AJAE appendix for Stochastic Rangeland Use under Capital Constraints

Mimako Kobayashi, Richard E. Howitt, Lovell S. Jarvis, and Emilio A. Laca

August, 2006

Note: The material contained herein is supplementary to the article named in the title and published in the American Journal of Agricultural Economics (AJAE).
Appendix 1: Details on Sheep Biology

Sheep biology relationships used in the current model are specified using information from a detailed sheep simulation model XLUMBE developed by Laca. XLUMBE is a Microsoft Excel implementation of the model by Finlayson, Cacho, and Bywater (1995), and it simulates daily changes in sheep body condition driven by daily nutrient intake levels. XLUMBE was parameterized for Kazakh Fine Wool sheep managed as generally indicated in figure A1 on a semi-desert rangeland in southeastern Kazakhstan. Forage production parameters used in XLUMBE are reviewed in Breuer (2000). For sheep parameters, the figures for wool-type sheep found in Finlayson, Cacho, and Bywater (1995) were used since this sheep breed is genetically close to Kazakh Fine Wool sheep. Following Degen and Young (2002), parameter adjustments were made for the impacts of climate factors such as weight losses due to cold weather.

Using XLUMBE, we first simulated sheep performance at different feeding levels. We then specified simplified biological equations and estimated the parameters that fit the data generated by that simulation model. Here, we present the simplified sheep biology equations used in the current model. All parameters were estimated by least squares and the estimates are listed in table A1. These parameters were then checked to ensure their plausibility in terms of observed livestock production technology in Kazakhstan.

Forage intake level by an individual animal is determined by forage production (more abundant, palatable, and nutritious forage tends to allow greater intake), the size of the animal (larger animals have greater energy demand and appetite), as well as how many other animals are put on the range (competition effect). We model the amount of
forage that an average ewe grazes in a day ($\text{FORAGE}_t$) as a function of forage production ($\text{FP}_\text{ROD}_t$), the total number of grazing animals ($\text{TOTGRAZE}_t$), and the bodyweight of the average ewe ($\text{BW}_t$). The grazing equation is specified as a quadratic function:

$$
\text{FORAGE}_t = \alpha_0 + \alpha_1 \text{FP}_\text{ROD}_t + \alpha_2 \text{TOTGRAZE}_t + \alpha_3 \text{BW}_t \\
\quad + \alpha_4 \text{FP}_\text{ROD}_t^2 + \alpha_5 \text{TOTGRAZE}_t^2 + \alpha_6 \text{BW}_t^2 \\
\quad + \alpha_7 \text{FP}_\text{ROD}_t \cdot \text{TOTGRAZE}_t + \alpha_8 \text{FP}_\text{ROD}_t \cdot \text{BW}_t + \alpha_9 \text{TOTGRAZE}_t \cdot \text{BW}_t.
$$

Note that the available rangeland acreage (10,000 ha) is embedded in the forage equation parameters.

Since a younger animal consumes less forage than a mature ewe, for which the data are generated with XLUMBE, the calculation of grazing pressure is adjusted for flock composition. We define the total number of grazing sheep in terms of “ewe units:”

$$
\text{TOTGRAZE}_t = \gamma^{\text{LAF}}(1 - \delta^{\text{LA}})\text{LAF}_t + \gamma^{\text{LAM}}(1 - \delta^{\text{LA}})\text{LAM}_t + \gamma^{\text{YRF}}(\text{YRF}_t - \text{SYRF}_1) \\
\quad + \gamma^{\text{YRM}}(\text{YRM}_t - \text{SYR}_1) + \gamma^{\text{EWE}}(\text{EWE}_t - \text{SEWE}_1) + \gamma^{\text{WE}}(\text{WE}_t - \text{SWE}_1),
$$

where $\gamma$’s are the coefficients to convert animals of each category into ewe units ($\gamma^{\text{LAF}} = \gamma^{\text{LAM}} = 0.2; \gamma^{\text{YRF}} = \gamma^{\text{YRM}} = 0.7; \gamma^{\text{EWE}} = \gamma^{\text{WE}} = 1.0$). By choosing the volume of spring sales of yearlings and mature sheep, grazing pressure on the range is adjusted.

We assume that nutrient intake in spring, summer, and fall is limited to grazing while supplementary feeding of grass hay ($\text{GHAY}_t$) and barley ($\text{BAR}_t$) is the only nutrient source during winter. For simplicity, we assume that the same amount is fed to a ewe every day throughout the winter and that a proportionate amount is fed to other age and sex categories. To determine the total annual feed supplementation, we consider the total number of animals to feed ($\text{TOTFED}_t$). Again, to account for differences in feed
requirements for animals of different size and sex, animals in the flock are aggregated in ewe units such that:

\[ TOTFED_i = \xi^{LA}(1 - \delta^L)AF_i - SLAF2_i) + \xi^{LM}(1 - \delta^L)AM_i - SLAM2_i) \]
\[ + \xi^{RF}(YRF_i - SYRF1_i - SYRF2_i) + \xi^{RM}(YRM_i - SYRM1_i - SYRM2_i) \]
\[ + \xi^{WE}(EWE_i - SEWE1_i - SEWE2_i) + \xi^{WET}(WET_i - SWET1_i - SWET2_i), \]

where the \( \xi \)'s are the conversion coefficients (\( \xi^{LA} = \xi^{LM} = 0.3; \xi^{RF} = \xi^{RM} = 0.7; \xi^{WE} = \xi^{WET} = 1.0 \)).

We model the fall bodyweight (\( FBW_i \)) of an average ewe as a function of the spring bodyweight (\( BW_i \)) and the level of nutrient intake from grazing (\( FORAGE_i \)). Since the sheep simulation model XLUMBE predicts large differences in bodyweight growth for lactating and non-lactating ewes, we treat them separately and their weighted average is used to determine the fall bodyweight of an average ewe. For both lactating (with superscript \( L \)) and non-lactating (\( NL \)) ewes, the fall bodyweight is specified as a logistic function:

\[ FBW^L_i = \alpha_0 + (MAXFBW^L_i - \alpha_0) / (1 + \alpha_1\exp\{-\alpha_2FORAGE_i\}), i = N, NL, \]

where \( MAXFBW^L_i \) denotes the maximum possible fall bodyweight given the spring bodyweight. The maximum fall bodyweight is specified for a lactating ewe as:

\[ MAXFBW^L_i = \beta_0 + (MAXBW - \beta_0) / (1 + \beta_1\exp\{-\beta_2BW^L_i\}) \]

and for a non-lactating ewe as:

\[ MAXFBW^{NL}_i = \beta_0 + \beta_1\ln BW^{NL}_i, \]

where \( MAXBW \) is the biological maximum bodyweight for the breed (set at 66.38 kg as in XLUMBE). The estimated \( \beta \) coefficients are listed in table A2. The weighted average of
fall bodyweight for lactating and non-lactating ewes is calculated using the birth rate \( \theta_t \) as the weight:

\[
(A7) \quad FBW_t = \tau_0 FBW^L_t + (1 - \tau_0) FBW^{NL}_t,
\]

where \( \tau \) is a coefficient to control for twinning and lamb mortality, which is set at 0.9.

The maximum possible spring bodyweight, which appears in the bodyweight equation of motion (14), is specified as:

\[
(A8) \quad \text{MAXSPBW}_t = \beta_0 + (\text{MAXBW} - \beta_0) / (1 + \beta_1 \exp\{-\beta_2 FBW_t\}).
\]

The estimated \( \beta \) coefficients are listed in table A2. In order to assure realistic values for the bodyweight state variable, a minimum spring bodyweight of 45 kg \( (\alpha_0=45) \) is imposed in estimating equation (14) in the main text.

The estimated bodyweight equation (14) (main text) only behaves well for a realistic range of winter feeding. Accordingly, upper and lower bounds are imposed on winter feeding levels in order to predict realistic bodyweight values. Using information from the sheep biology literature (Finlayson, Cacho, and Bywater 1995; Degen and Young 2002) and XLUMBE, we define the upper and lower bounds on total winter feeding per head per day such that:

\[
(A9) \quad 0.87 \leq GHAY_t + 1.4BAR_t \leq 2.5.
\]

As with other variables, mortality and birth rates are modeled with reference to an average ewe’s bodyweight but are applied to the entire flock. We define the mortality rate \( \delta \) as the proportion of animals that die at the end of each production year. In order to account for the impacts of both grazing and winter feeding, the mortality rate is modeled as a function of fall bodyweight \( FBW_t \) and spring bodyweight at the end of the year \( BW_{t+1} \). The mortality rate is specified as a logistic function:
Lamb mortality ($\delta^{LA}$) occurs between lambing and weaning. For simplicity, we fix the lamb mortality rate at 0.1 in the current model.

The birth rate ($\theta$) denotes the number of lambs born per average ewe. We model the birth rate as a function of ewe spring bodyweight ($BW_t$), which corresponds to ewe bodyweight at the time of lambing.\textsuperscript{3} The birth rate is specified as a logistic function:

\begin{equation}
\theta_t = \frac{\alpha_0}{1 + \alpha_1 \exp\left(-\alpha_2 BW_t\right)}.
\end{equation}

The number of lambs born each year ($LAF_t$ and $LAM_t$) is modeled as the product of birth rate ($\theta_t$) and the number of mature ewes ($EWE_t$). We assume that lambing takes place after flock size is adjusted through spring sales and that births of female and male lambs occur with an equal probability. The number of female and male lambs born in year $t$ is defined such that:

\begin{equation}
LAF_t = LAM_t = 0.5 \theta_t (EWE_t - SEWE_{1t}).
\end{equation}

Finally, wool production is modeled for an average ewe with reference to ewe bodyweight. Since shearing takes place in spring, we use the spring bodyweight ($BW_t$) to characterize the wool yield per head of an average ewe, which is specified as:

\begin{equation}
WLY_t = \alpha_0 + \alpha_1 \ln BW_t.
\end{equation}

To determine total wool production, we multiply $WLY_t$ by the total number of sheep to be shorn ($TOTSHORN_t$). To account for wool yield differences among age and sex categories, we define $TOTSHORN_t$ in terms of ewe units such that:

\begin{equation}
TOTSHORN_t = \omega^{YRF}(YRF_t - SYRF_{1t}) + \omega^{YRM}(YRM_t - SYRM_{1t})
+ \omega^{EWE}(EWE_t - SEWE_{1t}) + \omega^{WET}(WET_t - SWET_{1t}),
\end{equation}
where the $\omega$’s are the conversion coefficients based on relative bodyweight

$(\omega^\text{YRF} = \omega^\text{YRM} = 0.7; \omega^\text{EWE} = \omega^\text{WET} = 1.0)$. 
Appendix 2: Details on Revenue, Cost, and Objective

The ewe prices per head in spring ($P_1$) and fall ($P_2$) are specified such that:

\[(A15) \quad P_1t = 0.8 * 46 * \frac{BW_t}{AVGFBW} \]

\[(A16) \quad P_2t = 46 * \frac{FBW_t}{AVGFBW}. \]

Total sheep sales are calculated in terms of ewe units by adjusting for price differentials between ewes and other categories. Total sheep sales in spring ($TOTSALE_{1t}$) and fall ($TOTSALE_{2t}$) are defined such that:

\[(A17) \quad TOTSALE_{1t} = \rho^{YRF}SYRF_{1t} + \rho^{YRM}SYRM_{1t} + \rho^{EWE}SEWE_{1t} + \rho^{WET}SWET_{1t} \]

\[(A18) \quad TOTSALE_{2t} = \rho^{LAF}SLAF_{2t} + \rho^{LAM}SLAM_{2t} \]

\[+ \rho^{YRF}SYRF_{2t} + \rho^{YRM}SYRM_{2t} + \rho^{EWE}SEWE_{2t} + \rho^{WET}SWET_{2t}, \]

where the $\rho$'s are the conversion coefficients. Based on available information on relative prices, we set $\rho^{LAF}=\rho^{LAM}=0.53$, $\rho^{YRF}=\rho^{YRM}=0.67$, and $\rho^{EWE}=\rho^{WET}=1.0$.

The annual profit is defined such that:

\[(A19) \quad PROFIT_t = P_1,TOTSALE_{1t} + P_2,TOTSALE_{2t} + P^{WOOL}WLY_t,TOTSHORN_t \]

\[ - (P^{GHAY}_t,GHAY_t + P^{BAR}_t,BAR_t)TOTFED_t - P^{SHEAR}_t,TOTSHORN_t. \]
Appendix 3: VF Stage Procedure

As in Howitt *et al.* (2002), we closely follow the regression-based approximation procedure in Judd (1998, p. 223 and p. 434). Here, we present the procedure adapted for our five-state application. Suppose each of the Chebychev polynomial functions $\phi_j(\cdot)$ is evaluated at $m$ discrete points in the state space (state nodes), then each (mapped) state node is defined as:

$$\hat{x}_{kj} = -\cos((2k-1)\pi/2m), \quad k = 1, \ldots, m; \quad j = 1, \ldots, 5.$$  

(A20)

We define $x_K=(x_{1k}, x_{2k}, x_{3k}, x_{4k}, x_{5k})$ the vector of state variables evaluated at state node $kj$ and denote the nodes for the five state variables collectively as $K$.

The steps of value function approximation are the following:

[Step 0] Choose a set of initial Chebychev coefficients $\alpha^0$ to form $\hat{V}(x; \alpha^0)$.

[Step 1] Solve the maximization problem on the right hand side of equation (17) (main text) $m^5$ times with each current state level at each of the state nodes and with $\hat{V}(\cdot; \alpha) = \hat{V}(\cdot; \alpha^0)$. Let $u_{Kl}$ denote the current controls and $x^+_{kl}$ the state outcomes with the current states at node $K$ and under realization of random factor at node $l$, then each of the maximization problem and the resulting value function level $v_K$ are defined as:

(A21)  

$$v_K = \max_u \left\{ \sum_{l=1}^q p_l \left[ f(x_K, u_{Kl}; \sigma_l) + \beta \hat{V} (x^+_{kl}; \alpha) \right] \mid x^+_{kl} = g(x_K, u_{Kl}; \sigma_l) \right\}.$$  

[Step 2] A new set of Chebychev coefficients are fitted using the $m^5 v_K$ levels as data points. A formula analogous to a least squares estimator:
(A22) \[ \alpha_{i1,i2,i3,i4,i5} = \]
\[ \sum_{k1=1}^{m} \sum_{k2=1}^{m} \sum_{k3=1}^{m} \sum_{k4=1}^{m} \sum_{k5=1}^{m} \nu_{k1,k2,k3,k4,k5} \phi_1(\hat{x}_{1,k1}) \phi_2(\hat{x}_{2,k2}) \phi_3(\hat{x}_{3,k3}) \phi_4(\hat{x}_{4,k4}) \phi_5(\hat{x}_{5,k5}) / \]
\[ ( \sum_{k1=1}^{m} \phi_1(\hat{x}_{1,k1})^2 ) ( \sum_{k2=1}^{m} \phi_2(\hat{x}_{2,k2})^2 ) ( \sum_{k3=1}^{m} \phi_3(\hat{x}_{3,k3})^2 ) ( \sum_{k4=1}^{m} \phi_4(\hat{x}_{4,k4})^2 ) ( \sum_{k5=1}^{m} \phi_5(\hat{x}_{5,k5})^2 ) \]
is used to obtain a new Chebychev coefficient.

[Step 3] Calculate the difference between the new and old Chebychev coefficients and compare the difference to a pre-chosen precision level (\(\varepsilon\)). Repeat Steps 1-3 until the convergence criterion is satisfied.
Appendix 4: Accuracy of Value Function Approximation

As suggested by Judd (1998) and Miranda and Fackler (2002), the quality of the approximated value function is examined to determine whether the Chebychev approximation is a correct approximation of the Bellman equation. The right-hand (RHS) and left-hand (LHS) sides of the Bellman equation (17) (main text) are calculated, with Chebychev polynomial coefficients in both expressions fixed at the converged levels $\alpha^*$, and their differences are examined. Both expressions are evaluated at nine evenly-spaced grid points in each state space, for a total of $9^5=59,049$ state-level combinations. For each grid point for each state variable, the mean and the standard deviation of the percentage residuals $100\times (\text{LHS} - \text{RHS})/\text{RHS}$ are calculated over all the grid combinations of the other four state variables ($9^4=6,561$ values).

The means and the one standard deviation ranges are plotted in figure A2. The mean percentage residuals for sheep state variables ($YRF$, $YRM$, $EWE$, and $WET$) range between 1.3% and 6.6%. For the bodyweight state variable, the mean percentage residuals range between -16.7% and 19.8%; the residuals are the largest in absolute terms for the first three grid points, i.e., 45.0 kg, 47.7 kg, and 50.3 kg. Since sheep mortality increases rapidly as bodyweight declines below 50 kg, that part of the range is not expected to be observed in reality. For bodyweight 53.0 kg and higher, the average percentage residuals range between -1.4% and 4.2%. Given the complexity of a five-state Chebychev approximation, the fit over a value function surface is bound to be less precise than in a case of a single-state problem. Nonetheless, the calculated residuals are sufficiently small in the reasonable bodyweight range to have confidence that the Chebychev polynomial accurately approximates the five-state Bellman equation.
Footnotes

1 To scale the coefficient estimates, $FPROD_t$ and $TOTGRAZE_t$ are multiplied by 0.001 in this equation.

2 Without this restriction, a concavity of the value function was not obtained in terms of the bodyweight state variable.

3 In reality, however, conception (and hence later lambing) is significantly influenced by ewe body condition at the time of breeding. Since breeding takes place in fall, the use of fall bodyweight in the previous production year ($FBW_{t-1}$), as opposed to spring bodyweight ($BW_t$), may be appropriate in modeling the birth rate in the current year ($\theta_t$). However, the use of variables from a previous period is infeasible within the current framework. We retain the current specification while acknowledging its limitation.
References


Table A1. Parameter Estimates for Sheep Biology Equations

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>FORAGE</td>
<td>FBW₀</td>
<td>FBWₐ</td>
<td>FBWₐ₀</td>
<td>BW</td>
<td>δ</td>
<td>θ</td>
<td>WLY</td>
</tr>
<tr>
<td>α₀</td>
<td>-8.3639</td>
<td>25.7716</td>
<td>23.7185</td>
<td>45.0000</td>
<td>0.0247</td>
<td>1.1000</td>
<td>-0.6823</td>
</tr>
<tr>
<td>α₁</td>
<td>3.1199</td>
<td>448.7948</td>
<td>47.0875</td>
<td>1.01E+13</td>
<td>7.19E-08</td>
<td>2.81E+06</td>
<td>0.9932</td>
</tr>
<tr>
<td>α₂</td>
<td>0.00018</td>
<td>4.2064</td>
<td>4.6375</td>
<td>4.2197</td>
<td>-0.2524</td>
<td>0.3374</td>
<td></td>
</tr>
<tr>
<td>α₃</td>
<td>0.2734</td>
<td>6.2461</td>
<td>-0.1047</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>α₄</td>
<td>-1.3928</td>
<td>0.2324</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>α₅</td>
<td>-0.0110</td>
<td>0.1852</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>α₆</td>
<td>-0.0020</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>α₇</td>
<td>0.000033</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>α₈</td>
<td>-3.34E-08</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>α₉</td>
<td>-3.11E-10</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>obs</td>
<td>4065</td>
<td>1201</td>
<td>1403</td>
<td>1201</td>
<td>1201</td>
<td>1788</td>
<td>1848</td>
</tr>
<tr>
<td>R²</td>
<td>0.9274</td>
<td>0.9833</td>
<td>0.9774</td>
<td>0.8771</td>
<td>0.9590</td>
<td>0.9333</td>
<td>0.5976</td>
</tr>
<tr>
<td>unit</td>
<td>kg/head/day</td>
<td>kg/head</td>
<td>kg/head</td>
<td>kg/head</td>
<td>proportion</td>
<td>lamb/ewe</td>
<td>kg/head</td>
</tr>
</tbody>
</table>

Source: Own estimation
Table A2. Parameter Estimates for Maximum Bodyweight Equations

<table>
<thead>
<tr>
<th></th>
<th>(A6)</th>
<th>(A7)</th>
<th>(A8)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( MAXFBW^L )</td>
<td>( MAXFBW^{NL} )</td>
<td>( MAXSPBW )</td>
<td></td>
</tr>
<tr>
<td>( \beta_0 )</td>
<td>36.3326</td>
<td>-112.6200</td>
<td>45.0000</td>
</tr>
<tr>
<td>( \beta_1 )</td>
<td>66.3800</td>
<td>44.8050</td>
<td>66.3800</td>
</tr>
<tr>
<td>( \beta_2 )</td>
<td>2.44E+07</td>
<td></td>
<td>1925.8813</td>
</tr>
<tr>
<td>( \beta_3 )</td>
<td>0.3587</td>
<td></td>
<td>0.1301</td>
</tr>
</tbody>
</table>

<p>| | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>obs</td>
<td>16</td>
<td>13</td>
<td>25</td>
</tr>
<tr>
<td>( R^2 )</td>
<td>0.9959</td>
<td>0.9890</td>
<td>0.8023</td>
</tr>
</tbody>
</table>

unit kg/head kg/head kg/head

Source: Own estimation
Figure A1. Typical sheep management timing in southeastern Kazakhstan
Figure A2. Mean and one standard deviation range of percentage residuals at nine grid points for five state variables
(Note: each bar represents information of $9^4 = 6,561$ points)
Figure A3. Forage production and ewe bodyweight (base specification)