Determining Optimal Levels of Nitrogen Fertilizer Using Random Parameter Models

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Abstract
The parameters of yield response functions can vary by year. Past studies usually assume yield functions are nonstochastic or 'limited' stochastic. In this study, we estimate rye-ryegrass yield functions where all parameters are random. Optimal nitrogen recommendations are calculated for two yield response functions: linear response plateau and Spillman-Mitscherlich. Nonstochastic models are rejected in favor of stochastic parameter models. However, the economic benefits of using fully stochastic models are small since optimal nitrogen rates do not differ greatly between stochastic and nonstochastic models.

Key words: cereal rye-ryegrass, linear response plateau, Monte Carlo, nitrogen, random parameters

Introduction

Models predicting crop yield response to nitrogen (N) fertilizers are often used to make fertilizer recommendation rates (Lanzer and Paris 1981; Cerrato and Blackmer 1990; Babcock 1992; Makowski and Wallach 2002; Mooney et al. 2008). Unfortunately, model based nitrogen rate recommendations are vulnerable to misspecification of the yield response function. The objective of this study is to determine expected profit maximizing nitrogen rate recommendations for a winter cereal rye (*S.cereale*)/ryegrass (*Lolium multiflorum Lam*) forage crop based on models that differ in functional form and whether or not model parameters are assumed stochastic.

Previous work on crop response to nitrogen fertilizer has usually used either limiting nutrient response functions or polynomial models. Plateau functional forms tend to best fit data from field studies (Heady and Pesek 1954, Lanzer and Paris 1981, Grimm, Paris, and Williams 1987). Past studies have often assumed that the parameters of the yield function are nonstochastic or 'limited' stochastic (some parameters are considered stochastic and others are not), and that all model errors are independent. This assumption often leads to estimating the parameter values of the assumed yield function by ordinary least squares. Research suggests, however, that parameters of yield response functions can vary by year.

Random parameter models have been suggested by Berck and Helfand (1990), Paris (1992), Makowski and Wallach (2002), and Tembo et al (2008). Berck and Helfand (1990), and Paris (1992) consider linear response plateau models where the intercept and plateau parameters are random, but without random effects. Tembo et al (2008) adds uncorrelated random effects to the intercept and plateau, but not to the slope. Of these studies, only Makowski and Wallach (2002) treat all of the model parameters as random. Makowski and Wallach (2002) consider a linear-plus-plateau function in which wheat yield response is related to N uptake, and nitrogen uptake is related to applied nitrogen.

We consider three crop response functions: the linear response plateau (LRP), the Spillman-Mitscherlich, and the quadratic; and we make all model parameters random. Our random parameter model lets parameters vary stochastically by year. The data used are annual rye-ryegrass forage data collected from a long-term nitrogen fertilization experiment in south-central Oklahoma. We conduct nested likelihood ratio tests to choose between nonstochastic and stochastic models (Greene, 2008), and evaluate the economic value of using the alternative models by comparing expected profit. The ultimate goal is to make optimal nitrogen rate recommendations for cool season cereal rye (*S.cereale*)-ryegrass (*Lolium multiflorum Lam*) forage producers in southern Oklahoma.

Determining the Profit Maximizing Level of Nitrogen Fertilizer

Consider a risk-neutral forage producer whose objective is to maximize expected net returns from winter cereal rye-ryegrass forage. The producer seeks to maximize expected net return above nitrogen cost:

$$(1) \qquad \max_N E(R_t|N) = pE[y_t] - rN$$

$$\text{s.t.} \quad y_t = F(N), N \geq 0,$$

where R_t is the producer's net return at time *t*, y_t is the forage yield, *N* is the level of applied nitrogen, *r* is the price of applied nitrogen fertilizer, and *p* is the price of forage. Yield expectations are obtained through the production function *F*(*N*), which is stochastic due to weather and other factors. We consider the three production functional forms in turn.

Linear Response Plateau

A stochastic linear response plateau function is specified as

$$y_{it} = \min(\beta_0 + (s_t + \beta_1)N_{it},\ \mu_p + v_t) + u_t + \varepsilon_{it}, \tag{2}$$

where y_{it} is the forage yield of cereal rye-ryegrass from the i^{th} plot in year t, N_{it} is the level of nitrogen fertilizer, μ_p is mean plateau yield, s_t is the slope random effect, v_t is the plateau year random effect, u_t is the (intercept) year random effect, and ε_{it} is a random error term that is normally distributed and independent of the three random effects. The intercept random effect is added to the whole equation rather than just to β_0 so that the model of Tembo et al. (2008) is a special case**.** The variance parameters u_t, s_t, and v_t are correlated and normally distributed. Makowski and Wallach (2002) use a model where $(\beta_0, \beta_1, \mu_p) \sim N(\boldsymbol{\beta}, \boldsymbol{\Omega})$. Our model is parameterized differently, but is equivalent to Makowski and Wallach (2002).

The random effect u_t shifts the whole function up or down, which could be due to a variety of weather factors, insects or disease. The slope random effect s_t may be due to nitrogen losses from leaching, soil or weather characteristics, or weed pressure during critical growth periods. The plateau year random effect v_t shifts the yield potential from applying more nitrogen, which mostly varies due to rainfall in a given year. For example, when growing conditions are favorable in a given year, the plateau yield increases as does the amount of nitrogen that the plants can use. When the model is nonstochastic, the random variables v_t and s_t will be zero, but u_t may still be included.

The function is continuous, but its derivatives do not exist with respect to either its parameters or *N* at the knot point where the response and the plateau are joined, but the derivatives of expected yield do exist for the stochastic model. Choosing the level of nitrogen (*N**) that maximizes equation (1) follows the rule from economic theory that marginal factor/input cost (MFC) should equal marginal expected product value (MVP).With a nonstochastic linear response plateau function, equation (2) will exhibit constant positive marginal product when $\mu_p > \beta_0 + \beta_1 N$. If MVP > MFC, then nitrogen should be applied until MVP=MFC. Increasing *N* beyond the level required to reach μ_p will generate negative marginal returns. Therefore, with the nonstochastic LRP, *N** would either be the level required to reach the plateau (N_p) or zero:

$$(3) \qquad N^* = \begin{cases} N_p, & \text{if VMP} > MFC \\ 0\,, & \text{otherwise.} \end{cases}$$

For the stochastic LRP, the random variable u_t in equation (2) enters linearly, and therefore it drops out after taking expectations. Therefore, the expectation of *y* becomes

$$(4) \qquad E(y_t) = E\left[\min\left(\beta_0 + (\beta_1 + s_t)N, \mu_p + v_t\right)\right].$$

Since s_t and v_t are random and correlated, the expectation in (4) requires integrating with respect to s_t and v_t, which defines a double integral that must be solved numerically:

$$(5) \qquad E(y_t) = \iint\left[\min\left(\beta_0 + (\beta_1 + s_t)N, \mu_p + v_t\right)\right]\varphi(s_t, v_t)\partial s_t \partial v_t \,,$$

where $\varphi(s_t, v_t)$ is the multivariate normal probability density function. Tembo et al. (2008) use the approach developed for Tobit models and obtain *N** by evaluating a univariate normal probability density function since they do not allow the slope to be random. Makowski and Wallach (2002) solve the integral using Monte Carlo integration. The integration in (5) can also be solved using other numerical approximation methods such as Gaussian cubature (DeVuyst and Preckel 2007). We use Monte Carlo to solve the double integral. The optimal level of *N* is obtained by direct non-linear optimization (grid search would also work since there is only one choice variable).

Spillman-Mitscherlich

The Spillman-Mitscherlich yield response function is an exponential function (Spillman 1923). A univariate stochastic form of this function is

$$(5)\qquad y_{it} = a - (b + s_t)\exp\left((-c + v_t)N_{it}\right) + u_t + \varepsilon_{it},$$

where a is the maximum or potential yield obtainable by applying nitrogen under the conditions of the experiment; b is the increase in yield due to applied nitrogen; c is the ratio of successive increments in output a to total output *y*; u_t, s_t, and v_t are correlated random effects; and ε_{it} is the independent error term. When the model is nonstochastic, the random variables s_t and v_t are zero, but u_t is still included.

Equation (5) shows that as the application rate of nitrogen increases, the yield increases at a decreasing rate and asymptotically approaches a maximum as the application rate (theoretically) approaches infinity. The function does not strictly adhere to the law of the minimum like in the case of the linear response plateau (allows for convex rather than right-angled isoquants), but unlike the polynomial functions, it exhibits a plateau. The function exhibits sufficient flexibility to accommodate from near perfect substitution to near zero factor substitution if the data and production process so suggest (Frank, Beattie, and Embleton1990).

The optimal level of nitrogen is obtained by substituting (5) into (1) and then solving the optimization problem. For the nonstochastic Mitscherlich yield function, the optimal level of nitrogen (*N**) is obtained by solving the first order condition for *N*, which gives

$$(6)\qquad N^* = \frac{1}{-c}\left[\ln\left(\frac{(r/p)}{cb}\right)\right].$$

For the stochastic Mitscherlich, since the random variables s_t and v_t do not enter linearly in (5), the expectation of *y* is obtained by numerically solving the integral:

$$(7) \qquad E(y_t) = \iint [a + (b + s_t)\exp(-c + v_t)\,N]\varphi(s_t, v_t)\partial s_t \partial v_t.$$

The double integral is solved using Monte Carlo integration. Monte Carlo approximates (7) with a summation, which is then substituted into (1) and the optimal level of nitrogen is then obtained by nonlinear optimization.

Quadratic Response

A random parameter quadratic response model is specified as

$$(8) \qquad y_{it} = \beta_0 + (\beta_1 + v_t)N_{it} + (\beta_2 + s_t)N_{it}^2 + u_t + \varepsilon_{it},$$

where β_0 is the intercept parameter whose position (value) can be shifted up or down from year to year by the year random effect u_t, β_1 is the linear response coefficient with random effect parameter v_t, β_2 is the quadratic parameter whose value can be shifted up or down by the random effect s_t, and ε_{it} is the independent error term assumed to be normally distributed. The random effects v_t, s_t and u_t are correlated and normally distributed. When the model is nonstochastic, the random effects v_t and s_t would be zero, but u_t is still included.

Since (8) is continuously twice differentiable and all the random parameters enter in (8) linearly, (1) gives the same analytical solution for both stochastic and nonstochastic models. Note that for the nonstochastic model, the values of v_t, s_t and u_t are all zero. Hence the problem of calculating N* simplifies to the usual:

$$(9) \qquad N^* = (\beta_1 - r/p)/2\beta_2.$$

Model Fit and Selection Criteria

Likelihood ratio tests are used to choose between stochastic and nonstochastic models (Greene 2008). The calculated likelihood ratio statistics have a chi-square distribution under the null hypothesis. To choose between competing model functional forms, Davidson and Mackinnon

(1981) suggest using formal non-nested tests such as the J-test and P-test. These tests, however, cannot be used here since they can only be used when the nonoverlapping parameters are associated with fixed effects.

The literature on non-nested hypothesis tests provides a variety of criteria to select the model that best fits data based on the information of the true model with respect to the fitted model. When competing non-nested models are fully parameterized and estimated by maximum likelihood, a popular criterion is the adjusted model log-likelihood such as AIC (Akaike, 1974) and BIC (Schwarz 1978). However, these criteria do not take into account whether the differences in the penalized log-likelihoods are statistically significant or not. When observations are independent and identically distributed, a test can be done following Vuong (1989). Pollak and Wales (1991) introduced the Likelihood Dominance Criterion (LDC). The LDC provides rationale to compare two models based on the difference in estimated likelihoods, with adjustments for differences in the number of parameters, and for a given significance level (Pollak and Wales 1991; Grewal, Lilien, and Mallapragada 2006). The criterion involves a fictitious experiment where two competing hypothesis are nested in a composite and the concept of dominance ordering is used to choose among the two. This criterion is the one we use for testing hypothesis to choose between our non-nested models.

Let H_1 and H_2 be two models (hypotheses) with n_1 and n_2 parameters, respectively, and let L_1 and L_2 be the log likelihoods. Let $C(v)$ denote a critical value of the chi-square distribution with v degrees of freedom at significance level α. According to the LDC:

1. Select H_1 if $L_2 - L_1 < [C(n_2 + 1) - C(n_1 + 1)]/2$.

2. Select H_2 if $L_2 - L_1 > [C(n_2 - n_1 + 1) - C(1)]/2$.

3. Otherwise, model selection is indeterminate.

When $n_1 = n_2$ (our case), the indeterminate region reduces to zero and the criterion reduces to a simple comparison of estimated maximum likelihood values (Pollak and Wales 1991).

Data

Forage yield data are cross-sectional times-series from a long-term experiment conducted by the Agricultural Division of The Samuel Roberts Noble Foundation (1997-2008) at Red River demonstration and research station near Burneyville, in south-central Oklahoma. The experiment began in 1979 and was aimed at evaluating the effect of nitrogen fertilization rate and harvest timing on the annual rye-ryegrass forage production system, using a randomized complete block design. Details of the experimental set up are described in Altom et al. (1996) who analyzed the data from 1979 to 1992.

Our dataset covers 14 years from fall 1993 to spring 2007. Six treatment levels of nitrogen (34-0-0) were administered: 0, 100, 150, 200, 300, and 400 pounds per acre per year. Treatments were replicated three times for each level of nitrogen. Split applications were used. Ammonium nitrate was broadcast and incorporated prior to planting in the fall. Spring applications were not incorporated. Fall fertilization was done between September 24 and October 25. Spring fertilization was between February 20 and March 17. Phosphorous was banded with the seed at a rate of 50lbs P_2O_5/acre every year, Potassium was broadcast and incorporated prior to planting at an average rate of 100 lbs K_2O/acre. Lime was applied to the plots used in the study.

Forage yields were determined by clipping individual plots that were 12 by 13 ft. Plots were clipped multiple times to simulate grazing. Yearly dry matter forage yields were the sum of all clippings for that year. Average annual rye-ryegrass yield response to nitrogen fertilization is shown in figure 1.

Estimation

The models are estimated using NLMIXED procedure in SAS (SAS Institute Inc. 2003). The dependent variable is yield, and the independent variable is nitrogen. For the quadratic, nonstochastic LRP and nonstochastic Mitscherlich models, the error term and random effects enter the equation linearly. In the stochastic LRP and the stochastic Mitscherlich models, the two non-intercept random effects enter the equations nonlinearly. The random effects are estimated as free correlated parameters, but the error term is independent.

The NLMIXED procedure fits nonlinear mixed models by maximizing an approximation to the likelihood integrated over the random effects (SAS Institute Inc. 2003). As is common in nonlinear optimization, convergence can be difficult and computing the objective function and its derivatives can lead to arithmetic overflows (SAS Institute Inc. 2003). The models have no closed form and can only be approximated numerically. To achieve convergence, three efforts are employed: scaling, varying starting points, and using different optimization techniques available in SAS.

Pinheiro and Bates (1995) provide evidence that of the several different integrated likelihood approximations methods, adaptive Gaussian quadrature is one of the best. We use adaptive Gaussian quadrature to approximate the likelihood function integrals and maximize the function by the dual quasi-Newton optimization algorithm. Other optimization techniques that enabled convergence are the Newton-Raphson method with ridging and the Trust-Region Method (SAS Institute Inc. 2003). The quadratic and nonstochastic Mitscherlich models converge with less need of scaling and changing starting point values. Estimates obtained are then used to determine the optimal level of nitrogen.

For the stochastic LRP and stochastic Mitscherlich, the estimated parameters are used in Monte Carlo integration. The random vector $[\mathbf{s_t}, \mathbf{v_t}] \sim \mathbf{N}(\mathbf{0}, \mathbf{\Omega})$. We use the Cholesky decomposition, $\mathbf{\Omega} = \mathbf{P'P}$, where $\mathbf{P}$ is a lower triangular matrix. Let $\mathbf{Z}$ be a 2x1 vector of independent draws, then $\mathbf{PZ} \sim \mathbf{N}(\mathbf{0}, \mathbf{\Omega})$. With sufficient draws, the sample average of the function being integrated provides an approximation to the integral (Greene 2008). We use 10,000 draws for our approximation. To obtain the optimal level of N, we use the SAS PROC NLP procedure and maximize our objective function (1) using Newton-Raphson with ridging.

Results

Estimated parameters are reported for the quadratic model in table 1, linear response plateau in table 2, and Mitscherlich in table 3. For all models, the mean parameters and variance estimates are statistically significant at the 5% level based on Wald t-tests. Covariance parameters of the stochastic quadratic model are not statistically significant at the 5% level. Covariance parameters of the stochastic Mitscherlich and the covariance between the plateau and the slope in the stochastic LRP are statistically significant. The likelihood ratio (LR) statistic for the stochastic quadratic versus the nonstochastic quadratic model is 170; the LR for the stochastic linear response plateau versus the nonstochastic linear response plateau is 269.4; and the LR for the stochastic Mitscherlich versus the nonstochastic Mitscherlich is 262.8. All the LR statistics are greater than the critical chi-square (X_5^2) value[1] at any conventional significance level. Stochastic models fit our data better than the alternative non-stochastic models.

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[1] Note that there is a potential nuisance parameter problem with this hypothesis test since imposing that the two variances are zero also imposes that the three covariances are zero. We do not explore this issue since all null hypotheses are rejected even using the more conservative critical value.

Based on the LDC (Pollak and Wales, 1991), we choose the model functional form that fits our data best. The estimated maximum likelihood value for the stochastic LRP is 2295.1. The likelihood value for the stochastic quadratic is 2348.6, and for the Mitscherlich it is 2300.0. Both models have the same number of parameters (n=9). Hypothesis testing on model functional form according to the LDC ranking favors the stochastic LRP over the stochastic Mitscherlich and the stochastic Mitscherlich over the stochastic quadratic model. From the illustration in figure 1, a quadratic model may be considered a poor choice for this dataset on the basis that it assumes symmetry. It indicates that yield decreases past the peak at the same rate it increases before the peak. We base our optimal N rate recommendations on the LRP and the Mitscherlich models.

Profit maximizing level of nitrogen is evaluated at 2009 input and output prices. Although nitrogen 34-0-0 ammonium nitrate was used in the experiment, The Samuel Roberts Noble Foundation Agricultural Division currently recommends using 46-0-0 urea. The prices of 34-0-0 and 46-0-0 are $.51/lb of N and $.41/lb of N, respectively. We do a sensitivity analysis by determining nitrogen rate recommendations as input prices vary. The per pound price of forage is determined as the cost of beef gain per pound divided by the pounds of forage required by a stocker animal to produce a one-pound gain. Based on the National Research Council (1984) net energy equations used to estimate livestock requirements, Ishrat , Epplin, and Krenzer (2003) and Krenzer et al (1996), show that one pound of beef gain requires 10 lbs (dry matter) of standing forage. Within the south-central Great Plains, the cost per pound of gain has ranged from $0.32/lb since 2005 to $0.55/lb in 2008. Currently, due to decreased prices of corn and fertilizer, this cost has declined to $.45/lb, (which is approximately the mean across the period). Therefore, at the cost of beef gain per pound of $0.45, the price per pound of forage is $0.45/10=$0.045. Our optimal nitrogen rate recommendations are based on nitrogen prices of $0.41/lb and forage sale prices of $0.045/lb.

The estimated optimal nitrogen rates and their standard errors for the models are included in the respective tables of results. At current prices, the profit maximizing level of nitrogen obtained with the nonstochastic linear response plateau model is 182.3 lbs/acre, the level of nitrogen required to reach the plateau. Applied nitrogen increases yield at a rate of 13.8 lbs/acre until the plateau yield level of 8235.7 lbs/acre. At $0.045 sale price of forage, the marginal value product of nitrogen is $ 0.62 per pound, which is greater than the $ 0.41/lb price of nitrogen. The 95% confidence interval of the optimal level of nitrogen obtained the nonstochastic LRP is 209.4 lbs/acre to 154.6 lbs/acre. Maximum profits for the stochastic linear response plateau are achieved with nitrogen fertilization of 143.6 lbs/acre. The 95% confidence interval of this estimate is to apply 115.5 lbs/acre to 171.8lbs/acre of nitrogen. The expected profit function of the nonstochastic LRP is higher than that of the stochastic LRP (Figure 2a). The loss from using the nonstochastic LRP to predict optimal nitrogen levels when the stochastic LRP is the true model is approximately $9.0 per acre. This loss is small because the expected profit function of the stochastic LRP is relatively flat at current input and output prices.

Profit maximizing level of nitrogen obtained with a non-stochastic Mitscherlich is 113.5 lbs/acre. The 95% confidence interval of this estimate is 95.4 lbs/acre to 130.4 lbs/acre of nitrogen. The optimal level of nitrogen obtained with a stochastic Mitscherlich model is 107.4 lbs/acre. The 95% confidence interval for the optimal level of nitrogen obtained with the stochastic Mitscherliuch is 103 lbs/acre to 110.6 lbs/acre. The expected profit function of the non-stochastic Mitscherlich model is higher than that of the stochastic Mitscherlich (Figure 2b). The loss from using the non-stochastic Mitscherlich model to predict the optimal level of nitrogen when the stochastic Mitscherlich is the true model is approximately $1.0 per acre. The economic benefits of using fully stochastic models are small since optimal nitrogen rates do not differ greatly between stochastic and nonstochastic models. Profit maximizing level of nitrogen obtained with a nonstochastic quadratic model is 144.3 lbs/acre, and the optimal level of nitrogen obtained with a stochastic quadratic model is 171.4 lbs/acre.

We notice from figure 3 that fertilizer recommendations for the stochastic linear response plateau and the stochastic Mitscherlich can be less or more than fertilization rates recommended with the alternative nonstochastic model, depending on price ratios of the input and the output. The stochastic quadratic model consistently estimates higher optimal levels of nitrogen than the alternative nonstochastic model.

Summary and Conclusions

Models predicting crop yield response to nitrogen fertilizer are often used to recommend optimal fertilizer rates. Past studies usually assume the parameters of the yield function are nonstochastic or 'limited' stochastic, and that all model errors are independent. Given that research suggests that the parameters of the yield functions vary by year, estimating a random parameter model could give a more realistic model of producers' profit expectations. In this study, we consider yield functions where all parameters are random. The approach was applied to cereal rye/ryegrass forage data collected from a long-term nitrogen fertilization experiment in south-central Oklahoma to determine and compare the profitability of nitrogen estimated from stochastic models and the alternative nonstochastic models. The model functional forms considered are the linear response plateau, the quadratic, and the Spillman-Mitscherlich.

Constant parameter models are rejected in favor of random parameter models. The quadratic model fits the data poorly. At current prices, a nonstochastic LRP gives an optimal level of nitrogen that is 38.7 lbs/acre higher than the stochastic LRP. The loss from using a nonstochastic LRP instead of a stochastic LRP to predict optimal nitrogen level when the stochastic LRP is the true model is only $9.0 per acre. At the optimum, a non-stochastic Mitscherlich model gives an optimal level of nitrogen that is 6.1 lbs/acre of nitrogen higher than the stochastic Mitscherlich model. The loss from using a nonstochastic Mitscherlich model to estimate the optimal N rate when the stochastic Mitscherlich is the true model is only about $1.0 per acre. The finding by Makowski and Wallach (2002) that it pays to use a random parameter model to calculate nitrogen rates is supported but the loss from not using random parameters models to determine the optimal level of nitrogen is very small. The observation by Cerrato and Blackmer (1990) and other researchers that the quadratic model estimates a higher optimal nitrogen rate than a linear response plateau is supported for stochastic models but not for nonstochastic models.

Current recommendations of fertilizing annual cool season cereal rye-ryegrass pastures from the Noble Foundation are to apply 100 to 200 lbs/acre. Our estimated optimal rates are within this range. Based on the estimates from the stochastic LRP, the 95% confidence interval level is to apply between 115.5 lbs/acre to 171.8lbs/acre annually. Based on the estimates from the stochastic Mitscherlich, however, the 95% confidence interval for recommendations is between 103 lbs/acre to 110.6 lbs/acre annually.

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Table 1. Rye-Ryegrass Yield (1000lbs/acre) Response to Nitrogen (100lbs/acre) Using the Nonstochastic and Stochastic Quadratic Models

Parameter	Stochastic Quadratic		Nonstochastic Quadratic	
	Estimate	SE	Estimate	SE
Intercept (β_0)	5.74	0.54	5.77	1.15
Slope (β_1)	1.74	0.44	1.64	0.18
Quadratic term (β_2)	-0.24	0.10	-0.25	0.04
Variance of intercept random effect (σ_u^2)	13.46	3.29	19.32	7.08
Variance of error term (σ_ε^2)	1.89	0.11	2.43	0.14
Variance of slope random effect (σ_v^2)	1.93	0.35		
Variance of quadratic term random effect (σ_s^2)	0.47	0.20		
Covariance (σ_u^2, σ_s^2)	1.62	1.51		
Covariance (σ_s^2, σ_v^2)	-0.004	0.38		
Covariance (σ_u^2, σ_v^2)	-0.03	0.06		
Optimal level of N (100lbs/acre)	1.71	0.12	1.44	0.15
-2 Log Likelihood	2348.6		2433.6	

Table 2. Rye-Ryegrass Yield (1000lbs/acre) Response to Nitrogen Using the Nonstochastic and Stochastic Linear Response Plateau Models

Parameter	Stochastic Linear Plateau		Nonstochastic Linear Plateau	
	Estimate	SE	Estimate	SE
Intercept (β_0)	5.67	0.29	5.72	1.15
Slope (β_1)	1.62	0.31	1.38	0.17
Yield plateau (μ_p)	8.01	0.12	8.23	1.14
Intercept random effect (σ_u^2)	13.96	1.53	19.32	7.08
Variance of error term (σ_ε^2)	1.85	0.11	2.42	0.14
Plateau random effect (σ_v^2)	3.65	0.33		
Variance of slope random effect (σ_s^2)	0.89	0.16		
Covariance (σ_u^2, σ_s^2)	-1.41	0.74		
Covariance (σ_u^2, σ_v^2)	0.89	0.82		
Covariance (σ_s^2, σ_v^2)	1.54	0.18		
Optimal level of N (100lbs/acre)	1.44	0.14[a]	1.82	0.14[a]
-2 Log Likelihood	2295.10		2429.80	

[a] The standard error of N* for the stochastic LRP is obtained by Monte Carlo methods, while the standard error of N* for the nonstochastic LRP is obtained using the delta rule.

Table 3. Rye-Ryegrass Yield (1000lbs/acre) Response to Nitrogen Using the Nonstochastic and Stochastic Spillman-Mitscherlich Models

Parameter	Stochastic Mitscherlich		Nonstochastic Mitscherlich	
	Estimate	SE	Estimate	SE
Maximum (potential) yield (a)	7.91	0.12	8.47	1.15
Response due to nitrogen (b)	3.28	0.38	2.81	0.23
Ratio of successive increments (c)	1.31	0.26	0.89	0.16
Variance of error term (ε_t)	1.85	0.11	2.42	0.14
Intercept random effect (σ_u^2)	19.44	1.10	19.35	7.09
Variance of slope random effect (σ_s^2)	5.89	1.45		
Plateau random effect (σ_v^2)	0.37	0.15		
Covariance (σ_u^2, σ_s^2)	8.36	1.16		
Covariance (σ_s^2, σ_v^2)	1.67	0.36		
Covariance (σ_u^2, σ_v^2)	0.80	0.19		
Optimal level of N (100lbs/acre)	1.07	0.02[b]	1.13	0.09[b]
-2 Log Likelihood	2300.0		2431.4	

[b] The standard error of N* for the stochastic Mitscherlich is obtained by Monte Carlo methods, while the standard error of N* for the nonstochastic Mitscherlich is obtained using the delta rule.

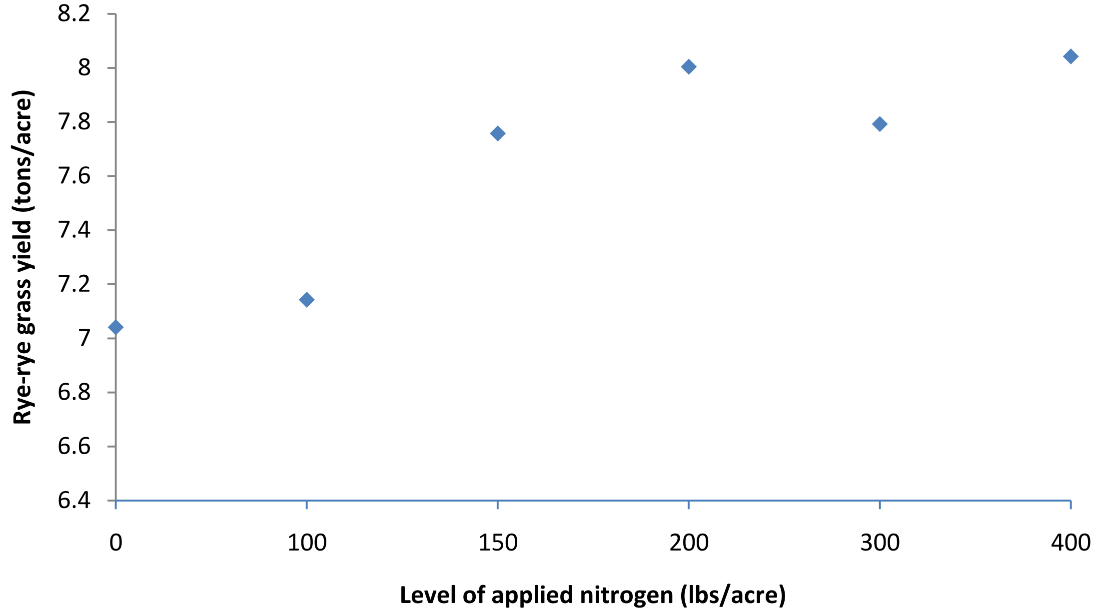
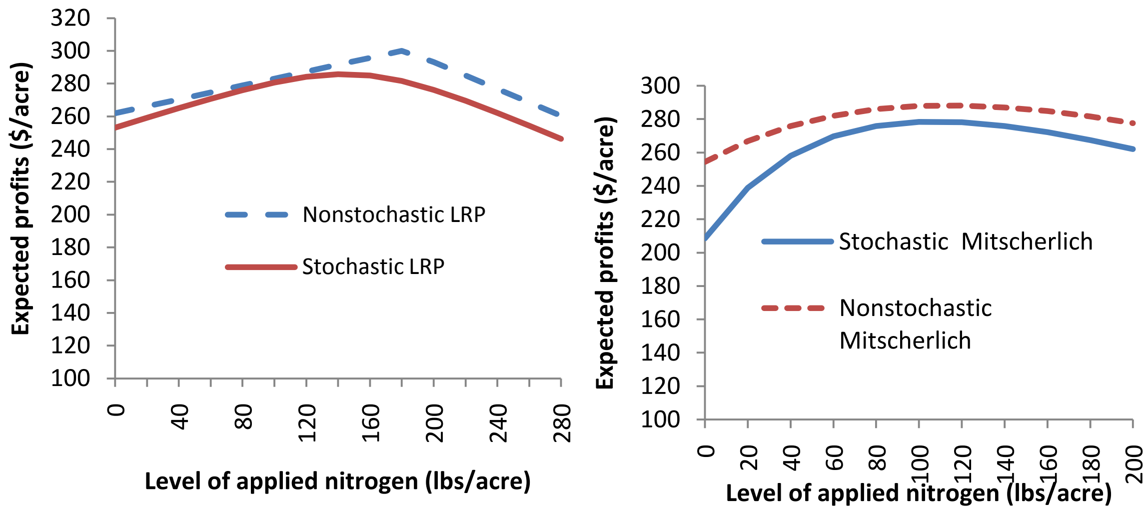


Figure 1. Ryegrass yield response to applied nitrogen



(a) **(b)**

Figure 2. Expected profit functions (Price of ryegrass =$.0450/lb, price of nitrogen=$.41/lb)

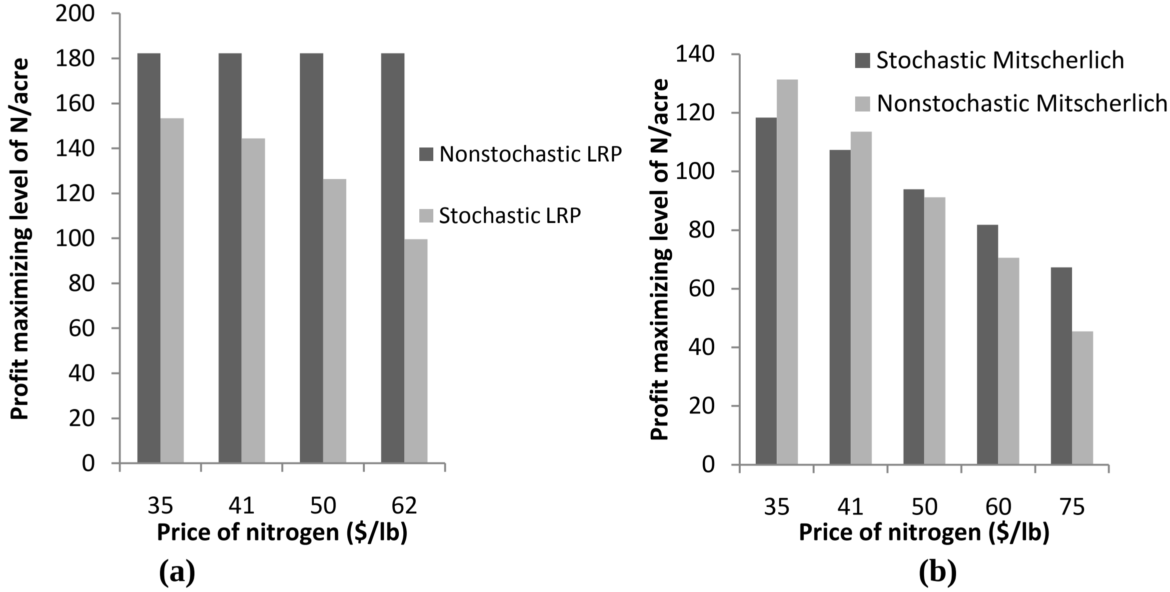


Figure 3. Optimal level of N at varying prices for the LRP models and quadratic models (price of ryegrass is constant at $ 0.045)