorrelation depending on the level of seasonal differencing. This autocorrelation is introduced even though there is no serial correlation in the series prior to seasonal differencing. Previous studies using seasonally differenced models do not fully account for this autocorrelation.

The econometric problem resulting from using overlapping data are that the error terms follow a moving average (MA) process. This autocorrelation results in inefficient estimates and biased hypothesis tests. Harri and Brorsen (2009) compare different estimators used with overlapping data in the context of single equations. They show that when lagged values of the dependent variables are not included as explanatory variables, the generalized least squares (GLS) estimator is the appropriate estimator. The covariance matrix for the GLS transformation can be derived analytically in the case of overlapping data.

In this paper, we show how to obtain consistent and asymptotically efficient estimates of a demand system using seasonally differenced data. Specifically, we propose a GLS estimator for estimating a system of equations with overlapping data as an extension of the GLS estimator for single equations developed by Harri and Brorsen (2009). In addition, we apply our approach to the estimation of a demand model. Monte Carlo simulations are used to compare the properties of the GLS estimator with overlapping data (annual differences) and the conventional seemingly unrelated regressions (SUR) estimator with disaggregate differenced data (monthly differences). The proposed GLS estimator is also compared with the first-order, autoregressive model considered by recent research (Muhammad, 2007; Muhammad, Jones, and Hahn, 2007; Seale, Marchant, and Basso, 2003).

The rest of the paper is organized as follows. First, the estimator combining SUR and GLS is derived. Then a Monte Carlo study is used to determine the bias and inefficiency of the various estimators. Then U.S. meat demand is estimated with the alternative methods to illustrate the differences between the estimators.

The Model

Start with the following system of M equations:

$$w_m = Z_m\beta_m + D_m\gamma_m + \varepsilon_m, \quad m=1,\ldots,M, \tag{1}$$

where $\boldsymbol{w}_m$ is a $(T \times 1)$ vector of the values of the dependent variable, T is the length of the time series, $\boldsymbol{Z}_m$ is a $(T \times l_m)$ matrix of the values of explanatory variables, $\boldsymbol{D}$ is a $(T \times p)$ matrix of the values of the p dummy variables with $p = 11$ for monthly data and $p = 3$ for quarterly data, $\boldsymbol{\beta}_m$ and $\boldsymbol{\gamma}_m$ are respectively $(l_m \times 1)$ and $(p \times 1)$ vectors of regression coefficients, and $\boldsymbol{\varepsilon}_m$ is a $(T \times 1)$ vector of the disturbances. Assume that $\boldsymbol{\varepsilon} = [\boldsymbol{\varepsilon}_1', \boldsymbol{\varepsilon}_2', \ldots, \boldsymbol{\varepsilon}_M']'$ has $E[\boldsymbol{\varepsilon}] = 0$ and $E[\boldsymbol{\varepsilon\varepsilon}'] = \boldsymbol{\Sigma}$. Further assume that disturbances are uncorrelated across observations, but have contemporaneous covariance $\boldsymbol{V}$. In other words, $E[\boldsymbol{\varepsilon}_{mt}\boldsymbol{\varepsilon}_{ns}] = \boldsymbol{\sigma}_{mn}$, if $t = s$ and zero otherwise. Therefore, we can also write $\Sigma = \boldsymbol{V} \otimes \boldsymbol{I}_T$.

If one instead uses annual differences, these annual differences represent an aggregation of level $k = 12$ for monthly differences or $k = 4$ for quarterly differences. The system with the aggregated variables can be represented as:

$$y_m = X_m\beta_m + u_m, \quad m=1,\ldots,M, \tag{2}$$

where each element of the vectors $\boldsymbol{y}$ and $\boldsymbol{X}$ of the aggregated variables and the vector $\boldsymbol{u}$ of the error term in Equation (2) is the sum of k elements of $\boldsymbol{w}$, $\boldsymbol{Z}$, and $\boldsymbol{\varepsilon}$ respectively, as presented below:

$$\begin{aligned} y_t &= \sum_{j=0}^{k-1} w_{t+j}; \quad X_t = \sum_{j=0}^{k-1} Z_{t+j}; \\ u_t &= \sum_{j=0}^{k-1} \varepsilon_{t+j}, \quad t=1,\ldots,T-k+1; \\ j &= 0,\ldots,k-1. \end{aligned} \tag{3}$$

Given the size of the original sample, T, the sample size for the aggregated data are $T - k + 1$. Note also that the seasonal dummy variables no longer appear in Equation (2). Thus, the aggregate model has the same degrees of freedom as the disaggregate model. The aggregation of the variables in Equation (3) induces an MA process of order $k - 1$ in the error term $\boldsymbol{u}_m$ in Equation (2).

The system in Equation (1) is the disaggregate model which, depending on the available data, can be estimated with either monthly differences or quarterly differences. In other words, Equation (1) can represent either a differenced nondifferential demand model or a differential demand model. Similar to the arguments of Davis (2004) in the production case, there is a difference in how the error term enters the model specification for these two cases. In the former case, the "append and difference" approach, an error term is first appended to the model and then the difference is taken. In the latter case, the "difference and append" approach, the difference is taken first and then an error term is appended to the model. It is important to emphasize here that both approaches, "append and difference" and particularly "difference and append" are applied to Equation (1). An erroneous application of the "difference and append" approach would be to extend it to aggregate and append, which would lead to a mistaken conclusion that the error term in Equation (2) is free of autocorrelation. As we show later when we discuss the seasonal unit root model, the only way that the error term in Equation (2) could be free of autocorrelation is for the error term in Equation (1) to follow a 12th difference process. However, this error structure is not supported by previous literature and our empirical results. As a result, our approach applies to differential demand models (e.g., the Rotterdam and Dutch Central Bureau of Statistics (CBS) models) and also differenced nondifferential demand models (e.g., the First-Difference Almost Ideal Demand System (FDAIDS) model). It does not apply to nondifferential demand models in levels like the Almost Ideal Demand System (AIDS) model. As our Monte Carlo simulations show, the same drawbacks of using seasonally differenced data apply to both a demand system that is linear in the parameters as well as demand systems that are nonlinear in the parameters.

From the assumption that the original error terms were uncorrelated with zero mean, it follows that:

$$(4)\qquad E[u_{tm}] = E\left[\sum_{j=0}^{k-1} \varepsilon_{(t+j)m}\right] = \sum_{j=0}^{k-1} E[\varepsilon_{(t+j)m}] = 0.$$

Also, since the successive values of ε_{jm} are homoskedastic and uncorrelated, the unconditional variance of u_{tm} is:

$$(5)\qquad \text{var}[u_{tm}] = \sigma^2_{um} = E[\varepsilon^2_{tm}] = k\sigma^2_{\varepsilon_m}.$$

Based on the fact that two different error terms, u_{tm} and $u_{(t+s)m}$, ($t = 1, \ldots, T$ and $s = t + 1, \ldots, T$), have $k - s$ common original error terms, ε_m, for any $k - s > 0$, the covariances between the error terms in Equation (2) are:

$$(6)\qquad \begin{aligned}\text{cov}[u_{tm}, u_{(t+s)m}] &= E[u_{tm}, u_{(t+s)m}] \\ &= (k-s)\sigma^2_{\varepsilon_m} \qquad \forall (k-s) > 0.\end{aligned}$$

Similarly, the contemporaneous covariances between the error terms in Equation (2) are:

$$(7)\qquad \begin{aligned}&\text{cov}[u_{tm}, u_{sn}] = E[u_{tm}, u_{sn}] \\ &= E\left[\sum_{j=0}^{k-1} \varepsilon_{(t+j)m}, \sum_{j=0}^{k-1} \varepsilon_{(s+j)n}\right] = k\sigma_{mn} \qquad \forall t = s.\end{aligned}$$

Dividing Equation (6) by Equation (5) gives the correlations between the two error terms, u_{tm} and $u_{(t+s)m}$ as:

$$(8)\qquad corr[u_{tm}, u_{(t+s)m}] = \begin{cases} \frac{k-s}{k}, & \forall (k-s) > 0 \\ 0, & otherwise \end{cases}.$$

Collecting terms from Equation (8), we have the correlation matrix of each $\boldsymbol{u}_m$, $\boldsymbol{\Omega}$ ($T - k + 1 \times T - k + 1$) as:

$$(9)\ \Omega = \begin{bmatrix} 1 & \frac{k-1}{k} & \ldots & \frac{k-s}{k} & \ldots & \frac{1}{k} & 0 & \ldots & 0 \\ \frac{k-1}{k} & 1 & \frac{k-1}{k} & \ldots & \frac{k-s}{k} & \ldots & \frac{1}{k} & \ldots & 0 \\ \ldots & \frac{k-1}{k} & 1 & \frac{k-1}{k} & \ldots & \frac{k-s}{k} & \ldots & \frac{1}{k} & \ldots \\ \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots \\ \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots \\ \ldots & \frac{1}{k} & \ldots & \frac{k-s}{k} & \ldots & \frac{k-1}{k} & 1 & \frac{k-1}{k} & \ldots \\ 0 & \ldots & \frac{1}{k} & \ldots & \frac{k-s}{k} & \ldots & \frac{k-1}{k} & 1 & \frac{k-1}{k} \\ 0 & \ldots & 0 & \frac{1}{k} & \ldots & \frac{k-s}{k} & \ldots & \frac{k-1}{k} & 1 \end{bmatrix}.$$

With $\boldsymbol{\Omega}$ defined as above, we can express the covariance matrix for

$$\boldsymbol{u} = [\boldsymbol{u}'_1, \boldsymbol{u}'_2, \ldots \boldsymbol{u}'_M]' \text{ as } \mathrm{E}[\boldsymbol{uu}'] = \boldsymbol{\Sigma_u} = kV \otimes \Omega.$$

To obtain efficient estimates, the generalized least squares (GLS) parameter estimates can be derived as follows:

$$(10a)\qquad \hat{\beta} = (X'\Sigma_u^{-1}X)^{-1}X'\Sigma_u^{-1}y,$$

$$(10b)\qquad \begin{aligned}\hat{\beta} = &(X'(k^{-1}V^{-1} \otimes \Omega^{-1})X)^{-1} \\ &\times X'(k^{-1}V^{-1} \otimes \Omega^{-1})y,\end{aligned}$$

or

$$(10c)\qquad \hat{\beta} = (X'V^{-1} \otimes \Omega^{-1}X)^{-1}(X'V^{-1} \otimes \Omega^{-1})y.$$

Let $\boldsymbol{P}' = \boldsymbol{C}\boldsymbol{\Lambda}^{-1/2}$, where $\boldsymbol{C}$ is the matrix of the eigenvectors of $\boldsymbol{\Omega}$, and $\boldsymbol{\Lambda}$ is the diagonal matrix containing the eigen values of $\boldsymbol{\Omega}$. Then, $\boldsymbol{\Omega}^{-1} = \boldsymbol{P}'\boldsymbol{P}$. Substituting this result into Equation (10c) and rearranging the equation, we obtain:

$$(11) \quad \hat{\beta} = (X'P'(V^{-1} \otimes I)PX)^{-1}X'P'(V^{-1} \otimes I)Py.$$

Let $\boldsymbol{X^*} = \boldsymbol{PX}$ and $\boldsymbol{y^*} = \boldsymbol{Py}$ we get:

$$(12) \quad \hat{\beta} = (X^{*\prime}(V^{-1} \otimes I)X^*)^{-1}X^{*\prime}(V^{-1} \otimes I)y^*,$$

which is the conventional SUR estimator with an unknown contemporaneous covariance matrix $\boldsymbol{V}$ with the transformed variables $\boldsymbol{X^*}$ and $\boldsymbol{y^*}$.

Similarly the variance-covariance matrix of the GLS estimates from Equation (12) is:

$$(13) \quad Var[\hat{\beta}] = \sigma_\varepsilon^2(X^{*\prime}(V^{-1} \otimes I)X^*)^{-1}.$$

Alternative Estimators

Among alternative estimators, one estimator is the maximum likelihood estimator developed by Beach and MacKinnon (1979). Studies that have used this model include Eales, Durham, and Wessells (1997), Muhammad (2007); Muhammad, Jones, and Hahn (2007), and Seale, Marchant, and Basso (2003). This estimator imposes the same AR(1) parameter for all **M** equations following Berndt and Savin (1975) and therefore is referred to as the AR(1) estimator from hereon. In the general case considered by Beach and MacKinnon (1979), the AR(1) parameter needs to be estimated. However in our case, this parameter can be derived analytically to be $(k - 1)/k$. The AR(1) estimator, however, is inefficient and hypothesis tests remain biased since it assumes an AR(1) process when the true process is a MA$(k - 1)$ process that declines linearly rather than exponentially as assumed with an AR(1).

Another estimator, the seasonal difference model of Box and Jenkins (1970), which is called a seasonal unit root model in more recent literature, uses data which are in some sense overlapping, but do not create an overlapping data problem if correctly specified. For annual data, the seasonal unit root model for the dependent variable ω_t is defined as:

$$(14) \quad \begin{aligned} \omega_t &= \alpha\kappa_t + \eta_t, \\ \eta_t &= \eta_{t-12} + \xi_t, \end{aligned}$$

where κ_t is an independent variable, η_t is the error term following a 12th difference process, ξ_t is independently and identically normally distributed, α is a parameter, and t represents observation frequency at monthly level. In this case, the model

$$(15) \quad \omega_t - \omega_{t-12} = \alpha(\kappa_t - \kappa_{t-12}) + \xi_t$$

has no autocorrelation. In this example, 12th differencing leads to a model that can be estimated using overlapping data and ordinary least squares. Seasonal unit roots have largely been used when the research objective was forecasting (Clements and Hendry, 1997). One problem with the seasonal unit root model is that it is often rejected in empirical work (McDougall, 1995). Another problem is that it implies each month's price can wander aimlessly away from the prices of the other months. Such a model seems implausible for most economic time series. Hylleberg et al. (1990) developed a general procedure that can test for unit roots at some seasonal frequencies without maintaining that unit roots are present at all seasonal frequencies. Beaulieu and Miron (1993) extend the Hylleberg et al. (1990) procedure to monthly data and test a number of U.S. aggregate data series for the presence of seasonal unit roots. They find that the presence of unit roots is rejected at most of the seasonal frequencies for a large fraction of the data series. Wang and Tomek (2007) present another challenge to the seasonal unit root model since they argue that commodity prices should not have any unit roots. While a seasonal unit root model may be an unlikely model, if it is the true model, no autocorrelated residuals would occur.

Monte Carlo Simulation Procedure

In this section we discuss the Monte Carlo study used to compare the properties of the proposed estimator and alternative estimators. We generate the data according to Equation (1). Given that Equation (1) can represent any differential demand model or a demand model in first-difference form and the findings of Bryant and Davis (2008) that the FDAID model outperforms the other models we use the FDAID functional form to generate our data. We use a system of three equations, and thus $\boldsymbol{Z}_m$ consists of three correlated log price series, P_1, P_2, P_3, an exogenous variable representing the log of the ratio of expenditures, $\log(X)$, and a price index, $\log(P)$. We use two different price indices in our simulations. The first price index, which leads to a FDAID model linear in parameters, is generated exogenously. The second price index, which leads to a FDAID model nonlinear in parameters, is generated as in Equation (16) (see also Deaton and Muellbauer, 1980),

$$\log(P) = \alpha_0 + \sum_m \alpha_m \log(p_m) + \frac{1}{2}\sum_m \sum_n \theta_{nm} \log(p_n) \log(p_m). \tag{16}$$

With the second price index used in the data generating process, we then estimate both the nonlinear FDAID model as well as the linear approximation FDAID model where, in the later case, the Divisia price index is used in the estimation.

$\boldsymbol{D}$ consists of four ($k = 4$) quarterly or twelve ($k = 12$) monthly-fixed dummy variables that satisfy the following conditions:

$$\sum_{j=1}^{k} d_{jm} = 0, \quad m = 1, \ldots, M, \quad j,k = 1, \ldots, 4, \quad or \quad j,k = 1, \ldots, 12, \tag{17}$$

where, d_i and d_j represent monthly dummy variables. To ensure the adding up restriction, we impose these four other conditions:

$$\sum_{i=1}^{m} d_{ij} = 0, \sum_{i=1}^{m} \alpha_i = 1, \quad \sum_{i=1}^{m} w_i = 1, \quad \text{and} \quad \sum_{i=1}^{m} \varepsilon_i = 0, \tag{18}$$

where α_i represents the intercept for the ith equation. If one of the three generated shares is negative, then that system observation is rejected and is replaced by a different draw of correlated random errors. Finally, the homogeneity and symmetry restrictions are imposed on the parameters of the system. We also simulated the results for the case of stochastic seasonality specified in Harvey and Scott (1994) and Fraser and Moosa (2002). The findings were the same as with deterministic seasonality. The main difference is that parameter estimates have larger standard errors under stochastic seasonality. This result is expected because of the additional error terms related to the stochastic seasonal components.

We generate 10,000 samples of 30, 60, 120, 250, and 500 observations according to Equation (1). Aggregate observations are obtained according to Equation (2), using two different levels of aggregation: $k = 12$ for monthly observations and $k = 4$ for quarterly observations. We estimate both the disaggregate Equation (1) and the overlapping Equation (2) (referred as the GLS/SUR model). Finally, we obtain the maximum likelihood estimates for the AR(1) model of Beach and MacKinnon (1979) in Equation (2) by imposing the same AR(1) parameter for each equation equal to $(k - 1)/k$.

Monte Carlo Results

Table 1 presents the actual slope parameters and the means and standard errors for the three estimators from Monte Carlo simulations for the case of a FDAID model linear in parameters with the exogenous price index. Results are reported only for one equation, since the results are very similar. Three main findings are to be noted. First, slope estimates from all models are consistent as expected. Second, the slope estimates and their standard errors are exactly the same for the disaggregate model and the GLS/SUR model. Third, the true standard errors of the AR(1) model are larger than those of the disaggregate model and the GLS/SUR model. On average, the standard errors of the AR(1) model are 18–30% larger. The loss in efficiency for the AR(1) model generally increases with sample size and aggregation level.

Table 2 reports rejection rates of the hypothesis that estimated parameters are equal to the actual values for the significance level of 5% for the case of a FDAID model linear in parameters with the exogenous price index. The rejection rate for the AR(1) model is twice and for larger sample sizes and aggregation

Table 1. Monte Carlo Simulation Results for the FDAID Model Linear in Parameters

				Disaggregate Model		GLS/SUR Model		AR(1) Model	
Sample Size	Aggregation Level	Variable	Actual Parameter Values	Parameter Estimates	Standard Error	Parameter Estimates	Standard Error	Parameter Estimates	Standard Error
30	4	P1	0.02	0.0203	0.0218	0.0203	0.0218	0.0203	0.0262
		P2	0.03	0.0294	0.0317	0.0293	0.0317	0.0302	0.0375
		P3	−0.05	−0.0500	0.0449	−0.0504	0.0449	−0.0499	0.0528
		ln(X/P)	0.025	0.0249	0.0142	0.0250	0.0142	0.0248	0.0170
30	12	P1	0.02	0.0207	0.0274	0.0207	0.0274	0.0208	0.0327
		P2	0.03	0.0293	0.0398	0.0293	0.0398	0.0307	0.0470
		P3	−0.05	−0.0504	0.0562	−0.0504	0.0562	−0.0503	0.0665
		ln(X/P)	0.025	0.0250	0.0178	0.0250	0.0178	0.0249	0.0211
60	4	P1	0.02	0.0200	0.0141	0.0200	0.0141	0.0201	0.0166
		P2	0.03	0.0301	0.0206	0.0301	0.0206	0.0301	0.0247
		P3	−0.05	−0.0501	0.0291	−0.0501	0.0291	−0.0510	0.0349
		ln(X/P)	0.025	0.0250	0.0092	0.0250	0.0092	0.0249	0.0109
60	12	P1	0.02	0.0200	0.0170	0.0200	0.0170	0.0202	0.0205
		P2	0.03	0.0301	0.0224	0.0301	0.0224	0.0302	0.0299
		P3	−0.05	−0.0502	0.0318	−0.0502	0.0318	−0.0504	0.0422
		ln(X/P)	0.025	0.0250	0.00997	0.0250	0.00997	0.0249	0.0133
120	4	P1	0.02	0.0201	0.0010	0.0201	0.0100	0.0201	0.0166
		P2	0.03	0.0301	0.0140	0.0301	0.0140	0.0301	0.0167
		P3	−0.05	−0.0502	0.0199	−0.0502	0.0199	−0.0502	0.0239
		ln(X/P)	0.025	0.0250	0.006	0.0250	0.006	0.0250	0.0074
120	12	P1	0.02	0.0202	0.0010	0.0202	0.0100	0.0202	0.0138
		P2	0.03	0.0301	0.0146	0.0301	0.0146	0.0302	0.0138
		P3	−0.05	−0.0502	0.0207	−0.0502	0.0207	−0.0501	0.0283
		ln(X/P)	0.025	0.0248	0.0065	0.0248	0.0065	0.0250	0.0088
250	4	P1	0.02	0.0199	0.0098	0.0199	0.0098	0.0198	0.0142
		P2	0.03	0.0301	0.0096	0.0301	0.0096	0.0299	0.0150
		P3	−0.05	−0.0501	0.0149	−0.0501	0.0149	−0.0509	0.0208
		ln(X/P)	0.025	0.0248	0.0046	0.0248	0.0046	0.0247	0.0065
250	12	P1	0.02	0.0200	0.0067	0.0200	0.0067	0.0201	0.087
		P2	0.03	0.0300	0.0068	0.0300	0.0068	0.0301	0.097
		P3	−0.05	−0.0500	0.0136	−0.0500	0.0136	−0.0501	0.0166
		ln(X/P)	0.025	0.0250	0.0032	0.0250	0.0032	0.0251	0.0045
500	4	P1	0.02	0.0201	0.0041	0.0201	0.0041	0.0201	0.0086
		P2	0.03	0.0300	0.0046	0.0300	0.0046	0.0300	0.0079
		P3	−0.05	−0.0501	0.0108	−0.0501	0.0108	−0.0501	0.0149
		ln(X/P)	0.025	0.0251	0.0042	0.0251	0.0042	0.0250	0.0059
500	12	P1	0.02	0.0200	0.0049	0.0200	0.0049	0.0200	0.0065
		P2	0.03	0.0300	0.0056	0.0300	0.0056	0.0300	0.0069
		P3	−0.05	−0.0499	0.0098	−0.0499	0.0098	−0.0499	0.0122
		ln(X/P)	0.025	0.0250	0.0030	0.0250	0.0030	0.0250	0.0038

levels even three times as large as the nominal level. This false significance may explain the popularity of this approach. In the meantime the rejection rates for the GLS/SUR model and the disaggregate model, which are not reported since they are the same as the ones for the GLS/SUR model, are very close to the nominal level.

Table 3 reports the simulation results for the case where the price index is generated according to Equation (15). Table 3 reports the

Table 2. Rejection Levels of the Hypothesis that Estimated Parameters Are Equal to Their Actual Values for the FDAID Model Linear in Parameters

Sample Size	Aggregation Level	Nominal Level	Variable	Rejection Level	
				GLS/SUR Model	AR(1) Model
30	4	0.05	P1	0.061	0.106
		0.05	P2	0.057	0.100
		0.05	P3	0.058	0.093
		0.05	ln(X/P)	0.058	0.097
30	12	0.05	P1	0.067	0.102
		0.05	P2	0.067	0.098
		0.05	P3	0.064	0.099
		0.05	ln(X/P)	0.064	0.102
60	4	0.05	P1	0.049	0.096
		0.05	P2	0.051	0.101
		0.05	P3	0.053	0.110
		0.05	ln(X/P)	0.048	0.101
60	12	0.05	P1	0.049	0.138
		0.05	P2	0.048	0.143
		0.05	P3	0.050	0.138
		0.05	ln(X/P)	0.049	0.138
120	4	0.05	P1	0.048	0.099
		0.05	P2	0.049	0.093
		0.05	P3	0.050	0.105
		0.05	ln(X/P)	0.049	0.099
120	12	0.05	P1	0.048	0.156
		0.05	P2	0.048	0.157
		0.05	P3	0.050	0.153
		0.05	ln(X/P)	0.047	0.149
250	4	0.05	P1	0.048	0.092
		0.05	P2	0.052	0.094
		0.05	P3	0.049	0.098
		0.05	ln(X/P)	0.048	0.097
250	12	0.05	P1	0.051	0.139
		0.05	P2	0.050	0.141
		0.05	P3	0.049	0.138
		0.05	ln(X/P)	0.048	0.137
500	4	0.05	P1	0.051	0.089
		0.05	P2	0.050	0.091
		0.05	P3	0.049	0.093
		0.05	ln(X/P)	0.049	0.095
500	12	0.05	P1	0.050	0.119
		0.05	P2	0.049	0.118
		0.05	P3	0.051	0.120
		0.05	ln(X/P)	0.050	0.117

mean parameter estimates and their standard errors from both the nonlinear and linear estimation for each of the three models. The standard errors from the disaggregate model and the GLS/SUR model are very similar with some notable differences for the case of the small sample size of 30. In addition, the standard errors for the AR(1) model are larger than those of the disaggregate and GLS/SUR models, confirming again the inefficiency of

Table 3. Monte Carlo Simulation Results for the FDAID Model Nonlinear in Parameters

				Disaggregate Model		GLS/SUR Model		AR(1) Model	
Sample Size	Aggregation Level	Variable	Actual Parameter Values	Nonlinear Estimation	Linear Estimation	Nonlinear Estimation	Linear Estimation	Nonlinear Estimation	Linear Estimation
30	4	P1	0.02	0.0209	0.0197	0.0202	0.0194	0.0204	0.0195
				(0.0231)	(0.0225)	(0.0217)	(0.0219)	(0.0249)	(0.0253)
		P2	0.03	0.0313	0.0287	0.0317	0.0297	0.0321	0.0299
				(0.0339)	(0.0350)	(0.0318)	(0.0305)	(0.0351)	(0.0355)
		P3	−0.05	−0.0522	−0.0478	−0.0519	−0.0483	−0.0525	−0.0483
				(0.0454)	(0.0452)	(0.0456)	(0.0449)	(0.0515)	(0.0525)
		ln(X/P)	0.025	0.0227	0.0209	0.0231	0.0218	0.0235	0.0216
				(0.0142)	(0.0144)	(0.0144)	(0.0144)	(0.0168)	(0.0166)
30	12	P1	0.02	0.0211	0.0198	0.0205	0.0194	0.0205	0.0192
				(0.0299)	(0.0294)	(0.0282)	(0.0285)	(0.0294)	(0.0296)
		P2	0.03	0.0319	0.0291	0.0323	0.0298	0.0325	0.0294
				(0.0438)	(0.0452)	(0.0408)	(0.0403)	(0.0421)	(0.0416)
		P3	−0.05	−0.0530	−0.0481	−0.0528	−0.0484	−0.0530	−0.0488
				(0.0576)	(0.0580)	(0.0592)	(0.0591)	(0.0619)	(0.0615)
		ln(X/P)	0.025	0.0235	0.0213	0.0238	0.0218	0.0242	0.0212
				(0.0183)	(0.0185)	(0.0189)	(0.0190)	(0.0195)	(0.0192)
60	4	P1	0.02	0.0203	0.0192	0.0197	0.0191	0.0199	0.0189
				(0.0144)	(0.0142)	(0.0140)	(0.0139)	(0.0165)	(0.0169)
		P2	0.03	0.0309	0.0287	0.0312	0.0294	0.0315	0.0293
				(0.0213)	(0.0224)	(0.0211)	(0.0197)	(0.0234)	(0.0239)
		P3	−0.05	−0.0512	−0.0477	−0.0509	−0.0482	−0.0513	−0.0484
				(0.0293)	(0.0292)	(0.0300)	(0.0290)	(0.0337)	(0.0349)
		ln(X/P)	0.025	0.0221	0.0211	0.0226	0.0218	0.0229	0.0217
				(0.0092)	(0.0093)	(0.0092)	(0.0092)	(0.0110)	(0.0109)

Table 3. Continued

Sample Size	Aggregation Level	Variable	Actual Parameter Values	Disaggregate Model Nonlinear Estimation	Disaggregate Model Linear Estimation	GLS/SUR Model Nonlinear Estimation	GLS/SUR Model Linear Estimation	AR(1) Model Nonlinear Estimation	AR(1) Model Linear Estimation
60	12	P1	0.02	0.0202	0.0192	0.0198	0.0192	0.0200	0.0192
				(0.0176)	(0.0175)	(0.0173)	(0.0173)	(0.0195)	(0.0196)
		P2	0.03	0.0313	0.0292	0.0313	0.0295	0.0316	0.0294
				(0.0244)	(0.0251)	(0.0241)	(0.0230)	(0.0260)	(0.0264)
		P3	−0.05	−0.0515	−0.0484	−0.0512	−0.0486	−0.0516	−0.0488
				(0.0336)	(0.0331)	(0.0341)	(0.0334)	(0.0378)	(0.0388)
		ln(X/P)	0.025	0.0226	0.0215	0.0227	0.0218	0.0230	0.0216
				(0.0104)	(0.0105)	(0.0106)	(0.0106)	(0.0121)	(0.0121)
120	4	P1	0.02	0.0201	0.0189	0.0194	0.0188	0.0195	0.0188
				(0.0086)	(0.0085)	(0.0084)	(0.0083)	(0.0098)	(0.0099)
		P2	0.03	0.0306	0.0284	0.0307	0.0291	0.0308	0.0291
				(0.0143)	(0.0152)	(0.0147)	(0.0135)	(0.0157)	(0.0161)
		P3	−0.05	−0.0507	−0.0480	−0.0501	−0.0485	−0.0504	−0.0485
				(0.0201)	(0.0201)	(0.0207)	(0.0196)	(0.0227)	(0.0240)
		ln(X/P)	0.025	0.0219	0.0209	0.0225	0.0217	0.0225	0.0216
				(0.0061)	(0.0061)	(0.0060)	(0.0061)	(0.0073)	(0.0073)
120	12	P1	0.02	0.0198	0.0189	0.0195	0.0189	0.0196	0.0189
				(0.0079)	(0.0076)	(0.0077)	(0.0075)	(0.0087)	(0.0090)
		P2	0.03	0.0308	0.0290	0.0307	0.0292	0.0309	0.0292
				(0.0139)	(0.0144)	(0.0141)	(0.0132)	(0.0151)	(0.0157)
		P3	−0.05	−0.0507	−0.0485	−0.0502	−0.0487	−0.0504	−0.0488
				(0.0222)	(0.0217)	(0.0227)	(0.0217)	(0.0245)	(0.0260)
		ln(X/P)	0.025	0.0223	0.0214	0.0225	0.0217	0.0227	0.0216
				(0.0068)	(0.0068)	(0.0068)	(0.0068)	(0.0082)	(0.0081)
250	4	P1	0.02	0.0200	0.0191	0.0194	0.0190	0.0194	0.0190
				(0.0074)	(0.074)	(0.0074)	(0.0074)	(0.0099)	(0.0101)
		P2	0.03	0.0304	0.0284	0.0305	0.0292	0.0304	0.0292
				(0.0128)	(0.0129)	(0.0128)	(0.0127)	(0.0140)	(0.0143)

Table 3. Continued

Sample Size	Aggregation Level	Variable	Actual Parameter Values	Disaggregate Model: Nonlinear Estimation	Disaggregate Model: Linear Estimation	GLS/SUR Model: Nonlinear Estimation	GLS/SUR Model: Linear Estimation	AR(1) Model: Nonlinear Estimation	AR(1) Model: Linear Estimation
		P3	−0.05	−0.0504	−0.0478	−0.0499	−0.0484	−0.0498	−0.0484
				(0.0163)	(0.0162)	(0.0162)	(0.0162)	(0.0191)	(0.0195)
		ln(X/P)	0.025	0.0218	0.0210	0.0224	0.0216	0.0223	0.0217
				(0.0050)	(0.0051)	(0.0050)	(0.0050)	(0.0062)	(0.0064)
250	12	P1	0.02	0.0198	0.0191	0.0195	0.0190	0.0195	0.0190
				(0.0066)	(0.0065)	(0.0064)	(0.0064)	(0.0080)	(0.0079)
		P2	0.03	0.0305	0.0289	0.0305	0.0292	0.0305	0.0293
				(0.0124)	(0.0125)	(0.0124)	(0.0123)	(0.0146)	(0.0148)
		P3	−0.05	−0.0504	−0.0485	−0.0500	−0.0487	−0.0500	−0.0487
				(0.0186)	(0.0185)	(0.0187)	(0.0183)	(0.0204)	(0.0208)
		ln(X/P)	0.025	0.0222	0.0215	0.0225	0.0216	0.0225	0.0218
				(0.0054)	(0.0055)	(0.0054)	(0.0054)	(0.0064)	(0.0063)
500	4	P1	0.02	0.0201	0.0190	0.0193	0.0189	0.0193	0.0189
				(0.0067)	(0.0065)	(0.0066)	(0.0064)	(0.0078)	(0.0080)
		P2	0.03	0.0306	0.0284	0.0303	0.0292	0.0304	0.0291
				(0.0118)	(0.0119)	(0.0120)	(0.0116)	(0.0127)	(0.0129)
		P3	−0.05	−0.0507	−0.0479	−0.0496	−0.0483	−0.0497	−0.0483
				(0.0156)	(0.0157)	(0.0160)	(0.0150)	(0.0177)	(0.0181)
		ln(X/P)	0.025	0.0218	0.0210	0.0224	0.0217	0.0223	0.0217
				(0.0041)	(0.0040)	(0.0040)	(0.0040)	(0.0053)	(0.0053)
500	12	P1	0.02	0.0198	0.0191	0.0195	0.0190	0.0195	0.0190
				(0.0059)	(0.058)	(0.0057)	(0.0057)	(0.0068)	(0.0069)
		P2	0.03	0.0306	0.0289	0.0305	0.0292	0.0305	0.0293
				(0.0114)	(0.0114)	(0.0111)	(0.0112)	(0.0125)	(0.0127)
		P3	−0.05	−0.0504	−0.0485	−0.0500	−0.0487	−0.0500	−0.0487
				(0.0148)	(0.0147)	(0.0142)	(0.0141)	(0.0165)	(0.0168)
		ln(X/P)	0.025	0.0222	0.0215	0.0225	0.0216	0.0225	0.0218
				(0.0048)	(0.0048)	(0.0048)	(0.0048)	(0.0062)	(0.0062)

Note: Standard errors are in parenthesis.

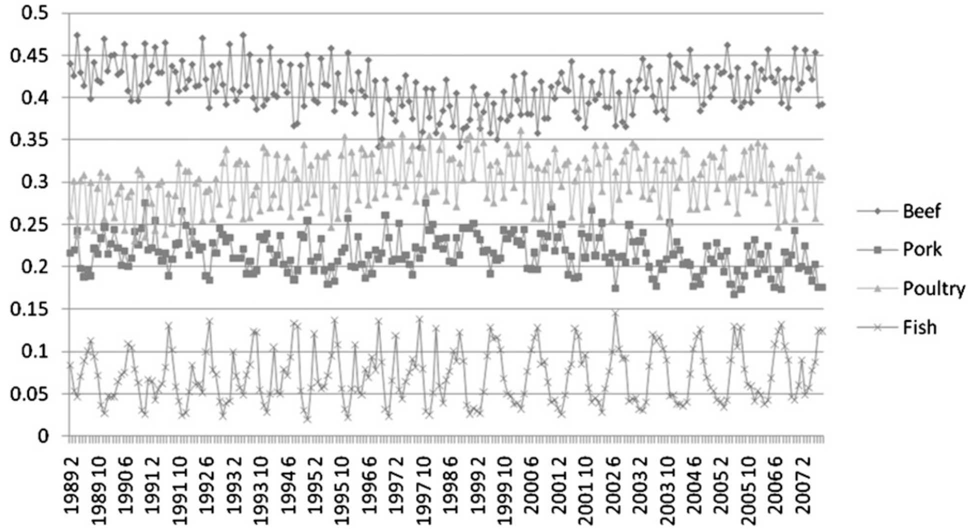

Figure 1. Monthly Expenditure Shares for Beef, Pork, Poultry and Fish

the AR(1) model. Results in Table 3 also allow for a comparison between the nonlinear and linear estimation of a demand model that is nonlinear in parameters. Results show that in particular for small samples there is some disadvantage in terms of both more bias in parameter estimates and higher standard errors when employing the nonlinear estimation versus the linear estimation.[1]

The Monte Carlo results show no advantage to using overlapping data over disaggregate data in the case of a demand model linear in parameters. There also is little or no advantage in small sample sizes to using the nonlinear "true" FDAIDS rather than the linear approximate version. Disaggregate data may be avoided by researchers since it is more likely to show significant lagged variables and significant autocorrelation. But, using aggregate data does not really remove the effects of lagged adjustments and autocorrelation; it only lessens their effects and reduces the power of the tests to detect such violations of standard assumptions (Harri and Brorsen, 2009).

[1] We also simulated the results for the case of four price series. The results with four price series, not reported here, are similar to the results we report for three price series. Therefore, our results are robust if more price series are included in the model.

Empirical Example

We estimate U.S. meat demand to compare the empirical performance of the different estimation models discussed above. Data are monthly observations from January 1989 to August 2007. Per capita beef, pork, and poultry quantities and retail prices are obtained from U.S. Department of Agriculture (1989) *Livestock and Poultry Situation and Outlook Reports*. Per capita fish quantities and retail prices are derived using the approach in Schmitz and Capps (1993, p. 10) and Kinnucan et al. (1997). Bryant and Davis (2008), using the Bayesian Averaging of Classical Estimates approach, find that the First-Difference Almost Ideal Demand model outperforms the other models considered in their analysis. Therefore, we use FDAID as our functional form. The fish equation is dropped from the estimation.

We first test the assumption of seasonal unit roots in the shares series. The simple plot of the monthly expenditure shares for beef, pork, poultry and fish presented in Figure 1 shows no patterns of annual seasonality. For a formal test, we employ the Beaulieu and Miron (1993) extension of the Hylleberg et al. (1990) procedure for monthly data. Beaulieu and Miron (1993) suggest that to show that no seasonal unit root exists at any frequency, π_k in Equation

Table 4. Tests for Seasonal Unit Roots

Expenditure Shares	π_2	$\{\pi_3\ \pi_4\}$	$\{\pi_5\ \pi_6\}$	$\{\pi_7\ \pi_8\}$	$\{\pi_9\ \pi_{10}\}$	$\{\pi_{11}\ \pi_{12}\}$
Beef	28.57[a]	8.04[b]	20.41[b]	13.32[b]	13.76[b]	14.73[b]
	(0.0001)[c]	(0.0004)[c]	(0.0001)[c]	(0.0001)[c]	(0.0001)[c]	(0.0001)[c]
Pork	21.65[a]	9.63[b]	18.90[b]	10.66[b]	11.49[b]	16.47[b]
	(0.0001)[c]	(0.0001)	(0.0001)[c]	(0.0001)[c]	(0.0001)[c]	(0.0001)[c]
Poultry	21.75[a]	10.80[b]	21.15[b]	13.12[b]	13.34[b]	12.98[b]
	(0.0001)[c]	(0.0001)	(0.0001)[c]	(0.0001)[c]	(0.0001)[c]	(0.0001)[c]
Fish	24.85[a]	4.85[b]	8.84[b]	3.81[b]	7.67[b]	4.43[b]
	(0.0001)[c]	(0.0088)	(0.0002)[c]	(0.0239)[c]	(0.0006)[c]	(0.0131)[c]

[a] These are two-sided *t*-statistics.
[b] These are F-statistics.
[c] These are *p*-values.

(19) must not equal zero for $k = 2$ and for at least one member of each of the sets {3, 4}, {5, 6}, {7, 8}, {9, 10}, {11, 12}.

$$B(L)^*W_{13t} = \sum_{k=1}^{12} \pi_k W_{k,t-1} + d_1 + \sum_{k=2}^{12} d_k S_{kt} + \zeta_t \tag{19}$$

where W_k are functions of current and lagged values of monthly shares, S_k are dummy variables and $B(L)$ is a polynomial in the lag operator. Therefore, we perform the test using a two-sided *t*-test for $k = 2$ and F-tests for the other sets. Equation (19) is estimated for the four different meat products. Results of the seasonal unit root testing using first differences of monthly expenditure shares are presented in Table 4. Table 4 presents the results of the hypotheses tests that π_2 (using a *t*-test) and at least one member of each of the sets $\{\pi_3, \pi_4\}$, $\{\pi_5, \pi_6\}$, $\{\pi_7, \pi_8\}$, $\{\pi_9, \pi_{10}\}$, and $\{\pi_{11}, \pi_{12}\}$ (using F-tests) are equal to zero. Based on the results in Table 4 we fail to reject the hypothesis that π_k equals zero for any frequency for all four meat product equations. Therefore, we conclude that there are no seasonal unit roots in any of the share series.

Next we test the symmetry hypothesis for the three different models. The results of these tests are reported in Table 5. Since the symmetry hypothesis is not rejected, we report the results with the symmetry restriction imposed. Table 6 reports parameter estimates and their standard errors for the three different models and for the three equations of beef, pork, and poultry.

Parameter estimates and their standard errors for the disaggregate model and the GLS/SUR model are very similar for the three estimated equations regardless of the restrictions imposed. This is to be expected based on our Monte Carlo results. Results for the AR(1) model show higher standard errors, sometimes twice as high as the standard errors for the GLS/SUR model. The AR(1) model also shows lower significance levels compared with the other two models. Finally, Table 6 reports the Durbin-Watson test for the presence of autocorrelation. Both the disaggregate model and the AR(1) model show some negative autocorrelation in the residuals for all three

Table 5. Tests of the Symmetry Hypothesis

Model	F-value	*p*-value	Degrees of Freedom: Numerator	Degrees of Freedom: Denominator
Disaggregate	0.19	0.9014	3	618
GLS/SUR	0.23	0.8734	3	618
AR(1)	0.534	0.6591	3	618

Table 6. Parameter Estimates of U.S. Meat Demand with Symmetry Imposed

	Beef Equation			Pork Equation			Poultry Equation		
Variable	Disagg. Model	GLS/SUR Model	AR(1) Model	Disagg. Model	GLS/SUR Model	AR(1) Model	Disagg. Model	GLS/SUR Model	AR(1) Model
PBeef	0.2727*	0.2746*	0.2141*						
	(0.074)	(0.074)	(0.076)						
PPork	−0.0415	−0.0406	0.0056	0.2382*	0.2383*	0.2472*			
	(0.184)	(0.042)	(0.088)	(0.042)	(0.042)	(0.054)			
PPoultry	−0.1845*	−0.1846*	−0.1659*	−0.147*	−0.1469*	−0.110*	0.2274*	0.2257*	0.2050*
	(0.050)	(0.050)	(0.060)	(0.034)	(0.034)	(0.037)	(0.053)	(0.053)	(0.055)
PFish	−0.0467	−0.0494	−0.0654	−0.0495	−0.0508	0.0133	0.1043**	0.1059**	−0.0592
	(0.075)	(0.076)	(0.135)	(0.049)	(0.049)	(0.083)	(0.053)	(0.053)	(0.124)
Expend	0.2870*	0.2874*	0.2667*	0.1611*	0.1613*	0.1498*	−0.4066*	−0.4069*	−0.379*
	(0.013)	(0.013)	(0.012)	(0.035)	(0.008)	(0.007)	(0.010)	(0.010)	(0.011)
Feb	0.010			0.0351*			−0.0506*		
	(0.007)			(0.004)			(0.006)		
Mar	−0.014**			0.0246*			0.0267*		
	(0.007)			(0.004)			(0.006)		
Apr	0.012***			0.0249*			−0.0259*		
	(0.007)			(0.004)			(0.006)		
May	−0.018*			−0.0047			0.0116**		
	(0.007)			(0.004)			(0.006)		
Jun	−0.004			0.0176*			−0.0192*		
	(0.007)			(0.004)			(0.006)		
Jul	−0.011			0.0202*			−0.0151*		
	(0.007)			(0.004)			(0.006)		
Aug	−0.017**			0.0243*			0.0053		
	(0.007)			(0.004)			(0.006)		
Sep	0.016**			0.0467*			−0.0342*		
	(0.007)			(0.004)			(0.006)		
Oct	−0.002			0.0351*			0.0089		
	(0.007)			(0.004)			(0.006)		

Table 6. Continued

	Beef Equation			Pork Equation			Poultry Equation		
Variable	Disagg. Model	GLS/SUR Model	AR(1) Model	Disagg. Model	GLS/SUR Model	AR(1) Model	Disagg. Model	GLS/SUR Model	AR(1) Model
Nov	0.023*			0.0509*			−0.0295*		
	(0.009)			(0.004)			(0.007)		
Dec	0.025*			0.0407*			−0.0406*		
	(0.007)			(0.004)			(0.006)		
DW	2.66	2.26	2.87	3.04	1.91	2.77	2.66	1.99	3.03

*,**, and *** denote respectively significance at 1%, 5%, and 10%.

equations. On the other hand, no autocorrelation is found for the GLS/SUR model.

Conclusions

Estimating a demand system using seasonally differenced data leads to autocorrelated residuals where the degree of correlation depends on the level of differencing. Ignoring this autocorrelation results in inefficient estimates and biased hypothesis tests. The Beach and MacKinnon estimator, used in some previous works, is also inefficient in this case.

We show how to obtain consistent and asymptotically efficient estimates of a demand system using seasonally differenced data. Monte Carlo simulations confirm that the theoretical derivation of the GLS estimator using an analytically derived correlation matrix produces consistent and efficient estimates.

We apply our combined GLS/SUR model and the alternative models to estimate U.S. meat demand. The empirical results confirm our Monte Carlo findings that the GLS/SUR estimator has lower standard errors than the AR(1) estimator when using seasonally differenced data. In addition, the empirical results show the GLS/SUR estimator with seasonally differenced data and no specification of seasonality yields nearly the same parameter estimates as the disaggregate estimation with the form of seasonality specified. We recommend against using seasonally differenced data to estimate demand systems since there is no gain to doing so if the correct estimation method is used.

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