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Variance Risk Premiums and Predictive Power of Alternative Forward Variances in the Corn Market

by

Zhiguang Wang, Scott W. Fausti, and Bashir A. Qasmi

SDSU

Department of Economics
South Dakota State University

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Abstract

We propose a fear index for corn using the variance swap rate synthesized from out-of-the-money call and put options as a measure of implied variance. Previous studies estimate implied variance based on Black (1976) model or forecast variance using the GARCH models. Our implied variance approach, based on variance swap rate, is model independent. We compute the daily 60-day variance risk premiums based on the difference between the realized variance and implied variance for the period from 1987 to 2009. We find negative and time-varying variance risk premiums in the corn market. Our results contrast with Egelkraut, Garcia, and Sherrick (2007), but are in line with the findings of Simon (2002). We conclude that our synthesized implied variance contains superior information about future realized variance relative to the implied variance estimates based on the Black (1976) model and the variance forecasted using the GARCH(1,1) model.

Key Words: Variance Risk Premium, Variance Swap, Model-free Variance, Implied Variance, Realized Variance, Corn VIX

JEL codes: Q13, Q14, G13, G14

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[1]Zhiguang Wang, Scott W. Fausti and Bashir A. Qasmi are Assistant Professor, Professor and Associate Professor at Department of Economics, South Dakota State University. We acknowledge the financial support from Agricultural Experiment Station at South Dakota State University (Project H363-10).
Corresponding Author: Zhiguang Wang, Box 504 Scobey Hall, Department of Economics, South Dakota State University, Brookings, SD 57007. Phone: 605-688-4861. Fax: 605-688-6386. Email address: zhiguang.wang@sdstate.edu

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Introduction

In a recent report by the European Commission (2009), the dramatic rise in historical price volatility in world commodity markets was addressed. The report highlighted the extreme and unpredictable increase in price volatility associated with commodity markets during the 2008-2009 credit crisis. The heightened and fluctuating volatility clearly evolves into a risk factor for commodity derivatives, which has received less attention than their counterpart derivatives in the financial market. Financial markets have tools, such as the VIX to gauge the level of derivative risk. Currently, there is no such tool for agricultural commodity markets. This shortcoming seems about to end. The Wall Street Journal recently reported that the CME Group Inc. plans to introduce such a tool for corn, soybean, gold and crude oil in partnership with the Chicago Board Options Exchange (Bunge, March 8, 2010).

The development of new risk management tools is necessary to improve the risk management capabilities of commodity market participants. In a recent study, Wilson and Dahl (2009) noted that increased grain price volatility has implications for reducing the effectiveness of traditional hedging techniques for producers. They conclude that producers may need to seek alternative risk management mechanisms like forward cash contracts to avoid the risk of increased price volatility. Accordingly, Wilson and Dahl argue that one would expect less reliance on traditional hedging with commodity futures and options, and an increased use of the cash forward contract sales as both sellers and buyers try to manage their price risk. The lack of hedging effectiveness due to stochastic volatility indicates that volatility should be treated as a risk factor in addition to the well-accepted price risk. The development of commodity price volatility indexes and the financial derivatives associated with these indexes would provide producers with a tool to directly manage price volatility risk when using traditional commodity hedging risk management strategies.

It is crucial for risk managers, commodity processors and commodity options traders to accurately gauge the level of price volatility and to: (1) understand how the market prices volatility (variance) risk which underlies the market value of commodity derivatives, and (2) develop a reliable forecast for commodity price volatility based on recent advancements in the finance literature. This paper addresses these two issues by evaluating price volatility in the corn futures and options markets.

The market premium for variance risk has received considerable attention in the finance literature. Traditionally, the difference between realized variance and risk-neutral variance is referred to as a *variance risk premium* (VRP). It has been well documented that risk-neutral variance, inferred from options prices, is greater than the realized variance in equity and energy markets (Bakshi and Kapadia 2003; Carr and Wu 2009; Doran and Ronn 2008), hence negative VRP. The occurrence of a negative VRP indicates that risk-averse investors in equity markets are willing to pay a higher premium or accept a loss to realize a lower variance in the future. It remains unknown whether such an observation in equity and energy markets applies to agricultural commodity markets.

The literature of agricultural commodity markets has not dealt with the variance risk premium, although several papers touch upon the difference between realized variance and implied risk-neutral variance. Implied variance estimation methods used in the past studies on commodities have relied upon the Black (1976) model. Manfredo and Sanders (2004) report a smaller realized variance compared to implied variance for live cattle options, as does Simon (2002) for grain options. However, Egelkraut, Garcia and Sherrick (2007) report "no systematic overprediction" of the implied variance[2] over the realized variance in the corn options market.

On the issue of predicting future variance, there are two traditional approaches: the ARCH model of conditional variance and the forward-looking options implied volatility model (based on Black 1976). The first approach is pioneered by Engle (1982) and Bollerslev (1986). Figlewski (2004) provides a comprehensive overview of its application in financial markets. However, the ARCH family of predicting future variance is still backward-looking, since the prediction is based on the past returns and variances. The second approach has been favored in recent commodity studies (Simon 2002; Giot 2003; Manfredo and Sanders 2004). Both Simon (2002) and Manfredo and Sanders (2004) find that the implied volatility from the Black (1976) formula encompasses all information of the GARCH(1,1) model (predicted volatility).

In this paper, we propose an implied variance measure different from the traditional Black (1976) framework. Carr and Wu (2009) developed a robust method for

[2] They estimate the implied volatility from all call and put options by assuming the lognormal distribution of returns on commodity futures. Although the implied volatility is not based on a single call or put option, it is still within the framework of Black (1976).

estimating the variance risk premium based on the concept of a "variance swap." Not only is this approach more robust than the Black method, it also overcomes the "small sample size" issue associated with commodity market implied volatility studies.[3] Our application of this approach is the first to address the variance risk premium issue in agricultural commodity markets.

Our approach is to model the implied variance using a variance swap rate on corn futures. This approach is similar to the logic underlying the creation of the VIX by the CBOE. The square of the VIX is a variance swap rate that can be synthesized from observable market option prices. Based on the variance swap rate estimate for the implied variance, we compute the variance risk premium for the period from 1987 to 2009. We further test its statistical significance for the whole sample and a subsample from 1987 to 2001 to replicate the results for the time period in the Egelkraut, Garcia, and Sherrick (2007) study. We evaluate our approach by first calculating and comparing the variance risk premium based on the Black-implied variance model to our variance swap model, and then assessing the forecasting efficiency. To test the forecasting performance of alternative forward variances, we run the encompassing regression on the three competing variances: variance swap rate, Black implied volatility and the GARCH (1,1) predicted variance.

[3] Both Simon (2002) and Egelkraut, Garcia, and Sherrick (2007) only calculate Black-implied volatility on a particular day. The former computes the volatility for options with four weeks to expiration, whereas the latter estimates the volatility for options two months before the beginning of the time interval of their interests. The sample size based on this approach will be inevitably small, although this approach avoids the overlapping issue raised by Christensen and Prabhala (1998).

Empirical results indicate that a negative variance risk premium does exist in the corn market and does vary across time. These results contrast with the previous finding by Egelkraut, Garcia, and Sherrick (2007) for the sample period from 1987-2001, but are consistent with Simon (2002). In both studies, the implied variance is computed using the traditional log-normality framework. However, we employ the model-independent variance swap rate on corn futures. We further conclude that the "variance swap" approach for estimating implied variance contains superior information about future realized variance relative to the traditional Black implied variance model and the variance forecasted by the GARCH(1,1) model. The economic implications of our study suggest that the CME introduction of the "Corn VIX" will improve volatility forecasting and enhance market participants' ability to more accurately gauge price risk in the corn market.

The next section provides a literature overview and defines the four variance measures to be evaluated. The data and methodology section provides the computation details of the variance estimators. The empirical results and discussions section presents evidence of the existence of the variance risk premium, and its variability across time. This section also compares the forecasting performance of these forward variances, and performs robustness analysis to validate the variance risk premium conclusion. A summary of results and the economic implications of the study for agricultural commodity markets are provided in the concluding section.

Measures of Variance and Variance Risk Premium

Following the literature on financial assets (Bakshi and Kapadia 2003; Carr and Wu 2009; and Todorov 2010), we define the variance risk premium for agricultural commodities as the difference between risk-neutral variance and realized variance. The realized variance is the variance calculated from returns for a given period of time. The risk-neutral variance is a forward variance implied from options. This paper explores both model-independent and model-dependent measures of forward variance. The model-independent measure proposed in this study resembles the VIX. The model-independent variance is essentially the variance swap rate synthesized from Corn option prices. This measure of forward variance is new to the literature of agricultural commodity markets. The classical model-dependent measure is the at-the-money or near-the-money implied variance inferred from the Black model. Another approach used to forecast the future variance is the GARCH model of variance (Bollerslev 1986). Although the GARCH variance is not a risk-neutral variance per se, it does share similar forecasting features with both types of risk-neutral variances. Day and Lewis (1993), Lamoureux and Lastrapes (1993), Jorion (1995), and Figlewski (2004), and Manfredo and Sanders (2004) have considered various forms of GARCH(1,1) variance to compare with Black-Scholes implied variance. We also include GARCH(1,1) for comparison with the other two measures.

Realized variance

Realized variance is defined as the variance for returns on commodity futures for a given period of time. Denote S_t the nearby commodity future price at time t, realized volatility from time 1 to time T is computed as follows:

$$RV_{t,T} = \sqrt{\frac{1}{T}\sum_{i=1}^{T}[\ln(\frac{S_{t+i}}{S_{t+i-1}}) - \overline{\ln(\frac{S_{t+i}}{S_{t+i-1}})}]^2} \quad (1)$$

where $\ln(\frac{S_{t+i}}{S_{t+i-1}})$ and $\overline{\ln(\frac{S_{t+i}}{S_{t+i-1}})}$ are one-period return and the average return for T periods, respectively.

Model-independent variance

The model-independent variance is the risk-neutral value of a variance swap contract, the long side of which accepts the payoff. The payoff is $(RV_{t,T}\text{-}SW_{t,T})L$, where $RV_{t,T}$ is the realized variance between t and T; $SW_{t,T}$ is the swap rate at present time t paid at the future time T; L is the notional dollar that converts variance points into dollar value. The absence of arbitrage indicates that variance swap rate is equal to risk-neutral conditional realized variance, since there is no cash flow before the expiration. Carr and Madan (1998), Demeterfi et al. (1999), and Britten-Jones and Neuberger (2000), among others, show that variance swap can be replicated by a span of out-of-the-money options and futures without assuming a specific stochastic process of a financial asset. Hence, the variance swap rate is called “the model-independent variance”. We refer the interested readers to Carr and Wu (2009) for the proof. The VIX products offered by Chicago Board of Exchange (CBOE) employ the model-independence modeling approach which uses the variance swap rate. The formula for computing the variance swap rate between time t and T is as follows:

$$SW_{t,T} = 2[\int_0^{F_t}\frac{1}{K^2}(K-S_T)^+ dK + \int_{F_t}^{\infty}\frac{1}{K^2}(S_T-K)^+ dK] + 2\int_t^T \frac{1}{F_{s-}-F_t}dF_s \quad (2)$$

where F_t is the forward price at time t, s- is the time immediately before time s, K is the strike price of an option, $(K-S_T)^+$ is the payoff for an out-of-the-money put option, $(S_T - K)^+$ is the payoff for an out-of-the-money call option.

Model-dependent Variances

The first measure of model-dependent variance is the square of at-the-money implied volatility calibrated from the Black (1976) model. The Black formula for a call option price on commodity futures on an underlying strike at K, expiring in T years is:

$$c = e^{-rT}[FN(d_1) - KN(d_2)] \tag{3}$$

The put price is:

$$p = e^{-rT}[KN(-d_2) - FN(-d_1)] \tag{4}$$

where $d_1 = \frac{\ln(F/K) + (\sigma^2/2)T}{\sigma\sqrt{T}}$, and $d_2 = \frac{\ln(F/K) - (\sigma^2/2)T}{\sigma\sqrt{T}}$, r is the risk-free interest rate and σ is the implied volatility.

The second measure of model-dependent variance is the 60-day forward value forecasted from the GARCH(1,1) model. The conditional variance GV_{t+1} is explained by the past returns $R_t = \ln(\frac{S_{t+i}}{S_{t+i-1}})$ and past variance GV_t.[4] The functional form is:

[4] As in Manfredo and Sanders (2004), we use daily returns, instead of demeaned return in the GARCH(1,1) model. This representation is equivalent to the traditional GARCH model of futures return $R_t = m_t + \varepsilon_t$ by assuming a zero daily mean return m_t. This is reasonable since the mean of daily futures return is 0.000027 in our sample.

$$GV_{t+1} = \alpha_0 + \alpha_1 R_t^2 + \beta_1 GV_t \tag{5}$$

Data and Methodology

Daily data of corn futures and options are collected from the CME group. The futures data cover the period from January 1987 through January 2010, with the options data spanning from January 1987 through November 2009.[5] We choose 1987 to be the beginning year as in Egelkraut, Garcia, and Sherrick (2007). Our computation of both variances is based on end-of-day settlement prices. Prevailing interest rates during the period were obtained from the Federal Reserve at St. Louis.

Realized Variance (RV)

We choose the prices of nearby futures contracts as the basis for calculation. If the nearby contract has less than 60 days to expiration, the nearby contract is replaced by the next contract. Figure 1 shows the selection of futures contracts. The first row shows the calendar year. The second row shows the calendar month, divided into two halves. Contract bars represent futures contracts with the delivery months, March, May, July, September, and December in year t and March in year t+1. The end of each contract bar is the expiration date of the contract, roughly the middle of the month. The shaded area in each contract bar represents the period for which the futures prices are used for calculation. Specifically, the March contract is used for the trading dates from 10/16

[5] The futures data is two-month longer than the options data, because the computation of the 60-day realized variance requires two more months of futures prices than options prices for the 60-day implied variance.

through 1/15. Similarly, May for 1/16 through 3/15, July for 3/16 through 5/15, September for 5/16 through 7/15, and December for 7/16 through10/15. The shaded areas jointly complete the respective calendar year.

[Figure 1 about here]

Implied Variance from Variance Swap Rate (SWIV)

In order to calibrate the 60-day implied variance, we need to interpolate it from two variances implied in the nearby and the more distant options contracts. Our procedure essentially follows the calculation of VIX® by CBOE, with two differences: (1) daily settlement prices, instead of the mid-quote of daily close prices, are used for calculation; and (2) options with positive settlement prices are included, instead of positive bid prices. Based on the variance swap rate given by Equation (2), we obtain implied variances for nearby contracts and more distant contracts. We then use the following formula to interpolate the 60-day implied variance.

$$IV = (T_1 IV_1 \frac{T_2 - T_{60}}{T_2 - T_1} + T_2 IV_2 \frac{T_{60} - T_1}{T_2 - T_1}) \times \frac{T_{365}}{T_{60}}, \tag{6}$$

where T_1 and T_2 are time-to-expirations for the nearby and more distant contracts, T_{60} and T_{365} are 60 days and 365 days of time, and IV_1 and IV_2 are annualized implied variances synthesized from the nearby and more distant contracts.

Implied Variance from the Black (1976) Model (BKIV)

Inferring implied volatility from the Black (1976) model is widely used in the literature of commodity pricing (Simon 2002; Giot 2003; Manfredo and Sanders 2004). On a given date, we invert implied volatilities from the Black (1976) formula for the nearby contract and the more distant contracts. The 60-day variance is interpolated in the same approach as for SWIV. Another approach to infer implied volatility, adopted by Egelkraut, Garcia, and Sherrick (2007), is to estimate the volatility parameter from all traded call and put options with the same time to expiration. The volatility parameter is obtained by minimizing the difference between the model price and the observed market price for a given maturity. In this method, all option prices are equally weighted in the estimation. The equal weight of option prices essentially makes the results bias toward in-the-money (ITM) and at-the-money (ATM) options, since ITM and ATM options have much higher premiums than out-of-the-money (OTM) options. Such a bias has been widely noted in the empirical options pricing studies (Bakshi, Cao and Chen 1997; Bates 2003). Adding to the bias in estimating implied volatility is the reduced liquidity associated with ITM options.[6] Given these biases, we simply compute the traditional Black implied volatility using more actively traded ATM options. We also compare our results to Egelkraut, Garcia and Sherrick (2007).

Forward Variance from the GARCH(1,1) Model

We select futures prices using the same method employed for calculating the realized variance. First, time series of daily corn futures returns are constructed. Based on the

[6] Etling and Miller (2000) find that the liquidity of index options is maximized at the money, higher out of the money than in the money. Ait-Sahalia and Lo (1998) point out that expensive in-the-money options are illiquid and should be excluded in estimating the state-price densities of the underlying financial assets.

GARCH(1,1) model in Equation (5), we estimate the three parameters (α_0, α_1, β_1) from the past 500 daily futures prices using maximum likelihood.[7] The selection of 500 daily sample points is adopted from Figlewski (2004). These estimated parameters are then used to forecast the 60-day forward variance:

$E_t[GV_{t+60}] = \alpha_0 \sum_{i=0}^{59} (\alpha_1 + \beta_1)^i + (\alpha_1 + \beta_1)^{60} GV_t$, where GV_t is conditional variance at time t.

Results and Discussions

In this section we report summary statistics of the four variances. We analyze the existence of the variance risk premium and its variability across time. The existence of variance risk premium is tested across different sample periods. The seasonality of the premium is also investigated for the whole sample period. OLS regressions of realized variance on forward volatilities are used for testing whether variance risk premiums are constant or time varying. To confirm that variance swap rate is the most appropriate implied variance to measure variance risk premium, we compare the predictive power of the alternative variances through encompassing and modified Diebold Mariano (MDM) tests. Lastly, the measurement errors in synthesized implied variance are considered to strengthen the initial conclusion on the existence and time-varying characteristics of variance risk premiums.

Summary Statistics of Variances

[7] For the sake of space, we do not report the parameter estimates for 5720 days, which are available upon request.

Summary statistics for the complete sample from January 1987 through November 2009 are reported in Table 1. There are 5720 trading days for which corn futures and options data is available and futures data have at least two-month worth of sample points for computation. The mean and standard deviation of realized variance, synthesized variance swap rate, Black-implied variance, and GARCH-model-based variance differ significantly from each other. All four measures of variance show positive skewness and excess kurtosis, indicating asymmetry and extremeness of variance in the corn market. However, their logarithm values are subject to significantly lower degree of skewness and kurtosis. Given the difference, we also investigate whether log variance risk premium exists and varies over time.

[Table 1 about here]

Existence of Variance Risk Premium

(1) Significance of VRP across Different Sample Periods

We test the existence of variance risk premiums based on swap rate (SWIV) and Black implied variance (BKIV) in corn options. The mean variance risk premiums for the whole sample period and two subsample periods are reported in Table 2. The first subsample period, 1/1/1987-12/31/2001, is chosen to allow comparison of our results with those of Egelkraut, Garcia, and Sherrick (2007). The second subsample period covers 1/1/2002-11/22/2009. Standard errors for the mean estimates are adjusted for serial autocorrelation with 60 and 30 lags according to Newey and West (1987).

Variance risk premiums based on swap rate are clearly negative across all sample periods at the significance level well below 1%. Variance risk premiums based on Black implied variance are negative and statistically significant for the whole sample at the 5% level, and the first subsample at the 1% level. However, they are no longer significant for the second subsample at the 5% level. The reason that variance risk premium is lower for the Black model is, the Black implied variance includes only at-the-money options, but excludes out-of-the-money options that often have active trading activities and higher implied variance. The exclusion of higher implied variance underestimates the variance risk premium. Nevertheless, we can safely conclude that variance risk premiums are negative throughout the sample periods. We also find that the logarithm of variance risk premium based on SWIV and BKIV is significantly negative, which agrees with Carr and Wu (2009) on index options.

[Table 2 about here]

(2) Seasonality of VRP

We group the variance risk premiums by month. The twelve monthly average premiums for the whole sample are plotted in Figure 2. The top panel of Figure 2 illustrates that both implied and realized variances exhibit a seasonal pattern: increasing in the first half of the year and decreasing in the second half. The 60-day forward variances in May and June are the two highest, which reflects the uncertainty during the corn growing season. This is consistent with the findings by Anderson (1985); Egelkraut, Garcia, and Sherrick (2007); and Goodwin and Schnepf (2000). The bottom panel of Figure 2 shows that the average VRPs are negative for all 12 months. The negative VRPs are statistically significant at the 5% level for every month except for September which is significant at the 8% level. Unlike the level of variance, VRP does not show any seasonal pattern. Our results based on variance swap rate are different from Egelkraut, Garcia, and Sherrick (2007) in that: (1) The overall VRP is negative; (2) VRPs during both growing and non-growing seasons are negative, whereas they find statistically significant positive VRP.[8] This finding is significant because a positive VRP would be inconsistent with a typical investor's aversion toward variance if it persists in the long-run. Such a risk-averse investor would be willing to pay a premium or bear a loss to accept a lower variance in the future. On average, such risk attitude will result in a higher expected variance ex ante than the actual realized variance ex post.[9]

[Figure 2 about here]

In order to compare with Egelkraut, Garcia, and Sherrick (2007), we also report variance risk premium by month based on the Black model for the same sample period (January1987 through December 2001) in Figure 3. The average VRP in May is 189.31, statistically significant at the 1% level. Note that VRP in May covers the period between May and July. This result is consistent with Egelkraut, Garcia, and Sherrick's positive variance for intervals of April-June and June-August. Egelkraut, Garcia, and Sherrick (2007) report realized and implied variances for five intervals Feb-Apr, Apr-Jun, Jun-Aug, Aug-Nov and Nov-Feb of each year. Our results of BKIV show positive VRPs in

[8] A further investigation of the subsample of 1987-2001 shows statistically significant negative premiums, except that VRP in May is not significantly different from zero.

[9] Nevertheless, we do not rule out the possibility of realized variance being higher than expected (implied) variance, hence positive variance risk premium for some short periods, during the extremely volatile period as in 2008-2009.

April (p-value = 0.47) and May (p-value = 0.02), which corroborates with Egelkraut, Garcia, and Sherrick (2007) to some extent.

[Figure 3 about here]

Constant or time-varying Variance Risk Premium

We run two OLS regressions to test whether the variance risk premium is constant or time-varying.

$$RV_t = \alpha + \beta^* \ SWIV_t + e_t \tag{7}$$

$$Ln(RV_t) = \alpha + \beta^* \ Ln(SWIV_t) + e_t \tag{8}$$

A nonzero β will show that variance risk premium is time varying and correlates with variance swap rate. A zero variance risk premium will imply α=0 and β=1. In view of autocorrelation in the residuals, we compute the Generalized Method of Moments (GMM) estimates using the weighting matrix with 60 lags according to Newey and West (1987). The GMM estimates and their standard errors are reported in Table 3.[10] T statistics for β estimate strongly reject the null hypothesis of β=0. Agreeing with Carr and Wu (2009), we can conclude that variance risk premiums in dollar terms are time varying in the corn options market. In contrast to their findings, our variance risk premiums in log dollar terms are time varying in the corn options market. The results show that corn

[10] Carr and Wu (2009) use the same method for equity options to address the autocorrelation issue, which is noted in Christensen and Prabhala (1998); and Christensen, Hansen and Prabhala (2001). The autocorrelation is attributed to the overlapping use of options when daily series of implied variance are computed. Jiang and Tian (2005) conclude that the overlapping issue is immaterial once autocorrelation is accounted for in the GMM estimation.

options exhibit similar but different variance dynamics compared to financial options. An F test (p value < 0.001) rejects the joint hypothesis $\alpha=0$ and $\beta=1$ in both regressions, indicating again a nonzero variance risk premium.

[Table 3 about here]

Encompassing Test

To validate our findings, we evaluate the robustness of three variance measures. We consider the implied variance estimate using the variance swap (SWIV) approach, the Black implied variance (BKIV), and variance estimate generated by the GARCH(1,1) model to forecast the forward variance. Market participants are interested in finding out which alternative possesses the most information about the forward variance. We implement a test strategy based on OLS regression. This encompassing test is proposed by Granger and Newbold (1973, 1986), Harvey, Leybourne and Newbold (1998) and adopted by Manfredo and Sanders (2004) for testing the information content of implied volatility in live cattle options against the GARCH variance.

We first run a set of regressions, regressing the realized variance on each of the three alternatives of forward variance and obtain residuals of each regression based on GMM estimates.

$$RV_t = \alpha + \beta^* \, SWIV_t + e_{1t} \tag{10}$$

$$RV_t = \alpha + \beta^* \, BKIV_t + e_{2t} \tag{11}$$

$$RV_t = \alpha + \beta^* \, GARCH_t + e_{3t} \tag{12}$$

We then test whether one residual can help explain the other residual for each pair of variances: (SWIV, BKIV), (SWIV, GARCH), and (BKIV, GARCH) by performing a test on the coefficient *b* in the following regression:

$$e_{it}= a + b^* (e_{it}-e_{jt}) + \varepsilon_t \quad (13)$$

where e_{it} and e_{jt} are the preferred residuals and the competing residuals, respectively. [11]

If the null hypothesis H0 (b=0) holds, it implies that the competing residuals do not contribute to explaining the preferred residuals. In other words, the information of the preferred variance encompasses that of the competing variance. Table 4 presents the testing results of three pairs of variance forecasts: (SWIV, BKIV), (SWIV, GARCH), and (BKIV, GARCH). Standard errors of all estimates are reported according to Newey and West (1987) with 60 lags. The first variance of each pair is the preferred variance, whereas the second is the competing variance. For example, the first two columns report the results for SWIV and BKIV. The coefficient b = 0.44 with BKIV as the competing variance cannot be rejected from zero at the 3% significance level. The reverse regression shows the coefficient b = 0.56 is statistically different from zero at a better than 1% significance. A higher R-square of the reverse regression also points to a better explanatory power of SWIV. The third and fourth columns report the pair of SWIV and GARCH. The coefficient b = 0.02 with GARCH as the competing variance cannot be rejected from zero at any traditional significance. The coefficient b = 0.98 (p-value <

[11] This test avoids the interpretation problem associated with the traditional test, based on the following formulation: $RV_t = \alpha + \beta_1 * IV_t + \beta_2 * FV_t + e_t$, where RV_t, IV_t, FV_t are realized volatility, implied volatility and alternative future volatility, respectively. In fact, this equation tests both forecast unbiasedness ($\alpha = 0$ and $\beta_1 = 1$), and encompassing ($\alpha = 0$, $\beta_1 = 1$ and $\beta_2 = 0$). Simon (2002) uses this traditional test procedure.

0.0001) for the reverse regression clearly shows that SWIV contains all information from GARCH. The last two columns report the pair of BKIV and GARCH. Similar to the pair of SWIV and GARCH, we also conclude that BKIV is superior to GARCH in forecasting variance based on the coefficients of 0.01 and 0.99 in the two competing regressions.[12]

[Table 4 about here]

MDM Test

To further compare the forecasting power of forward variance, we perform the Modified Diebold Mariano (MDM) test proposed by Harvey, Leybourne and Newbold (1997). The MDM procedure based on testing the equality of squared forecast errors is robust to autocorrelation, heteroskedasticity and heavy tails in the error term. The MDM test statistic is defined as

$$MDM = \sqrt{\frac{(T-1)}{\frac{1}{T}\sum_{1}^{T}(d_t/\overline{d}-1)^2}} \tag{14}$$

where d_t is the time series of the difference between two competing percentage errors: SWIV and BKIV, $\overline{d}$ is the average of all d_t for all T periods. Two measures of percentage errors are defined: a) absolute percentage errors (APE) by $(|e_{it}|-|e_{jt}|)/RV$, in which e_{it} are e_{jt} are errors from Equations (11) through (13); and b) squared percentage errors (SPE) which are the square of APE.

[12] Different from our results, Simon (2002) cannot confirm that Black-implied volatility encompasses GJR-GARCH volatility for corn, although he finds that implied volatility has more forecast power for realized volatility than the GARCH volatility. A reason for the difference is that we employ different testing procedure.

Table 5 reports the MDM test results. APE for SWIV has a smaller mean (0.483) and standard deviation (0.479) than their counterparts for APE for BKIV (mean, 0.530 and standard deviation, 0.564). The results for SPE also confirm the finding. The p-values for APE and SPE are less than 0.001, further indicating that the forecasting errors for SWIV are significantly less than those for BKIV and GARCH.

[Table 5 about here]

Treatment of Errors in Variables

Since the synthetic swap rate (SWIV) is a proxy for the unobservable 60-day implied variance (IV), we need to deal with the measurement errors in IV in Equation (7). Following the treatment of the same issue for S&P500 index in Carr and Wu (2009), we assume that the unobservable implied variance follows an AR(1) process as specified in Equation (15). Realized variance (RV) and swap rate (SWIV) are both observable and described by Equations (16) and (17), respectively. Equations 15-17 collectively defines a classical Kalman filtering problem, with (15) being the state transition equation and (16) and (17) being measurement equations. Assuming the residuals in the three equations to be independent from each other, we can estimate the eight parameters (θ, ρ, α, β, σ_ε, σ_e, σ_ξ) using maximum likelihood.

$$IV_{t+1} = \theta(1- \rho) + \rho * IV_t + \varepsilon_{t+1} \tag{15}$$

$$RV_t = \alpha + \beta * IV_t + e_t \tag{16}$$

$$SWIV_t = IV_t + \xi_t \tag{17}$$

To test the null hypothesis β =1, we estimate two versions of the system: with free parameters and with β being restricted to 1. Parameter estimates and their standard errors are reported in Table 6. The estimates of α and β in Equation (16) are corrected for errors-in-variable bias that exists in the original regression of Equation (7). After correcting for the bias, the slope coefficient β increases from 0.76 to 1.34. A simple T test shows that both estimates are significantly different from both 0 and 1. We also perform likelihood ratio test on whether β is equal to 1. The likelihood ratio is 1600, clearly rejecting the null hypothesis based on the critical value of χ_1^2 (chi-square distribution with degree of freedom 1). Our earlier conclusion remains valid that variance risk premiums are time varying.

[Table 6 about here]

Conclusions

Motivated by dramatic increase in commodity price volatility and the success of the VIX in financial markets as a predictor of volatility in the index for S&P 500 options, we propose to calibrate the forward variance for corn from the model-independent variance swap rate. This approach represents a clear contribution to the literature discussing price volatility in commodity markets. We compute two traditional forward variances from the Black (1976) model and the GARCH(1,1) model as comparative benchmarks. Based on the new implied variance approach we adopted, we find that variance risk premiums, defined as the difference between the realized variance and the implied variance, are negative and time-varying for the whole sample from January 1987 to November 2009. The variance risk premiums are also found to be significantly negative for the subsample period from January 1987 to December 2001, which contrasts with the findings in Egelkraut, Garcia, and Sherrick (2007) for the same period. Nevertheless, the negative variance risk premiums are consistent with the findings in the finance literature and commodity studies by Simon (2002) on grain options and Manfredo and Sanders (2004) on live cattle options. In addition to using a less efficient Black model for estimating implied variance, the two studies neither address the issue of variance risk premium nor test their statistical significance. The presence of time-varying variance risk premium is confirmed by an OLS regression of the realized variance on the variance swap rate. We adjust the regression analysis for the potential measurement error in variance swap rate. The conclusion of time-varying variance risk premium remains unchanged. We also conclude that variance swap rate is a better forecasting tool for forward variance in terms of encompassing more information and generating less forecasting errors than the other two alternatives. The superior performance of variance swap rate also supports the measurement of variance risk premium using the model-independent approach relative to the Black implied variance estimation method (an innovation to the commodity derivatives literature).

The existence of variance risk premium suggests that investors of agricultural commodity options should be concerned with hedging variance risk. In view of the superior forecasting performance of variance swap rate, the square root of the variance swap rate calculated in this research can serve as the underlying volatility index for corn, which can be named as "CornVIX".[13] Futures and options on CornVIX can be used by

[13] The CME group and CBOE jointly announced a plan to introduce such index to the investors on March 15th 2010, while the paper is in the final stage of preparation.

investors to hedge volatility risk in corn options.

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Figures and Tables

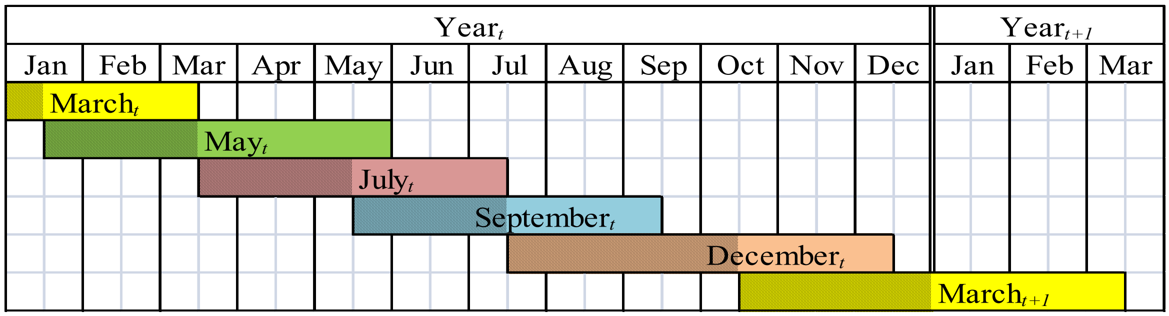

Figure 1 Selection of Futures Contracts for Computing Realized Variance

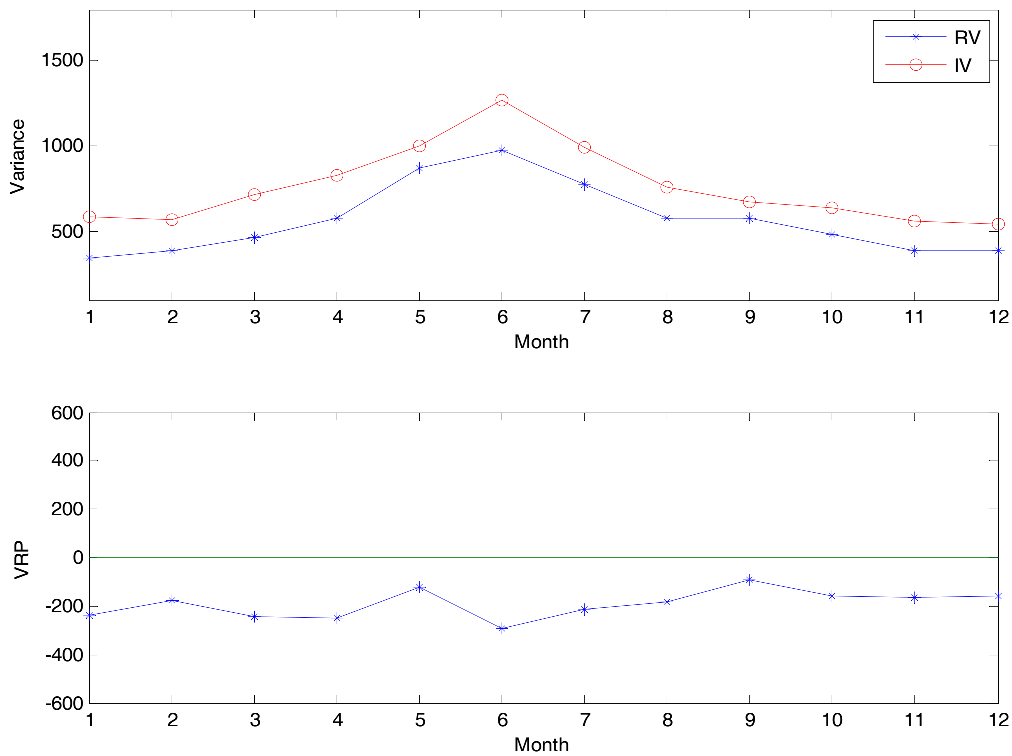

Figure 2 Variance Risk Premiums and Variances by Month

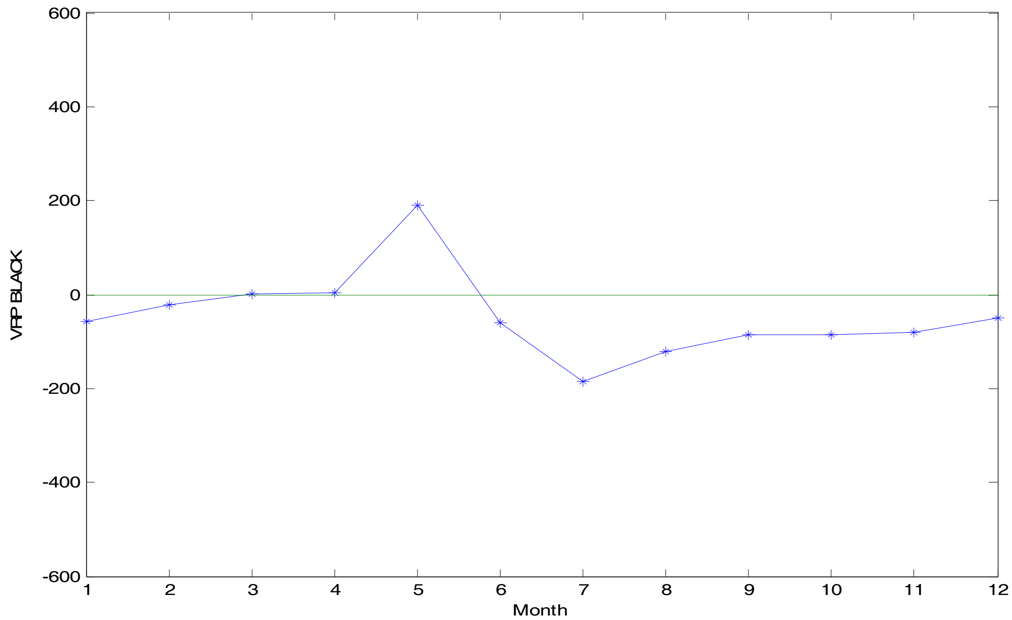

Figure 3 Black (1976) Variance Risk Premium by Month

Table 1 Summary Statistics of Variances

Variable	# of obs.	Mean	Std Dev	Median	Skewness	Kurtosis
RV60	5720	572.38	534.04	385.73	2.45	7.86
SWIV60	5720	764.42	522.08	602.21	1.87	4.21
BIV60	5720	656.85	787.67	432.02	8.72	145.42
GV60	5720	1406.73	3126.12	645.67	11.25	212.15
LOGHV60	5720	6.01	0.82	5.96	0.05	-0.18
LOGSWIV60	5720	6.45	0.59	6.40	0.41	-0.34
LOGBIV60	5720	6.16	0.77	6.07	0.31	0.65
LOGGV60	5720	6.66	0.91	6.47	1.03	1.71
VRP60	5720	-192.04	376.72	-178.51	0.66	8.83
VRPB60	5720	-84.46	715.40	-50.47	-11.03	224.95

Note: This table reports summary statistics of 60-day variances. RV, SWIV, BIV, GV, LOG, VRP, and VRPB are abbreviations of realized variance, swap-rate-based implied variance, Black-implied variance, GARCH-model-based variance, logarithm, swap-rate-based variance risk premium and Black-model-based variance risk premium, respectively.

Table 2 Existence of Variance Risk Premium in Corn Options

Sample Period	Stat.	VRP (60-lag/30-lag)		LOGVRP (60-lag/30-lag)		VRP-BK (60/30-lag)		LOGVRP-BK (60/30-lag)	
1/1/1987-12/31/2001	Mean	-178.75		-0.51		-46.50		-0.19	
	s.e.	23.25	21.52	0.04	0.04	20.68	20.58	0.04	0.04
	P-value	*	*	*	*	0.01	0.01	*	*
1/1/2002-11/22/2009	Mean	-217.32		-0.31		-50.94		-0.12	
	s.e.	61.34	50.68	0.05	0.05	60.25	49.86	0.05	0.05
	P-value	*	*	*	*	0.10	0.07	0.009	0.003
1/1/1987-11/22/2009	Mean	-192.04		-0.44		-48.03		-0.17	
	s.e.	26.11	22.48	0.04	0.03	24.97	21.84	0.03	0.03
	P-value	*	*	*	*	0.03	0.01	*	*

Note: This table reports the mean value of the variance risk premiums for corn options for the whole sample period 1/1/1987-11/22/2009 and two subsample periods: 1/1/1987-12/31/2001 and 1/1/2002-11/22/2009. (LOG)VRP is the 60-day (log) variance risk premium based on variance swap rate. (LOG)VRP-BK is the 60-day (log) variance risk premium based on Black implied variance. "s.e." denotes standard error of variance risk premium, which is adjusted for serial autocorrelation with 60 and 30 lags according to Newey and West (1987). "*" indicates a p-value less than 0.001.

Table 3 Regression of Realized Variance on Implied Variance

Model	α	s.e.	t-stat	p-value	SSE	MSE	Root MSE
RV vs. SWIV	-10.74	51.71	-0.21	0.84	7.24E+08	1.27E+05	355.80
	β	**s.e.**	**t-stat**	**p-value**	**R-square**	**Adj-R**	
	0.76	0.09	8.73	<.0001	0.56	0.56	
Ln(RV) vs. Ln(SWIV)	**α**	**s.e.**	**t-stat**	**p-value**	**SSE**	**MSE**	**Root MSE**
	-1.28	0.37	-3.50	0.00	1309.5	0.229	0.48
	β	**s.e.**	**t-stat**	**p-value**	**R-square**	**Adj-R**	
	1.13	0.06	20.16	<.0001	0.66	0.66	

Note: This table reports parameter estimates and statistics of two OLS regressions: realized variance vs. implied variance, and logarithm of realized variance vs. logarithm of implied variance.
$RV_t = \alpha + \beta * SWIV_t + e_t$
$Ln(RV_t) = \alpha + \beta * Ln(SWIV_t) + e_t$
s.e., t-stat, and p-value stand for standard error, t statistic and p value of the corresponding estimate. SSE, MSE, R-square and Adj-R are sum of squared errors, mean squared error, R square and adjusted R square of the regression, respectively. Standard errors are reported according to Newey and West (1987) with 60 lags. T-stat and p-value are calculated under the null hypothesis $\alpha = 0$ and $\beta=0$.

Table 4 Encompassing Test of Alternative Variance Forecasts

Model	SWIV vs. BKIV	BKIV vs. SWIV	SWIV vs. GARCH	GARCH vs. SWIV	BKIV vs. GARCH	GARCH vs. BKIV
a	0.00	0.00	0.00	0.00	0.00	0.00
s.e.	25.96	25.96	26.42	26.42	26.12	26.12
t-stat	0.00	0.00	0.00	0.00	0.00	0.00
p-value	1.00	1.00	1.00	1.00	1.00	1.00
b	0.44	0.56	0.02	0.98	0.01	0.99
s.e.	0.20	0.20	0.12	0.12	0.12	0.12
t-stat	2.16	2.73	0.13	8.37	0.13	8.47
p-value	0.031	0.006	0.894	<.0001	0.900	<.0001
SSE	7.11E+08	7.11E+08	7.24E+08	7.24E+08	7.32E+08	7.32E+08
MSE	1.24E+05	1.24E+05	1.27E+05	1.27E+05	1.28E+05	1.28E+05
Root MSE	352.70	352.70	355.80	355.80	357.70	357.70
R-square	0.018	0.028	0.000	0.550	0.000	0.546
Adj-R	0.018	0.028	0.000	0.550	0.000	0.546

Note: This table reports the regression results of three pairs of variance forecasts: (SWIV, BKIV), (SWIV, GARCH), and (BKIV, GARCH) with the following formulation:
$RV_t = \alpha + \beta * SWIV_t + e_{1t}$
$RV_t = \alpha + \beta * BKIV_t + e_{2t}$
$RV_t = \alpha + \beta * GARCH_t + e_{3t}$
$e_{it} = a + b * (e_{it} - e_{jt}) + \varepsilon_t$
The notations in this table follow those in Table 3.

Table 5 MDM Test of Forecasting Errors by SWIV, BKIV and GARCH

	N	MEAN	STD	MDM TEST	P VALUE
APE_SW	5720	0.483	0.479		
APE_BK	5720	0.530	0.564		
APE_G	5720	1.195	1.592		
APE_DIFF	5720	-0.047	0.300	-11.872	<0.001
APEG_DIFF	5720	-0.712	1.369	-39.342	<0.001
SPE_SW	5720	0.463	1.060		
SPE_BK	5720	0.599	1.480		
SPE_G	5720	3.961	12.173		
SPE_DIFF	5720	-0.136	1.168	-8.803	<0.001
SPEG_DIFF	5720	-3.499	11.711	-22.591	<0.001

Note: This table compares the forecasting errors of Black implied variance (BKIV) and variance forecast by the GARCH(1,1) model to those of the benchmark swap rate implied variance (SWIV). The comparison is based on the MDM test. The differences in forecasting errors between the two select measures are defined as follows:
APE_DIFF = APE_SW-APE_BK; SPE_DIFF = SPE_SW-SPE_BK;
APEG_DIFF = APE_SW-APE_G; APEG_DIFF = APE_SW-APE_G,
where SW, BK, and G following APE and SPE represent variance swap, Black and GARCH forecasting models. APE, SPE, and DIFF are the abbreviations for absolute percentage error, squared percentage error and difference in forecasting error.

Table 6 Maximum Likelihood Estimates of Regression of RV on SWIV

Parameters	α	β	θ	P	σ_ε	σ_e	σ_ξ	Loglik
Free	-449.421	1.337	773.412	0.995	37.227	60.173	339.027	-70041
s.e.	12.137	0.014	98.876	0.001	0.831	0.892	3.260	
Restricted	-192.087	1	776.930	0.996	48.438	61.595	361.241	-70441
s.e.	3.486	-	8.530	0.001	0.986	0.937	3.390	

Note: This table reports maximum likelihood estimates via Kalman filtering for two versions of the regression model of RV and SWIV. The model is specified as follows:

$IV_{t+1} = \theta(1-\rho) + \rho * IV_t + \varepsilon_{t+1}$

$RV_t = \alpha + \beta * IV_t + e_t$

$SWIV_t = IV_t + \xi_t$

The first version allows for free parameters, whereas the second version restricts the value of β to be 1. Standard errors (s.e.) and log likelihood values are reported for both regressions.