Option Pricing on Renewable Commodity Markets

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Introduction

OPTION MODELS by Black and Scholes (1973) and Schwartz (1997) assume spot prices follow a geometric Brownian, which implies that spot price volatility increases in proportion to the square root of time. Most agricultural commodity prices seem to exhibit mean reversion, which would greatly reduce price volatility in the long term. If this is true, the Black-Scholes and Schwartz models overestimate the fair value of long-term options. We propose an alternative model where spot prices are allowed to be mean reverting. The proposed model contains Schwartz's model as a special case. Seasonality in spot prices is also introduced, by allowing some parameters to be periodic functions of calendar time.

Examples

Figures 1, 2, and 3 are a graphical representation of the three assumptions that underlie the three models.

Figure 1 illustrates spot prices under the standard Black-Scholes assumption of geometric Brownian motion. The confidence interval for prices increases in proportion to the square root of time.

Figure 2 depicts spot prices under the Schwartz model. The long-term confidence interval for prices behaves like the Black-Scholes model.

Figure 3 shows spot prices under the proposed model. In the long run, the confidence interval is almost parallel to the x-axis.

The upper and lower confidence intervals are directly related to the fair option price. Therefore, intuitively models that incorporate mean reversion will generate lower option prices.

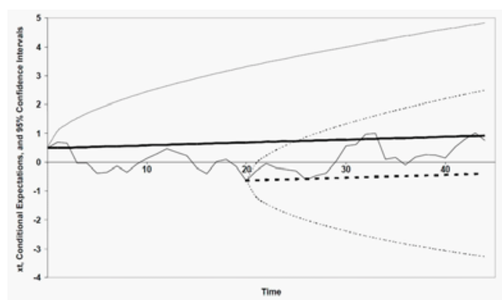

Figure 1. Behavior of xt, conditional expectations, and 95 percent confidence intervals under Brownian motion

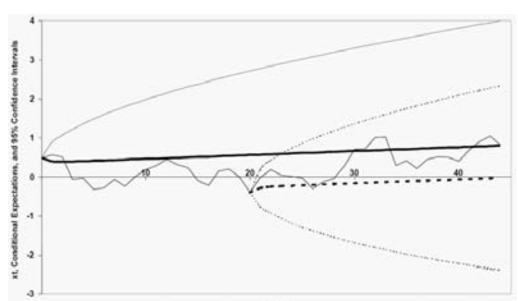

Figure 2. Behavior of xt, conditional expectations, and 95 percent confidence intervals under mean reversion in yt but not in xt

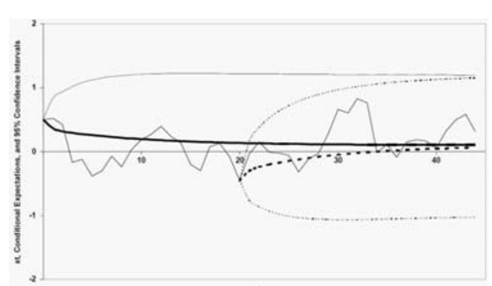

Figure 1. Behavior of xt, conditional expectations, and 95 percent confidence intervals under Brownian motion

Model

WE GENERALIZED the Schwartz two-factor model and added seasonality by using a truncated Fourier series. The first factor is the logarithm of the spot price (x), and is assumed to be a mean-reverting process with seasonality in the drift term

$$dx_t = \frac{du_x(t)}{k_x} + k_x\left[\frac{u_x(t)}{k_x} - x_t\right]dt + \sigma_x dw_x$$

where k_x is the speed of mean-reverting, $u_x(t)$ k_x is the long-run mean, and $u_x(t)$ has the functional form

$$u_x(t) = u_{x,0} + \sum_{h=1}^{H}\left[u_{x,h,cos}Cos\left(\frac{2\pi h}{s}t\right) + u_{x,h,sin}Sin\left(\frac{2\pi h}{s}t\right)\right]$$

The second factor is the convenience yield (c). Since convenience yield is positively correlated with spot prices, it is postulated to satisfy the equation $c_t = y_t + k_x x_t$, where y is a mean reverting process with seasonality,

$$dy_t = \frac{du_y(t)}{k_y} + k_y\left[\frac{u_y(t)}{k_y} - y_t\right]dt + \sigma_y\, dw_y.$$

And the functional form of u_y is like that of u_x.

If $k_x = 0$ and $u_x(t)$ and $u_y(t)$ are constant, the proposed model is identical to Schwartz's model. Analytical futures and option pricing formulas can be obtained by solving ordinary differential equations by the method proposed by Duffie, Pan and Singleton. Seasonality complicates the solution, but we still get closed-form pricing formulas.

Data and Estimation

MODELS WERE fitted using historical soybean futures and option prices from the Chicago Board of Trade. The model was estimated under a Bayesian Markov chain Monte Carlo algorithm. We employ four chains with different initial values, which mix adequately after 2 million iterations.

Empirical Findings

The empirical evidence supports a mean reversion process for spot prices. The proposed model generates lower root mean square errors than Schwartz's model. A Bayesian procedure was then applied to Schwartz's model and the proposed model to predict futures prices with maturity dates up to sixty months. Predictions exhibit clear seasonality patterns.

Figure 5 shows the term structure of at-the-money call option. Within the sample's length to maturity, the differences predicted by the two models are negligible. However, as the length to maturity increases the differences become very obvious, with Schwartz's model predicting consistently higher options premiums.

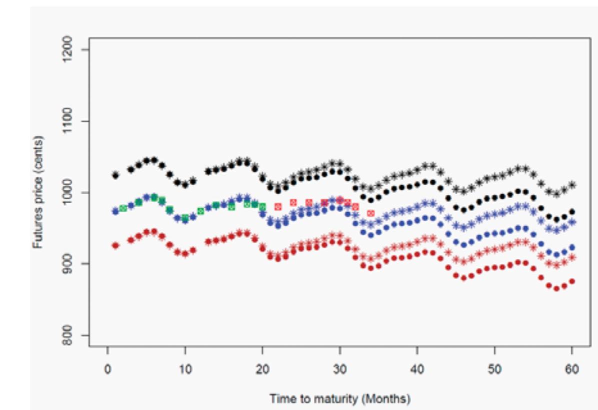

Figure 4: Projection of futures price by Schwartz model and proposed model on Jan/15/2010

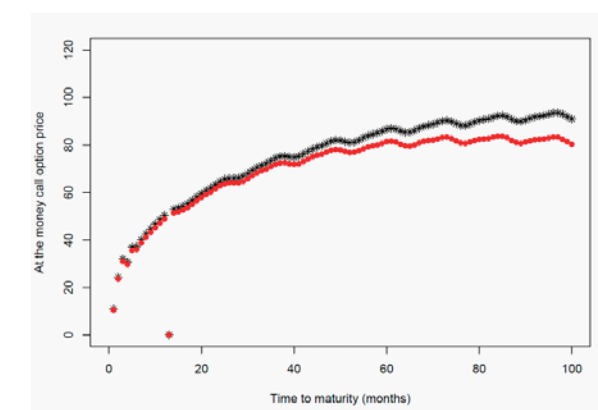

Figure 5: Projection of at the money call option price by Schwartz model and proposed model on Jan/15/2010