Forecasting Volatilities of Corn Futures at Distant Horizons

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Abstract

Accurately forecasting volatility at distant horizons is critical for managing long-term risk in agriculture. Given the poor performance of GARCH-type models in long-term volatility forecast, we develop a risk-adjusted implied volatility forecast model, which adjust the risk-neutral implied volatility by correctly accounting for the volatility risk premium. The paper evaluates the performance of the new implied volatility forecast in the corn futures market relative to two alternative forecasts - a three-year moving average forecast and a naïve forecast. The finding from the study is that the new implied volatility forecasts have better or at least equal predictive power compared to alternative predicting approaches.

Keywords: Risk-neutral, Volatility risk premium, Forecast, Corn options
Accurately forecasting volatility at distant horizons is critical for measuring, monitoring, and managing long-term risk in agriculture. Consider a crop merchant who buys and stores corn at harvest for later resale at a price which is unknown at the time of purchase. Clearly he needs volatility forecasts to evaluate his future profit margin. If he maintains a hedging program, then he needs volatility forecasts to determine effective hedge ratios and evaluate potential cash flow demands. Additionally, forecasting volatility can also help crop extension educator to develop simulation tools such as AgRisk™ program, which needs volatility as an input to develop harvest-time revenue distribution of grain farms with and without using a variety of risk management strategies (Manfredo, Leuthold, and Irwin, 2001).

Volatility forecasts have received considerable attention in the literature because of its importance in risk management. There is a large body of literature on forecasting performance of various volatility models, including econometric models using time series data and option implied volatility (IV) models. The econometric method such as using ARCH-type models or stochastic volatility models is backward-looking, using the historical information. Generally the method could provide accurate predictions for volatility at short horizons. But it is inappropriate when used to predict long-term volatilities as more-distant predictions generally revert to the unconditional mean (Egelkraut, Garcia, and Sherrick, 2007). The IV method is obtained from observed options premiums by inverting a theoretical option pricing method. It incorporates all market participants’ expectations based on all available historical information, and thus is forward-looking. It is believed to provide a better forecast for short and long-term volatilities. The predictive power of these models in agricultural commodities markets has also been scrutinized. Garcia and Leuthold (2004) offer an early review of the literature and conclude that IV method provides more reasonable forecasts for agricultural commodity price volatilities of nearby futures contracts.
However, whether the IV approach could provide superior forecast power for distant prices is unclear. Research on this issue is scarce. To date, only two known studies have investigated forecast power, but both of them find that the implied volatility approach provides biased forecasts for the volatility of distant agricultural commodity prices (Szakmary et al., 2003 and Egelkraut, Garcia, and Sherrick, 2007).

Hence, it is of interest to find out the source of the bias and the way to correct it. The bias may be caused by factors such as measurement errors and/or volatility risk premiums. Poteshman (2000) shows that the bias in the implied volatility does not disappear after measurement errors are corrected by using last-period implied and realized volatilities as instrumental variables. Recent research has turned to other explanations, particularly a volatility risk premium. In a Monte Carlo simulation, Doran and Ronn (2008) demonstrate that the volatility risk premium is the only parameter that generates the disparity between implied and realized volatilities in energy markets, even in the presence of jumps and a jump premium. It is believed that volatility risk premium also plays a significant role in the forecast of long term volatility: the longer the horizon, the less predictable the volatility, and thus investors would demand a risk premium for bearing the volatility risk. Therefore, addressing the volatility risk premium when inferring the implied volatility may improve forecast of volatility at distant horizons.

In this study we estimate a volatility risk premium using the GMM approach and adjust the implied volatility for the volatility risk premium accordingly. We further evaluate whether the adjusted implied volatility forecasts are superior to other forecasts. We compare the forecasts with the forecasts derived from time series models: a moving average forecast and a naïve forecast. In the empirical application we focus on corn futures contracts and study the price volatilities in 2-month and 3-month horizons in view of the structure of corn futures options contract expiration.
Methodology

Heston model

The existing literature that uses the B-S model assumes that volatility of the logarithm of the underlying asset price is constant over the life of the option. Since this contradicts the empirical findings that volatility is time-varying, more generalized option pricing models are developed to incorporate stochastic volatility. In this study volatility is assumed to follow Heston’s (1993) one-factor mean-reverting square root process.

\[
\begin{align*}
    dp_t &= \mu_t dt + \sqrt{V_t} dB_{1t} \\
    dV_t &= k (\theta - V_t) dt + \sigma_t dB_{2t}, \\
    corr(dB_{1t}, dB_{2t}) &= \rho
\end{align*}
\]

where \( p_t \) is the logarithmic corn futures price \( (p_t = \log(S_t)) \) and \( V_t \) is the instantaneous volatility. The instantaneous correlation between the two separate Brownian motions driving the price and volatility processes is generally negative in empirical analysis, or \( \rho < 0 \).

Given this dynamic specification for the underlying process and assumptions of no arbitrage and a linear volatility risk premium, standard pricing arguments can be transformed into the following equivalent martingale measure, or risk-neutral distribution:

\[
\begin{align*}
    dp_t &= r_t dt + \sqrt{V^*_t} dB^*_{1t} \\
    dV^*_t &= k^*(\theta^* - V^*_t) dt + \sigma^* dB^*_t, \\
    corr(dB^*_{1t}, dB^*_t) &= \rho
\end{align*}
\]

where \( r_t \) is the risk-free interest rate. The risk-neutral parameters in (2) is directly associated with the parameters in the actual price process in eq. (1) by the relationships, \( k^* = k + \lambda \) and \( \theta^* = k\theta/(k + \lambda) \), where \( k^* \) means the speed of mean reversion for the risk-neutral volatility process, \( \theta^* \) is the long-term mean volatility, and \( \lambda \) refers to the stochastic volatility risk premium. Since volatility risk premium is generally empirically estimated to be negative, the
degree of mean reversion for the risk-neutral volatility process, as determined by $k^*$, is slower than the mean reversion for the actual process, as determined by $k$.

**Moment Conditions**

Doran and Ronn (2008) recommend a two-step procedure to estimate volatility risk premiums based on the Heston’s volatility specification. But the limitation of the method is that it places strong restrictions on the data-generating process, e.g. the correlation between the volatility and the asset return is zero ($\rho = 0$). Following Bollerslev, Gibson, and Zhou (2008), we apply a more generalized estimation method - GMM. Two moment conditions are derived from the Heston model under a risk-neutral probability measure and a physical probability measure, respectively.

First, we define an integrated volatility from date $t$ to date $t + \Delta$ as

$$v_{{t, t + \Delta}} = \int_{t}^{t + \Delta} V_s ds,$$

From model (1), one relationship between the integrated volatility and its lag value can be derived (Bollerslev and Zhou, 2002)

$$v_{{t + \Delta, t + 2\Delta}} = \alpha \cdot v_{{t, t + \Delta}} + \beta + \sigma e^{-k\Delta} \int_{t}^{t + \Delta} e^{k\gamma} dB_{s}$$

where $\alpha = e^{-\lambda}$ and $\beta = \theta \cdot (1 - e^{-k\Delta})$. Therefore the first conditional moment can be reached by taking the expectation under the physical measure at time $t$

$$E(v_{{t + \Delta, t + 2\Delta}}|F_t) = \alpha \cdot E(v_{{t, t + \Delta}}|F_t) + \beta,$$

where $F_t$ is available information at time $t$. The first conditional moment equation establishes the link between the objective expectation of the integrated volatility and its lag.
The second moment condition can be derived from the model (2). As formally shown by Bollerslev and Zhou (2006), the relationship between the objective expectation of the integrated volatility with its first moment under the risk-neutral dynamics is

\[(6) E(v_{t,t+\Delta}|\mathcal{F}_t) = a \cdot E^*(v_{t,t+\Delta}|\mathcal{F}_t) + b,\]

where \( E^*(\cdot) \) refers to the expectation under the risk-neutral measure, \( a = \frac{1-e^{-k\Delta}}{k}/\frac{1-e^{-k^*\Delta}}{k^*} \), and \( b = \theta \left( \Delta - \frac{1-e^{-k\Delta}}{k} \right) - \frac{(1-e^{-h\Delta})/k}{(1-e^{-k^*\Delta})/k^*} \theta^*[\Delta - \frac{1-e^{-k^*\Delta}}{k^*}] \).

**Volatility Measures**

If integrated volatilities under the risk-neutral and physical measures can be observed, we can construct a standard GMM type estimator for parameters in model (2) using the moment conditions (5) and (6). However the integrated volatilities entering the moment conditions are not directly observable. A proxy measure or an approximation to the unobserved needs to be found. Much of the recent literature has suggested that the model-free realized volatility measures can afford an integrated volatility to any desired degree of accuracy (Anderson et al., 2008). The model-free realized volatility can be computed by summing the squared high-frequency returns over the time interval.\(^1\) We let \( v_{t,t+\Delta}^n \) be the realized volatility computed by aggregating intra-day returns over the interval \([t, t + \Delta]\):

\[(7) v_{t,t+\Delta}^n = \sum_{i=1}^{n}[p_{t+\frac{i+1}{n}(\Delta)} - p_{t+\frac{i}{n}(\Delta)}]^2\]

Based on the theory of quadratic variation, \( v_{t,t+\Delta}^n \) becomes an increasingly accurate measure of integrated volatility when \( n \) increases asymptotically. Empirically, we construct the realized volatility from the summation of the five-minute squared return within the time interval.

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\(^1\) Bollerslev and Zhou (2006) points out that a host of market microstructure frictions invalidate the underlying martingale assumption for the returns at the ultra highest frequencies. Therefore, we need test the sensitivity of parameter estimator to the different frequency approximation of model free realized volatility.
Using option prices, it is also possible to construct a model-free measure of the risk-neutral expectation of the integrated volatility. Suppose that call options with a continuum of strike price ($K$) for a given maturity $t + \Delta$ are traded on an underlying asset. Britten-Jones and Neuberger (2000) define model-free implied volatility as the integral of call option prices over an infinite range of strike prices,

$$
IV_{t,t+\Delta}^* = 2 \int_0^\infty \frac{C(t+\Delta,K) - \max(0,S_t-K)}{K^2} dK
$$

where $C(t,K)$ denotes the price of a European call option maturing at time $t$ with an exercise price $K$. They show that the model free implied volatility can equals the true risk-neutral expectation of the integrated volatility,\(^2\)

$$
IV_{t,t+\Delta}^* = E^*(v_{t,t+\Delta}|\mathcal{F}_t).
$$

If option prices are available for all strike prices, the model free volatility can be calculated using numerical integration method. But in fact only a finite number of strike prices are traded in the market so that it can lead to truncation errors in the calculation process. In addition, the implementation also involves discretization errors due to numerical integration. Thus the empirical measure is

$$
IV_{t,t+\Delta}^* \approx 2 \int_{K_{\min}}^{K_{\max}} \frac{C(t+\Delta,K) - \max(0,S_t-K)}{K^2} dK
$$

$$
\approx \sum_{i=1}^{m} \left[ g(t + \Delta, K_i) + g(t + \Delta, K_{i-1}) \right] \Delta K
$$

where $\Delta K = (K_{\max} - K_{\min})/m$, $K_i = K_{\min} + \Delta K$ for $0 \leq i \leq m$, and $g(t + \Delta, K_i) = \frac{C(t+\Delta,K_i) - \max(0,S_t-K_i)}{K_i^2}$. $K_{\max}$ and $K_{\min}$ are upper and lower bounds of strike prices. Jiang and Tian (2005) show that these measurement errors can be ignored and the volatility can be accurately approximated from a finite number of options empirically. Therefore, we can

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\(^2\)Jiang and Tian (2005) also show that the same result can be derived even when the price process contains jump diffusions.
approximate the risk-neutral expectation of integrated volatility by the above numerical integration.

**Estimation**

Based on the two derived moment conditions and approximations to the dependent and independent variables, the standard GMM estimation procedure is used. An additional moment condition that uses the lag of realized volatility as instrument is used for more efficient estimation, taking the advantage of over-identified restrictions. The system of equations is

\[
(f_t(\xi)) = \begin{bmatrix}
  v_{t+\Delta,t+2\Delta} - \alpha \cdot v_{t,t+\Delta} - \beta \\
  (v_{t+\Delta,t+2\Delta} - \alpha \cdot v_{t,t+\Delta} - \beta) v_{t-\Delta,t} \\
  v_{t,t+\Delta} - a \cdot IV_{t,t+\Delta} - b \\
  (v_{t,t+\Delta} - a \cdot IV_{t,t+\Delta} - b) v_{t-\Delta,t}
\end{bmatrix}
\]

where \( \xi \) is the parameter vector \((k, \theta, \lambda)'\). Under the GMM framework, parameters are estimated by setting sample moments to be close to population counterpart. In population moment, we know that \( E(f_t(\xi) | \mathcal{F}_t) = 0 \). Thus we define its sample average \( g_T(\xi) = \frac{1}{T-3} \sum_{t=2}^{T-2} f_t(\xi) \). Since we have more moment conditions than parameters to estimate, we will not obtain a unique solution for \( \xi \). The corresponding GMM estimator is defined as the one that minimizes \( g_T(\xi) W^{-1} g_T(\xi) \), where \( W \) denotes a weighting, positive definitive matrix. To be efficient, \( W \) is generally taking the asymptotic covariance matrix of \( g_T(\xi) \) (Hansen 1982). So

\[
(12) \quad \hat{\xi} = \arg\min_{\xi} [g_T(\xi) W^{-1} g_T(\xi)]
\]

Under the null that the moment restrictions hold, the criterion function evaluated at the estimated \( \hat{\xi} \) has the chi-squared distribution with degrees of freedom equal to the number of independent moment conditions less than the number of estimated parameters. The criterion function is the minimized value of the objective function \( g_T(\xi) W^{-1} g_T(\xi) \) multiplied by the
sample size. We use the statistic to test the validity of the instruments. We follow Bollerslev, Gibson, and Zhou (2008) and employ an autocorrelation and heteroscedasticity robust covariance matrix $W$ to do statistical inference. Once the volatility risk premium is estimated, a risk-adjusted implied volatility can be derived from the second moment condition.

**Evaluating forecasts**

A popular norm to evaluate different forecast models is the minimization of a particular statistical loss function. However, there does not exist a unique loss function to evaluate the quality of forecast. In much of the previous literature, the general use of loss function is a particular set of statistical loss of function of the MSE-type. In the present work, instead of choosing a particular statistical loss function as the best and unique criterion, we follow Marcucci (2005) to adopt six different loss functions, which have different interpretations and can lead to a more complete forecast evaluation of the competing models.\(^3\) These statistical loss functions are:

\[
(13) \quad MSE_1 = \frac{\sum_{t=1}^{T}(v_{t,t+\Delta}^{1/2} - f_{t,t+\Delta}^{1/2})^2}{T}
\]

\[
(14) \quad MSE_2 = \frac{\sum_{t=1}^{T}(v_{t,t+\Delta} - f_{t,t+\Delta})^2}{T}
\]

\[
(15) \quad MAD_1 = \frac{\sum_{t=1}^{T}|v_{t,t+\Delta}^{1/2} - f_{t,t+\Delta}^{1/2}|}{T}
\]

\[
(16) \quad MAD_2 = \frac{\sum_{t=1}^{T}|v_{t,t+\Delta} - f_{t,t+\Delta}|}{T}
\]

\[
(17) \quad QLIKE = \frac{\sum_{t=1}^{T}(log(f_{t,t+\Delta}) + v_{t,t+\Delta}f_{t,t+\Delta}^{-1})}{T}
\]

\[
(18) \quad R2LOG = \frac{\sum_{t=1}^{T}(log(v_{t,t+\Delta}f_{t,t+\Delta}^{-1}))^2}{T}
\]

(13) and (14) are the typical mean squared error (MSE) metrics, while (15) and (16) are mean absolute deviation (MAD) criteria. The latter are more robust to the presence of outliers than the former, although they impose the same penalty on over- and under-prediction. The loss

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\(^3\)Marcucci (2005) adopts seven different loss functions. We exclude heteroscedasticity-adjusted MSE (HMSE) because the HMSE loss is not particularly suitable for evaluating different volatility forecasts.
function in (17) suggested by Bollerslev et al. (1994) is originated from a Gaussian likelihood. The criteria in (18) is similar to (14), using the $R^2$ metric in the regression of $\log(v_{t,t+\Delta})$ on a constant and $\log(f_{t,t+\Delta})$ given that the forecasts are unbiased.

These loss measures are statistically compared using the Modified Diebold Mariano (MDM) test (Harvey, Leybourne, and Newbold 1997). Diebold and Mariano (1995) propose a test of equal predictive ability of two competing models based on the null hypothesis of no difference in the accuracy of the two forecasts. The procedure first specifies one loss function $l(e)$ of the forecast error $e$. And then the loss differential between the two competing forecasts can be defined as $d_t = l(e_{i,t}) - l(e_{j,t})$, where $i$ and $j$ are two competing models. Assuming the sequence $\{d_t\}_{t=1}^T$ is covariance stationary and has a short memory, Diebold and Mariano (1995) show that the asymptotic distribution of the sample mean loss differential $\bar{d} = \frac{1}{T} \sum_{t=1}^T d_t$ is $\sqrt{T}(\bar{d} - \mu) \rightarrow N(0, V(\bar{d}))$. The DM statistic is

$$\text{DM} = \frac{\bar{d}}{\sqrt{V(\bar{d})}},$$

where $V(\bar{d})$ is an estimate of the asymptotic variance. Under the null hypothesis of $E(d_t) = 0$, the DM test statistic has a standard normal distribution asymptotically. The modified DM test suggested by Harvey, Leybourne, and Newbold (1997) takes into account over-sized problem met by DM test in small samples and long forecast horizons. Its statistic is generated by multiplying DM statistic by the factor $\sqrt{T^{-1}[T + 1 - 2m + T^{-1}m(m-1)]}$, where $m$ is the forecast horizon. The statistic is compared to critical value from the t-distribution with $T - 1$ degrees of freedom rather than the normal distribution as with the DM test. The MDM test can be applicable to a variety of loss functions and is asserted to be the best available method for determining differences in competing forecasts (Harvey, Leybourne, and Newbold 1997).
Data

Our empirical analysis is based on 2-month and 3-month implied and realized volatilities for corn futures. The intervals for which volatilities are computed are relevant to the structure of expiration for corn futures and options. Corn futures contracts expire five times a year, that is, March, May, July, September, and December. And the corresponding options contracts mature about one-month ahead of futures expiration. The intervals can be settled at the same time each year because corn futures and options contracts are listed repeatedly each year. The fixed five intervals over which volatilities are examined at each year are November to February, February to April, April to June, June to August, August to November.

Implied volatilities are extracted from call options contracts traded from 25 June 2001 to 22 April 2010. These contract prices are obtained from Bloomberg Terminal. We calculate the model-free implied volatility with the discrete version of (10). Striking prices on corn options are integer multiples of five and ten cents per bushel. Since trading volume of corn options with strike price interval of 5 points is too small to permit extraction of sufficient implied volatilities, we discretize the range of integration onto a grid of 10 points. Meanwhile, the call options prices are filtered to exclude options that are: (1) no contract volume; (2) options premium is smaller than its intrinsic value.

Our realized volatilities are calculated based on the summation of the five-minute squared returns on logarithmic corn futures within the corresponding intervals. Intra-day corn futures prices are purchased from an outside financial company. The realized volatilities based on high-frequency data should provide a good approximation to the unobserved integrated volatilities, especially, a better approximation than the one calculated from the sum of the daily squared returns.

Figure 1 plots realized volatility, and risk-neutral implied volatility. The graph shows three features about volatilities in the sample period: (a) both of the volatility measures were
higher during the latter half of the sample, which resulted from the commodities boom associated with bad weather and increasing demand for corn from biofuel production. They have also decreased more recently because the commodities bull is coming to an end; (b) volatilities have strong seasonality. They are higher at corn critical growth stages each year (July to August) than other stages such as harvest or planting. Corn is considered a determinant crop. That is, if the loss is brought about during key growth periods, it cannot be compensated by new growth from the later good weather. Thus crop-growing conditions, such as moisture and temperature in critical growth periods are important factors in affecting corn production. The periods are also characterized with high price volatilities; (c) the two measures have the similar movement over the time. Summary statistics are reported in Table 1. The risk-neutral implied volatility is systematically higher than realized volatility. Meanwhile, their unconditional distributions deviate from the normal.

**Empirical Results**

We separate the data into two parts to do an estimation and an out-of-sample evaluation. The estimation period contains data from 25 June 2001 to 24 Aug 2007. Table 2 reports the GMM estimation results. As can be seen from the table, when we specify a linear risk premium, the estimated $\lambda$ is -3.0 and is statistically significant at 1% level. This finding is consistent with other papers that have found a negative risk premium on stochastic volatility. For example, Doran and Ronn (2008) found the strong statistical significance for the negative volatility risk premium in energy markets even after accounting for the price risk premium and leverage effect. Volatility risk premium estimates are -1.1, -0.5 and -0.6 for natural gas, heating oil, and crude oil by using the sample from Jan 1994 through Apr 2004. Other parameters, such as the long-term mean volatility $\theta$ is positive and significant at the 1% level, while the adjustment speed $k$ is not significant even if it is positive. Finally, the Sargan test shows that
the overidentifying restrictions cannot be rejected at the traditional level and thus validates the choice of instrumental variables.

Alternative forecasts compared with our adjusted implied volatility include a naïve forecast and a moving average forecast. The naïve forecast is defined as the volatility realized during the interval last year while the moving average forecast is developed by averaging past three-year realized volatilities during the respective intervals. Table 3 reports the out-of-sample evaluation of different volatility forecasts according to the statistical loss functions. Considering all the alternative loss functions examined over these three volatility forecasts, we found that risk-adjusted implied volatility always has the minimum forecast errors compared to other two forecasts, regardless of which loss function is adopted. The ranking order of other two forecasts is complex, depending on specific loss functions. For example, forecast errors for the moving average forecast as measured by the MSEs or MADs are smaller than for the naïve forecast while the results are opposite if other functions are adopted.

Table 4 reports the MDM test when the benchmark is the best forecast (risk-adjusted implied volatility), compared to each of the other models. Also, the comparison is made taking into account all the loss functions. Significant difference can be found for the majority of loss functions between the implied volatility and the naïve forecast. We reject the null of equal forecast accuracy between the implied volatility and the naïve forecast at the 10% level for all loss functions but QLIKE, for which we only reject it at the 15% level. Differences between the implied volatility and the moving-average forecast are not significant for most of loss functions. Under the MSE loss function, the risk-adjusted implied volatility has a p-value of more than 0.15 for DMD test, failing to reject the null hypothesis. Under MAD, and R2LOG Loss functions, both p-values are below 10%, suggesting the significant improvement over the moving-average model.
As the IV from an efficient options market incorporates past information and future expectation - a larger information set, such a forecast should be able to perform at least as well as models based purely on historical data (Becker, Clements and Coleman-Fenn 2009). The results of this study are consistent with this conclusion. We further found when more historical information is incorporated into the time series forecast, the forecast ability will improve.

**Conclusion**

This paper proposes to adjust the volatility risk premium in pricing options on distant futures contracts and investigates whether the adjustment can improve the implied volatility forecasts. The volatility risk premium explains a significant portion of the bias in implied volatility forecasts found in the existing literature, thus reducing the bias in volatility forecasts. In this study we found that the new forecasts of implied volatilities have greater or at least as good predictive power as alternative predicting approaches. The framework developed in this paper can be used to forecast volatilities of futures contracts of longer horizons. It can also be used to derive the volatilities of non-corn agricultural commodities futures.
Reference


Figure 1: Realized Volatility and Risk-Neutral Implied Volatility
### Table 1: Summary Statistics for all-Sample Volatilities

<table>
<thead>
<tr>
<th>Statistics</th>
<th>Realized Volatility</th>
<th>Implied Volatility</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>0.0038</td>
<td>0.0164</td>
</tr>
<tr>
<td>Std. Dev.</td>
<td>0.0024</td>
<td>0.0080</td>
</tr>
<tr>
<td>Skewness</td>
<td>2.2648</td>
<td>1.1801</td>
</tr>
<tr>
<td>Kurtosis</td>
<td>10.7836</td>
<td>5.0489</td>
</tr>
<tr>
<td>Minimum</td>
<td>0.0009</td>
<td>0.0050</td>
</tr>
<tr>
<td>Maximum</td>
<td>0.0147</td>
<td>0.0451</td>
</tr>
</tbody>
</table>

### Table 2: Estimation of Parameters

<table>
<thead>
<tr>
<th>Parameters</th>
<th>estimator</th>
<th>Standard Error</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$k$</td>
<td>0.5782</td>
<td>0.7837</td>
<td>0.4675</td>
</tr>
<tr>
<td>$\theta$</td>
<td>0.0031</td>
<td>0.0006</td>
<td>0.0001</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>-3.0036</td>
<td>0.5193</td>
<td>0.0001</td>
</tr>
<tr>
<td>$\chi^2$(D.O.F=1)</td>
<td>1.3819</td>
<td></td>
<td>0.3988</td>
</tr>
</tbody>
</table>
### Table 3: Out-of-sample Evaluation of Different Volatility Forecasts

<table>
<thead>
<tr>
<th>Fun.</th>
<th>$MSE_1$</th>
<th>$MSE_2$</th>
<th>$MAD_1$</th>
<th>$MAD_2$</th>
<th>$QLIKE$</th>
<th>$R2LOG$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Implied Vol.</td>
<td>0.0001</td>
<td>0.000001</td>
<td>0.0073</td>
<td>0.0009</td>
<td>-4.2800</td>
<td>0.0920</td>
</tr>
<tr>
<td>Mov. Ave.</td>
<td>0.0003</td>
<td>0.000009</td>
<td>0.0124</td>
<td>0.0018</td>
<td>-4.1929</td>
<td>0.2389</td>
</tr>
<tr>
<td>Naive</td>
<td>0.0003</td>
<td>0.000012</td>
<td>0.0127</td>
<td>0.0020</td>
<td>-4.2106</td>
<td>0.2098</td>
</tr>
</tbody>
</table>

### Table 4: Modified Diebold-Mariano Test

(Benchmark: Risk-Adjusted Implied Volatility Forecast)

<table>
<thead>
<tr>
<th>Fun.</th>
<th>$MSE_1$</th>
<th>$MSE_2$</th>
<th>$MAD_1$</th>
<th>$MAD_2$</th>
<th>$QLIKE$</th>
<th>$R2LOG$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mov. Ave.</td>
<td>-1.5227</td>
<td>-1.2625</td>
<td>-1.8067</td>
<td>-1.5761</td>
<td>-1.6173</td>
<td>-1.8026</td>
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<tr>
<td><em>p-value</em></td>
<td>0.1518</td>
<td>0.2289</td>
<td>0.0940</td>
<td>0.1390</td>
<td>0.1298</td>
<td>0.0947</td>
</tr>
<tr>
<td>Naive</td>
<td>-1.9101</td>
<td>-1.8799</td>
<td>-2.0496</td>
<td>-2.1527</td>
<td>-1.5392</td>
<td>-1.9218</td>
</tr>
<tr>
<td><em>p-value</em></td>
<td>0.0784</td>
<td>0.0827</td>
<td>0.0611</td>
<td>0.0507</td>
<td>0.1477</td>
<td>0.0768</td>
</tr>
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</table>