Agricultural Arbitrage, Adjustment Costs, and the Intensive Margin

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Selected Paper prepared for presentation at the Southern Agricultural Economics Association Annual Meeting, Orlando, FL, February 6-9, 2010

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Abstract: Farmland and capital are an important and rapidly expanding component of the agricultural economy, and empirical evidence suggests that these assets are quasi-fixed in that adjustment costs are incurred when holdings are altered. Increased interest in the rate of return for investing in farmland suggests that an important consideration is the effect of adjustment costs on this return. A novel theoretical model is developed that ties together contributions from the farmland pricing and adjustment cost literatures, and the first order conditions for a utility maximizing decision maker are rearranged into intertemporal arbitrage equations that are similar in spirit to traditional finance models. The common assumptions that land and capital are quasi-fixed assets, and that production is characterized by constant returns to scale are tested and the evidence supports these assumptions. An empirical application of the arbitrage equations provides evidence that risk aversion and adjustment costs are jointly significant components of agricultural production, and that adjustment costs generate significant changes in the rate of return to farmland. The results have important policy implications as sluggish supply response due to quasi-fixity can lead to dramitically inflated commidity prices, and an accurate measure of the farmland return can help determine how far the extensive margin will expand or contract in response to a variety of policy scenarios, such as the subsidization of corn for ethanol, an increase in the variety of subsidized crop insurance products, or the introduction of new revenue support programs such as ACRE.

1 Introduction

1.1 Motivation

The United States Department of Agriculture's Economic Research Service (ERS) reports that near-record harvests in 2008, coupled with improved farm product demand and high commodity prices, should result in increased spending on farm real estate and capital investment. Since 2000, holdings of these assets at the national level has more than doubled from $1.03 trillion to $2.16 trillion, and currently represent 92% of all farm assets. These figures indicate that farmland and capital are an important and rapidly expanding component of the agricultural economy.

Empirical evidence suggests that farmland and capital are quasi-fixed in that adjustment costs are incurred when holdings are altered. A paper by de Fontnouvelle and Lence (2002) states that transaction costs for buying/selling farmland are as high as 15% of the land price to cover brokerage fees, legal fees, appraisals, and surveys.[1] Research analyzing the quasi-fixity of capital is common in the agricultural production literature (e.g. Vasavada and Chambers, 1986; Oude Lansink and Stefanou, 1997; and Gardebroek and Oude Lansink, 2004), where examples of adjustment costs include lending fees, learning costs, building and environmental licensing fees, and the value of time spent preparing investments. As a fraction of investment expenditures for a sample of Dutch pig farms, Gardebroek and Oude Lansink (2004) find that adjustment costs for buildings and machinery are as high as 1.6% and 5.1%, respectively.

There is considerable interest in the rate of return for investing in an acre of farmland. From 1960-1999, sample data suggests that this return has averaged 6.2% in the United States. Compared to the 2.9% average return on a relatively riskless security over the same period, this implies an equity premium just over 3%. While not as large as the premium for investing in the S&P index, it has attracted attention from investors outside of the agricultural sector. A recent New York Times article titled "Food is Gold, so Billions Invested in Farming," reports that huge investment funds have poured hundreds of billions of dollars into commodity markets and big private investors are buying farmland. However, an important question that has not been addressed is how the presence of adjustment costs might affect the farmland rate of return.

Answering this question is not as simple as with other investment opportunities, where the rate of return is determined exogenously (i.e. stocks, bonds, etc.). Investment in agriculture is composed of many simultaneous decisions such as how much land to hold, how much capital and variable inputs to use, and what farm products to produce. Additionally, farmers make these decision with an eye toward smooth wealth accumulation over time (Jensen and Pope, 2004), and the costs and benefits of farming likely include

[1] The authors note that in the absence of a broker, land adjustments still carry implicit costs for services that include search, advertising, showing of property, and provision of land market information.

many latent measures that are not observable to the researcher. These include the marginal effects of land, capital, and outputs on total variable inputs (intensive margin effects), and adjustment costs associated with quasi-fixed inputs.

To address these issues, a theoretical model is developed that incorporates life-cycle household consumption, agricultural production, financial economics, and adjustment costs for quasi-fixed inputs in one coherent framework. This model ties together contributions from the farmland pricing and adjustment cost literatures, and the first order conditions for a utility maximizing decision maker are rearranged into intertemporal arbitrage equations that are similar in spirit to traditional finance models.

The derivation of the theoretical arbitrage model relies on the assumptions that land and capital are quasi-fixed assets, and that production is characterized by constant returns to scale. These assumptions are empirically tested using parameter estimates from a system of variable input demands that is derived from a flexible specification for variable costs of production. This specification utilizes a new and innovative approach for overcoming aggregation and unobservable variable issues in econometric models of production, and was recently developed in LaFrance and Pope (2008a, 2008b).

An empirical application of the arbitrage system utilizes a flexible specification for variable costs of production and an explicit representation of adjustment costs to estimate the parameters of the model. These estimates are then used to address the following questions: Are risk aversion and adjustment costs jointly significant components of agricultural production? If adjustment costs are significant, how is the rate of return to farmland affected?

The main contributions of this paper are: the development of a novel theoretical model that synthesizes approaches from the farmland pricing and adjustment cost literatures; empirical evidence from the demand system estimation supporting the assumptions of constant returns to scale and quasi-fixity of land and capital; evidence from the arbitrage system estimation suggesting that risk aversion and adjustment costs are jointly significant components of agricultural production, and that adjustment costs generate significant changes in the rate of return to farmland. The findings for the joint significance of risk aversion and adjustment costs represent a novel contribution, as this joint hypothesis has never been tested in the literature. The adjusted rate of return findings are also novel, as the effects of adjustment costs on farmland returns have not been previously calculated.

These findings have important implications for the agricultural economy and associated policy instruments. The presence of non-zero adjustment costs for land and capital imply slower adjustments (i.e. not instantaneous) to their the optimal levels. Since both assets are necessary components of output production, slower adjustments will lead to lagged supply response to agricultural policy. This can result in dramatically inflated commodity prices as recently seen in the world economy. Understanding the effect that adjustment

costs have on the farmland rate of return has considerable policy relevance as well. This rate of return determines how far the extensive margin will expand or contract in response to a variety of policy scenarios, such as the subsidization of corn for ethanol, an increase in the variety of subsidized crop insurance products, or the introduction of new revenue support programs such as ACRE. It can also be used to calibrate payments for a variety of existing government programs, such as the conservation reserve program.[2]

Mishra et. al. (2004) note that the issue of farmland valuation for agricultural purposes is a perennial topic of interest for agricultural policy makers and farmers. Indeed, a recent document from the ERS (ERS, 2008) reports that the Food, Conservation, and Energy Act of 2008 will govern the bulk of Federal agriculture and related programs for the next five years, and includes programs that cover income and commodity price support, farm credit, and risk management. The effectiveness of these programs crucially depends on an accurate understanding of farmland valuation, and the empirical results presented in this paper support previous findings that farmland valuation is driven by several factors including risk aversion, adjustment costs, and the opportunity cost of on-farm investment (see Just and Miranowski, 1993; Lence and Miller, 1999; and references therein).

1.2 Literature Review

Starting with Just and Miranowski (1993) and followed by Lence and Miller (1999) and Chavas and Thomas (1999), the farmland pricing literature was extended to include adjustment costs (referred to as transaction costs in this literature) as an additional explanatory factor for the notorious boom/bust cycles of land prices. In these papers, farmland is the only quasi-fixed input and the adjustment costs associated with the buying/selling of farmland are modelled as a linear function of the land price or the change in acreage. Importantly, these models were not extended to the case of multiple quasi-fixed inputs and nonlinear adjustment costs, which are common assumptions in the adjustment cost literature. The model developed in this paper is similar to the above farmland pricing models, but includes multiple quasi-fixed inputs and a nonlinear, everywhere continuous structural specification for adjustment costs.[3]

The adjustment cost literature is reviewed in Gardebroek and Oude Lansink (2004). In the 1950's two theories explaining adjustment of quasi-fixed factors were developed, Cochrane's (1955) treadmill theory and Johnson's fixed asset theory (1956). Recently, Chavas (1994) reformulated the fixed asset model into a formal model starting from the farmers' long-run objective function. Beginning in the 1980's, many agricultural

[2]It should be noted that the revenue data used in this analysis to construct the farmland rate of return, discussed more fully in Section 3.2, includes the value of government payments. This approach is empirically beneficial as it, at least implicitly, incorporates the impacts of farm policy on the distribution of net returns in asset management decisions.

[3]Assuming that the nonlinear adjustment costs are everywhere continuous avoids derivation of unwieldy first order conditions pointed out in Lence's (2001) critique of the modelling approaches of Just and Miranowski (1993) and Chavas and Thomas (1999).

economic studies have applied the adjustment cost framework to explain adjustment of quasi-fixed inputs (e.g. Vasavada and Chambers, 1986; Howard and Shumway, 1988; Stefanou et. al., 1992; Thijssen, 1996; and Gardebroek and Oude Lansink, 2004).

It is worth distinguishing between two common approaches for modeling quasi-fixed input adjustments in the literature. Mundlak (2001) distinguishes between what he calls the primal and dual approaches to endogenous dynamics. The primal approach specifies production as a function of the level of the quasi-fixed input (among other arguments) and explicitly represents adjustment costs as a *separate* function of the change in the level of the quasi-fixed input (i.e. the investment). The dual approach specifies production as a function of the level of the quasi-fixed input and implicitly represents adjustment costs by including investment as an additional argument in the *same* function (see McLaren and Cooper, 1980; Epstein, 1981; and Epstein and Denny, 1983). Since one of the objectives of this paper is to understand how adjustment costs affect the rate of return to farmland, the primal approach is preferred because its provides a generalized representation of the adjusted return that nests the unadjusted rate.

A popular specification of the adjustment cost function is the symmetric quadratic, implying strictly convex adjustment costs over the whole range of adjustment. If the adjustment cost function is convex, then the increasing marginal investment costs provide a rationale for investment spreading over time. A number of studies have employed more general specifications to allow for asymmetry, including Chang and Stefanou (1988), Pfann and Verspagen (1989), and Pfann and Palm (1993). In addition, theoretical and empirical models have been developed that combine asset fixity theory and adjustment cost theory. Building on the work of Rothschild (1971), Hsu and Chang (1990) show that an adjustment cost function that is non differentiable at the origin causes thresholds for investment and disinvestment. Abel and Eberly (1994) present a unified model of investment that includes a difference between the purchase and selling prices of capital, asymmetric adjustment costs, and a fixed cost of adjustment. Empirical applications based on these studies include Oude Lansink and Stefanou (1997) and Pietola and Myers (2000).

An important shortcoming of the models presented in the adjustment cost literature is the omission of risk averse decision makers whose objective is to smooth wealth/consumption over time. This is an important consideration for modeling adjustment costs in the U.S. agricultural sector for two reasons. First, a recent ERS report on the structure and finances of U.S. farms, Hoppe et. al. (2007), finds that 98% of U.S. farms remained family farms as of 2004, and that these family farms account for 85% of the total value of production and own 93% of all farm assets. In a family farm, investment, consumption, and production decisions are made with an eye toward smooth wealth accumulation over time. This assumption is supported by Jensen and Pope (2004), who find that Kansas farm households facing greater uncertainty in income maintain larger stocks of wealth in order to smooth consumption. Second, empirical evidence presented in Whited (1998)

and Gardebroek and Oude Lansink (2004) suggests that inclusion of financial variables can help explain capital investments over time in an adjustment cost framework.

The theoretical model presented in this paper synthesizes common approaches in the farmland pricing and adjustment cost literatures. While farmland pricing models account for risk averse decision makers whose objective is to smooth wealth/consumption over time, the inclusion of multiple quasi-fixed inputs and nonlinear adjustment cost specifications are omitted. Conversely, adjustment cost models account for multiple quasi-fixed inputs and nonlinear adjustment costs, but pull up short of including risk averse decision makers that smooth wealth/consumption over time.

It is worth noting that the system of arbitrage equations derived in the next section are similar in spirit to traditional finance models, especially Merton's (1973) intertemporal capital asset pricing model (ICAPM). A review of theoretical and empirical asset pricing models in the finance literature is provided in Cochrane (2005) and Singleton (2006), and specifically for the ICAPM in Brennan et. al. (2005).

Since the ICAPM provides a convenient framework for synthesizing the farmland pricing and adjustment cost models, it is briefly presented here. In general, the ICAPM states that the optimal holding of a risky asset in period t (measured in dollars) is defined by

$$E_t \left[m_{t+1} \left(r_{F,t+1} - r \right) \right] = 0, \tag{1}$$

where $E_t(\cdot)$ is the conditional expectation at the beginning of period t given available information, m_{t+1} is the stochastic intertemporal marginal rate of substitution of wealth between periods t and $t+1$, $r_{F,t+1}$ is the stochastic rate of return realized in period $t+1$ for a dollar invested in the risky asset in period t, r is the time-constant, known rate of return for investing in an alternative risk-free asset (typically assumed to be some form of government bond or T-bill), and $r_{F,t+1} - r$ is the excess return of the risky asset over the alternative return. The intertemporal marginal rate of substitution m_{t+1} is stochastic because it depends on wealth in period $t+1$, which is not known with certainty at time t when the investment decision is made. The excess return is also stochastic, and its covariance with m_{t+1} generates a risk correction for the risky asset. To see this, rewrite the above expression as

$$E_t \left(r_{F,t+1} - r \right) = -\frac{COV \left(m_{t+1}, r_{F,t+1} - r \right)}{E_t \left(m_{t+1} \right)}, \tag{2}$$

which implies that a negative covariance generates a positive excess return to reward the reduction in wealth variance. A significant strength of the ICAPM is its generality, as the arbitrage equation (2) holds *for any risky asset.*

The remainder of this paper is organized as follows. Section 2 develops the theoretical model of agricultural arbitrage that synthesizes the farmland pricing and adjustment cost models, and an estimable system of arbitrage equations for on- and off-farm investments is derived from the model's first order conditions. Secion 3 describes the data that is used in the estimation of the variable input demand system (Section 4) and the estimation of the arbitrage system (Section 5). Section 6 concludes.

2 Theoretical Model of Agricultural Arbitrage

The model developed in this section incorporates life-cycle household consumption, agricultural production, financial economics, and adjustment costs for quasi-fixed inputs in one coherent framework. The agent's decision process is representative of a family farm, in which the agent controls the means of production and makes investment decisions to generate wealth used for consumption good purchases. The first order conditions from the model are used to construct arbitrage equations for on- and off-farm investments similar to the ICAPM framework.

2.1 Set Up

A list of the variable definitions used in the model is provided in Table 1. All financial returns, farm asset gains/losses, and revenues from the production of farm outputs are assumed to be realized at the end of each period. Variable inputs are assumed to be committed to farm production activities at the beginning of each decision period and the current period market prices for the variable inputs are known when these decisions are made.

The farm's production technology is represented by a Pope and Just (1996) *ex ante* variable cost function

$$C_t\left(\check{\mathbf{w}}_t, A_t, K_t, \bar{\mathbf{Y}}_t\right), \tag{3}$$

which is a function of an n_V-vector of variable input prices $\check{\mathbf{w}}_t$ (the non-standard notation is used to simplify the specification for the cost function in Section 4), total farmland A_t, total value of capital measured in dollars K_t, and an n_Y-vector of expected outputs $\bar{\mathbf{Y}}_t$. Moschini (2001) notes that the standard cost function (where *ex post* output $\mathbf{Y}_t$ replaces *ex ante* expected output $\bar{\mathbf{Y}}_t$) is not relevant when farmers make input decisions before realizing production shocks, as is the case here. Including A_t and K_t as arguments of $C_t(\cdot)$ implies that land and capital are treated as fixed when variable input decisions are made. Thus, the static minimization of variable costs given input prices, land, capital, and expected output is subsumed in the model.

For this analysis it is useful to work with a per acre version of the cost function which is derived from (3) by imposing linear homogeneity in land, capital, and expected output (i.e. imposing constant returns to scale). This approach is supported by Chavas (2001), which reports there is no strong evidence that diseconomies of scale exist for large farms, and there is a fairly wide range of farm sizes where average cost is approximately constant (e.g. Kislev and Peterson, 1996). Defining the $N \times N$ diagonal matrix $\mathbf{\Delta}(x_i)$ such that x_i is the ith main diagonal element for each $i = 1, ..., N$, the per acre cost function is defined as

$$c_t\left(\check{\mathbf{w}}_t, k_t, \mathbf{\Delta}\left(\bar{y}_{q,t}\right)\mathbf{s}_t\right) \equiv \frac{1}{A_t} C_t\left(\check{\mathbf{w}}_t, A_t, K_t, \bar{\mathbf{Y}}_t\right) = C_t\left(\check{\mathbf{w}}_t, 1, k_t, \bar{\mathbf{Y}}_t/A_t\right), \tag{4}$$

where k_t, $\bar{y}_{q,t}$, and $\mathbf{s}_t$ are capital per acre, expected yield, and shares of land devoted to farm outputs $q = 1, ..., n_Y$, respectively. Shares are defined as $s_{q,t} \equiv a_{q,t}/A_t$ for all $q = 1, ..., n_Y$ where $a_{q,t}$ is acreage allocated to output q. Note that $\mathbf{\Delta}\left(\bar{y}_{q,t}\right)\mathbf{s}_t$ is the product of an $n_Y \times n_Y$ diagonal matrix of expected yields and an n_Y-vector of output shares, and is related to the n_Y-vector of total expected output by

$$\bar{\mathbf{Y}}_t = \mathbf{\Delta}\left(\bar{y}_{q,t}\right)\mathbf{a}_t = A_t\mathbf{\Delta}\left(\bar{y}_{q,t}\right)\mathbf{a}_t/A_t = A_t\mathbf{\Delta}\left(\bar{y}_{q,t}\right)\mathbf{s}_t. \tag{5}$$

The empirical specification for the per acre cost function $c_t(\cdot)$ used in Section 5 requires data on expected outputs divided by total farmland, $\bar{\mathbf{Y}}_t/A_t$, as a consequence of imposing linear homogeneity. Replacing this argument with $\mathbf{\Delta}\left(\bar{y}_{q,t}\right)\mathbf{s}_t$ in (4) is done for two reasons. First, since *ex ante* planted acreage and total farmland data is available for the outputs considered in the empirical application, a measure for output shares is readily available. Second, forecasting expected yields using *ex post* yield data is more effective than using *ex post* output data to forecast expected output since the share of farmland devoted to each output can vary widely over time. Thus, replacing $\bar{\mathbf{Y}}_t/A_t$ with $\mathbf{\Delta}\left(\bar{y}_{q,t}\right)\mathbf{s}_t$ as the argument in the per acre cost function is warranted.

To distinguish between quasi-fixed and variable inputs, adjustment costs for land and capital are included. It is assumed that when farmers change land and capital holdings, adjustment costs for these quasi-fixed inputs are incurred. Specifically, the costs of adjusting land and capital from (A_{t-1}, k_{t-1}) to (A_t, k_t) in period t are represented by the adjustment cost function $\Psi_t\left(A_{t-1}, A_t, k_{t-1}, k_t\right)$, which is everywhere continuous and differentiable.

Initial wealth in period t, W_t, is allocated at the beginning of the period to off-farm assets, quasi-fixed inputs, production costs, and consumption expenditures according to

$$W_t = \underbrace{B_t + F_t}_{\text{off-farm assets}} + \underbrace{\left(p_{L,t} + k_t\right)A_t}_{\text{quasi-fixed inputs}} + \underbrace{A_t c_t + \Psi_t}_{\text{production costs}} + \underbrace{M_t}_{\text{consumption}}. \tag{6}$$

Off-farm assets include risk-free bonds and risky financial instruments, total monetary holdings being B_t and F_t. Quasi-fixed inputs are land A_t and capital per acre k_t, with $p_{L,t}$ representing the price of land. Production costs include both variable costs per acre c_t and adjustment costs Ψ_t. Although some production costs occur at or near harvest (i.e. near $t+1$), all costs are included in (2.4) at time t because they are incurred before revenues are received. M_t represents total expenditure on consumption goods.

The *ex post* actual yield for farm output q, $y_{q,t+1}$, is realized stochastically at the end of the period such that

$$y_{q,t+1} = \bar{y}_{q,t}\left(1+\varepsilon_{q,t+1}\right),\ q = 1,\ldots,n_Y \tag{7}$$

where $\bar{y}_{q,t}$ is expected yield and $\varepsilon_{q,t+1}$ is a random shock with $E(\varepsilon_{q,t+1}) = 0$. Total revenue realized from production of farm outputs at the end of the period is

$$R_{t+1} \equiv \sum_{q=1}^{n_Y} \underbrace{\overbrace{p_{Y_q,t+1} y_{q,t+1}}^{\text{rev per acre}} s_{q,t} A_t}_{\text{revenue farm product } i} = \underbrace{\mathbf{p}'_{Y,t+1} \mathbf{\Delta}\left(y_{q,t+1}\right) \mathbf{s}_t A_t}_{\text{total revenue farm products}}, \tag{8}$$

where the $n_Y \times 1$ price vector $\mathbf{p}_{Y,t+1}$ is realized at the end of the period. The second term on the right hand side is exactly the same as the first, but written in matrix notation.

End-of-period wealth W_{t+1} is the sum of gross returns on off-farm assets, the end-of-period value of quasi-fixed inputs, and revenues from farm products,

$$W_{t+1} = \underbrace{\left(1+r\right) B_t + \left(1+r_{F,t+1}\right) F_t}_{\text{gross return on off-farm assets}} + \underbrace{\left[p_{L,t+1} + \left(1+\rho_{K,t+1}\right) k_t\right] A_t}_{\text{end-of-period value quasi-fixed inputs}} + \underbrace{R_{t+1}}_{\text{farm rev}}. \tag{9}$$

Here r is the risk-free rate of return on bonds, $r_{F,t+1}$ is the rate of return for the risky financial instrument, $p_{L,t+1}$ is the price of land at the end of the period, and $\rho_{K,t+1}$ is the percentage change in the value of capital realized at the end of the period.

The farm household's objective is the maximization of the expected stream of utility flows from consumption at time t,

$$V_t = E_t\left[\sum_{j=0}^{\infty} \rho^{-j} \upsilon_{t+j}\right],\ \upsilon_{t+j} \equiv \upsilon\left(\mathbf{p}_{Q,t+j}, M_{t+j}\right), \tag{10}$$

where V_t is the expected present value in period t, $E_t\left(\cdot\right)$ is the conditional expectation at the beginning of period t given available information, ρ^{-1} is the single period discount factor, and $\upsilon_{t+j} \equiv \upsilon\left(\mathbf{p}_{Q,t+j}, M_{t+j}\right)$ is the periodic indirect utility function for consumption in period $t+j$ given total consumption expenditures M_{t+j} and consumption good prices $\mathbf{p}_{Q,t+j}$. Including the indirect utility function implies that the static optimization of utility from consumption goods given expenditures and prices is subsumed in the model.

The problem given by the set of equations (6)-(10) is summarized as a dynamic programming problem with corresponding Bellman equation

$$V_t\left(W_t, A_{t-1}, k_{t-1}; \cdot\right) = \max_{M,B,F,k,A,\mathbf{s},\bar{\mathbf{y}}} \upsilon_t + \rho^{-1} E_t\left[V_{t+1}\left(W_{t+1}, A_t, k_t; \cdot\right)\right], \tag{11}$$

subject to the constraints (6)-(9). Note that the present value in period t depends upon the given states of initial wealth W_t, land A_{t-1}, and capital k_{t-1}. Additional arguments of the value function are denoted by $\cdot$ and are not explicitly represented here as the empirical analysis focuses on wealth, capital, and farmland.[4] This maximization problem is equivalently expressed using the Lagrangian

$$\begin{aligned}
\mathcal{L} \quad = \quad & \upsilon_t + \rho^{-1} E_t\left[V_{t+1}\left(\left(1+r\right)B_t + \left(1+r_{F,t+1}\right)F_t\right.\right. \\
& \left. + \left[p_{L,t+1} + \left(1+\rho_{K,t+1}\right)k_t\right]A_t + \mathbf{p}'_{Y,t+1}\boldsymbol{\Delta}\left(\bar{y}_{q,t}\left(1+\varepsilon_{q,t+1}\right)\right)\mathbf{s}_t A_t, A_t, k_t; \cdot\right)\right] \\
& + \lambda_t\left[W_t - B_t - F_t - \left(p_{L,t} + k_t\right)A_t - A_t c_t - \Psi_t - M_t\right],
\end{aligned} \tag{12}$$

where λ_t is the shadow value of an additional dollar of wealth in period t.

[4] It is important to note that this omission is a limiting factor in the analysis, especially if historical prices, rates of return, and/or asset pricing errors contain information on expected values in the next period.

2.2 Derivation of Arbitrage Equations

The Kuhn-Tucker conditions for optimal $(M_t, B_t, F_t, k_t, A_t, \mathbf{s}_t, \bar{\mathbf{y}}_t)$ are:

$$\frac{\partial \mathcal{L}}{\partial M} = \frac{\partial \upsilon_t}{\partial M} - \lambda_t \leq 0, M \geq 0, M\frac{\partial \mathcal{L}}{\partial M} = 0; \tag{13}$$

$$\frac{\partial \mathcal{L}}{\partial B} = \rho^{-1} E_t \left[\frac{\partial V_{t+1}}{\partial W}(1+r)\right] - \lambda_t \leq 0, B \geq 0, B\frac{\partial \mathcal{L}}{\partial B} = 0; \tag{14}$$

$$\frac{\partial \mathcal{L}}{\partial F} = \rho^{-1} E_t \left[\frac{\partial V_{t+1}}{\partial W}(1+r_{F,t+1})\right] - \lambda_t \leq 0, F \geq 0, F\frac{\partial \mathcal{L}}{\partial F} = 0; \tag{15}$$

$$\begin{aligned}\frac{\partial \mathcal{L}}{\partial k} &= \rho^{-1} E_t \left[\frac{\partial V_{t+1}}{\partial W}\left(1+\rho_{K,t+1}\right) A_t + \frac{\partial V_{t+1}}{\partial k}\right] \\ &-\lambda_t \left(A_t + \frac{\partial c_t}{\partial k} A_t + \frac{\partial \Psi_t}{\partial k_t}\right) \leq 0, k \geq 0, k\frac{\partial \mathcal{L}}{\partial k} = 0;\end{aligned} \tag{16}$$

$$\begin{aligned}\frac{\partial \mathcal{L}}{\partial A} &= \rho^{-1} E_t \left\{\frac{\partial V_{t+1}}{\partial W}\left[p_{L,t+1} + \left(1+\rho_{K,t+1}\right) k_t + \mathbf{p}'_{Y,t+1}\mathbf{\Delta}\left(\bar{y}_{q,t}\left(1+\varepsilon_{q,t+1}\right)\right)\mathbf{s}_t\right] + \frac{\partial V_{t+1}}{\partial A}\right\} \\ &-\lambda_t \left(p_{L,t} + k_t + c_t + \frac{\partial \Psi_t}{\partial A_t}\right) \leq 0, A \geq 0, A\frac{\partial \mathcal{L}}{\partial A} = 0;\end{aligned} \tag{17}$$

$$\begin{aligned}\frac{\partial \mathcal{L}}{\partial \mathbf{s}} &= \rho^{-1} E_t \left[\frac{\partial V_{t+1}}{\partial W}\mathbf{\Delta}\left(\bar{y}_{q,t}\left(1+\varepsilon_{q,t+1}\right)\right)\mathbf{p}_{Y,t+1} A_t\right] \\ &-\lambda_t \left(\frac{\partial c_t}{\partial \mathbf{s}} A_t\right) \leq 0, \mathbf{s} \geq 0, \mathbf{s}'\frac{\partial \mathcal{L}}{\partial \mathbf{s}} = 0;\end{aligned} \tag{18}$$

$$\frac{\partial \mathcal{L}}{\partial \bar{\mathbf{y}}} = \rho^{-1} E_t \left[\frac{\partial V_{t+1}}{\partial W}\mathbf{\Delta}\left(p_{Y_i,t+1} s_{i,t}\right)\left(\mathbf{i} + \boldsymbol{\varepsilon}_{t+1}\right) A_t\right] - \lambda_t \left(\frac{\partial c_t}{\partial \bar{\mathbf{y}}}\right) \leq 0, \bar{\mathbf{y}} \geq 0, \bar{\mathbf{y}}'\frac{\partial \mathcal{L}}{\partial \bar{\mathbf{y}}} = 0. \tag{19}$$

The following relationships are implications of the envelope theorem and hold for all t:

$$\frac{\partial V_t}{\partial W} = \lambda_t, \ \frac{\partial V_t}{\partial A} = -\lambda_t \frac{\partial \Psi_t}{\partial A_{t-1}}, \text{ and } \frac{\partial V_t}{\partial k} = -\lambda_t \frac{\partial \Psi_t}{\partial k_{t-1}}, \tag{20}$$

where the variables (λ_t, A_t, k_t) are all evaluated at their optimal choices. Interior solutions for all variables are assumed for what follows.

2.2.1 Consumption

Using $\lambda_t = \frac{\partial V_t}{\partial W}$, the first order condition for consumption expenditures is rewritten as

$$\frac{\partial \upsilon_t}{\partial M} = \frac{\partial V_t}{\partial W}, \tag{21}$$

which implies the marginal value of a dollar must be the same in any use. This equivalence allows one to construct the intertemporal marginal rate of substitution as a function of consumption expenditures or wealth. The latter is used in what follows since the empirical application in Section 5 utilizes a long time series of state-level wealth (which is not available for consumption or consumption expenditures of

agricultural households).

2.2.2 Bonds

Using $\lambda_t = \frac{\partial V_t}{\partial W}$, the first order condition for bonds is rewritten as

$$\rho^{-1} E_t \left[\frac{\partial V_{t+1}}{\partial W} (1+r) \right] = \frac{\partial V_t}{\partial W}. \tag{22}$$

This is the standard marginal condition for the optimal holding of bonds under the ICAPM. The right hand side is the opportunity cost of a marginal reduction in current consumption (since $\frac{\partial V_t}{\partial W} = \frac{\partial \upsilon_t}{\partial M}$ above) due to a marginal increase in the investment in bonds in period t. The left hand side $\rho^{-1} E_t \left(\frac{\partial V_{t+1}}{\partial W} (1+r) \right)$ is the marginal increase in discounted expected value the farmer obtains from the extra payoff of $(1+r)$ received at the end of the period. The farmer continues to buy or sell bonds until the marginal loss equals the marginal gain.

2.2.3 Risky Financial Instrument

Similarly, the first order condition for the risky financial instrument is rewritten as

$$\rho^{-1} E_t \left[\frac{\partial V_{t+1}}{\partial W} (1+r_{F,t+1}) \right] = \frac{\partial V_t}{\partial W}. \tag{23}$$

This is the standard marginal condition for the optimal holding of a risky asset under the ICAPM. The right hand side is the same as in (22), and the left hand side $\rho^{-1} E_t \left[\frac{\partial V_{t+1}}{\partial W} (1+r_{F,t+1}) \right]$ is the marginal increase in discounted expected value the farmer obtains from the extra payoff of $1 + r_{F,t+1}$ received at the end of the period.

The first order conditions for bonds and the risky financial instrument are rewritten as

$$\rho^{-1} E_t \left[\frac{\frac{\partial V_{t+1}}{\partial W}}{\frac{\partial V_t}{\partial W}} (1+r) \right] = 1 \tag{24}$$

and

$$\rho^{-1} E_t \left[\frac{\frac{\partial V_{t+1}}{\partial W}}{\frac{\partial V_t}{\partial W}} (1+r_{F,t+1}) \right] = 1. \tag{25}$$

Subtracting the first from the second yields the standard ICAPM arbitrage equation for investing in the risky instrument relative to investing in bonds

$$E_t \left[m_{t+1} \left(r_{F,t+1} - r \right) \right] = 0, \tag{26}$$

where $m_{t+1} \equiv \rho^{-1}\left(\frac{\partial V_{t+1}}{\partial W}/\frac{\partial V_t}{\partial W}\right)$ is the stochastic intertemporal marginal rate of substitution of wealth, and $r_{F,t+1} - r$ is the excess return of the risky instrument over bonds. Thus, the theoretical model incorporates the ICAPM for off-farm risky investments.

2.2.4 Land

Using $\lambda_t = \frac{\partial V_t}{\partial W}$ and $\frac{\partial V_{t+1}}{\partial A} = -\lambda_{t+1}\frac{\partial \Psi_{t+1}}{\partial A_t} = -\frac{\partial V_{t+1}}{\partial W}\frac{\partial \Psi_{t+1}}{\partial A_t}$, the first order condition for land is rewritten as

$$\rho^{-1}E_t\left\{\frac{\partial V_{t+1}}{\partial W}\left[p_{L,t+1} + \left(1+\rho_{K,t+1}\right)k_t + \mathbf{p}'_{Y,t+1}\mathbf{\Delta}\left(\bar{y}_{q,t}\left(1+\varepsilon_{q,t+1}\right)\right)\mathbf{s}_t - \frac{\partial \Psi_{t+1}}{\partial A_t}\right]\right\}$$
$$= \frac{\partial V_t}{\partial W}\left(p_{L,t} + k_t + c_t + \frac{\partial \Psi_t}{\partial A_t}\right). \tag{27}$$

The right hand side is the opportunity cost of a marginal reduction in current consumption due to a marginal increase in the investment in land in period t. The farmer pays $p_{L,t}$ for the additional acre, $k_t + c_t$ for the additional capital and variable costs to farm the acre, and incurs additional adjustment costs $\frac{\partial \Psi_t}{\partial A_t}$.[5] Thus, the extra investment associated with the additional acre is $p_{L,t} + k_t + c_t + \frac{\partial \Psi_t}{\partial A_t}$. The left hand side is the marginal increase in discounted expected value the farmer obtains from the extra payoff of $p_{L,t+1} + \left(1+\rho_{K,t+1}\right)k_t + \mathbf{p}'_{Y,t+1}\mathbf{\Delta}\left(\bar{y}_{q,t}\left(1+\varepsilon_{q,t+1}\right)\right)\mathbf{s}_t - \frac{\partial \Psi_{t+1}}{\partial A_t}$ received at the end of the period. $p_{L,t+1}$ is the end-of-period value of the additional acre, $\left(1+\rho_{K,t+1}\right)k_t$ is the end-of-period value of the additional capital used to farm the acre, $\mathbf{p}'_{Y,t+1}\mathbf{\Delta}\left(\bar{y}_{q,t}\left(1+\varepsilon_{q,t+1}\right)\right)\mathbf{s}_t$ is the additional revenue from production of farm outputs, and $\frac{\partial \Psi_{t+1}}{\partial A_t}$ is the marginal adjustment cost given that acreage will be adjusted to A_{t+1} in the next period.[6][7] The farmer continues to invest in land until the marginal loss equals the marginal gain.

As with the risky financial instrument, the arbitrage equation for increasing land in period t by an additional acre relative to investing in bonds is

$$E_t\left[m_{t+1}\left(r_{L,t+1} - r\right)\right] = 0, \tag{28}$$

[5] In general, the sign of $\frac{\partial \Psi_t}{\partial A_t}$ depends on the initial level of land A_{t-1} and can be either positive or negative. For example, under a quadratic specification $\Psi_t\left(A_{t-1}, A_t\right) = \frac{1}{2}\gamma\left(A_t - A_{t-1}\right)^2$, this implies $\frac{\partial \Psi_t}{\partial A_t} = \gamma\left(A_t - A_{t-1}\right)$, which is positive when $A_t > A_{t-1}$ and negative when $A_t < A_{t-1}$.

[6] Using the notation for total revenues defined above, $R_{t+1} \equiv \mathbf{p}'_{Y,t+1}\mathbf{\Delta}\left(\bar{y}_{i,t}\left(1+\varepsilon_{i,t+1}\right)\right)\mathbf{s}_t A_t$, the marginal revenue per acre is $\frac{\partial R_{t+1}}{\partial A_t} = \mathbf{p}'_{Y,t+1}\mathbf{\Delta}\left(\bar{y}_{i,t}\left(1+\varepsilon_{i,t+1}\right)\right)\mathbf{s}_t$.

[7] In general, the sign of $\frac{\partial \Psi_{t+1}}{\partial A_t}$ depends on the future level of land A_{t+1} and can be either positive or negative. Under a quadratic specification, $\frac{\partial \Psi_{t+1}}{\partial A_t} = \gamma\left(A_{t+1} - A_t\right)$, which is positive when $A_{t+1} > A_t$ and negative when $A_{t+1} < A_t$.

where m_{t+1} is defined as above and the rate of return is defined by

$$r_{L,t+1} \equiv \frac{\text{extra payoff - extra investment}}{\text{extra investment}}$$
$$= \frac{p_{L,t+1} - p_{L,t} + \rho_{K,t+1} k_t + \mathbf{p}'_{Y,t+1} \mathbf{\Delta} \left(\bar{y}_{q,t} \left(1 + \varepsilon_{q,t+1}\right)\right) \mathbf{s}_t - c_t - \left(\frac{\partial \Psi_{t+1}}{\partial A_t} + \frac{\partial \Psi_t}{\partial A_t}\right)}{p_{L,t} + k_t + c_t + \frac{\partial \Psi_t}{\partial A_t}}. \tag{29}$$

2.2.5 Capital

Using $\lambda_t = \frac{\partial V_t}{\partial W}$ and $\frac{\partial V_{t+1}}{\partial k} = -\lambda_{t+1} \frac{\partial \Psi_{t+1}}{\partial k_t} = -\frac{\partial V_{t+1}}{\partial W} \frac{\partial \Psi_{t+1}}{\partial k_t}$, the first order condition for capital is rewritten as

$$\rho^{-1} E_t \left\{ \frac{\partial V_{t+1}}{\partial W} \left[\left(1 + \rho_{K,t+1}\right) A_t - \frac{\partial \Psi_{t+1}}{\partial k_t} \right] \right\} = \frac{\partial V_t}{\partial W} \left(A_t + \frac{\partial c_t}{\partial k} A_t + \frac{\partial \Psi_t}{\partial k_t} \right). \tag{30}$$

The right hand side is the opportunity cost of a marginal reduction in current consumption due to a marginal increase in the investment in capital per acre in period t. Since this increase is made across all acres, the farmer pays A_t for the additional capital, receives a reduction in total variable costs $\frac{\partial c_t}{\partial k} A_t$ (assuming additional capital reduces variable costs, $\frac{\partial c_t}{\partial k} < 0$), and incurs additional adjustment costs $\frac{\partial \Psi_t}{\partial k_t}$.[8] Thus, the extra investment implied by this increase is $A_t + \frac{\partial c_t}{\partial k} A_t + \frac{\partial \Psi_t}{\partial k_t}$. The left hand side is the marginal increase in discounted expected value the farmer obtains from the extra payoff of $\left(1 + \rho_{K,t+1}\right) A_t - \frac{\partial \Psi_{t+1}}{\partial k_t}$ received at the end of the period. Since the increase in capital is made across all acres, $\left(1 + \rho_{K,t+1}\right) A_t$ is the end-of-period value of the additional capital, and $\frac{\partial \Psi_{t+1}}{\partial k_t}$ is the marginal adjustment cost given that capital will be adjusted to k_{t+1} in the next period.[9] The farmer continues to invest in capital per acre until the marginal loss equals the marginal gain.

As with land, the arbitrage equation for increasing capital per acre in period t by an additional dollar relative to investing in bonds is

$$E_t \left[m_{t+1} \left(r_{K,t+1} - r \right) \right] = 0, \tag{31}$$

where m_{t+1} is defined as above and the rate of return is defined by

$$r_{K,t+1} \equiv \frac{\text{extra payoff - extra investment}}{\text{extra investment}}$$
$$= \frac{\rho_{K,t+1} A_t - \frac{\partial c_t}{\partial k} A_t - \left(\frac{\partial \Psi_{t+1}}{\partial k_t} + \frac{\partial \Psi_t}{\partial k_t}\right)}{A_t + \frac{\partial c_t}{\partial k} A_t + \frac{\partial \Psi_t}{\partial k_t}}. \tag{32}$$

[8] As with land, the sign of $\frac{\partial \Psi_t}{\partial k_t}$ depends on the initial level of capital k_{t-1} and can be either positive or negative.

[9] As with land, the sign of $\frac{\partial \Psi_{t+1}}{\partial k_t}$ depends on the future level of capital k_{t+1} and can be either positive or negative.

2.2.6 Shares

In a similar manner, the arbitrage equations for increasing the share of output i in period t by an additional unit relative to investing in bonds are

$$E_t\left[m_{t+1}\left(r_{S_q,t+1}-r\right)\right]=0,\ q=1,...,n_Y \tag{33}$$

where m_{t+1} is defined as above and the rate of return for this investment is defined by

$$\begin{aligned} r_{S_q,t+1} &\equiv \frac{\text{extra payoff - extra investment}}{\text{extra investment}} \\ &= \frac{\bar{y}_{q,t}\left(1+\varepsilon_{q,t+1}\right)p_{Y_q,t+1}A_t-\frac{\partial c_t}{\partial s_q}A_t}{\frac{\partial c_t}{\partial s_q}A_t}. \end{aligned} \tag{34}$$

Since the first term in the numerator is just the extra payoff for increasing the share by an additional unit and the second is the the extra investment, this is the ratio of the (extra payoff - extra investment) to the extra investment. Thus, $r_{S_q,t+1}$ is the rate of return for increasing the share of output q by an additional unit, and $r_{S_q,t+1}-r$ is the associated excess return over bonds. Note that the extra investment is solely a function of increased variable costs. In reality, there are likely adjustment costs associated with altering crop shares, in which case a marginal adjustment cost function would become part of the investment and payoff expressions as with farmland and capital above.[10]

The equations (26), (28), (31), and (33) together form a system of arbitrage equations. Empirical counterparts to this system of equations are derived and estimated in Section 5 using specifications for the marginal value of wealth function, the per acre variable cost function, the adjustment cost function, and an assumption on the expectations process. As mentioned in the introduction, a system of variable input demands is estimated in Section 4 and the parameter estimates are used to test the constant returns to scale and quasi-fixed input assumptions used in this section. The data used for these applications is discussed in the following section.

3 Data

State-level panel data is used in the empirical applications presented in Sections 4 and 5. The same states are included in both applications. The limiting factor for which states are included is the choice of farm outputs included in the system of arbitrage equations. The choice of crops is in turn limited by the availability

[10] One can easily extend the adjustment cost function to include vectors of current and lagged crop shares, i.e. $\Psi_t\left(A_{t-1},A_t,k_{t-1},k_t,\mathbf{s}_{t-1},\mathbf{s}_t\right)$. This is a focus of ongoing research.

of planted acreage data. A preliminary analysis of data availability indicated that, for any four crops, the number of states that planted all four in all years between 1960 and 1999 was largest for corn, oats, soybeans, and wheat.[11] Thus, these farm outputs are used and the 17 states that fit the inclusion criteria are Georgia, Iowa, Illinois, Indiana, Kansas, Michigan, Minnesota, Missouri, North Carolina, North Dakota, Nebraska, Ohio, Oklahoma, Pennsylvania, South Carolina, South Dakota, and Wisconsin.

3.1 Data for Demand System Estimation

The state-level variables required to estimate the system of demand equations in Section 4 are: variable input expenditures and prices, total variable cost, total capital, total farmland, and instruments to account for the joint decision making nature of the farm. The majority of the data is from Eldon Ball's state-level panel that spans 1960-1999. This data has been compiled by the USDA/ERS and is described in detail in Ball et. al. (1999). It is the most comprehensive and consistent data on revenues and costs for the agricultural sector available at this time.

The specific aggregate variable input categories considered in this analysis are labor, energy, agricultural chemicals, and other materials. To construct these accounts, the ERS collects state-level nominal expenditure and price data for a collection of subaggregate variable inputs which are discussed in more detail below. The aggregate nominal variable input expenditure accounts are constructed by summing across subaggregate nominal expenditure data, and Divisia price indexes for each aggregate category (except labor) are constructed from the subaggregate prices using expenditure shares as weights.

Each of the four aggregate variable input categories are constructed using subaggregate expenditure and price data, which was provided by Eldon Ball. The provision of the subaggregate data allowed for reconstruction of aggregate variable input expenditure and price data where deemed necessary. Unless indicated otherwise, the methodology for constructing the expenditure and price aggregates presented below is attributed to Eldon Ball.

The labor subaggregates are hired labor and self employed (unpaid family) labor. Labor hours worked are reported for both hired and self employed (unpaid family) workers, and the compensation for hired labor includes the value of provided housing and contributions to social insurance, and is quality-adjusted to account for quality changes over time. The compensation for self employed labor from the ERS data was not used in this analysis, as it is an inferred opportunity cost of working on the farm. The hired labor wage rate more accurately measures the replacement cost of self employed workers, and is used instead for this reason. Labor expenditures were calculated as the sum of total hours worked across both subaggregates,

[11] The period 1960-1999 was selected because it coincides with the years of Eldon Ball's panel data set, the merits of which are discussed below. The data has been recently extended through 2004, but was not available in time for this analysis.

multiplied by the hired labor wage rate. The price of labor used is the hired labor wage rate.

The ERS energy accounts are constructed using state-level electricity and fuel (includes petroleum fuel and natural gas) expenditure and price data, which are then used to construct a Divisia price index for energy. The ERS agricultural chemical accounts are constructed using fertilizer and pesticide expenditure and price data, which are adjusted for changes in quality over time. Specifically, constant quality price indexes for fertilizer and pesticide were constructed under the hedonic regression technique, which are then used to construct a Divisia price index for agricultural chemicals.[12] The ERS other materials account is constructed using expenditure and price data for the following (quite exhaustive) subaggregate categories: purchases of seed, feed, and livestock, on-farm use of crops and livestock, machine and building maintenance and repairs, custom machine services, contract labor, transportation and storage services, shop equipment, veterinarian services, and irrigation from public sellers of water. As with the other categories, subaggregate expenditures and prices are used to construct a Divisia price index for materials. To construct the deflated per acre measures that are used in the empirical application, each of the nominal expenditure variables for energy, agricultural chemicals, and other materials are divided by the product of hired labor wage rate and total farmland (discussed below).

The nominal total variable cost measure is constructed as the sum of the nominal variable input expenditures for labor, energy, agricultural chemicals, and other materials. The deflated per acre measure is constructed in the same way as the deflated per acre expenditure variables.

The ERS capital subaggregates include separate categories for automobiles, buildings, trucks, tractors, and other machinery. For each subaggregate, a measure of the productive stock (in nominal dollars) is constructed as the cumulation of past investments adjusted for discards of worn-out assets and loss of efficiency over the service life. Both rental rates and price indices for each subaggregate were provided, which provides flexibility for measuring the service flow versus stock of capital. Since capital is being measured as a quasi-fixed input, the value of the stock of each subaggregate was constructed by multiplying the productive stock by the price index. A nominal value of capital was constructed by summing across the subaggregate categories, and the deflated per acre measure is constructed in the same way as the deflated per acre expenditure variables.

A commonly used measure for total farmland is provided by the ERS (referred to as the total land in farms variable), utilizes a cubic interpolation method to fill in data points between census years. This measure is usable for census years, but artificially smooths out the underlying data in between census years. An alternative measure, used in this analysis, is constructed using the National Agricultural Statistics Service's

[12] The price index for fertilizers is formed by regressing the prices of single nutrient and multigrade fertilizer materials on the proportion of nutrients contained in the materials. Similarly, prices for pesticides are regressed on differences in physical characteristics such as toxicity, persistence in the environment, and leaching potential.

(NASS) Acreage Reports, which provide a measure of planted acreage for principal crops in each state using an annual survey. Define A_t^T to be the total land in farms variable in period t, and A_t^C to be the planted acreage for principal crops in period t. The algorithm for constructing the farmland measure used in this analysis, A_t^F, is

$$A_t^F = \begin{cases} A_t^T \text{ if } t \text{ is a census year} \\ A_t^C + \frac{1}{t_u - t_l} \sum_{t=t_l}^{t_u} \left(A_t^T - A_t^C\right) \text{ if } t \text{ is not a census year} \end{cases},$$

where t_l is the first census year less than t and t_u is the first census year greater than t.[13] Assuming that farmland outside of principal crops does not change dramatically from year to year, this variable provides a more reasonable measure of farmland.

The instruments used are deflated variable cost per acre, deflated capital per acre, and deflated variable input prices averaged across the 17 states and then lagged two periods, and the following general economy variables observed at the national level: real per capita disposable personal income, the real rate of return on AAA corporate 30-year bonds, the real manufacturing wage rate, the real index of prices paid by manufacturers for materials and components, and the real index of prices paid by manufacturers for fuel, energy and power. Per capita disposable personal income is deflated by the consumer price index for all items, while the aggregate wholesale price variables are deflated by the implicit price deflator for gross domestic product. The variable cost, capital, and price instruments are lagged two periods to create predetermined, strictly exogenous variables (Engel et. al., 1983). The general economy instruments are not lagged because they are exogenous to the agricultural economy. In addition, a constant and time trend are included in the instrument set.

3.2 Additional Data for Arbitrage System Estimation

The additional state-level variables required to estimate the system of arbitrage equations in Section 5 are: the real interest rate, the rate of return on the risky financial instrument, price of land, *ex post* total farm revenue, wealth, number of farms, and acreage, *ex post* revenues, and *ex ante* expected yields for corn, oats, soy, and wheat. In addition, instrumental variables are required to account for the joint decision making nature of the farm.

The measures used for the rate of return on bonds and the risky financial instrument come from Robert Shiller's financial indicators data set. The chosen measures are the real interest rate and the real return on the S&P index, respectively.

An accurate measure of the price of farmland is very difficult to obtain. The ERS has an estimated service flow for land that is commonly used in demand analysis if land is assumed to be variable, and is

[13] The Ag Census years relevant to the 1960-1999 time period are 1959, 1964, 1969, 1974, 1978, 1982, 1987, 1992, 1997, and 2002.

constructed as a Divisia price index which makes it unserviceable here since a measure expressed in \$/acre is required. Another commonly used price comes from NASS's Agricultural Statistics annuals, which report state-level averages for the value of land and buildings per acre. This price has the advantage of being measured in \$/acre, but includes the value of buildings which is problematic. A third option is to use an annual state-level land-to-building ratio provided by ERS that, when multiplied by the value of land and buildings per acre, provides a reasonable farmland price expressed in \$/acre. This is the approach taken here, and has the advantage of being in \$/acre and specific to land.

The measures for *ex post* total farm revenue and *ex post* revenues for corn, oats, soy, and wheat come from Eldon Ball's state-level panel. This measure is the best available measure for total farm revenues, as it consists of primary production (excluding production for intermediate consumption) and production of non-agricultural goods and services where they cannot be distinguished from the primary agricultural activity and non-agricultural (or secondary) output. The measure of secondary output aggregates revenues from a diverse set of income sources, including machine and labor services, livestock feeding, owner provided housing, and soilbank and conservation reserve program rental payments. The primary production accounts for total farm revenue and individual crop revenues are constructed from the producer's perspective; that is, important government payments, including deficiency and diversion/set aside payments for crops as well as wool and mohair payments, are included.

The measures for wealth and the number of farms come from the ERS balance sheet data. Wealth is a notoriously difficult variable to measure in agriculture. The best available data on wealth are found in the Agricultural Resource Management Survey and the U.S. Census of Agriculture, which are conducted by NASS (Pope et. al., 2009). However, both resources are unserviceable here since neither is a balanced panel. Another option that does not suffer from this shortcoming is to use the equity variable, measured as net assets minus liabilities, from the ERS balance sheet data. This is the approach taken here.

Acreage measures for corn, oats, soy, and wheat are obtained from the (NASS) Acreage Reports. Since these decisions are made prior to realization of the production shock, it is important to use planted rather than harvested acreage. These data are readily available for the crops used in this analysis.

Measures for *ex ante* expected yields for corn, oats, soy, and wheat are not available and are typically treated as unobservable to the researcher in practice. A common approach that overcomes this limitation is to predict period t yields using period $t-1$ *ex post* actual yield data. NASS yield data is used to fit the system of equations

$$y_{ijt} = \alpha_{ij} + \beta_{ij} y_{ijt-1} + \tau_{ij} t \; i = 1, ..., I \; j = C, O, S, W \; t = 1, ..., T$$

for the $I = 17$ states using state-level yield data for corn (C), oats (O), soy (S), and wheat (W) obtained from NASS. The period used to estimate the parameters is 1950-2005, and estimates for *ex ante* expected yields used in this analysis are constructed by

$$\bar{y}_{ijt} = \hat{\alpha}_{ij} + \hat{\beta}_{ij} y_{ijt-1} + \hat{\tau}_{ij} t \; i = 1, ..., 17 \; j = C, O, S, W \; t = 1960, ..., 1999.$$

Since all but the wealth and farmland variables are measured per acre, a representative agent model is estimated by replacing the wealth and farmland variables with per-farm averages in each state. That is, average wealth and land per farm are used instead of state-level aggregates. It is important to note that all monetary variables are converted to real 1999 dollars using the BLS CPI.

The data used to construct the instruments used are: (i) across state averages of wealth per farm, land per farm, capital per acre, shares of farmland allocated to corn, oats, soybeans, and wheat, the rate of return for the S&P index, and the following general economy variables observed at the national level: (ii) real per capita disposable personal income, the real rate of return on AAA corporate 30-year bonds, the unemployment rate, the real manufacturing wage rate, the real index of prices paid by manufacturers for materials and components, and the real index of prices paid by manufacturers for fuel, energy and power. Per capita disposable personal income is deflated by the consumer price index for all items, while the aggregate wholesale price variables are deflated by the implicit price deflator for gross domestic product.

Because of nonstationarity concerns, the use of first differences and/or ratios of the instruments is warranted (Cochrane, 2005). For the variables in (i), lagged first differences and ratios (i.e. $z_{t-1} - z_{t-2}$ and z_{t-1}/z_{t-2}) of the 17-state averages are used to construct predetermined, strictly exogenous instruments (Engel et. al., 1983). These instruments are complemented with simple differences and ratios (i.e. $z_t - z_{t-1}$ and z_t/z_{t-1}) of the variables in (ii). The instruments from (ii) are not lagged because they are general economy variables that are exogenous to the agricultural economy. In addition, a constant and time trend are included in the instrument set.

4 Demand System Estimation

As mentioned in the introduction, the derivation of the theoretical arbitrage model relies on the assumptions that land and capital are quasi-fixed assets, and that production is characterized by constant returns to scale. These assumptions are empirically tested in this section using parameter estimates from a system of variable input demands that is derived from a flexible specification for variable costs of production.

4.1 Set Up

The specification for the per acre cost function follows LaFrance and Pope (2008a, 2008b), which address two common problems in econometric models of production, aggregation and unobservable variables. The authors identify the necessary and sufficient restrictions on technology and cost that generate conditional factor demands that are functions of input prices, quasi-fixed inputs, and cost.[14] Demands have this characteristic if and only if outputs are weakly separable from variable input prices in the cost function; that is, the cost function can be written as $c\left(\mathbf{w}, k, \theta\left(k, \mathbf{\Delta}\left(\bar{y}_q\right)\mathbf{s}\right)\right)$ where $\theta\left(\cdot\right)$ is the constant of integration associated with integrating the conditional factor demands over variable input prices. The authors also derive the subset of this class of models that satisfies exact aggregation with respect to costs, which is especially important for this study as state-level accounts are used to construct the rates of return for land, capital, and output shares. A list of additional variable definitions used in the model is provided in Table 2.

As noted in Ball et. al. (2008), the neoclassical model of conditional demands for variable inputs with joint production, fixed inputs, and production uncertainty is

$$\mathbf{x}(\check{\mathbf{w}}, \mathbf{\Delta}\left(\bar{y}_q\right)\mathbf{s}, k) = \arg\min\{\check{\mathbf{w}}^\top \mathbf{x} : F(\mathbf{x}, \mathbf{\Delta}\left(\bar{y}_q\right)\mathbf{s}, k) \leq 0\}, \tag{35}$$

where $\mathbf{x}$ is an n_V–vector of variable inputs per acre, $\check{\mathbf{w}}$ is an n_V–vector of variable input prices (the non-standard notation is used to simplify the specification for the cost function below), $\mathbf{\Delta}\left(\bar{y}_q\right)\mathbf{s}$ is an n_Y–vector of expected outputs over total farmland, k is capital per acre, and $F(\mathbf{x}, \mathbf{\Delta}\left(\bar{y}_q\right)\mathbf{s}, k)$ is the per-acre joint production function.

The essential problem is that inputs are applied *ex ante* under stochastic production, while the vector of *ex ante* outputs $\mathbf{\Delta}\left(\bar{y}_q\right)\mathbf{s}$ is not observable (in practice). The solution derived by LaFrance and Pope (2008a, 2008b), identifies the necessary and sufficient condition to consistently estimate conditional input demands as functions of variables that are observable when the inputs are committed to production – prices of the inputs, the level of capital per acre, and the per-acre variable cost of production – so that variable input demands can be written as

$$\mathbf{x}(\check{\mathbf{w}}, \mathbf{\Delta}\left(\bar{y}_q\right)\mathbf{s}, k) = \mathbf{g}(\check{\mathbf{w}}, k, c\left(\check{\mathbf{w}}, \mathbf{\Delta}\left(\bar{y}_q\right)\mathbf{s}, k\right)). \tag{36}$$

The authors prove that the variable input demand equations have this structure if and only if the variable

[14] Importantly, the demands are not a function of unobservable expected output.

cost function has the structure

$$c\left(\check{\mathbf{w}}, \boldsymbol{\Delta}\left(\bar{y}_q\right)\mathbf{s}, k\right) \equiv c\left(\check{\mathbf{w}}, k, \theta\left(k, \boldsymbol{\Delta}\left(\bar{y}_q\right)\mathbf{s}\right)\right), \tag{37}$$

if and only if the joint production function has the structure

$$F(\mathbf{x}, \boldsymbol{\Delta}\left(\bar{y}_q\right)\mathbf{s}, k) \equiv F(\mathbf{x}, k, \theta\left(k, \boldsymbol{\Delta}\left(\bar{y}_q\right)\mathbf{s}\right)). \tag{38}$$

In other words, outputs are weakly separable from the variable inputs. Although this result is somewhat restrictive in outputs, it is quite flexible in the inputs (Ball et. al., 2008).[15]

The specification of the per acre cost function is a full rank 3 model that follows Ball et. al. (2008),

$$f\left(\tilde{c}\right) = \eta\left(\mathbf{g}, \tilde{k}\right) - \left(\frac{\varphi\left(\mathbf{g}\right)}{\boldsymbol{\delta}^{\top}\mathbf{g} + \sqrt{\varphi\left(\mathbf{g}\right)}\theta\left(\tilde{k}, \boldsymbol{\Delta}\left(\bar{y}_q\right)\mathbf{s}\right)}\right), \tag{39}$$

where

$$\begin{aligned}
&f\left(\tilde{c}\right) = \frac{\tilde{c}^{\kappa} + \kappa - 1}{\kappa}, \tilde{c} = \frac{c}{w_{n_V}}, c = c\left(\mathbf{w}, k, \theta\right), g_v\left(\tilde{w}_v\right) = \frac{\tilde{w}_v^{\lambda} + \lambda - 1}{\lambda}, \\
&\tilde{\mathbf{w}}\left(\mathbf{w}, w_{n_V}\right) = \frac{1}{w_{n_V}}\mathbf{w}, \mathbf{w} = \left[w_1, ..., w_{n_V - 1}\right]^{\top}, \boldsymbol{\alpha}_0 = \left[\alpha_{0,1}, ..., \alpha_{0,n_V - 1}\right]^{\top}, \\
&\boldsymbol{\alpha}_1 = \left[\alpha_{1,1}, ..., \alpha_{1,n_V - 1}\right]^{\top}, \eta\left(\mathbf{g}, \tilde{k}\right) = \left(\boldsymbol{\alpha}_0 + \boldsymbol{\alpha}_1 k\right)^{\top}\mathbf{g} + \alpha_{0,n_V} + \alpha_{1,n_V}\tilde{k}, \\
&\tilde{k} = \frac{k}{w_{n_V}}, \text{ and } \varphi\left(\mathbf{g}\right) = \mathbf{g}^{\top}\mathbf{B}\mathbf{g} + 2\boldsymbol{\gamma}^{\top}\mathbf{g} + 1.
\end{aligned} \tag{40}$$

Here, $\boldsymbol{\alpha}_0$, α_{0,n_V}, $\boldsymbol{\alpha}_1$, α_{1,n_V}, $\mathbf{B}$, $\boldsymbol{\gamma}$, and $\boldsymbol{\delta}$ are parameters. $\mathbf{B}$ is an $(n_V - 1) \times (n_V - 1)$ matrix, $\boldsymbol{\delta}$ is an n_V-vector, $\boldsymbol{\alpha}_0$, $\boldsymbol{\alpha}_1$, and $\boldsymbol{\gamma}$ are $(n_V - 1)$-vectors, and α_{0,n_V} and α_{1,n_V} are scalars.

Note that linear homogeneity of the cost function in prices has been imposed by deflating prices by w_{n_V}, and that the n_V^{th} input is treated asymmetrically from the other inputs, both in the first- (conditional mean) and second-order (variance-covariance) components of the model. The authors note that the translated Box-Cox functions f and $\mathbf{g}$ are observationally equivalent to standard Box-Cox transformations. If $\kappa = 1$, then $f(\tilde{c}) = \tilde{c}$, while if $\kappa = 0$, then $f(\tilde{c}) = 1 + \ln\tilde{c}$. The same results apply to $g_v\left(\tilde{w}_v\right)$ for $\lambda = 1$ or 0, respectively. All other values of $(\kappa, \lambda) \in \mathbb{R}_+^2$ yield functional forms of the PIGL class in input prices and cost, allowing one to nest this class of demand models with a rank three generalized translog and a rank three generalized quadratic production model.

[15] Among other things it implies that marginal rates of product transformation are independent of the variable inputs and factor intensities. Of course, if these restrictions are deemed too strong, then an alternative approach to formulating the variable cost function becomes necessary .

Solving for the cost function yields

$$c\left(\mathbf{w},k,\theta\right)=w_{n_V}f^{-1}\left(y\left(\mathbf{g}\left(\tilde{\mathbf{w}}\left(\mathbf{w}\right)\right),\theta\right)\right), \tag{41}$$

where

$$\begin{aligned} y\left(\mathbf{g},\theta\right) &\equiv \eta\left(\mathbf{g},\tilde{k}\right)-\left(\frac{\varphi\left(\mathbf{g}\right)}{\boldsymbol{\delta}^{\top}\mathbf{g}+\sqrt{\varphi\left(\mathbf{g}\right)}\theta}\right), \\ f^{-1}(z) &= \left(z\kappa+1-\kappa\right)^{1/\kappa}. \end{aligned} \tag{42}$$

This implies that the n_V-1 variable input demands are given by (derivation in Appendix C)

$$\mathbf{q}\equiv\frac{\partial c}{\partial\mathbf{w}}=\tilde{c}^{1-\kappa}\boldsymbol{\Delta}\left(\tilde{w}_v^{\lambda-1}\right)\frac{\partial y}{\partial\mathbf{g}}, \tag{43}$$

where the expression

$$\frac{\partial y}{\partial\mathbf{g}}=\left\{\begin{array}{c}\boldsymbol{\alpha}+\left[1-\boldsymbol{\delta}^{\top}\mathbf{g}\left(\frac{y-\eta\left(\mathbf{g},\tilde{k}\right)}{\varphi(\mathbf{g})}\right)\right] \\ *\left(\frac{y-\eta\left(\mathbf{g},\tilde{k}\right)}{\varphi(\mathbf{g})}\right)\left(\mathbf{B}\mathbf{g}+\boldsymbol{\gamma}\right)+\frac{\left[y-\eta\left(\mathbf{g},\tilde{k}\right)\right]^2}{\varphi(\mathbf{g})}\boldsymbol{\delta}\end{array}\right\} \tag{44}$$

is derived in LaFrance et. al. (2005).

In order for this n_V-1 equation incomplete demand system to satisfy integrability, demands need to be positive and homogenous of degree 0 in prices, and the cost function's Hessian matrix needs to be symmetric and negative semidefinite. The cost function's $n_V\times n_V$ Hessian matrix is (derivation in Appendix C)

$$\breve{\mathbf{H}}=\begin{bmatrix}\mathbf{H} & -\mathbf{H}^{\top}\tilde{\mathbf{w}} \\ -\tilde{\mathbf{w}}^{\top}\mathbf{H} & \tilde{\mathbf{w}}^{\top}\mathbf{H}\tilde{\mathbf{w}}\end{bmatrix}, \tag{45}$$

where

$$\begin{aligned} \mathbf{H} &= \frac{1}{w_{n_V}}\left[\begin{array}{c}\left(\lambda-1\right)\boldsymbol{\Delta}\left(\tilde{w}_v^{-1}\right)\boldsymbol{\Delta}\left(q_v\right)+\tilde{c}^{1-\kappa}\boldsymbol{\Delta}\left(\tilde{w}_v^{\lambda-1}\right)\frac{\partial^2 y}{\partial\mathbf{g}\partial\mathbf{g}^{\top}}\boldsymbol{\Delta}\left(\tilde{w}_v^{\lambda-1}\right) \\ +\left(1-\kappa\right)\tilde{c}^{-1}\mathbf{q}\mathbf{q}^{\top}\end{array}\right], \\ \frac{\partial^2 y}{\partial\mathbf{g}\partial\mathbf{g}^{\top}} &= \left[1-\boldsymbol{\delta}^{\top}\mathbf{g}\left(\frac{y-\eta}{\varphi}\right)\right]\left(\frac{y-\eta}{\varphi}\right)\left[\mathbf{B}-\frac{\left(\mathbf{B}\mathbf{g}+\boldsymbol{\gamma}\right)\left(\mathbf{B}\mathbf{g}+\boldsymbol{\gamma}\right)^{\top}}{\varphi}\right] \\ &\quad +2\frac{\left(y-\eta\right)^3}{\varphi^2}\left[\mathbf{I}-\frac{1}{\varphi}\left(\mathbf{B}\mathbf{g}+\boldsymbol{\gamma}\right)g^{\top}\right]\boldsymbol{\delta}\boldsymbol{\delta}^{\top}\left[\mathbf{I}-\frac{1}{\varphi}\mathbf{g}\left(\mathbf{B}\mathbf{g}+\boldsymbol{\gamma}\right)^{\top}\right]. \end{aligned} \tag{46}$$

LaFrance et. al. (2005, 2006) show that $1-\boldsymbol{\delta}^{\top}\mathbf{g}\left[\left(y-\eta\right)/\varphi\right]>0$, $y-\eta<0$, $\varphi>0$, and $\mathbf{B}=\mathbf{L}\mathbf{L}^{\top}+\boldsymbol{\gamma}\boldsymbol{\gamma}^{\top}$, where $\mathbf{L}$ is lower triangular, are necessary and sufficient for the matrix $\mathbf{H}$ (and thus $\breve{\mathbf{H}}$ due to adding up)

to be symmetric, negative semidefinite in an open neighborhood of $\kappa = \lambda = 1$. Note that for $\kappa = \lambda = 1$, the first and third matrices in expression (46) vanish, in which case the above conditions are necessary and sufficient for $\partial^2 y/\partial \mathbf{g} \partial \mathbf{g}^\top$ (and correspondingly $\mathbf{H}$ and $\breve{\mathbf{H}}$) to be symmetric, negative semidefinite. While $\mathbf{B} = \mathbf{L}\mathbf{L}^\top + \boldsymbol{\gamma}\boldsymbol{\gamma}^\top$ is sufficient for symmetry of $\breve{\mathbf{H}}$ for all values of κ and λ, the conditions above are not sufficient for negative semidefiniteness for values of κ and λ outside of the open neighborhood and less than 1.

Since sufficient conditions for negative semidefiniteness across all values of $\kappa, \lambda \in [0, 1]$ are not apparent, the econometric model discussed below proceeds without imposing negative semidefiniteness. This and the positive demands property are checked after estimation using in-sample data and the findings are discussed in the results section. Note that homogeneity of degree 0 has been directly imposed in (43), and the remaining requirement for integrability of the demand system, symmetry of $\breve{\mathbf{H}}$, is also directly imposed using $\mathbf{B} = \mathbf{L}\mathbf{L}^\top + \boldsymbol{\gamma}\boldsymbol{\gamma}^\top$ (discussed below).

4.2 Econometric Structure and Estimation

Let $i = 1, ..., I$ index states, $j = 1, ..., J$ index equations, and $t = 1, ..., T$ index time. The $J - 1$ vector of per-acre variable inputs is written in deflated expenditure format as

$$\begin{aligned}
&\tilde{\mathbf{e}}_{i,t} \equiv \tilde{\mathbf{w}}_{i,t}^\top \mathbf{q}_{i,t} \\
&= \tilde{c}_{i,t}^{1-\kappa} \boldsymbol{\Delta}\left(\tilde{w}_{i,j,t}^{\lambda}\right) \left\{ \boldsymbol{\alpha}_{0,i} + \boldsymbol{\alpha}_1 k_{i,t} + \left[1 - \boldsymbol{\delta}^\top \mathbf{g}_{i,t} \left(\frac{f\left(\tilde{c}_{i,t}\right) - \eta_i\left(\mathbf{g}_{i,t}, \tilde{k}_{i,t}\right)}{\varphi\left(\mathbf{g}_{i,t}\right)} \right) \right] \right. \\
&\left. \left(\frac{f\left(\tilde{c}_{i,t}\right) - \eta_i\left(\mathbf{g}_{i,t}, \tilde{k}_{i,t}\right)}{\varphi\left(\mathbf{g}_{i,t}\right)} \right) \left(\mathbf{B}\mathbf{g}_{i,t} + \boldsymbol{\gamma}\right) + \frac{\left[f\left(\tilde{c}_{i,t}\right) - \eta_i\left(\mathbf{g}_{i,t}, \tilde{k}_{i,t}\right) \right]^2}{\varphi\left(\mathbf{g}_{i,t}\right)} \boldsymbol{\delta} \right\},
\end{aligned} \tag{47}$$

where

$$\begin{aligned}
&f\left(\tilde{c}_{i,t}\right) = \frac{\tilde{c}_{i,t}^{\kappa} + \kappa - 1}{\kappa}, \tilde{c}_{i,t} = \frac{c_{i,t}}{w_{i,J,t}}, g_{i,j,t}\left(\tilde{w}_{i,j,t}\right) = \frac{\tilde{w}_{i,j,t}^{\lambda} + \lambda - 1}{\lambda}, \\
&\tilde{\mathbf{w}}_{i,t}\left(\mathbf{w}_{i,t}, w_{i,J,t}\right) = \frac{1}{w_{i,J,t}} \mathbf{w}_{i,t}, \mathbf{w}_{i,t} = \left[w_{i,1,t}, ..., w_{i,J-1,t}\right]^\top, \tilde{k}_{i,t} = \frac{k_{i,t}}{w_{i,J,t}}, \\
&\eta_i\left(\mathbf{g}_{i,t}, \tilde{k}_{i,t}\right) = \left(\boldsymbol{\alpha}_{0,i} + \boldsymbol{\alpha}_1 \tilde{k}_{i,t}\right)^\top \mathbf{g}_{i,t} + \alpha_{0,J,i} + \alpha_{1,J} \tilde{k}_{i,t}, \boldsymbol{\alpha}_{0,i} = \left[\alpha_{0,1,i}, ..., \alpha_{0,J-1,i}\right]^\top, \\
&\boldsymbol{\alpha}_1 = \left[\alpha_{1,1}, ..., \alpha_{1,J-1}\right]^\top, \text{and } \varphi\left(\mathbf{g}_{i,t}\right) = \mathbf{g}_{i,t}^\top \mathbf{B} \mathbf{g}_{i,t} + 2\boldsymbol{\gamma}^\top \mathbf{g}_{i,t} + 1.
\end{aligned} \tag{48}$$

The only restriction imposed on the parameters during estimation is that the matrix of second-order price effects,

$$\mathbf{B}^* = \begin{bmatrix} \mathbf{B} & \boldsymbol{\gamma} \\ \boldsymbol{\gamma}^\top & 1 \end{bmatrix}, \tag{49}$$

is positive semidefinite. Lemma 5 in the Appendix of LaFrance et. al. (2005) proves that a sufficient condition for this is that

$$\mathbf{B} = \mathbf{L}\mathbf{L}^\top + \boldsymbol{\gamma}\boldsymbol{\gamma}^\top, \tag{50}$$

where $\mathbf{L}\mathbf{L}^\top$ is a (possibly reduced rank) Choleski factorization

$$\mathbf{L} = \begin{bmatrix} l_{1,1} & 0 & 0 \\ \vdots & \ddots & 0 \\ l_{J-1,1} & \cdots & l_{J-1,J-1} \end{bmatrix}. \tag{51}$$

Thus, the submatrix of second-order price effects $\mathbf{B}$ is replaced with $\mathbf{L}\mathbf{L}^\top + \boldsymbol{\gamma}\boldsymbol{\gamma}^\top$ everywhere in the right-hand side of (47).

The estimator developed in Ball et. al. (2008) is used to estimate the incomplete system of demand equations. This estimator includes a 3-dimensional error covariance matrix (across states, equations, and time) and accounts for the fact that quasi-fixed inputs, expected/planned outputs, and variable input prices are all almost certainly jointly determined with the variable input demands.

Following Ball et. al. (2008), the $N = J - 1$ state-level variable input demand equations are

$$\tilde{e}_{ijt} = f_{ijt}\left(\tilde{\mathbf{w}}_{it}, \tilde{k}_{it}, \tilde{c}_{it}; \boldsymbol{\theta}\right) + u_{ijt}, i = 1, ..., I, j = 1, ..., N, t = 1, ..., T \tag{52}$$

where $\tilde{\mathbf{w}}_{it}$ is the $N \times 1$ vector of normalized input prices, $\tilde{k}_{it}$ is normalized capital per acre, $\tilde{c}_{it}$ is normalized variable cost per acre, $\boldsymbol{\theta}$ is a $K \times 1$ vector of parameters to be estimated, and u_{ijt} is a mean zero random error term. Suppose the errors are intertemporally correlated,

$$u_{ijt} = \sum_{j'=1}^{N} \phi_{jj'} u_{ij't-1} + v_{ijt},\ i = 1, ..., I,\ j = 1, ..., N,\ t = 1, ..., T \tag{53}$$

while the mean zero random variables v_{ijt} are uncorrelated across time, but correlated across inputs within each state, $E\left(\mathbf{v}_{i\cdot t}\mathbf{v}_{i\cdot t}'\right) = \boldsymbol{\Sigma}_i$, $\mathbf{v}_{i\cdot t} = [v_{i1t}, ..., v_{iNt}]'$. Let $\boldsymbol{\Sigma}_i^{-1} = \mathbf{L}_i\mathbf{L}_i'$ be a lower triangular Choleski factorization of the ith state's $N \times N$ inverted covariance matrix. Then the typical element of $\boldsymbol{\varepsilon}_{i\cdot t} = \boldsymbol{\Sigma}_i^{-1/2}\mathbf{v}_{i\cdot t} = \mathbf{L}_i'\mathbf{v}_{i\cdot t}$ is $\varepsilon_{ijt} = \sum_{j'=1}^{N} l_{ijj'} v_{ij't}$. The mean zero, unit variance random variables, ε_{ijt}, now are uncorrelated across

inputs and time, but are assumed to be correlated across states depending on how far apart the states are from each other. That is, $E\left(\varepsilon_{ijt}\varepsilon_{ij't}\right) = \rho\left(d_{ii'}\right)$, $j = 1, ..., J$, where $d_{ii'}$ is the geographic distance between states i and i'. The $I \times I$ matrix,

$$R = \begin{bmatrix} 1 & \rho\left(d_{12}\right) & & \rho\left(d_{1I}\right) \\ \rho\left(d_{12}\right) & 1 & & \\ \vdots & \vdots & \ddots & \\ \rho\left(d_{1I}\right) & \rho\left(d_{2I}\right) & & 1 \end{bmatrix}, \tag{54}$$

is symmetric, positive definite, and for simplicity, is assumed to be constant across j.

Consistent estimation and inferences are made using the following semi-parametric GMM estimator. Let $\mathbf{Z}$ denote the $T \times N_Z$ matrix of instruments common across states and let $\mathbb{N} = \mathbf{Z}\left(\mathbf{Z}'\mathbf{Z}\right)^{-1}\mathbf{Z}'$ denote the associated $T \times T$ projection matrix. First, stack (52) by equations and time, and use nonlinear two-stage least squares (NL2SLS) to estimate $\boldsymbol{\theta}$ consistently,

$$\hat{\boldsymbol{\theta}}_{2SLS} = \arg\min_{\boldsymbol{\theta}} \sum_{i=1}^{I} \mathbf{u}_{i\cdot\cdot}^{\top}\left(\mathbb{N} \otimes \mathbf{I}_N\right)\mathbf{u}_{i\cdot\cdot}, \tag{55}$$

where

$$\mathbf{u}_{i\cdot\cdot} = \tilde{\mathbf{e}}_{i\cdot\cdot} - \mathbf{f}_{i\cdot\cdot}\left(\tilde{\mathbf{w}}_{i\cdot}, \tilde{k}_{i\cdot}, \tilde{c}_{i\cdot}; \boldsymbol{\theta}\right) \tag{56}$$

and $\mathbf{I}_N$ is an $N \times N$ identity matrix. This consistent estimator of $\boldsymbol{\theta}$ is then used to generate consistent estimates of the errors,

$$\hat{u}_{ijt} = \tilde{e}_{ijt} - f_{ijt}\left(\tilde{\mathbf{w}}_{it}, \tilde{k}_{it}, \tilde{c}_{it}; \hat{\boldsymbol{\theta}}_{2SLS}\right), \; i = 1, ..., I, \; j = 1, ..., N, \; t = 1, ..., T. \tag{57}$$

Second, for $t = 2, ...T$, estimate the $N \times N$ intertemporal correlation matrix, $\boldsymbol{\Phi}$, by linear seemingly unrelated regression (SUR) using the identity matrix $\mathbf{I}_N$ as the weighting matrix,

$$\hat{\boldsymbol{\Phi}} = \arg\min_{\boldsymbol{\Phi}} \sum_{i=1}^{I}\sum_{t=2}^{T}\left(\hat{\mathbf{u}}_{i\cdot t} - \boldsymbol{\Phi}\hat{\mathbf{u}}_{i\cdot t-1}\right)^{\top}\mathbf{I}_N\left(\hat{\mathbf{u}}_{i\cdot t} - \boldsymbol{\Phi}\hat{\mathbf{u}}_{i\cdot t-1}\right). \tag{58}$$

Third, construct consistent estimates of the spatially correlated error terms,

$$\hat{\varepsilon}_{ijt} = \sum_{j'=1}^{N} \hat{l}_{ijj'}\hat{v}_{ij't}, \tag{59}$$

where $\hat{v}_{ijt} = \hat{u}_{ijt} - \sum_{j'=1}^{N} \hat{\phi}_{jj'} \hat{u}_{ij't-1}$ and $\hat{\mathbf{L}}_i = \left[\hat{l}_{ijj'}\right]_{j,j'=1,\ldots,N}$ satisfies $\hat{\mathbf{\Sigma}}_i^{-1} = \hat{\mathbf{L}}_i \hat{\mathbf{L}}_i^{\top}$. Then calculate consistent sample estimates for the across-state spatial correlations as,

$$\hat{\rho}_{ii'} = \sum_{j=1}^{N} \sum_{t=2}^{T} \frac{\hat{\varepsilon}_{ijt} \hat{\varepsilon}_{ij't}}{N(T-1)}, i, i' = 1, \ldots, I. \tag{60}$$

Then use the $\frac{1}{2}I(I-1)$ spatial correlations to estimate the relationship between the spatial correlations and the geographic distance between states using robust nonlinear least squares to obtain $\hat{\mathbf{R}} = [\hat{\rho}(d_{ii'})]$. A third-order exponential specification for the correlation function,

$$\hat{\rho}(d_{ii'}) = \exp\left(\eta_0 + \sum_{k=1}^{N} \eta_k d_{ii'}^k\right). \tag{61}$$

Fourth, let $\hat{\mathbf{R}}^{-1} = \mathbf{Q}\mathbf{Q}^{\top}$, where $\mathbf{Q}$ is a lower triangular Choleski factorization of the inverse spatial correlation matrix, and write $\omega_{ijt} = \sum_{i'=1}^{I} q_{ii'} \varepsilon_{i'jt}$. Now the random variables are mean zero, unit variance, and uncorrelated across equations, states, and time. Replacing the unknown parameters and error terms with the consistent estimates developed with the above estimation steps, and substituting backwards recursively yields

$$\begin{aligned}
\hat{\omega}_{ijt} &= \sum_{i'=1}^{I} \hat{q}_{ii'} \hat{\varepsilon}_{i'jt} \\
&= \sum_{i'=1}^{I} \hat{q}_{ii'} \sum_{j'=1}^{N} \hat{l}_{i'jj'} \hat{v}_{i'j't} \\
&= \sum_{i'=1}^{I} \hat{q}_{ii'} \sum_{j'=1}^{N} \hat{l}_{i'jj'} \left(\hat{u}_{i'j't} - \sum_{j''=1}^{N} \hat{\phi}_{j'j''} \hat{u}_{i'j''t-1}\right) \\
&\xrightarrow{P} \omega_{ijt},
\end{aligned} \tag{62}$$

with $E(\omega_{ijt}) = 0$, $E\left(\omega_{ijt}^2\right) = 1$, and $E(\omega_{ijt}\omega_{i'j't'}) = 0$ for $(i,j,t) \neq (i',j',t')$. A final NL3SLS step of the form,

$$\hat{\boldsymbol{\theta}}_{3SLS} = \arg\min_{\boldsymbol{\theta}} \sum_{i=1}^{I} \left[\hat{\boldsymbol{\omega}}_{i\cdot\cdot}\left(\tilde{\mathbf{w}}_{i\cdot}, \tilde{k}_{i\cdot}, \tilde{c}_{i\cdot}; \boldsymbol{\theta}\right)\right]^{\top} (\mathbb{N} \otimes \mathbf{I}_N) \left[\hat{\boldsymbol{\omega}}_{i\cdot\cdot}\left(\tilde{\mathbf{w}}_{i\cdot}, \tilde{k}_{i\cdot}, \tilde{c}_{i\cdot}; \boldsymbol{\theta}\right)\right], \tag{63}$$

gives consistent, efficient, asymptotically normal estimates of $\boldsymbol{\theta}$. White's heteroskedasticity consistent covariance estimator is used for robustness to heteroskedasticity beyond the state specific, across equation covariance matrices.

4.3 Results

4.3.1 Data

The data used to estimate the model is described in Section 3.1. It is important to note that labor is omitted from the system, so an incomplete system of demands is estimated for energy, agricultural chemicals, and other materials. As discussed in 3.1, two period lags of the variable cost, capital, and price variables are included in the instrument set. This leads to omission of the first two years, 1960 and 1961, from the empirical sample. Thus, 646 ($= 17 * 38$) observations on 17 states from 1962-1998 are used in the final NL3SLS step.

4.3.2 Structural Breaks

A recent paper, Gutierrez et. al. (2007), investigates the role that structural breaks in the agricultural economy play in finding a stable cointegration relationship between farmland prices and rents. The authors utilize panel data for 31 U.S. states between 1960 and 2000 and find that all states have at some point been subject to breaks.

Empirically, evidence of structural breaks is found by testing whether the model's parameters are stable across time. There are many diagnostic procedures for testing parameter stability (e.g. Brown et. al., 1975, and Ploberger and Krämer, 1992), however a testing procedure presented in LaFrance (2008b) is better suited for this analysis since it was developed for large nonlinear simultaneous equation systems with a small sample size.

The procedures presented here are attributed to LaFrance (2008b). Let $\hat{\varepsilon}_{jt}$ denote the jth equation's estimated residual for period t and $\hat{\sigma}_j^2 = \sum_{t=1}^{T} \hat{\varepsilon}_{jt}^2 / T$ denote estimated variance of the residual for the jth equation. Define the test statistic

$$Q_{jT} = \sup_{z \in [0,1]} |B_{jT}(z)| \tag{64}$$

where

$$B_{jT}(z) \equiv \frac{1}{\sqrt{T}\hat{\sigma}_j} \sum_{t=1}^{zT} \left(\hat{\varepsilon}_{jt} - \bar{\hat{\varepsilon}}_j\right) \xrightarrow{D} B(z). \tag{65}$$

Here, $z \in [0,1]$ represents the break point, $[zT]$ is the largest integer that does not exceed zT, $\bar{\hat{\varepsilon}}_j$ is the sample mean for $\hat{\varepsilon}_{j1}, ..., \hat{\varepsilon}_{jT}$, and $B(z)$ is a standard Brownian bridge. The statistic Q_{jT} is a single equation first-order parameter stability statistic, and the corresponding system-wide first-order parameter stability statistic is defined by

$$Q_T = \sup_{z \in [0,1]} |B_T(z)| \tag{66}$$

where

$$B_T(z) \equiv \frac{1}{\sqrt{T}} \sum_{t=1}^{zT} \left[\frac{1}{\sqrt{N}} \sum_{j=1}^{N} \left(\hat{\xi}_{jt} - \overline{\hat{\xi}} \right) \right] \xrightarrow{D} B(z). \tag{67}$$

Here, the $\hat{\xi}_{jt}$ are elements of the tth estimated standardized error vector $\hat{\boldsymbol{\xi}}_t = \hat{\boldsymbol{\Sigma}}^{-1/2}\hat{\boldsymbol{\varepsilon}}_t$, and $\overline{\hat{\xi}} = \sum_{t=1}^{T}\sum_{j=1}^{J}\hat{\xi}_{jt}/JT$. Both test statistics have an asymptotic 5% critical value of 1.36 (Ploberger and Krämer, 1992).

These test statistics are used to test for parameter stability in the estimated model and the results are given under Model A in Table 3. Note that the null hypothesis of parameter stability is rejected at the 5% significance level a total of eight times across the three equations, and another three times for the overall system. These findings suggest parameter instability is a problem that needs to be addressed.

Since all 17 states that are used in this analysis are a subset of the states analyzed in Gutierrez et. al. (2007), their findings are used to guide the parameter instability issues. Looking at their Table 2, there is considerable evidence of structural breaks around the years 1973 and 1986, which are the starting and ending years of the notorious boom/bust period of agricultural land prices.[16] The authors note that the U.S. agricultural economy experienced oil price shocks, an unusually large farm income following the growth of agricultural exports due to devaluation of the dollar, and bad weather conditions in competing production regions overseas in the mid 1970's, and experienced increased uncertainty in expected returns on farmland investments, high real interest rates, and low commodity prices during the second half of the 1980's.

To account for these findings, additional parameters are included in the demand system specification given by (4.14) above. Specifically, the $\boldsymbol{\alpha}_{0,i}$ vector is now specified as

$$\boldsymbol{\alpha}_{0,i} = \left[\alpha_{0,1,i} + \tau_{i,1,73}D73 + \tau_{i,1,86}D86, ..., \alpha_{0,J-1,i} + \tau_{i,J-1,73}D73 + \tau_{i,J-1,86}D86\right]^{\top}, \tag{68}$$

and $\eta_i\left(\mathbf{g}_{i,t}, \tilde{k}_{i,t}\right)$ is now

$$\eta_i\left(\mathbf{g}_{i,t}, \tilde{k}_{i,t}\right) = \left(\boldsymbol{\alpha}_{0,i} + \boldsymbol{\alpha}_1\tilde{k}_{i,t}+\right)^{\top}\mathbf{g}_{i,t} + \alpha_{0,J,i} + \alpha_{1,J}\tilde{k}_{i,t} + \tau_{i,J,73}D73 + \tau_{i,J,86}D86, \tag{69}$$

where the τ's are parameters to be estimated and the dummy variables $D73$ and $D86$ are defined as

$$D73 = \begin{cases} 1 \text{ if } t \leq 1973 \\ 0 \text{ otherwise} \end{cases} \text{ and } D86 = \begin{cases} 1 \text{ if } t \leq 1986 \\ 0 \text{ otherwise} \end{cases}. \tag{70}$$

Note that the τ's are included in a flexible way, as they are not restricted to be equal across either states or

[16] Of the 31 total states studied in Gutierrez et. al. (2007), over half registered a break within ±2 years of 1973 and nearly two thirds registered a break within ±2 years of 1986. Of the 17 states included in this study, over half registered a break within ±2 years of 1973 and all but 2 registered a break within ±2 years of 1986.

equations.

Looking again at Table 3, the results of the parameter stability tests using this modified specification are given under Model B. Note that the null hypothesis of parameter stability fails to be rejected across all equations and for the system as a whole at a 5% significance level. In addition, state-specific Wald tests for null hypotheses of the form

$$H_0 : \tau_{i,1,73} = ... = \tau_{i,J,73} = \tau_{i,1,86} = ... = \tau_{i,J,86} = 0 \tag{71}$$

support rejection of the null for all but three states at the 1% significance level and all but two states at the 5% significance level. These results suggest inclusion of the structural change parameters, so the following results are reported for Model B.

4.3.3 Error Term Properties

The estimated 3×3 intertemporal autocorrelation matrix, with White/Huber robust asymptotic standard errors in parentheses and ***, **, and * denoting statistical significance at the 1%, 5%, and 10% levels respectively, is:

$$\hat{\mathbf{\Phi}} = \begin{bmatrix} \underset{(.0669952)^{**}}{.1458964} & \underset{(.2018368)^{*}}{.3519273} & \underset{(.1053752)^{*}}{-.1875367} \\ \underset{(.0154423)}{.0119240} & \underset{(.0520242)^{***}}{.5614300} & \underset{(.0357891)}{.0158372} \\ \underset{(.0346364)^{**}}{.0720567} & \underset{(.1092367)^{**}}{-.2588840} & \underset{(.0850879)^{***}}{.4656813} \end{bmatrix}. \tag{72}$$

Rows and columns 1 through 3 are the other materials, energy, and agricultural chemicals equations. Note that neither symmetry nor positive semidefiniteness have been directly been imposed on $\mathbf{\Phi}$, both of which are common (yet perhaps overly restrictive) assumptions when modeling singular AR(1) systems (see Holt, 1998 for examples). The basis for including the fully unrestricted $\mathbf{\Phi}$ is to increase flexibility of the system, and the properties of this approach are discussed in LaFrance (2008a). The parameter estimates imply positive semidefiniteness and stable dynamics, as the Eigen values for the 4×4 difference equation are all non-negative and less than 1 (largest Eigen value is 0.55). The F-test that all parameter estimates are jointly zero is firmly rejected at the 1% significance level, and the Durbin-Watson statistics do not suggest higher order serial correlation (average Durbin-Watson statistics across states for each equation are 1.848, 1.853, and 1.899).

The estimated spatial correlation function, with White/Huber robust standard errors in parentheses, is:

$$\hat{\rho}\left(d_{ii'}\right) = \exp\left(\underset{(.303)}{-.303} - \underset{(.161 \times 10^{-2})^{**}}{.342 \times 10^{-2} d_{ii'}} + \underset{(.252 \times 10^{-5})^{*}}{.442 \times 10^{-5} d_{ii'}^2} - \underset{(.120 \times 10^{-8})^{*}}{.212 \times 10^{-8} d_{ii'}^3} \right), \tag{73}$$

where i and i' index states $1, ..., 17$. A Wald test for the joint hypothesis that all parameters are equal to zero yields a *p-value* of 0.000, suggesting that spatial correlation across states is prevalent in the system. A 2-dimensional plot of the empirical data, estimated correlation function, and 95% confidence band are presented in Figure 1. Note that the predicted correlation decreases steadily, flattens out, then decreases again as distance increases. The predicted correlation remains strictly positive throughout the sample, suggesting that the error of states as far as $1,387$ miles away (largest distance in the sample) remain positively correlated.

4.3.4 Parameter Estimates

Recall that the submatrix of second-order price effects $\mathbf{B}$ was replaced with $\mathbf{L}\mathbf{L}^\top + \boldsymbol{\gamma}\boldsymbol{\gamma}^\top$ during estimation. The Choleski factor $\mathbf{L}$ was found to be reduced rank, as the lower right diagonal element $l_{3,3}$ could not be identified empirically. During estimation, this parameter was held fixed at 0.03 as in LaFrance (2008b). The parameter estimates $\hat{\mathbf{L}}$ and $\hat{\boldsymbol{\gamma}}$ are

$$\hat{\mathbf{L}} = \begin{bmatrix} .08784 & 0 & 0 \\ -.1263 & .03114 & 0 \\ -.01686 & -.09076 & 0.03 \end{bmatrix} \text{ and } \hat{\boldsymbol{\gamma}} = \begin{bmatrix} .07662 \\ -.005905 \\ -.007685 \end{bmatrix}, \tag{74}$$

which generate an estimate of the full 4×4 symmetric matrix of second-order price effects,

$$\begin{aligned} \hat{\mathbf{B}}^* &= \begin{bmatrix} \hat{\mathbf{L}}\hat{\mathbf{L}}^\top + \hat{\boldsymbol{\gamma}}\hat{\boldsymbol{\gamma}}^\top & \hat{\boldsymbol{\gamma}} \\ \hat{\boldsymbol{\gamma}}^\top & 1 \end{bmatrix} \\ &= \begin{bmatrix} .01358 & -.01155 & -.002070 & .07662 \\ -.01155 & .01697 & -.0006495 & -.005905 \\ -.002070 & -.0006495 & .008681 & -.007685 \\ .07662 & -.005905 & -.007685 & 1 \end{bmatrix}. \end{aligned} \tag{75}$$

The Eigen values of $\hat{\mathbf{B}}^*$ are all positive, the smallest being 3.43×10^{-6}, and a Wald test for the null hypothesis that the nine lower diagonal elements of $\hat{\mathbf{B}}^*$ are jointly zero is rejected at the 1% significance level.

Table 4 presents the estimates of the κ, λ, $\alpha_{1,1}$-$\alpha_{1,4}$ and δ_1-δ_4 parameters. Perhaps the most interesting hypotheses with regard to functional form have to do with the transformations of prices and variable costs. The viability of the industry standards of logarithmic or linear transformations are investigated by testing the following null hypotheses: (1) linear-linear, $H_0 : \kappa = \lambda = 1$, $\chi^2(2) = 11.03$, *p-value* $= 0.004$; (2) log-log, $H_0 : \kappa = \lambda = 0$, $\chi^2(2) = 291.5$, *p-value* $= 0.000$; (3) log-linear, $H_0 : \kappa = 0, \lambda = 1$, $\chi^2(2) = 1,389$,

p-value $= 0.000$; and (4) linear-log, $H_0 : \kappa = 1, \lambda = 0,$, $\chi^2(2) = 826.8$, *p-value* $= 0.000$. All four null hypotheses are rejected at the 1% significance level. Note that even though the singular null hypothesis $\lambda = 0$ has *p-value* $= 0.149$ (from Table 4), the log-log and linear-log transformations are not supported by the data.

At first look, the extension to rank three appears to seriously over fit this data, as evidenced by the high *p-values* for three of the δ estimates. However, a Wald test and a GMM-LR test (i.e. Newey and West's, 1987, D Statistic) for the joint significance of these coefficients produces different results. For the null hypothesis $H_0 : \boldsymbol{\delta} = \mathbf{0}$, the Wald statistic is 3.48 (*p-value* $= 0.479$) while the GMM-LR statistic is $1,216$ (*p-value* $= 0.000$).[17] It is not clear which of these tests is preferred, but the GMM-LR test does support the rank 3 extension.

Recall that the *ex-ante* cost function was assumed to include both land and capital as quasi-fixed inputs, and to be homogeneous of degree 1 in land, capital, and expected output (i.e. constant returns to scale). These assumptions were used to derive a per acre cost function by dividing cost by total farmland, and implied that per acre cost is a function of capital per acre. If these assumptions are correct, an implication of the assumptions is that per acre variable input demands are a function capital per acre as well. Empirically, this amounts to testing the null hypothesis $H_0 : \alpha_{1,1} = \alpha_{1,2} = \alpha_{1,3} = \alpha_{1,4} = 0$. As with the rank 3 parameters, both a Wald and GMM-LR test were conducted. Both tests reject the null at standard significance levels as the Wald statistic is 11.34 (*p-value* $= 0.022$) and the GMM-LR statistic is $1,558$ (*p-value* $= 0.000$).[18] Both tests support the inclusion of capital per acre in the demand model, which is taken as evidence in favor of the quasi-fixed input and constant returns to scale assumptions.

The remaining parameter estimates are the $\hat{\alpha}_{0,j,i}$ (recall that the $\hat{\tau}$'s were reported above). While there are far too many estimates to report here, it should be noted that state-specific Wald tests for null hypotheses of the form $H_0 : \alpha_{0,1,i} = ... = \alpha_{0,4,i} = 0$ reject the null for all but five states at a 1% significance level, all but two states at a 5% significance level, and all but one state at a 10% significance level. These results suggest inclusion of the $\alpha_{0,j,i}$ parameters.

The across-state averaged Durbin-Watson statistics do not suggest remaining serial correlation (averages

[17] For the GMM-LR test, the citerion

$$q_N \equiv \sum_{i=1}^{I} \left[\hat{\boldsymbol{\omega}}_{i\cdot\cdot}\left(\cdot;\hat{\boldsymbol{\theta}}_{3SLS}\right)\right]^{\top} (\mathbb{N} \otimes \mathbf{I}_N) \left[\hat{\boldsymbol{\omega}}_{i\cdot\cdot}\left(\cdot;\hat{\boldsymbol{\theta}}_{3SLS}\right)\right]$$

increased from 411 (unrestricted model) to 443 (restricted model). Thus resulting in the large test statistic $1,216$ $(= 38 * (443 - 411))$.

[18] For the GMM-LR test, the citerion

$$q_N \equiv \sum_{i=1}^{I} \left[\hat{\boldsymbol{\omega}}_{i\cdot\cdot}\left(\cdot;\hat{\boldsymbol{\theta}}_{3SLS}\right)\right]^{\top} (\mathbb{N} \otimes \mathbf{I}_N) \left[\hat{\boldsymbol{\omega}}_{i\cdot\cdot}\left(\cdot;\hat{\boldsymbol{\theta}}_{3SLS}\right)\right]$$

increased from 411 (unrestricted model) to 452 (restricted model). Thus resulting in the large test statistic $1,558$ $(= 38 * (452 - 411))$.

for each equation are 1.972, 1.855, and 1.722). Testing for mean zero residuals for each state and equation is conducted as in LaFrance (2008b), where

$$z_{ij} = \frac{\sqrt{T}\hat{\varepsilon}_{ij}}{\hat{\sigma}_{ij}} \sim^{a} n(0,1) \tag{76}$$

is an asymptotic test statistic for mean zero residuals for each state and equation. All of the tests fail to reject the null of mean zero residuals at a 1% significance level (highest test statistic is 0.459).

Since zero degree homogeneity in prices and Hessian symmetry were directly imposed during estimation, the remaining integrability conditions that need to be checked are positive demands and Hessian negative semidefiniteness. Using the model's parameter estimates, predicted demands for all four variable inputs are found to be strictly positive across all states, equations, and years in the sample. Negative semidefiniteness of the complete 4×4 Hessian matrix $\breve{\mathbf{H}}$ was checked by calculating the Eigen values for each 3×3 Hessian submatrix $\hat{\mathbf{H}}_{it}$, the predicted Hessian submatrix for each state in each year.[19] For the $\hat{\mathbf{H}}_{1,1960}, ..., \hat{\mathbf{H}}_{17,1999}$, 89% of the Eigen values are negative, which implies that the assumption of negative semidefiniteness is consistent with the empirical model for a large majority of the data points. In conjunction with the evidence supporting parameter stability, no remaining serial correlation, and mean zero residuals presented above, these findings suggest that this is a reasonable and coherent model of variable input demands that is consistent with economic theory.

5 Arbitrage System Estimation

The purpose of this section is to estimate the following system of arbitrage system equations that were derived in Section 2,

$$E_t\left[m_{i,t+1}e_{i,j,t+1}\right] = 0, i = 1, ..., I \;\; j = L, K, C, O, S, W, F \;\; t = 1, ..., T \tag{77}$$

where $m_{i,t+1}$ is the intertemporal marginal rate of substitution of wealth for farmers in state i across periods t and $t+1$, and $e_{i,j,t+1} = r_{i,j,t+1} - r$ is the excess return for farmers in state i on asset j realized at the end of the period. The equation indexes stand for farmland (L), capital (K), corn (C), oats (O), soybeans (S), wheat (W), and the risky financial instrument (F). All farm-related rates of return, i.e. $r_{i,L,t+1}$, $r_{i,K,t+1}$, $r_{i,C,t+1}$, $r_{i,O,t+1}$, $r_{i,S,t+1}$, and $r_{i,W,t+1}$, are state specific and constructed using aggregate data. The rates of return on stocks and bonds are assumed to be exogenous to the farm sector, thus they are constant across

[19] For each $\breve{\mathbf{H}}_{it}$, exactly one Eigen value is necessarily zero due to adding up. Thus, the non-zero Eigen values associated with the submatrix $\mathbf{H}_{it}$ determine the definiteness of $\breve{\mathbf{H}}_{it}$.

states and given by $r_{F,t+1}$ and r.

5.1 Aggregation

Since the rates of return for farmland, capital, and crop shares are constructed using state-level data, an aggregation over farm-level micro data, it is important to understand how this issue affects the equilibrium arbitrage conditions (77). Following the illustration of aggregation bias in Pope et. al. (2009), consider an arbitrary arbitrage condition for farm h in state i for asset j in period t,

$$E_t \left[m \left(\mathbf{x}_{h,i,t+1} \right) e \left(\mathbf{y}_{h,i,j,t+1} \right) \right] = 0. \tag{78}$$

The intertemporal marginal rate of substitution is written as a function of an arbitrary data vector $\mathbf{x}_{h,i,t+1}$ and the the excess return is a function of an arbitrary data vector $\mathbf{y}_{h,i,j,t+1}$ for ease of exposition. Summing over the H_j farms in state j yields the implications of first-order conditions at the state level,

$$E_t \left[\sum_{h=1}^{H_j} \frac{1}{H_j} m \left(\mathbf{x}_{h,i,t+1} \right) e \left(\mathbf{y}_{h,i,j,t+1} \right) \right] = 0. \tag{79}$$

This is not the condition imposed by a representative household approach with average state-level data, $E_t \left[m \left(\bar{\mathbf{x}}_{i,t+1} \right) e(\bar{\mathbf{y}}_{i,j,t+1}) \right] = 0$, where overbars denote averaging across farms within state i. However, suppose that state-level excess returns are related to farm excess returns by $e(\mathbf{y}_{h,i,j,t+1}) = e(\bar{\mathbf{y}}_{i,j,t+1}) + u_{h,i,j,t+1}$, and that the farm rate of substitution is related to the state-level average by $m \left(\mathbf{x}_{h,i,t+1} \right) = m \left(\bar{\mathbf{x}}_{i,t+1} \right) + v_{h,i,t+1}$. Then using state-level average data yields

$$\begin{aligned} E_t \left[m \left(\mathbf{x}_{h,i,t+1} \right) e(\mathbf{y}_{h,i,j,t+1}) \right] &= E_t \left\{ \left[m \left(\bar{\mathbf{x}}_{i,t+1} \right) + v_{h,i,t+1} \right] \left[e(\bar{\mathbf{y}}_{i,j,t+1}) + u_{h,i,j,t+1} \right] \right\} \\ &= E_t [m \left(\bar{\mathbf{x}}_{i,t+1} \right) e(\bar{\mathbf{y}}_{i,j,t+1})] + E_t \left[m \left(\bar{\mathbf{x}}_{i,t+1} \right) u_{h,i,j,t+1} \right] \\ &\quad + E_t [v_{h,i,t+1} e(\bar{\mathbf{y}}_{i,j,t+1})] + E_t [v_{h,i,t+1} u_{h,i,j,t+1}]. \end{aligned} \tag{80}$$

The authors note that none of the last three terms on the last line need vanish in general, and that it is appropriate to include fixed effects of the form

$$E_t [m \left(\bar{\mathbf{x}}_{i,t+1} \right) e(\bar{\mathbf{y}}_{i,j,t+1})] - \left(\alpha_i + \phi_j \right) = 0, \tag{81}$$

where the α_i and ϕ_j are parameters to be estimated. Since a representative agent approach is taken in the following application, fixed effects of this form are included in the econometric specification.

5.2 Empirical Specifications

The empirical counterparts to the arbitrage equations given by (77) above are derived using specifications for the marginal value of wealth function, the per acre variable cost function, the adjustment cost function, and an assumption on the expectations process.

The specification for the marginal value of wealth for state i is given by

$$\frac{\partial V\left(W_t, A_{t-1}, k_{t-1}; \cdot\right)}{\partial W} = 1 - \beta_W W_t - \beta_{WL} A_{t-1} - \beta_{WK} k_{t-1}, \tag{82}$$

where β_W, β_{WL}, and β_{WK} are parameters to be estimated. This specification is consistent with a quadratic approximation of the value function in (W_t, A_{t-1}, k_{t-1}),

$$\begin{aligned} V\left(W_t, A_{t-1}, k_{t-1}; \cdot\right) &= \alpha_0\left(\cdot\right) + \begin{bmatrix} 1 & \alpha_L & \alpha_K \end{bmatrix} \begin{bmatrix} W_t \\ A_{t-1} \\ k_{t-1} \end{bmatrix} \\ &- \frac{1}{2} \begin{bmatrix} W_t & A_{t-1} & k_{t-1} \end{bmatrix} \begin{bmatrix} \beta_W & \beta_{WL} & \beta_{WK} \\ \beta_{WL} & \beta_L & \beta_{LK} \\ \beta_{WK} & \beta_{LK} & \beta_K \end{bmatrix} \begin{bmatrix} W_t \\ A_{t-1} \\ k_{t-1} \end{bmatrix}, \end{aligned} \tag{83}$$

where the general, unspecified and unidentifiable $\alpha_0\left(\cdot\right)$ is a function of other arguments in the value function beside wealth, land, and capital (see footnote 6). The associated state-specific intertemporal marginal rates of substitution are

$$\begin{aligned} m_{i,t+1} &\equiv \rho^{-1}\left(\frac{\partial V\left(W_{i,t+1}, A_{i,t}, k_{i,t}; \cdot\right)/\partial W}{\partial V\left(W_{i,t}, A_{i,t-1}, k_{i,t-1}; \cdot\right)/\partial W}\right) \\ &= \rho^{-1}\left(\frac{1 - \beta_W W_{i,t+1} - \beta_{WA} A_{i,t} - \beta_{WK} k_{i,t}}{1 - \beta_W W_{i,t} - \beta_{WA} A_{i,t-1} - \beta_{WK} k_{i,t-1}}\right). \end{aligned} \tag{84}$$

Using the $v = 1, ..., n_V$ to index variable inputs and $q = C, O, S, W$ to index crops, the specifications for the marginal cost of capital and crop shares are derived from the same cost function specified in Section 3,

$$f\left(\tilde{c}_{i,t}\right) = \eta_i\left(\mathbf{g}_{i,t}, k_{i,t}\right) - \left(\frac{\varphi\left(\mathbf{g}_{i,t}\right)}{\boldsymbol{\delta}^\top \mathbf{g}_{i,t} + \sqrt{\varphi\left(\mathbf{g}_{i,t}\right)}\theta\left(\tilde{k}_{i,t}, \boldsymbol{\Delta}\left(\bar{y}_{i,q,t}\right)\mathbf{s}_{i,t}\right)}\right), \tag{85}$$

where

$$
\begin{aligned}
&f\left(\tilde{c}_{i,t}\right)=\frac{\tilde{c}_{i,t}^{\kappa}+\kappa-1}{\kappa},\tilde{c}_{i,t}=\frac{c_{i,t}}{w_{i,n_V,t}},g_{i,v,t}\left(\tilde{w}_{i,v,t}\right)=\frac{\tilde{w}_{i,v,t}^{\lambda}+\lambda-1}{\lambda},\\
&\tilde{w}_{i,v,t}\left(w_{i,v,t},w_{i,n_V,t}\right)=\frac{w_{i,v,t}}{w_{i,n_V,t}}\text{ for }v=1,...,n_V-1,\tilde{k}_{i,t}=\frac{k_{i,t}}{w_{i,n_V,t}},\\
&\eta_i\left(\mathbf{g}_{i,t},k_{i,t}\right)=\left(\boldsymbol{\alpha}_{0,i}+\boldsymbol{\alpha}_1\tilde{k}_{i,t}+\right)^{\top}\mathbf{g}_{i,t}+\alpha_{0,n_V,i}+\alpha_{1,n_V}\tilde{k}_{i,t}+\tau_{i,n_V,73}D73+\tau_{i,n_V,86}D86,\\
&\boldsymbol{\alpha}_{0,i}=\left[\alpha_{0,1,i}+\tau_{i,1,73}D73+\tau_{i,1,86}D86,...,\alpha_{0,n_V-1,i}+\tau_{i,n_V-1,73}D73+\tau_{i,n_V-1,86}D86\right]^{\top},\\
&\boldsymbol{\alpha}_1=\left[\alpha_{1,1},...,\alpha_{1,n_V-1}\right]^{\top},\text{ and }\varphi\left(\mathbf{g}_{i,t}\right)=\mathbf{g}_{i,t}^{\top}\mathbf{B}\mathbf{g}_{i,t}+2\boldsymbol{\gamma}^{\top}\mathbf{g}_{i,t}+1.
\end{aligned}
\tag{86}
$$

Note that both the marginal cost of capital and the marginal cost of shares will depend on $\partial\theta/\partial k$ and $\partial\theta/\partial\mathbf{s}$, the partial derivatives of theta with respect to capital and shares. Using the following parsimonious specification for theta,

$$
\theta\left(\tilde{k}_{i,t},\boldsymbol{\Delta}\left(\bar{y}_{i,q,t}\right)\mathbf{s}_{i,t}\right)=\boldsymbol{\pi}_S^{\top}\boldsymbol{\Delta}\left(\bar{y}_{i,q,t}\right)\mathbf{s}_{i,t}-\pi_K\tilde{k}_{i,t} \tag{87}
$$

where $\boldsymbol{\pi}_S=\left[\pi_{S_C}\ \pi_{S_O}\ \pi_{S_S}\ \pi_{S_W}\ \pi_{s\text{-}}\right]^{\top}$ is a parameter vector for corn, oats, soy, wheat, and other outputs, the marginal cost expressions are (derivation in Appendix C)

$$
\frac{\partial c_{i,t}}{\partial k}=\tilde{c}_{i,t}^{1-\kappa}\left(\alpha_{1,n_V}+\boldsymbol{\alpha}_1^{\top}\mathbf{g}_{i,t}\right)-\tilde{c}_{i,t}^{1-\kappa}\frac{\left[f\left(\tilde{c}_{i,t}\right)-\eta_i\left(\mathbf{g}_{i,t},k_{i,t}\right)\right]^2}{\sqrt{\varphi\left(\mathbf{g}_{i,t}\right)}}\pi_K\text{ and} \tag{88}
$$

$$
\frac{\partial c_{i,t}}{\partial\mathbf{s}}=w_{i,n_V,t}\tilde{c}_{i,t}^{1-\kappa}\frac{\left[f\left(\tilde{c}_{i,t}\right)-\eta_i\left(\mathbf{g}_{i,t},k_{i,t}\right)\right]^2}{\sqrt{\varphi\left(\mathbf{g}_{i,t}\right)}}\boldsymbol{\Delta}\left(\bar{y}_{i,q,t}\right)\boldsymbol{\pi}_S. \tag{89}
$$

Estimates of $\tilde{c}_{i,t}^{1-\kappa}\left(\alpha_{1,n_V}+\boldsymbol{\alpha}_1^{\top}\mathbf{g}_{i,t}\right)$ and $\tilde{c}_{i,t}^{1-\kappa}\left[f\left(\tilde{c}_{i,t}\right)-\eta_i\left(\mathbf{g}_{i,t},k_{i,t}\right)\right]^2/\sqrt{\varphi\left(\mathbf{g}_{i,t}\right)}$ are constructed using the estimates $\hat{\kappa}$, $\hat{\lambda}$, $\hat{\boldsymbol{\alpha}}_{0,i}$, $\hat{\boldsymbol{\alpha}}_1$, $\hat{\alpha}_{0,J,i}$, $\hat{\alpha}_{1,J}$, $\hat{\mathbf{B}}$, and $\hat{\boldsymbol{\gamma}}$ reported in the Section 3. Call these $\hat{\varsigma}_{1,i,t}$ and $\hat{\varsigma}_{2,i,t}$ respectively. The specifications (88) and (89) are rewritten as

$$
\frac{\partial c_{i,t}}{\partial k}=\hat{\varsigma}_{1,i,t}-\hat{\varsigma}_{2,i,t}\pi_K\text{ and} \tag{90}
$$

$$
\frac{\partial c_{i,t}}{\partial\mathbf{s}}=w_{i,n_V,t}\hat{\varsigma}_{2,i,t}\boldsymbol{\Delta}\left(\bar{y}_{i,q,t}\right)\boldsymbol{\pi}_S. \tag{91}
$$

This approach simplifies estimation as the only parameters remaining in the marginal cost specifications are π_K and $\boldsymbol{\pi}_S$. However, fixing a subset of parameters during estimation confounds the standard errors. An approach for correcting this is presented in the following section, and implemented during estimation.

The specification of the adjustment cost function is given by

$$
\Psi\left(A_{i,t},A_{i,t-1},k_{i,t},k_{i,t-1}\right)=\frac{1}{2}\psi_L\left(A_{i,t}-A_{i,t-1}\right)^2+\frac{1}{2}\psi_K\left(k_{i,t}-k_{i,t-1}\right)^2A_{i,t}, \tag{92}
$$

where ψ_A and ψ_K are parameters. The corresponding marginal adjustment functions are

$$\frac{\partial \Psi\left(A_{i,t}, A_{i,t-1}, k_{i,t}, k_{i,t-1}\right)}{\partial A_t} = \psi_L\left(A_{i,t} - A_{i,t-1}\right) + \frac{1}{2}\psi_K\left(k_{i,t} - k_{i,t-1}\right)^2, \tag{93}$$

$$\frac{\partial \Psi\left(A_{i,t+1}, A_{i,t}, k_{i,t+1}, k_{i,t}\right)}{\partial A_t} = -\psi_L\left(A_{i,t+1} - A_{i,t}\right), \tag{94}$$

$$\frac{\partial \Psi\left(A_{i,t}, A_{i,t-1}, k_{i,t}, k_{i,t-1}\right)}{\partial k_t} = \psi_K\left(k_{i,t} - k_{i,t-1}\right) A_{i,t}, \text{ and} \tag{95}$$

$$\frac{\partial \Psi\left(A_{i,t+1}, A_{i,t}, k_{i,t+1}, k_{i,t}\right)}{\partial k_t} = -\psi_K\left(k_{i,t+1} - k_{i,t}\right) A_{i,t+1}. \tag{96}$$

Following Gardebroek and Oude Lansink (2004), expectations in the model are assumed to be formed rationally, implying that farmers know the underlying process that generates values of period $t+1$ variables. This assumption allows the unobserved expected values of $t+1$ variables in the arbitrage equations to be replaced by their realized counterparts. A mean zero expectation error that captures the difference between the expected and realized values at time $t+1$ is then added to each equation, and the econometric approach described in the following section describes the nonlinear instrumental variable (GMM) estimator used to minimize these errors.

5.3 Econometric Structure and Estimation

The econometric structure is similar to Pope et. al. (2009) and the estimator follows Ball et. al. (2008), outlined in Section 4.2. Let $i = 1, ..., I$ index states and $t = 1, ..., T$ index time. In general, the implicit state-level system of arbitrage equations is

$$g_{ijt}\left(\mathbf{x}_{ijt}; \boldsymbol{\theta}\right) = u_{ijt},\ i = 1, ..., I\ j = L, K, C, O, S, W, F\ t = 1, ..., T \tag{97}$$

where $\mathbf{x}_{ijt}$ is a matrix of the data, $\boldsymbol{\theta}$ is a $k \times 1$ parameter vector to be estimated, $g_{ijt}\left(\mathbf{x}_{ijt}; \boldsymbol{\theta}\right) \equiv m_{i,t+1} e_{i,j,t+1} - \left(\alpha_i + \phi_j\right)$ is the arbitrage condition, $m_{i,t+1}$ is given by (84), $e_{i,j,t+1} = r_{i,j,t+1} - r$ is the excess return, $\left(\alpha_i + \phi_j\right)$ are the fixed effects that account for aggregation errors as discussed in Section 5.1. The rates of return for farmland, capital, corn, oats, soy, and wheat are constructed using the specifications for the variable and

adjustment cost functions,

$$
\begin{aligned}
r_{i,L,t+1} &= \frac{p_{L,i,t+1} - p_{L,i,t} + \rho_{K,i,t+1} k_{i,t} + \mathbf{p}'_{Y,i,t+1} \boldsymbol{\Delta}\left(y_{i,q,t}\right) \mathbf{s}_{i,t} - c_{i,t} - \left(\frac{\partial \Psi_{i,t+1}}{\partial A_t} + \frac{\partial \Psi_{i,t}}{\partial A_t}\right)}{p_{L,i,t} + k_{i,t} + c_{i,t} + \frac{\partial \Psi_{i,t}}{\partial A_t}}, \\
r_{i,K,t+1} &= \frac{\rho_{K,i,t+1} A_{i,t} - \frac{\partial c_{i,t}}{\partial k} A_{i,t} - \left(\frac{\partial \Psi_{i,t+1}}{\partial k_t} + \frac{\partial \Psi_{i,t}}{\partial k_t}\right)}{A_{i,t} + \frac{\partial c_{i,t}}{\partial k} A_{i,t} + \frac{\partial \Psi_{i,t}}{\partial k_t}}, \text{ and} \\
r_{i,S_q,t+1} &= \frac{p_{Y_q,i,t+1} y_{i,q,t} A_{i,t} - \frac{\partial c_{i,t}}{\partial s_q} A_{i,t}}{\frac{\partial c_{i,t}}{\partial s_q} A_{i,t}} \quad q = C, O, S, W,
\end{aligned} \tag{98}
$$

where the expressions for $\partial c_{i,t}/\partial k$, $\partial c_{i,t}/\partial s_q$, $\partial \Psi_{i,t}/\partial A_t$, $\partial \Psi_{i,t+1}/\partial A_t$, $\partial \Psi_{i,t}/\partial k_t$, and $\partial \Psi_{i,t+1}/\partial k_t$ are given by (90)-(96) above, and the unobserved expected values of $t+1$ variables have been replaced by their realized counterparts. Due to multicollinearity, the fixed effect for the risky financial instrument, ϕ_F, is omitted during estimation. This implies that the fixed effects $-\left(\alpha_i + \phi_j\right)$ represent mean departures from the risky financial instrument arbitrage condition within each state.

All variables that are measured in dollars (i.e. p_L, k, $\mathbf{p}_Y$, and c), the marginal costs for capital and shares (i.e. $\partial c/\partial k$ and $\partial c/\partial \mathbf{s}$), and the marginal adjustment cost for land (i.e. $\partial \Psi_t/\partial A_t$, and $\partial \Psi_{t+1}/\partial A_t$) are converted to 1999 dollars using the BLS consumer price index. Note that the marginal adjustment cost for capital expressions, $\partial \Psi_t/\partial k_t$ and $\partial \Psi_{t+1}/\partial k_t$, are now a function of capital measured in 1999 dollars, so no additional conversion is necessary.

The estimation approach proceeds exactly as (52) through (63), except that the initial set up is given by (97) and $g_{ijt}(\cdot)$ replaces $\tilde{e}_{ijt} - f_{ijt}(\cdot)$ everywhere. Using a superscript A, the final NL3SLS step for the arbitrage system is defined as

$$
\hat{\boldsymbol{\theta}}^A_{3SLS} = \arg\min_{\boldsymbol{\theta}} \sum_{i=1}^{I} \left[\hat{\boldsymbol{\omega}}^A_{i\cdot\cdot}\left(\mathbf{x}_{ijt}; \boldsymbol{\theta}^A\right)\right]^\top \left(\mathbb{N} \otimes \mathbf{I}_{N^A}\right) \left[\hat{\boldsymbol{\omega}}^A_{i\cdot\cdot}\left(\mathbf{x}_{ijt}; \boldsymbol{\theta}^A\right)\right]. \tag{99}
$$

Compare this to the final NL3SLS step for the demand system (using superscript D)

$$
\hat{\boldsymbol{\theta}}^D_{3SLS} = \arg\min_{\boldsymbol{\theta}} \sum_{i=1}^{I} \left[\hat{\boldsymbol{\omega}}^D_{i\cdot\cdot}\left(\tilde{\mathbf{w}}_{i\cdot}, \tilde{k}_{i\cdot}, \tilde{c}_{i\cdot}; \boldsymbol{\theta}^D\right)\right]^\top \left(\mathbb{N} \otimes \mathbf{I}_{N^D}\right) \left[\hat{\boldsymbol{\omega}}^D_{i\cdot\cdot}\left(\tilde{\mathbf{w}}_{i\cdot}, \tilde{k}_{i\cdot}, \tilde{c}_{i\cdot}; \boldsymbol{\theta}^D\right)\right], \tag{100}
$$

and recall that the parameters in $\boldsymbol{\theta}^A$ common to $\boldsymbol{\theta}^D$ are held fixed during estimation. This approach yields consistent estimates for $\boldsymbol{\theta}^A$, however the standard errors need to be corrected. To address this issue, define the joint arbitrage and demand systems as $\hat{\boldsymbol{\omega}}^J_{i\cdot\cdot}\left(\cdot; \boldsymbol{\theta}^A, \boldsymbol{\theta}^D\right) = \left[\hat{\boldsymbol{\omega}}^A_{i\cdot\cdot}\left(\mathbf{x}_{ijt}; \boldsymbol{\theta}^A\right) \quad \hat{\boldsymbol{\omega}}^D_{i\cdot\cdot}\left(\tilde{\mathbf{w}}_{i\cdot}, \tilde{k}_{i\cdot}, \tilde{c}_{i\cdot}; \boldsymbol{\theta}^D\right)\right]$ and

do a single iteration using the objective function

$$\sum_{i=1}^{I}\left[\hat{\boldsymbol{\omega}}_{i\cdot\cdot}^{J}\left(\cdot;\hat{\boldsymbol{\theta}}_{3SLS}^{A},\hat{\boldsymbol{\theta}}_{3SLS}^{D}\right)\right]^{\top}\left(\mathbb{N}\otimes\mathbf{I}_{N^{A}+N^{D}}\right)\left[\hat{\boldsymbol{\omega}}_{i\cdot\cdot}^{J}\left(\cdot;\hat{\boldsymbol{\theta}}_{3SLS}^{A},\hat{\boldsymbol{\theta}}_{3SLS}^{D}\right)\right]. \tag{101}$$

This approach yields consistent, asymptotically efficient parameter estimates and consistent standard errors for the parameters of interest in $\boldsymbol{\theta}^{A}$ (Rothenberg and Leenders, 1964). As before, White/Huber's heteroskedasticity consistent covariance estimator is used for robustness to heteroskedasticity.

5.4 Results

5.4.1 Data

The data used to estimate the model is described in Sections 3.1 and 3.2. As discussed in 3.2, lagged first differences and ratios of the wealth, farmland, capital, share, and S&P return variables are included in the instrument set. This leads to omission of the first two years, 1960 and 1961, from the empirical sample. In addition, construction of the arbitrage equations (97) requires period $t+1$ data. This leads to omission of the last year, 1999, from the empirical sample. Thus, 629 ($= 17 * 37$) observations on 17 states from 1962-1998 are used in the final NL3SLS step.

5.4.2 Structural Breaks

The system-wide first-order parameter stability statistics and the mean zero test statistics, defined in Section 4.3.2, are used to evaluate the appropriateness of including structural change parameters as in the demand system estimation. Looking at Table 5, Model A does not include any structural change parameters. The system-wide first-order parameter stability statistics, Q_T, suggest that parameter instability is not a serious problem as the null of parameter stability is only rejected for a single state at the 5% significance level. Even though the average of the mean zero test statistics, $\bar{z}_i$, appear reasonable, it is important to note that the number of test statistics that support rejection of the mean zero null hypothesis (column labeled "# reject 1%") is quite high. Of the 119 test statistics across 17 states and 7 equations, 34 fail to reject the null.

As noted in Section 4.3.2, there exists both theoretical and empirical evidence of structural breaks in the agricultural economy around the years 1973 and 1986. To empirically test for structural breaks in the arbitrage system, an additional fixed effect of the form $\tau_{i,j,73}D73+\tau_{i,1,86}D86$ is included in the system (97). As before, the τ's are parameters to be estimated and the dummy variables $D73$ and $D86$ are defined as

$$D73=\begin{cases}1 \text{ if } t\leq 1973\\ 0 \text{ otherwise}\end{cases} \text{ and } D86=\begin{cases}1 \text{ if } t\leq 1986\\ 0 \text{ otherwise}\end{cases}. \tag{102}$$

Note that the τ's are quite flexible as they are not restricted to be equal across either states or equations.

Looking again at Table 5, the results of the parameter stability and mean zero tests under this alternative specification are given under Model B. While the Q_T and $\bar{z}_i$ columns do not suggest much improvement, the number of test statistics that support rejection of the mean zero null hypothesis has dropped dramatically from 34 to 8. Whereas there were several states that had 4 or more rejections under the initial model, now there is only a single state with more than 1. In addition, state-specific Wald tests for null hypotheses of the form

$$H_0 : \tau_{i,1,73} = ... = \tau_{i,7,73} = \tau_{i,1,86} = ... = \tau_{i,7,86} = 0 \tag{103}$$

support rejection of the null for all 17 states at the 1% significance level. These results suggest inclusion of the structural change parameters, so the following results are reported for Model B.

5.4.3 Error Term Properties

The estimated 7×7 intertemporal autocorrelation matrix, with White/Huber robust asymptotic standard errors in parentheses and ***, **, and * denoting statistical significance at the 1%, 5%, and 10% levels respectively, is

$$\hat{\mathbf{\Phi}} = \begin{bmatrix} \begin{matrix} 0.293 \\ (0.0857)^{***} \end{matrix} & \begin{matrix} 0.291 \\ (0.0516)^{***} \end{matrix} & \begin{matrix} 0.0707 \\ (0.0961) \end{matrix} & \begin{matrix} -0.139 \\ (0.110) \end{matrix} & \begin{matrix} 0.0604 \\ (0.197) \end{matrix} & \begin{matrix} -0.0655 \\ (0.127) \end{matrix} & \begin{matrix} -0.0272 \\ (0.0568) \end{matrix} \\ \begin{matrix} 0.372 \\ (0.0787)^{***} \end{matrix} & \begin{matrix} 0.444 \\ (0.0849)^{***} \end{matrix} & \begin{matrix} 5.63\times10^{-3} \\ (0.0884) \end{matrix} & \begin{matrix} -0.120 \\ (0.0817) \end{matrix} & \begin{matrix} 0.0501 \\ (0.119) \end{matrix} & \begin{matrix} -0.0535 \\ (0.105) \end{matrix} & \begin{matrix} 0.0922 \\ (0.0801) \end{matrix} \\ \begin{matrix} 0.286 \\ (0.212) \end{matrix} & \begin{matrix} -0.341 \\ (0.159)^{**} \end{matrix} & \begin{matrix} -0.0313 \\ (0.207) \end{matrix} & \begin{matrix} -0.284 \\ (0.176) \end{matrix} & \begin{matrix} 0.542 \\ (0.249)^{**} \end{matrix} & \begin{matrix} -0.192 \\ (0.207) \end{matrix} & \begin{matrix} 0.0235 \\ (0.216) \end{matrix} \\ \begin{matrix} 0.296 \\ (0.216) \end{matrix} & \begin{matrix} -0.357 \\ (0.159)^{**} \end{matrix} & \begin{matrix} -0.395 \\ (0.176)^{**} \end{matrix} & \begin{matrix} 0.296 \\ (0.175)^{*} \end{matrix} & \begin{matrix} 0.308 \\ (0.231) \end{matrix} & \begin{matrix} -0.176 \\ (0.172) \end{matrix} & \begin{matrix} -8.69\times10^{-3} \\ (0.217) \end{matrix} \\ \begin{matrix} 0.246 \\ (0.214) \end{matrix} & \begin{matrix} -0.380 \\ (0.161)^{**} \end{matrix} & \begin{matrix} -0.345 \\ (0.221) \end{matrix} & \begin{matrix} -0.173 \\ (0.179) \end{matrix} & \begin{matrix} 0.616 \\ (0.262)^{**} \end{matrix} & \begin{matrix} -0.0543 \\ (0.193) \end{matrix} & \begin{matrix} -6.35\times10^{-3} \\ (0.220) \end{matrix} \\ \begin{matrix} 0.300 \\ (0.214) \end{matrix} & \begin{matrix} -0.340 \\ (0.165)^{**} \end{matrix} & \begin{matrix} -0.167 \\ (0.180) \end{matrix} & \begin{matrix} -0.214 \\ (0.171) \end{matrix} & \begin{matrix} 0.311 \\ (0.258) \end{matrix} & \begin{matrix} 0.0980 \\ (0.231) \end{matrix} & \begin{matrix} 0.0203 \\ (0.218) \end{matrix} \\ \begin{matrix} 0.0713 \\ (0.135) \end{matrix} & \begin{matrix} 0.295 \\ (0.0641)^{***} \end{matrix} & \begin{matrix} -0.0136 \\ (0.170) \end{matrix} & \begin{matrix} 0.0652 \\ (0.124) \end{matrix} & \begin{matrix} -0.433 \\ (0.216)^{**} \end{matrix} & \begin{matrix} 0.276 \\ (0.111)^{**} \end{matrix} & \begin{matrix} -0.0962 \\ (0.148) \end{matrix} \end{bmatrix}. \tag{104}$$

Rows and columns 1 through 7 are the land, capital, corn, oat, soy, wheat, and S&P arbitrage equations. The implied dynamics for the fully unrestricted $\mathbf{\Phi}$ are stable, with the largest $|Eigen\ value|$ for the 7×7 difference equation is equal to 0.753, indicating no evidence of nonstationarity. The F-test that all parameter estimates are jointly zero is firmly rejected at the 1% significance level, and the Durbin-Watson statistics do not suggest higher order serial correlation (average Durbin-Watson statistics across states for each equation are 2.06, 1.76, 1.80, 1.77, 1.70, 1.47, and 1.89).

The estimated spatial correlation function, with White/Huber robust standard errors in parentheses, is:

$$\hat{\rho}\left(d_{ii'}\right) = \exp\left(-\underset{(.348)}{.498} - \underset{(.175\times10^{-2})}{.233\times10^{-2}d_{ii'}} + \underset{(.256\times10^{-5})}{.215\times10^{-5}d_{ii'}^2} - \underset{(.111\times10^{-8})}{.697\times10^{-9}d_{ii'}^3}\right), \tag{105}$$

where i and i' index states $1, ..., 17$. While none of the parameter estimates are individually significantly different from zero at a 10% level, a Wald test for the hypothesis that all parameters are jointly equal to zero yields a *p-value* of 0.000. This suggests that spatial correlation across states is prevalent in the system. A 2-dimensional plot of the empirical data, estimated correlation function, and 95% confidence band are presented in Figure 2. Note that the predicted correlation decreases steadily as distance increases, but becomes very flat from a distance of approximately 800 miles out, so that the error of states as far apart as $1,387$ miles away (largest distance in the sample) remain positively correlated. Failing to account for this property would lead to biased and inconsistent statistical inferences.

5.4.4 Parameter Estimates

The parameter estimates for the system of arbitrage equations are reported in Table 6. The β_W, β_L, and β_K are the marginal value of wealth coefficients for wealth, land, and capital; ψ_L and ψ_K are the coefficients for land and capital adjustment costs; and π_K, π_{S_C}, π_{S_O}, π_{S_S}, and π_{S_W} are the parameters for the marginal cost of capital and output shares.

The parameter estimates for the marginal value of wealth, $\hat{\beta}_W$,$\hat{\beta}_{WL}$, and $\hat{\beta}_{WK}$, are all individually statistically different from zero at a 1% significance level (*p-values* of 0.000 for all three). Furthermore, a Wald test for the null hypothesis that they are jointly zero is rejected at a 1% level (*p-value* of 0.000). Since $-\beta_W < 0$ is necessary and sufficient for concavity of the value function with respect to wealth, the point estimate of $.7189577 \times 10^{-6}$ is inconsistent with the null hypothesis of risk neutrality (i.e. $\beta_W = 0$) and suggests risk averse behavior.

The parameter estimates for the adjustment cost function, $\hat{\psi}_L$ and $\hat{\psi}_K$, are individually statistically different from zero at a 1% significance level (*p-values* of 0.000 and 0.002 respectively). Furthermore, a Wald test for the null hypothesis that they are jointly zero is rejected at a 1% significance level (*p-value* of 0.000). Since $\psi_L > 0$ and $\psi_K > 0$ are necessary and sufficient for convexity of the adjustment cost function, the point estimates of 0.1001551 and 0.8794872×10^{-3} are inconsistent with the null hypothesis of zero adjustment costs (i.e. $\psi_L = 0$ and $\psi_K = 0$) and suggest that convex adjustment costs for land and capital are a significant component of agricultural production.

The joint hypothesis of risk neutrality and zero adjustment costs is tested using the null hypothesis $H_0 : \beta_W = \psi_L = \psi_K = 0$. A Wald test for this null is rejected at a 1% significance level (*p-value* of 0.000). The positive point estimates for all three parameters suggest that risk aversion and convex adjustment costs are jointly significant components of agricultural production.

Recall that many of the parameters of the cost function were held fixed at their estimates from the demand system estimation reported in Section 4.3. The parameters that are not recoverable from the demand

system are exactly the parameters associated with the constant of integration, $\theta\left(\tilde{k}, \mathbf{\Delta}\left(\bar{y}_q\right)\mathbf{s}\right)$. These are the estimates reported for π_K, π_{S_C}, π_{S_O}, π_{S_S}, and π_{S_W} in Table 6. All estimates are individually statistically different from zero at a 5% significance level (*p-values* of 0.000, 0.021, 0.021, 0.021, and 0.020 respectively)). Furthermore, a Wald test for the null hypothesis that they are jointly zero is rejected at a 5% significance level (*p-value* of 0.0143). An interesting property to note is that the nearly identical estimates for π_{S_C}, π_{S_O}, π_{S_S}, and π_{S_W} imply that the marginal rates of product transformation are all near 1.[20] This provides empirical evidence supporting the near-perfect (if not perfect) substitutability of corn, oats, soy, and wheat.

The extent to which each state's arbitrage equation over or underestimates full arbitrage relative to the S&P can investigated by constructing estimates for the state and equation fixed effects, $\hat{\mu}_{ij} = \hat{\alpha}_i + \hat{\phi}_j$. Of the 102 $\hat{\mu}_{ij}$'s (17 states and 6 equations), only three were not statistically different from zero at a 1% significance level ($\hat{\mu}_{7,6}$, $\hat{\mu}_{8,1}$, and $\hat{\mu}_{13,6}$). This result suggests there are significant departures from the full arbitrage conditions, likely due to the use of aggregate data (see fixed effect discussion in Section 5.1). It is interesting to note that 83% of $\hat{\mu}_{ij}$'s are negative, which implies a strong undershooting of the farm-related arbitrage conditions relative to the S&P. One possible explanation for this is that investment in agriculture is not subject to the equity premium puzzle associated with stockmarket-related investment opportunities (see Mehra and Prescott, 1985).

The parameter estimates in Table 6 are used to construct estimates of marginal adjustment costs for each state,

$$\frac{\widehat{\partial\Psi}_{i,t}}{\partial A_t} = \hat{\gamma}_L\left(A_{i,t} - A_{i,t-1}\right) + \frac{1}{2}\hat{\gamma}_K\left(k_{i,t} - k_{i,t-1}\right)^2 \text{ and} \tag{106}$$

$$\frac{\widehat{\partial\Psi}_{i,t+1}}{\partial A_t} = -\hat{\gamma}_L\left(A_{i,t+1} - A_{i,t}\right), \tag{107}$$

which are then used to construct estimates of the rate of return to farmland in the presence of adjustment costs (call this the adjusted rate of return) for each state,

$$\hat{r}_{i,L,t+1} = \frac{p_{i,L,t+1} - p_{i,L,t} + \rho_{i,K,t+1}k_{i,t} + \mathbf{p}'_{Y,i,t+1}\mathbf{\Delta}\left(y_{i,q,t}\right)\mathbf{s}_{i,t} - c_{i,t} - \left(\frac{\widehat{\partial\Psi}_{i,t+1}}{\partial A_t} + \frac{\widehat{\partial\Psi}_{i,t}}{\partial A_t}\right)}{p_{L,i,t} + k_{i,t} + c_{i,t} + \frac{\widehat{\partial\Psi}_{i,t}}{\partial A_t}}. \tag{108}$$

[20] As stated in LaFrance and Pope (2008b), the variable input demand equations have the structure

$$\mathbf{x}(\mathbf{w}, \mathbf{\Delta}\left(\bar{y}_i\right)\mathbf{s}, k) = \mathbf{g}(\mathbf{w}, k, c\left(\mathbf{w}, \mathbf{\Delta}\left(\bar{y}_i\right)\mathbf{s}, k\right))$$

if and only if

$$c\left(\mathbf{w}, \mathbf{\Delta}\left(\bar{y}_i\right)\mathbf{s}, k\right) \equiv c\left(\mathbf{w}, k, \theta\left(k, \mathbf{\Delta}\left(\bar{y}_i\right)\mathbf{s}\right)\right),$$

if and only if

$$F(\mathbf{x}, \mathbf{\Delta}\left(\bar{y}_i\right)\mathbf{s}, k) \equiv F(\mathbf{x}, k, \theta\left(k, \mathbf{\Delta}\left(\bar{y}_i\right)\mathbf{s}\right)).$$

This implies that the marginal rates of production transformation will be equal to the ratio of the elements of $\boldsymbol{\pi}_S$ under the assumed specification of $\theta\left(k, \mathbf{\Delta}\left(\bar{y}_i\right)\mathbf{s}\right)$.

Column 1 of Table 7 reports the average value of these estimates across time for each state, $\bar{r}_{i,L} = \frac{1}{T}\sum_{t=1}^{T}\hat{r}_{i,L,t+1}$. The average adjusted rates of return vary widely across states from a low of 3.06% in Michigan to a high of 9.76% in Georgia. Overall, the average adjusted return across states is 6.12% with a standard deviation of 1.99%. Two states, Georgia and North Carolina, have an average adjusted rate of return to farmland that has outperformed the S&P index (average return of 8.28% for the time period considered). It is worthwhile to note that all states have overperformed relative to the real interest rate, which has averaged an annual return of 2.88% for the period considered.

For comparison purposes, column 2 reports time-averaged unadjusted rates of return to farmland in the absence of adjustment costs (i.e. setting $\widehat{\partial\Psi}_{i,t}/\partial A_t = \widehat{\partial\Psi}_{i,t+1}/\partial A_t = 0$ in 5.32), $\bar{r}^0_{i,L} = \frac{1}{T}\sum_{t=1}^{T}\hat{r}^0_{i,L,t+1}$, and column 3 reports the difference between these estimates, $\Delta\bar{r}_{i,L} = \bar{r}_{i,L,i} - \bar{r}^0_{i,L}$. The difference between the adjusted and unadjusted estimates is negative for all but one state, Oklahoma, indicating that not accounting for adjustment costs leads to rate of return estimates that are biased upward on average. Column 4 reports the percentage change to the unadjusted rate of return when adjustment costs are taken into account, $\Pi\bar{r}_{i,L,i} = 100 * \Delta\bar{r}_{i,L}/\bar{r}^0_{i,L}$. Across states, the average markdown is only 0.635%, and is no greater than 2.76% for any state.

The simple average difference, $\Delta\bar{r}_{i,L}$, is not a very good measure of the effect of adjustment costs on the rate of return because the estimated difference is actually positive for 43% of the sample data points. Column 5 reports the mean absolute difference for each state, $\Delta\overline{ar}_{i,L} = \frac{1}{T}\sum_{t=1}^{T}\left|\hat{r}_{i,L,t+1} - \hat{r}^0_{i,L,t+1}\right|$, which are substantially higher than the $\Delta\bar{r}_{i,L}$. Column 6 reports the mean absolute deviation as a percentage of the unadjusted rate of return, $\Pi\bar{r}_{i,L,i} = 100 * \Delta\overline{ar}_{i,L}/\bar{r}^0_{i,L}$. Across states, the mean absolute deviation is 5.60% and is near 22% for two states (North and South Dakota). This suggests that not accounting for adjustment costs can lead to severely misrepresented rates of return to farmland.

The statistical significance of the differences between the adjusted and unadjusted rates reported in column 3 is tested using Wald tests. For each state and year in the data set, a Wald test of the null hypothesis that $\hat{r}_{i,L,t+1} - \hat{r}^0_{i,L,t+1} = 0$ was conducted. Column 6 reports the fraction of Wald tests that support rejection of the null at a 1% significance level within each state. As an example, the reported value of 0.97 for Georgia means that 97% of the Wald tests support rejection of the null hypothesis that the adjusted and unadjusted rates of return are equal. Looking at column 6, the lowest fraction of "reject" conclusions is 0.81 (Pennsylvania) and four states failed to reject in all years (Iowa, North Dakota, Oklahoma, and South Dakota). The average across states is .94. This suggests that the results reported in column 3 represent statistically significant differences between the adjusted and unadjusted rates of return to farmland. Thus, the presence of adjustment costs generates statistically significant differences in the rate of return to farmland that are negative on average and have a mean absolute deviation that is 5.60% of the unadjusted

rate.

While it is beyond the scope of this paper, one could solve the system of arbitrage equations for optimal farmland, capital, and share allocations and use the estimated parameters to simulate the effects of different agricultural policy instruments. As mentioned in the introduction, one could use the parameter estimates to investigate the effect of policies targeting lower food prices, taking into account the supply lag driven by partial adjustment of land and capital over time. In addition, one could use the adjusted farmland return estimates to investigate how far the extensive margin will expand or contract in response to a variety of policy scenarios including subsidization of corn for ethanol, an increase in the variety of subsidized crop insurance products, and the introduction of new revenue support programs such as ACRE.

6 Conclusion

The focus of this research is whether risk aversion and adjustment costs are jointly significant components of agricultural production, and the effect that adjustment costs have on rate of return for investing in farmland. Investment in agriculture is composed of many simultaneous decisions and both costs and benefits of farming likely include many latent measures that are not observable to the researcher. To address these issues, a theoretical model is developed that incorporates life-cycle household consumption, agricultural production, financial economics, and adjustment costs in one coherent framework. This model ties together contributions from the farmland pricing and adjustment cost literatures, and the first order conditions for a utility maximizing decision maker are rearranged into intertemporal arbitrage equations that are similar in spirit to traditional finance models.

The derivation of the theoretical arbitrage model relies on the assumptions that land and capital are quasi-fixed assets, and that production is characterized by constant returns to scale. These assumptions are empirically tested using parameter estimates from a system of variable input demands that is formulated using a new and innovative approach for overcoming aggregation and unobservable variable issues in econometric models of production. The estimated model provides evidence of structural breaks in the agricultural economy around the years 1973 and 1986, which are the starting and ending years of the notorious boom/bust period of agricultural land prices. Accounting for these breaks led to parameter estimates that showed no evidence of instability across the sample period 1960-1999.

The econometric approach utilizes a nonlinear instrumental variable (GMM) estimator that accounts for correlation of the residuals across time, space, and equations. The results from the spatial correlation analysis suggest that the errors from states more than $1,000$ miles apart remain positively correlated. In addition, the demand specification is flexible enough to nest the industry standards of logarithmic or linear

transformations of pertinent variables, none of which are supported by the data. Finally, the results of the model are consistent with the constant returns to scale and quasi-fixity assumptions mentioned above, as capital per acre is found to be a statistically significant explanatory variable in the per acre variable input demand specifications.

An empirical application of the arbitrage system utilizes a flexible specification for variable costs of production and an explicit representation of adjustment costs to estimate the parameters of a system of arbitrage equations that describe optimal investment in an off-farm risky financial instrument, farmland, capital, and output shares for corn, oats, soybeans, and wheat. Further evidence of structural breaks in the agricultural economy around the years 1973 and 1986 is found, and the importance of accounting for spatial correlation of the errors across states is reinforced.

The estimated parameters are used to test the joint hypothesis of risk neutrality and zero adjustment costs. Results strongly support the rejection of this hypothesis, and are consistent with risk averse decision making and convex adjustment costs. This is a novel result as the joint hypothesis had not been tested in the literature. The estimates were also used to construct estimates of the rate of return to farmland in the presence of adjustment costs. These adjusted rates were found to be statistically significantly different from the unadjusted rates. The mean absolute deviation from the unadjusted rate is 5.60% across all states and is near 22% for two states. This finding suggests that not accounting for adjustment costs can lead to severely misrepresented rates of return to farmland.

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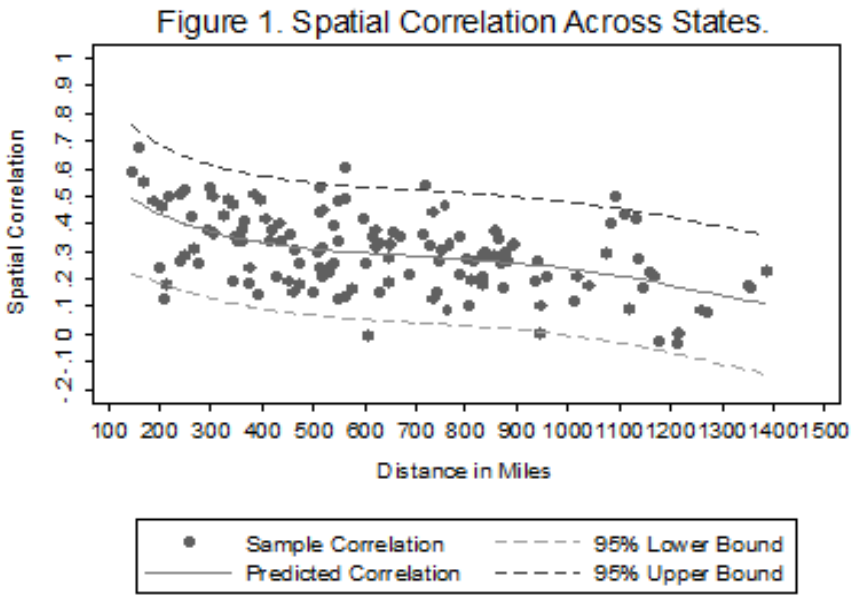
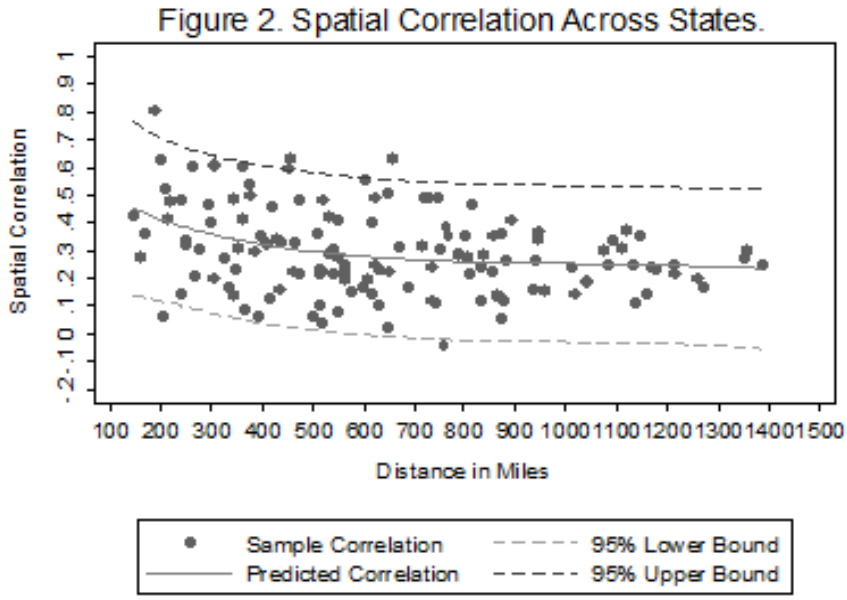
Appendix

A Figures





B Tables

The following abbreviations for the 17 states are used: Georgia (GA), Iowa (IA), Illinois (IL), Indiana (IN), Kansas (KS), Michigan (MI), Minnesota (MN), Missouri (MO), North Carolina (NC), North Dakota (ND), Nebraska (NE), Ohio (OH), Oklahoma (OK), Pennsylvania (PA), South Carolina (SC), South Dakota (SD), and Wisconsin (WI).

Table 1. Variable Definitions for Theoretical Arbitrage Model

Variable	Definition
W_t	beginning-of-period total wealth in dollars
B_t	current holding of bonds with a risk free rate of return r
F_t	current holding of a risky financial asset in dollars
$r_{F,t+1}$	dividend plus capital gains rate on the financial asset
$a_{q,t}$	current allocation of land to q^{th} farm output, $q = 1, \ldots, n_Y$
A_t	total acres of farmland
$s_{q,t} = a_{q,t}/A_t$	current share of land for q^{th} farm output, $q = 1, \ldots, n_Y$
$p_{L,t}$	beginning-of-period market price of land
K_t	total value of capital in dollars
$k_t = K_t/A_t$	total value of capital per acre in dollars
$\rho_{K,t+1}$	percentage change in the value of capital
$w_{v,t}$	variable input prices, $v = 1, \ldots, n_V$
$\mathbf{w}_t$	vector of variable input prices $w_{1,t},\ldots,w_{n_V-1,t}$
$\bar{Y}_{q,t}$	expected output for q^{th} farm output, $q = 1, \ldots, n_Y$
$\bar{y}_{q,t}$	expected yield per acre for q^{th} farm output, $q = 1, \ldots, n_Y$
$\varepsilon_{q,t+1}$	production disturbance for q^{th} farm output, $q = 1, \ldots, n_Y$
$y_{q,t+1}$	end-of-period actual yield of qth farm output, $q = 1, \ldots, n_Y$
$p_{Y_q,t+1}$	end-of-period market price for q^{th} farm output, $q = 1, \ldots, n_Y$
$\mathbf{p}_{Q,t}$	vector of market prices for consumption goods
M_t	total consumption expenditures
ρ^{-1}	single period discount factor

Table 2. Additional Variable Definitions for Cost Function

Variable	Definition
$\mathbf{x}_t$	vector of variable input quantities per acre
$w_{v,t}$	variable input prices, $v = 1, \ldots, n_V$
$\mathbf{w}_t$	vector of variable input prices $w_{1,t},\ldots,w_{n_V-1,t}$
$\check{\mathbf{w}}_t$	vector of variable input prices $w_{1,t},\ldots,w_{n_V,t}$
c_t	total variable cost per acre

Table 3. Brownian Bridge Tests for Structural Break

	Model A				Model B			
	Q_{1T}	Q_{2T}	Q_{3T}	Q_T	Q_{1T}	Q_{2T}	Q_{3T}	Q_T
GA	0.581	0.895	0.548	0.610	0.460	0.651	0.830	0.683
IA	0.783	0.930	1.442*	1.107	0.568	0.574	0.681	0.868
IL	0.842	1.581*	1.160	0.729	0.441	0.630	0.834	0.988
IN	0.510	1.341	1.134	0.785	0.593	0.856	0.694	0.546
KS	0.883	1.036	0.869	0.654	0.612	0.899	1.036	0.817
MI	1.064	1.293	0.748	1.503*	0.603	0.628	0.636	0.674
MN	0.725	1.183	1.266	1.523*	0.695	1.064	0.520	0.688
MO	0.611	1.235	0.772	0.448	0.348	0.877	0.577	0.623
NC	1.679*	0.538	0.739	0.836	0.519	0.862	0.709	0.360
ND	0.534	1.924*	1.088	0.448	0.599	0.928	0.982	0.929
NE	0.790	1.290	1.596*	1.328	0.777	0.630	0.615	0.821
OH	0.692	1.678*	0.735	0.385	0.480	0.875	0.723	0.497
OK	1.664*	1.010	1.725*	0.917	0.526	0.723	1.085	0.709
PA	0.823	0.586	0.692	1.053	0.284	0.709	0.665	0.724
SC	1.723*	0.658	0.995	1.018	0.685	0.712	0.481	0.670
SD	0.640	1.292	1.179	0.553	0.679	0.695	1.072	0.474
WI	1.171	0.913	1.757*	1.579*	0.672	0.829	0.718	0.780

Model A does not include structural change parameters for 1973 and 1986, Model B does. Columns labeled Q_{1T}, Q_{2T}, and Q_{3T} report single equation first-order tied Brownian bridge test statistics for materials, energy, and ag chemicals, while the column labeled Q_T reports the system of equations test statistic (see text).

* indicates statistically different from null hypothesis at the 5% significance level.

Table 4. Estimated Coefficients and Robust Standard Errors

Coefficient	Estimate	Standard Error	T-Ratio	P-Value
κ	.1687992	.0604251	2.793527	[.005]
λ	.0782715	.0541744	1.444806	[.149]
$\alpha_{1,1}$	$.6972398 \times 10^{-2}$	$.2893635 \times 10^{-2}$	2.409563	[.016]
$\alpha_{1,2}$	$-.2136856 \times 10^{-3}$	$.1444549 \times 10^{-3}$	-1.479255	[.139]
$\alpha_{1,3}$	$-.2163776 \times 10^{-3}$	$.2377874 \times 10^{-3}$	$-.9099623$	[.363]
$\alpha_{1,4}$	.0239923	$.7520395 \times 10^{-2}$	3.190295	[.001]
δ_1	.0148734	$.9205614 \times 10^{-2}$	1.615689	[.106]
δ_2	$-.9977413 \times 10^{-3}$	$.7150425 \times 10^{-3}$	-1.395359	[.163]
δ_3	$-.1188801 \times 10^{-2}$	$.8387804 \times 10^{-3}$	-1.417297	[.156]
δ_4	.0554081	.0314798	1.760119	[.078]

Table 5. Brownian Bridge Tests for Structural Break

	Model A			Model B		
	Q_T	$\bar{z}_i = \frac{1}{J}\sum_{j=1}^{J} z_{ij}$	# reject 1%	Q_T	$\bar{z}_i = \frac{1}{J}\sum_{j=1}^{J} z_{ij}$	# reject 1%
GA	1.407*	−0.9113	3	0.679	0.0957	0
IA	0.944	−0.2471	0	0.550	0.0760	0
IL	0.531	−0.0167	0	0.523	0.2015	0
IN	1.273	0.4022	1	0.787	0.4713	1
KS	0.486	0.4277	0	1.09	0.3333	1
MI	0.961	1.250	2	1.22	0.2664	1
MN	0.905	−0.2673	0	0.705	0.1753	0
MO	1.258	0.5481	1	0.897	−0.6405	0
NC	0.752	0.2024	6	1.06	0.3027	1
ND	0.891	0.3049	3	1.21	−0.5361	1
NE	0.652	−0.0336	1	0.826	0.7332	1
OH	0.489	0.5324	1	1.01	0.3704	0
OK	1.005	−0.4293	5	1.42*	−0.0982	2
PA	0.634	0.1963	2	0.786	0.1547	0
SC	1.045	−0.4765	4	0.418	0.0655	0
SD	0.954	0.7513	4	0.436	0.4177	0
WI	0.976	−0.6812	1	1.06	0.1406	0

Model A does not include structural change params for 1973 and 1986, Model B does.

Q_T reports tied Brownian bridge system of equations test statistic (see text).

$z_{ij}=\sqrt{T}\hat{\varepsilon}_{ij}/\hat{\sigma}_{ij}\sim^{a} n(0,1)$ is mean zero test stat for state i equation j residuals, $\bar{z}_i$ is avg of test statistics across equations. "# rejects 1%" is number of $|z_{ij}|$ greater than 2.33.

* indicates statistically different from null hypothesis at 5% significance level.

Table 6. Estimated Coefficients and Robust Standard Errors

Coefficient	Estimate	Standard Error	T-Ratio	P-Value
β_W	$.7189577 \times 10^{-6}$	$.6964200 \times 10^{-7}$	10.32362	[.000]
β_{WL}	$-.3153465 \times 10^{-3}$	$.6399488 \times 10^{-4}$	-4.927683	[.000]
β_{WK}	$.2453637 \times 10^{-2}$	$.4114235E \times 10^{-4}$	59.63775	[.000]
ψ_L	.1001551	.0186678	5.365121	[.000]
ψ_K	$.8794872 \times 10^{-3}$	$.2442608 \times 10^{-3}$	3.600607	[.000]
π_K	$.1006472 \times 10^{-3}$	$.2812572 \times 10^{-4}$	3.578474	[.000]
π_{S_C}	.0173540	$.7541878 \times 10^{-2}$	2.301016	[.021]
π_{S_O}	$.5262389 \times 10^{-2}$	$.2276083 \times 10^{-2}$	2.312038	[.021]
π_{S_S}	.0245156	.0106022	2.312323	[.021]
π_{S_W}	.0312612	.0133934	2.334072	[.020]

State and equation fixed effects were included in the model. Due to multicollinearity, the fixed effect for the risky financial instrument arbitrage equation (S&P) was omitted. Durbin-Watsons are 1.70, 1.76, 1.51, 2.08, 1.79, 1.84, and 1.67 for the land, capital, corn, oat, soy, wheat, and S&P equations respectively.

Table 7. Rate of Return to Farmland Estimates

	$\bar{r}_{L,i}$	$\bar{r}^0_{L,i}$	$\Delta\bar{r}_{L,i}$	$\Pi\bar{r}_{L,i}$	$\Delta\overline{ar}_{L,i}$	$\Pi\overline{ar}_{L,i}$	Wald, 1%
GA	0.0976	0.0980	−0.000347	−0.354	0.00250	2.55	0.97
IA	0.0664	0.0668	−0.000366	−0.548	0.00146	2.19	1.00
IL	0.0719	0.0722	−0.000324	−0.448	0.00113	1.57	0.92
IN	0.0550	0.0553	−0.000345	−0.623	0.000987	1.78	0.86
KS	0.0601	0.0603	−0.000220	−0.365	0.00418	6.94	0.97
MI	0.0306	0.0310	−0.000426	−1.37	0.000971	3.12	0.89
MN	0.0622	0.0624	−0.000258	−0.413	0.00261	4.17	0.95
MO	0.0465	0.0467	−0.000169	−0.362	0.00141	3.03	0.97
NC	0.0910	0.0913	−0.000306	−0.336	0.000705	0.772	0.92
ND	0.0799	0.0822	−0.00227	−2.76	0.0184	22.3	1.00
NE	0.0647	0.0650	−0.000290	−0.446	0.00493	7.58	0.97
OH	0.0357	0.0361	−0.000361	−0.999	0.000821	2.27	0.86
OK	0.0367	0.0367	0.0000252	0.0685	0.00356	9.70	1.00
PA	0.0351	0.0354	−0.000291	−0.822	0.000526	1.48	0.81
SC	0.0715	0.0718	−0.000312	−0.435	0.00169	2.35	0.97
SD	0.0797	0.0806	−0.000920	−1.14	0.0177	21.9	1.00
WI	0.0547	0.0550	−0.000245	−0.445	0.000723	1.31	0.92

Estimates constructed using sample data from 1962-1998. See text for definitions of column headers.

C Derivations of Equations

C.1 Derivation of Equation (43)

Since $c(\mathbf{w}, k, \theta) = w_{n_V} f^{-1}(y(\mathbf{g}(\tilde{\mathbf{w}}(\mathbf{w})), \theta))$, the composite function theorem implies

$$\begin{aligned}
\mathbf{q} &\equiv \frac{\partial c}{\partial \mathbf{w}} = \frac{\partial}{\partial \mathbf{w}} w_{n_V} f^{-1}(y(\mathbf{g}(\tilde{\mathbf{w}}(\mathbf{w})), \theta)) \\
&= w_{n_V} \frac{\partial \tilde{\mathbf{w}}}{\partial \mathbf{w}} \frac{\partial \mathbf{g}}{\partial \tilde{\mathbf{w}}} \frac{\partial y}{\partial \mathbf{g}} \frac{\partial f^{-1}}{\partial y} \\
&= w_{n_V} \tilde{c}^{1-\kappa} \mathbf{\Delta}\left(\frac{1}{w_{n_V}}\right) \mathbf{\Delta}\left(\tilde{w}_i^{\lambda-1}\right) \frac{\partial y}{\partial \mathbf{g}} \\
&= \tilde{c}^{1-\kappa} \mathbf{\Delta}\left(\tilde{w}_v^{\lambda-1}\right) \frac{\partial y}{\partial \mathbf{g}}. \qquad (109)
\end{aligned}$$

C.2 Derivation of Equations (45) and (46)

First note that the composite function theorem implies that the $(n_V - 1) \times (n_V - 1)$ elements of the Hessian are

$$
\begin{aligned}
\mathbf{H} &\equiv \frac{\partial^2 c}{\partial \mathbf{w} \partial \mathbf{w}^\top} = \frac{\partial}{\partial \mathbf{w}^\top} \left[\frac{\partial c}{\partial \mathbf{w}} \right] \\
&= \frac{\partial}{\partial \mathbf{w}^\top} \left[w_{n_V} \frac{\partial \tilde{\mathbf{w}}}{\partial \mathbf{w}} \frac{\partial \mathbf{g}}{\partial \tilde{\mathbf{w}}} \frac{\partial y}{\partial \mathbf{g}} \frac{\partial f^{-1}}{\partial y} \right] \\
&= w_{n_V} \left\{ \frac{\partial}{\partial \mathbf{w}^\top} \frac{\partial \tilde{\mathbf{w}}}{\partial \mathbf{w}} \right\} \frac{\partial \mathbf{g}}{\partial \tilde{\mathbf{w}}} \mathbf{\Delta} \left(\frac{\partial y}{\partial g_v} \right) \frac{\partial f^{-1}}{\partial y} + w_{n_V} \frac{\partial \tilde{\mathbf{w}}}{\partial \mathbf{w}} \left\{ \frac{\partial}{\partial \mathbf{w}^\top} \left[\frac{\partial \mathbf{g}}{\partial \tilde{\mathbf{w}}} \frac{\partial y}{\partial \mathbf{g}} \frac{\partial f^{-1}}{\partial y} \right] \right\} \\
&= w_{n_V} \frac{\partial^2 \tilde{\mathbf{w}}}{\partial \mathbf{w} \partial \mathbf{w}^\top} \frac{\partial \mathbf{g}}{\partial \tilde{\mathbf{w}}} \mathbf{\Delta} \left(\frac{\partial y}{\partial g_v} \right) \frac{\partial f^{-1}}{\partial y} \\
&\quad + w_{n_V} \frac{\partial \tilde{\mathbf{w}}}{\partial \mathbf{w}} \left\{ \left[\frac{\partial}{\partial \mathbf{w}^\top} \frac{\partial \mathbf{g}}{\partial \tilde{\mathbf{w}}} \right] \mathbf{\Delta} \left(\frac{\partial y}{\partial g_v} \right) \frac{\partial f^{-1}}{\partial y} + \frac{\partial \mathbf{g}}{\partial \tilde{\mathbf{w}}} \frac{\partial}{\partial \mathbf{w}^\top} \left[\frac{\partial y}{\partial \mathbf{g}} \frac{\partial f^{-1}}{\partial y} \right] \right\} \\
&= w_{n_V} \frac{\partial^2 \tilde{\mathbf{w}}}{\partial \mathbf{w} \partial \mathbf{w}^\top} \frac{\partial \mathbf{g}}{\partial \tilde{\mathbf{w}}} \mathbf{\Delta} \left(\frac{\partial y}{\partial g_v} \right) \frac{\partial f^{-1}}{\partial y} + w_{n_V} \frac{\partial \tilde{\mathbf{w}}}{\partial \mathbf{w}} \frac{\partial^2 \mathbf{g}}{\partial \tilde{\mathbf{w}} \tilde{\mathbf{w}}^\top} \frac{\partial \tilde{\mathbf{w}}}{\partial \mathbf{w}^\top} \mathbf{\Delta} \left(\frac{\partial y}{\partial g_v} \right) \frac{\partial f^{-1}}{\partial y} \\
&\quad + w_{n_V} \frac{\partial \tilde{\mathbf{w}}}{\partial \mathbf{w}} \frac{\partial \mathbf{g}}{\partial \tilde{\mathbf{w}}} \frac{\partial^2 y}{\partial \mathbf{g} \partial \mathbf{g}^\top} \frac{\partial \mathbf{g}}{\partial \tilde{\mathbf{w}}^\top} \frac{\partial \tilde{\mathbf{w}}}{\partial \mathbf{w}^\top} \frac{\partial f^{-1}}{\partial y} + w_{n_V} \frac{\partial \tilde{\mathbf{w}}}{\partial \mathbf{w}} \frac{\partial y}{\partial \mathbf{g}} \frac{\partial^2 f^{-1}}{\partial y \partial y} \frac{\partial y}{\partial \mathbf{g}^\top} \frac{\partial \mathbf{g}}{\partial \tilde{\mathbf{w}}^\top} \frac{\partial \tilde{\mathbf{w}}}{\partial \mathbf{w}^\top} \\
&= w_{n_V} \left\{ \frac{\partial^2 \tilde{\mathbf{w}}}{\partial \mathbf{w} \partial \mathbf{w}^\top} \frac{\partial \mathbf{g}}{\partial \tilde{\mathbf{w}}} \mathbf{\Delta} \left(\frac{\partial y}{\partial g_v} \right) \frac{\partial f^{-1}}{\partial y} + \frac{\partial \tilde{\mathbf{w}}}{\partial \mathbf{w}} \frac{\partial^2 \mathbf{g}}{\partial \tilde{\mathbf{w}} \tilde{\mathbf{w}}^\top} \frac{\partial \tilde{\mathbf{w}}}{\partial \mathbf{w}^\top} \mathbf{\Delta} \left(\frac{\partial y}{\partial g_v} \right) \frac{\partial f^{-1}}{\partial y} \right. \\
&\quad \left. + \frac{\partial \tilde{\mathbf{w}}}{\partial \mathbf{w}} \frac{\partial \mathbf{g}}{\partial \tilde{\mathbf{w}}} \frac{\partial^2 y}{\partial \mathbf{g} \partial \mathbf{g}^\top} \frac{\partial \mathbf{g}}{\partial \tilde{\mathbf{w}}^\top} \frac{\partial \tilde{\mathbf{w}}}{\partial \mathbf{w}^\top} \frac{\partial f^{-1}}{\partial y} + \frac{\partial \tilde{\mathbf{w}}}{\partial \mathbf{w}} \frac{\partial \mathbf{g}}{\partial \tilde{\mathbf{w}}} \frac{\partial y}{\partial \mathbf{g}} \frac{\partial^2 f^{-1}}{\partial y \partial y} \frac{\partial y}{\partial \mathbf{g}^\top} \frac{\partial \mathbf{g}}{\partial \tilde{\mathbf{w}}^\top} \frac{\partial \tilde{\mathbf{w}}}{\partial \mathbf{w}^\top} \right\} \\
&= w_{n_V} \left\{ \mathbf{0} + \frac{1}{w_{n_V}} \tilde{c}^{1-\kappa} \frac{\partial^2 \mathbf{g}}{\partial \tilde{\mathbf{w}} \tilde{\mathbf{w}}^\top} \mathbf{\Delta} \left(\frac{\partial y}{\partial g_v} \right) + \frac{1}{w_{n_V}} \tilde{c}^{1-\kappa} \mathbf{\Delta} \left(\tilde{w}_v^{\lambda-1} \right) \frac{\partial^2 y}{\partial \mathbf{g} \partial \mathbf{g}^\top} \mathbf{\Delta} \left(\tilde{w}_v^{\lambda-1} \right) \right. \\
&\quad \left. + \frac{1}{w_{n_V}} \mathbf{\Delta} \left(\tilde{w}_v^{\lambda-1} \right) \frac{\partial y}{\partial \mathbf{g}} \frac{\partial^2 f^{-1}}{\partial y \partial y} \frac{\partial y}{\partial \mathbf{g}^\top} \mathbf{\Delta} \left(\tilde{w}_v^{\lambda-1} \right) \right\} \\
&= \frac{1}{w_{n_V}} \left\{ \tilde{c}^{1-\kappa} \frac{\partial^2 \mathbf{g}}{\partial \tilde{\mathbf{w}} \tilde{\mathbf{w}}^\top} \mathbf{\Delta} \left(\frac{\partial y}{\partial g_v} \right) + \tilde{c}^{1-\kappa} \mathbf{\Delta} \left(\tilde{w}_v^{\lambda-1} \right) \frac{\partial^2 y}{\partial \mathbf{g} \partial \mathbf{g}^\top} \mathbf{\Delta} \left(\tilde{w}_v^{\lambda-1} \right) \right. \\
&\quad \left. + \mathbf{\Delta} \left(\tilde{w}_v^{\lambda-1} \right) \frac{\partial y}{\partial \mathbf{g}} \frac{\partial^2 f^{-1}}{\partial y \partial y} \frac{\partial y}{\partial \mathbf{g}^\top} \mathbf{\Delta} \left(\tilde{w}_v^{\lambda-1} \right) \right\} \\
&= \frac{1}{w_{n_V}} \left\{ \tilde{c}^{1-\kappa} (\lambda - 1) \mathbf{\Delta} \left(\tilde{w}_v^{\lambda-2} \right) \mathbf{\Delta} \left(\frac{\partial y}{\partial g_v} \right) + \tilde{c}^{1-\kappa} \mathbf{\Delta} \left(\tilde{w}_v^{\lambda-1} \right) \frac{\partial^2 y}{\partial \mathbf{g} \partial \mathbf{g}^\top} \mathbf{\Delta} \left(\tilde{w}_v^{\lambda-1} \right) \right. \\
&\quad \left. + (1 - \kappa) \tilde{c}^{1-2\kappa} \mathbf{\Delta} \left(\tilde{w}_v^{\lambda-1} \right) \frac{\partial y}{\partial \mathbf{g}} \frac{\partial y}{\partial \mathbf{g}^\top} \mathbf{\Delta} \left(\tilde{w}_v^{\lambda-1} \right) \right\}. \qquad (110)
\end{aligned}
$$

Equation (109) implies

$$
\frac{\partial y}{\partial \mathbf{g}} = \tilde{c}^{\kappa-1} \mathbf{\Delta} \left(\tilde{w}_v^{1-\lambda} \right) \mathbf{q} \qquad (111)
$$

and

$$
\mathbf{\Delta} \left(\frac{\partial y}{\partial g_v} \right) = \mathbf{\Delta} \left(\tilde{c}^{\kappa-1} \tilde{w}_i^{1-\lambda} q_i \right), \qquad (112)
$$

which are plugged into the above yielding

$$
\begin{aligned}
\mathbf{H} &= \frac{1}{w_{n_V}}\left[\tilde{c}^{1-\kappa}(\lambda-1)\,\boldsymbol{\Delta}\left(\tilde{w}_v^{\lambda-2}\right)\boldsymbol{\Delta}\left(\frac{\partial y}{\partial g_v}\right)+\tilde{c}^{1-\kappa}\boldsymbol{\Delta}\left(\tilde{w}_v^{\lambda-1}\right)\frac{\partial^2 y}{\partial\mathbf{g}\partial\mathbf{g}^\top}\boldsymbol{\Delta}\left(\tilde{w}_v^{\lambda-1}\right)\right.\\
&\quad\left.+(1-\kappa)\,\tilde{c}^{1-2\kappa}\boldsymbol{\Delta}\left(\tilde{w}_v^{\lambda-1}\right)\frac{\partial y}{\partial\mathbf{g}}\frac{\partial y}{\partial\mathbf{g}^\top}\boldsymbol{\Delta}\left(\tilde{w}_v^{\lambda-1}\right)\right]\\
&= \frac{1}{w_{n_V}}\left[\tilde{c}^{1-\kappa}(\lambda-1)\,\boldsymbol{\Delta}\left(\tilde{w}_v^{\lambda-2}\right)\boldsymbol{\Delta}\left(\tilde{c}^{\kappa-1}\tilde{w}_v^{1-\lambda}q_v\right)\right.\\
&\quad+\tilde{c}^{1-\kappa}\boldsymbol{\Delta}\left(\tilde{w}_v^{\lambda-1}\right)\frac{\partial^2 y}{\partial\mathbf{g}\partial\mathbf{g}^\top}\boldsymbol{\Delta}\left(\tilde{w}_v^{\lambda-1}\right)\\
&\quad\left.+(1-\kappa)\,\tilde{c}^{1-2\kappa}\boldsymbol{\Delta}\left(\tilde{w}_v^{\lambda-1}\right)\tilde{c}^{\kappa-1}\boldsymbol{\Delta}\left(\tilde{w}_v^{1-\lambda}\right)\mathbf{q}\left[\tilde{c}^{\kappa-1}\boldsymbol{\Delta}\left(\tilde{w}_v^{1-\lambda}\right)\mathbf{q}\right]^\top\boldsymbol{\Delta}\left(\tilde{w}_v^{\lambda-1}\right)\right]\\
&= \frac{1}{w_{n_V}}\left[\tilde{c}^{1-\kappa}\tilde{c}^{\kappa-1}(\lambda-1)\,\boldsymbol{\Delta}\left(\tilde{w}_v^{\lambda-2}\right)\boldsymbol{\Delta}\left(\tilde{w}_v^{1-\lambda}\right)\boldsymbol{\Delta}(q_v)\right.\\
&\quad+\tilde{c}^{1-\kappa}\boldsymbol{\Delta}\left(\tilde{w}_v^{\lambda-1}\right)\frac{\partial^2 y}{\partial\mathbf{g}\partial\mathbf{g}^\top}\boldsymbol{\Delta}\left(\tilde{w}_v^{\lambda-1}\right)\\
&\quad\left.+(1-\kappa)\,\tilde{c}^{1-2\kappa}\tilde{c}^{\kappa-1}\tilde{c}^{\kappa-1}\boldsymbol{\Delta}\left(\tilde{w}_v^{\lambda-1}\right)\boldsymbol{\Delta}\left(\tilde{w}_v^{1-\lambda}\right)\mathbf{q}\mathbf{q}^\top\boldsymbol{\Delta}\left(\tilde{w}_v^{1-\lambda}\right)\boldsymbol{\Delta}\left(\tilde{w}_v^{\lambda-1}\right)\right]\\
&= \frac{1}{w_{n_V}}\left[(\lambda-1)\,\boldsymbol{\Delta}\left(\tilde{w}_v^{-1}\right)\boldsymbol{\Delta}(q_v)+\tilde{c}^{1-\kappa}\boldsymbol{\Delta}\left(\tilde{w}_v^{\lambda-1}\right)\frac{\partial^2 y}{\partial\mathbf{g}\partial\mathbf{g}^\top}\boldsymbol{\Delta}\left(\tilde{w}_v^{\lambda-1}\right)\right.\\
&\quad\left.+(1-\kappa)\,\tilde{c}^{-1}\mathbf{q}\mathbf{q}^\top\right]. \qquad (113)
\end{aligned}
$$

The remaining piece is $\partial^2 y/\partial\mathbf{g}\partial\mathbf{g}^\top$, which is given by equation 94 in LaFrance et. al. (2005),

$$
\begin{aligned}
\frac{\partial^2 y}{\partial\mathbf{g}\partial\mathbf{g}^\top} &= \left[1-\boldsymbol{\delta}^\top\mathbf{g}\left(\frac{y-\eta}{\varphi(\mathbf{g})}\right)\right]\left(\frac{y-\eta}{\varphi}\right)\left[\mathbf{B}-\frac{(\mathbf{B}\mathbf{g}+\boldsymbol{\gamma})(\mathbf{B}\mathbf{g}+\boldsymbol{\gamma})^\top}{\varphi}\right]\\
&\quad+2\frac{(y-\eta)^3}{\varphi^2}\left[\mathbf{I}-\frac{1}{\varphi}(\mathbf{B}\mathbf{g}+\boldsymbol{\gamma})\,g^\top\right]\boldsymbol{\delta}\boldsymbol{\delta}^\top\left[\mathbf{I}-\frac{1}{\varphi}\mathbf{g}(\mathbf{B}\mathbf{g}+\boldsymbol{\gamma})^\top\right]. \qquad (114)
\end{aligned}
$$

The adding up condition $c=\mathbf{w}^\top\mathbf{q}+w_{n_V}q_{n_V}$ implies that the demand for the n_V^{th} input is given by

$$
q_{n_V}=\frac{1}{w_{n_V}}\left[c-\mathbf{w}^\top\mathbf{q}\right]=\tilde{c}-\tilde{\mathbf{w}}^\top\mathbf{q}. \qquad (115)
$$

This implies that the $(n_V, 1), ..., (n_V, n_V - 1)$ elements of the Hessian are

$$\begin{aligned}
\mathbf{h} &\equiv \frac{\partial q_{n_V}}{\partial \mathbf{w}^\top} = \frac{\partial}{\partial \mathbf{w}^\top}\left[\tilde{c} - \tilde{\mathbf{w}}^\top \frac{\partial c}{\partial \mathbf{w}}\right] \\
&= \frac{1}{w_{n_V}} \frac{\partial}{\partial \mathbf{w}^\top}\left[c - \mathbf{w}^\top \frac{\partial c}{\partial \mathbf{w}}\right] \\
&= \frac{1}{w_{n_V}}\left[\frac{\partial c}{\partial \mathbf{w}^\top} - \frac{\partial}{\partial \mathbf{w}^\top}\mathbf{w}^\top \frac{\partial c}{\partial \mathbf{w}}\right] \\
&= \frac{1}{w_{n_V}}\left[\frac{\partial c}{\partial \mathbf{w}^\top} - \left[\frac{\partial c}{\partial \mathbf{w}^\top}\frac{\partial \mathbf{w}}{\partial \mathbf{w}^\top} + \mathbf{w}^\top \frac{\partial}{\partial \mathbf{w}^\top}\frac{\partial c}{\partial \mathbf{w}}\right]\right] \\
&= \frac{1}{w_{n_V}}\left[\frac{\partial c}{\partial \mathbf{w}^\top} - \frac{\partial c}{\partial \mathbf{w}^\top} - \mathbf{w}^\top \frac{\partial^2 c}{\partial \mathbf{w}\mathbf{w}^\top}\right] \\
&= -\tilde{\mathbf{w}}^\top \frac{\partial^2 c}{\partial \mathbf{w}\mathbf{w}^\top} \\
&= -\tilde{\mathbf{w}}^\top \mathbf{H}, \qquad (116)
\end{aligned}$$

which are the weighted sums of the columns of $\mathbf{H}$ due to adding up. The (n_V, n_V) element of the Hessian is given by

$$\begin{aligned}
h_{n_V n_V} &\equiv \frac{\partial q_{n_V}}{\partial w_{n_V}} = \frac{\partial}{\partial w_{n_V}} \frac{1}{w_{n_V}}\left[c - \mathbf{w}^\top \frac{\partial c}{\partial \mathbf{w}}\right] \\
&= \frac{1}{w_{n_V}} \frac{\partial}{\partial w_{n_V}}\left[c - \mathbf{w}^\top \frac{\partial c}{\partial \mathbf{w}}\right] + \left[c - \mathbf{w}^\top \frac{\partial c}{\partial \mathbf{w}}\right] \frac{\partial}{\partial w_{n_V}} \frac{1}{w_{n_V}} \\
&= \frac{1}{w_{n_V}}\left[\frac{\partial}{\partial w_{n_V}} c - \mathbf{w}^\top \left[\frac{\partial}{\partial w_{n_V}} \frac{\partial c}{\partial \mathbf{w}}\right]\right] + \left[c - \mathbf{w}^\top \frac{\partial c}{\partial \mathbf{w}}\right](-1)\left(\frac{1}{w_{n_V}}\right)^2 \\
&= \frac{1}{w_{n_V}}\left[\frac{\partial c}{\partial w_{n_V}} - \mathbf{w}^\top \frac{\partial^2 c}{\partial \mathbf{w} \partial w_{n_V}}\right] - \left[c - \mathbf{w}^\top \frac{\partial c}{\partial \mathbf{w}}\right]\left(\frac{1}{w_{n_V}}\right)^2 \\
&= \frac{1}{w_{n_V}}\left[q_{n_V} - \mathbf{w}^\top \mathbf{h}^\top\right] - w_{n_V} q_{n_V}\left(\frac{1}{w_{n_V}}\right)^2 \\
&= \frac{1}{w_{n_V}} q_{n_V} - \frac{1}{w_{n_V}} \mathbf{w}^\top \mathbf{h}^\top - \frac{1}{w_{n_V}} q_{n_V} \\
&= -\tilde{\mathbf{w}}^\top \mathbf{h}^\top \\
&= -\tilde{\mathbf{w}}^\top \left(-\tilde{\mathbf{w}}^\top \mathbf{H}\right)^\top \\
&= \tilde{\mathbf{w}}^\top \mathbf{H} \tilde{\mathbf{w}}, \qquad (117)
\end{aligned}$$

which is a quadratic form in the elements of $\mathbf{H}$ due to adding up. Thus, the full $n_V \times n_V$ Hessian is

$$\breve{\mathbf{H}} = \begin{bmatrix} \mathbf{H} & -\mathbf{H}^\top \tilde{\mathbf{w}} \\ -\tilde{\mathbf{w}}^\top \mathbf{H} & \tilde{\mathbf{w}}^\top \mathbf{H} \tilde{\mathbf{w}} \end{bmatrix}. \qquad (118)$$

C.3 Derivation of Equations (88) and (89)

Since $c(\mathbf{w}, k, \theta) = w_{n_V} f^{-1}\left(y\left(\mathbf{g}\left(\tilde{\mathbf{w}}\left(\mathbf{w}\right)\right), \theta\left(\tilde{k}, \boldsymbol{\Delta}\left(\bar{y}_q\right)\mathbf{s}\right)\right)\right)$, the composite function theorem implies

$$\begin{aligned}
\frac{\partial c}{\partial k} &= \frac{\partial}{\partial k} w_{n_V} f^{-1}\left(y\left(\mathbf{g}\left(\tilde{\mathbf{w}}\left(\mathbf{w}\right)\right), \theta\left(\tilde{k}, \boldsymbol{\Delta}\left(\bar{y}_q\right)\mathbf{s}\right)\right)\right) \\
&= w_{n_V}\left[\frac{\partial \tilde{k}}{\partial k}\frac{\partial y}{\partial \tilde{k}} + \frac{\partial \theta}{\partial k}\frac{\partial y}{\partial \theta}\right]\frac{\partial f^{-1}}{\partial y} \\
&= w_{n_V}\tilde{c}^{1-\kappa}\left[\frac{1}{w_{n_V}}\left(\alpha_{1,n_V} + \boldsymbol{\alpha}_1^\top \mathbf{g}\right) + \frac{\partial \theta}{\partial k}\frac{\partial y}{\partial \theta}\right], \qquad (119)
\end{aligned}$$

and

$$\begin{aligned}
\frac{\partial c}{\partial \mathbf{s}} &= \frac{\partial}{\partial \mathbf{s}} w_{n_V} f^{-1}\left(y\left(\mathbf{g}\left(\tilde{\mathbf{w}}\left(\mathbf{w}\right)\right), \theta\left(\tilde{k}, \boldsymbol{\Delta}\left(\bar{y}_q\right)\mathbf{s}\right)\right)\right) \\
&= w_n \tilde{c}^{1-\kappa}\frac{\partial \theta}{\partial \mathbf{s}}\frac{\partial y}{\partial \theta}. \qquad (120)
\end{aligned}$$

The expression for $\partial y/\partial \theta$ is

$$\begin{aligned}
\frac{\partial y}{\partial \theta} &= \frac{\partial}{\partial \theta}\left[\eta\left(\mathbf{g}, \tilde{k}\right) - \left(\frac{\varphi\left(\mathbf{g}\right)}{\boldsymbol{\delta}^\top \mathbf{g} + \sqrt{\varphi\left(\mathbf{g}\right)}\theta}\right)\right] \\
&= -\left[\varphi\left(\mathbf{g}\right)\frac{\partial}{\partial \theta}\left(\boldsymbol{\delta}^\top \mathbf{g} + \sqrt{\varphi\left(\mathbf{g}\right)}\theta\right)^{-1}\right] \\
&= \frac{\varphi\left(\mathbf{x}\right)^{3/2}}{\left(\boldsymbol{\delta}^\top \mathbf{x} + \sqrt{\varphi\left(\mathbf{x}\right)}\theta\right)^2} \\
&= \frac{1}{\sqrt{\varphi\left(\mathbf{x}\right)}}\left(\frac{\varphi\left(\mathbf{x}\right)}{\boldsymbol{\delta}^\top \mathbf{x} + \sqrt{\varphi\left(\mathbf{x}\right)}\theta}\right)^2. \qquad (121)
\end{aligned}$$

From equation (85),

$$\left(\frac{\varphi\left(\mathbf{x}\right)}{\boldsymbol{\delta}^\top \mathbf{x} + \sqrt{\varphi\left(\mathbf{x}\right)}\theta}\right)^2 = \left[y - \eta\left(\mathbf{x}\right)\right]^2 \qquad (122)$$

which implies

$$\frac{\partial y}{\partial \theta} = \frac{\left[y - \eta\left(\mathbf{x}\right)\right]^2}{\sqrt{\varphi\left(\mathbf{x}\right)}}. \qquad (123)$$

Since the specification for theta

$$\theta\left(\tilde{k}, \boldsymbol{\Delta}\left(\bar{y}_q\right)\mathbf{s}\right) = \boldsymbol{\pi}_S^\top \boldsymbol{\Delta}\left(\bar{y}_q\right)\mathbf{s} - \pi_K \tilde{k}, \qquad (124)$$

implies

$$\frac{\partial \theta}{\partial k} = -\frac{1}{w_{n_V}}\pi_K \text{ and } \frac{\partial \theta}{\partial \mathbf{s}} = \boldsymbol{\Delta}\left(\bar{y}_{iq}\right)\boldsymbol{\pi}_S, \qquad (125)$$

the marginal cost expressions are

$$\frac{\partial c}{\partial k} = \tilde{c}^{1-\kappa}\left(\alpha_{1,n_V} + \boldsymbol{\alpha}_1^{\top}\mathbf{g}\right) - \tilde{c}^{1-\kappa}\frac{\left[f\left(\tilde{c}\right) - \eta\left(\mathbf{g}, k\right)\right]^2}{\sqrt{\varphi\left(\mathbf{g}\right)}}\pi_K, \text{ and} \tag{126}$$

$$\frac{\partial c}{\partial \mathbf{s}} = w_{n_V}\tilde{c}^{1-\kappa}\frac{\left[f\left(\tilde{c}\right) - \eta\left(\mathbf{g}, k\right)\right]^2}{\sqrt{\varphi\left(\mathbf{g}\right)}}\boldsymbol{\Delta}\left(\bar{y}_q\right)\boldsymbol{\pi}_S, \tag{127}$$

where the substitution $y = f\left(\tilde{c}\right)$ has been made. Adding i and t subscripts where appropriate completes the derivation.