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Assessing the Relationship between Probability of Default and Loss Given Default in an Agricultural Loan Portfolio

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The authors are former Graduate Student and Professor, respectively, in the Department of Applied Economics. They wish to acknowledge the data assistance provided by AgriBank, FCB and AgStar Financial Services, ACA.

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The Agricultural Lender's Problem

Agricultural lenders are particularly concerned with credit risk since their credit portfolios may be relatively under-diversified and adverse market-wide (or systematic) conditions may pose a significant threat to the performance of their loan portfolio and their business. So, how should an agricultural lender adjust its capital reserves to guard against unexpected downturns in the business cycle?

The Basel Accord recognizes the existence of fluctuations in the economy and the impact on both default and loss rates. The Accord makes broad recommendations on how a bank should adjust its capital holdings in response to market fluctuations. The majority of agricultural lenders, being small-to-medium sized institutions, typically do not have the resources to adequately explore and implement this requirement. Our objective in this study is to identify a consistent methodology for addressing this problem and develop an empirical tool that agricultural lenders might use to evaluate the implications of these fluctuations for the capital positions they hold.

We explicitly model the interdependency between loan default rates and loan loss rates by applying the framework established by Miu and Ozdemir (2006). That framework enables us to develop a model which captures the correspondence between the probability of default and the loss given default. Miu and Ozdemir propose a stylized model that decomposes the correlations between loss and default rates into their systematic and nonsystematic components. This allows us to isolate and forecast the impact of fluctuations in the business cycle on the optimal level of economic capital for a lender. Specifically, we project the portfolio value-at-risk that is conditional on the phase of the agricultural business cycle. Economic capital projections are simulated using @Risk, a software program that is an add-on to Microsoft Excel. This allows us to develop a simulation tool that can be utilized and replicated by associations in the Farm Credit System. We examine a sub-portfolio of agricultural mortgage loans originated by AgStar Financial Services, ACA, which serves rural producers in Minnesota and Wisconsin.

The Miu-Ozdemir Framework

The Miu-Ozdemir model decomposes the relationship between probability of default (PD) and loss given default (LGD) for a given borrower according to the sensitivity of the borrower's credit risks to a common (or shared) systematic factor and the sensitivity to borrower-specific (random or idiosyncratic) factors. These “dependencies” are categorized into four distinct types of correlations that exist between: (i) the systematic risk factors of PD and LGD for a given borrower, (ii) the idiosyncratic risk factors of PD and LGD for a given borrower, (iii) the PD risk drivers across different borrowers, and (iv) the LGD risk drivers across different borrowers (see Figure 1).

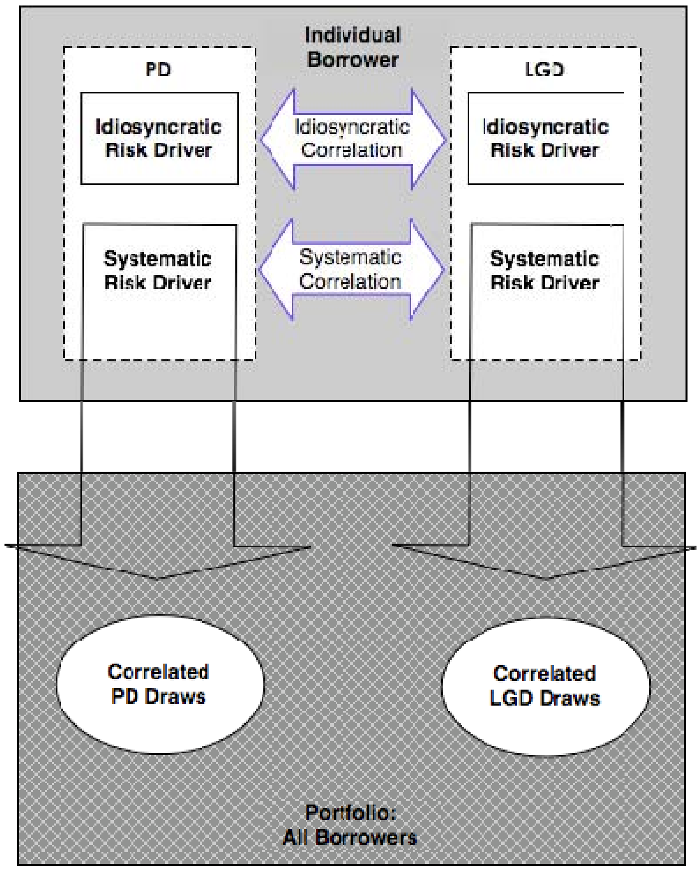

Figure 1. Systematic and Idiosyncratic Risks. (Source: Miu and Ozdemir, 2006).

It is expected that, during a recession (or downturn in the business cycle), unsecured or under-secured loans would exhibit higher rates of default and losses as a result of being more vulnerable to market-wide conditions. This phenomenon is implied by correlations (i) and (ii) identified above. Further, individual borrowers are susceptible to their own borrower-specific risks that may reduce their asset values independent of changes in economic conditions at large.

In the model description that follows we draw from the model presented in Miu and Ozdemir (2006) and use their notation. We begin by defining the systematic PD and

LGD risks, both of which are driven by the systematic risk factor X_t[1], via equations (1) and (2).

$$P_t = \beta_{PD} \times X_t + \varepsilon_{PD,t}, \quad (1)$$

$$L_t = \beta_{LGD} \times X_t + \varepsilon_{LGD,t}, \quad (2)$$

$\beta_{PD,t}$ represents the sensitivity of the systematic PD risk P_t to the market-wide risk factor X_t and $\beta_{LGD,t}$ represents the sensitivity of the systematic LGD risk L_t to the market-wide risk factor. By assumption, the residual changes ($\varepsilon_{PD,t}$ and $\varepsilon_{LGD,t}$) are mutually independent, they are independent of X_t, and they are normally distributed so that both of the systematic risks (P_t and L_t) are standard-normally distributed.

The borrower-specific (idiosyncratic) risks ($e_{PD,t}$ and $e_{LGD,t}$) are assumed to be not independent for any given borrower and we define them in (3) and (4).

$$e^i_{PD,t} = \theta^i_{PD} \times x^i_t + \varepsilon^i_{PD,t}, \quad (3)$$

$$e^i_{LGD,t} = \theta^i_{LGD} \times x^i_t + \varepsilon^i_{LGD,t}, \quad (4)$$

$\theta^i_{PD,t}$ represents the sensitivity of the idiosyncratic PD risk $e^i_{PD,t}$ to the standard-normally distributed borrower-specific credit risk factor x^i_t. $\theta^i_{LGD,t}$ represents the sensitivity of the idiosyncratic LGD risk $e^i_{LGD,t}$ to the borrower-specific credit risk factor. The mutually-independent residual changes $\varepsilon^i_{PD,t}$ and $\varepsilon^i_{LGD,t}$ are assumed to be normally distributed with standard deviation such that the idiosyncratic risks $e^i_{PD,t}$ and $e^i_{LGD,t}$ are standard-normally distributed.

Individual PD risk is governed by both the systematic PD risk, P_t, and the borrower-specific PD risk, $e^i_{PD,t}$, and is assumed to follow a standard normal distribution.

$$p^i_t = R_{PD} \times P_t + \sqrt{1 - R^2_{PD}} \times e^i_{PD,t} \quad (5)$$

Individual LGD risk is similarly defined and is determined by both the systematic LGD risk, L_t, and the borrower-specific LGD risk, $e^i_{PD,t}$, and it is assumed to have a standard normal distribution.

$$l^i_t = R_{LGD} \times L_t + \sqrt{1 - R^2_{LGD}} \times e^i_{LGD,t} \tag{6}$$

R_{PD} represents the correlation between these individual PD risks and the systematic risk factor P_t. Likewise R_{LGD} represents the correlation between the individual LGD risks and the systematic risk factor L_t. The authors also further specify that both systematic correlations, R_{PD} and R_{LGD}, can be shown to be pair wise correlations that represent the correlations between any given pair of individual PD or LGD risks, respectively.

In (5) and (6) we implicitly define the correlation between PD and LGD for a given borrower due to the systematic risk factor(s).

$$Corr(p_t, l_t) = \beta_{PD} \beta_{LGD} R_{PD} R_{LGD} \tag{7}$$

By adding the correlation attributed to the idiosyncratic risk factors for each borrower, it follows that the complete correlation structure between PD and LGD for each representative borrower *i* is equal to (8).

$$Corr(p^i_t, l^i_t) = \beta_{PD} \beta_{LGD} R_{PD} R_{LGD} + \theta^i_{PD} \theta^i_{LGD} \sqrt{1 - R^2_{PD}} \sqrt{1 - R^2_{LGD}} \tag{8}$$

The first term in (8) represents the correlation due to systematic risk factors. The second term in (8) represents the correlation due to idiosyncratic risk factors.

The lower the value of the systematic risk factor, the more adverse the state of the market. Thus, a lower value of the systematic PD sensitivity implies a *higher* likelihood of default, as adverse market-wide effects negatively impact an obligor's ability to repay debt.

R^2_{PD} and *PD* are the necessary inputs to the systematic PD function in (9). In (9) and (10), $\Phi^{-1}(\cdot)$ is the inverse of the cumulative standard normal distribution.

$$\log(\ell) \cong \sum_{t=1}^{T} \left(\frac{1}{2} \log\left(\frac{1-\hat{R}_{PD}^2}{\hat{R}_{PD}^2} \right) + \frac{1}{2} \left(\Phi^{-1}\left(\frac{k_t}{n_t} \right) \right)^2 - \frac{\left(\sqrt{1-\hat{R}_{PD}^2}\,\Phi^{-1}(k_t / n_t) - DP \right)^2}{2\hat{R}_{PD}^2} \right) \tag{9}$$

Having solved for R_{PD}^2 and *DP* in (9), we can solve for expected systematic PD risk, P_t, in each time period as shown in (10).

$$E\left(P_t \mid k_t, n_t; DP, R_{PD}\right) = \frac{DP - \Phi^{-1}(k_t / n_t) \cdot \sqrt{1-R_{PD}^2}}{R_{PD}} \tag{10}$$

We assume that the observed losses follow a beta distribution and map the observed LGDs onto their respective cumulative probabilities using the cumulative beta distribution function. We first rescale the actual LGD observations LGD_t^i by using the *range-normalization* transformation in (11).

$${}_R LGD_t^i = \frac{LGD_t^i - LGD_t^{i,MIN}}{LGD_t^{i,MAX} - LGD_t^{i,MIN}} \tag{11}$$

LGD_t^i is the actual observed LGD, ${}_R LGD_t^i$ is the rescaled LGD observation, $LGD_t^{i,MIN}$ is the minimum value of all the actual LGD observations and $LGD_t^{i,MAX}$ is the maximum value of the observed LGDs. By using the unconditional mean and variance of our range-normalized LGD observations ${}_R LGD_t^i$ we can solve for the parameters α and β in (12) and (13), where μ and σ^2 are the unconditional mean and variance of the range-normalized LGD observations ${}_R LGD_t^i$, respectively.

$$\mu = \frac{\alpha}{\alpha + \beta} \tag{12}$$

$$\sigma^2 = \frac{\alpha\beta}{(\alpha + \beta + 1)(\alpha + \beta)^2} \tag{13}$$

Solving for α and β allows us to map each of the normalized LGD observations onto their corresponding standard-normally distributed equivalent values. Because the Miu-Ozdemir model requires that our representation of individual LGD risk l_t^i is standard-normally distributed, we map each cumulative probability onto its corresponding value on the standard-normal distribution.

$$l_t^i = \Phi^{-1}(B({}_R LGD_t^i), \alpha, \beta)) \tag{14}$$

LGD is then obtained by transforming the standard-normally distributed individual LGD risks appropriately using the four-parameter specification of the beta distribution,[1] where *a* and *b* are the minimum and the maximum of the observed LGD values.

$$LGD_t^i = B^{-1}(\Phi(l_t^i), \alpha, \beta, a, b) \tag{15}$$

Because the standard deviation of the individual LGD risks l_t^i in each year is equal to $\sqrt{1-\hat{R}_{LGD}^2}$, we can derive the pair wise LGD correlation by calculating the "pooled estimate" of the standard deviation of l_t^i. Here n_t is the number of LGD observations in time period *t*, $\hat{\sigma}_t^2$ is the standard deviation of l_t^i in time period *t* and *T* is the total number of time periods available for the analysis.

$$\sqrt{1-\hat{R}_{LGD}^2} = \sqrt{\frac{\sum_{t=1}^{T}(n_t - 1)\hat{\sigma}_t^2}{\sum_{t=1}^{T} n_t - T}} \tag{16}$$

The systematic LGD risk is simply the mean of all the individual risks during a given year, divided by the square root of our estimated pair wise LGD correlation.

[1] Numerous statistical packages offer the four-parameter specification of the cumulative beta distribution. It performs the inverse of a range-normalization transformation after determining the inverse of a cumulative beta probability on the standard [0, 1] domain. This allows for further flexibility when applying the beta distribution, as our domain does not have to be bounded between 0 and 1.

$$L_t = \frac{\sum_{i=1}^{n_t} l_t^i}{n_t}, \text{ for all time periods } t \quad (17)$$

At this point we ask, if loss and default rates are modeled in an *acyclical* manner – where all correlations are set to zero – by how much would these *acyclical* LGD and economic capital estimates need to be increased in order to arrive at the same value-at-risk measures where these correlations *aren't* ignored? Miu and Ozdemir's simulation of different combinations of systematic and borrower-specific sensitivities shows that even with a moderate level of idiosyncratic PD/LGD correlation, LGD needs to be increased by as much as 37% when compared to estimates of economic capital that do not account for these correlations. In other words, *acyclically*-evaluated LGD estimates may severely understate the economic capital requirement, and these estimates need to be "marked up" accordingly.

Application of the Miu-Ozdemir Model

We develop the analysis of a loan portfolio by using @*Risk*, a simulation add-in for Microsoft Excel, and use it to incorporate the specific features of the Miu-Ozdemir model. For example, we take care to differentiate between the systematic and idiosyncratic components of the model during implementation. We are subjecting a portfolio of borrowers to the same systematic risk, while allowing their respective idiosyncratic risks to change. We use the flexibility of Visual Basic for Applications (VBA) macros to enhance @Risk's basic functionality in order to do so.

As shown in this section, the user of the model inputs the model correlation and sensitivity parameters into a spreadsheet template. These parameters are then used to perform the model calculations and display the results. @Risk allows for further flexibility to analyze simulation results by enabling users to customize the formatting and content of reports.

Model Description Window

The model description window as shown in Figure 2 is descriptive in nature and provides the user with a summary of the Miu-Ozdemir equations and variables, their respective distributions, and how they are related to each other. For example, it is a quick way to verify that the systematic risk factor, X_t, is standard-normally distributed and is related to the systematic PD and LGD risks by way of the sensitivity parameters β_{PD} and β_{LGD}.

Formula	Element	Description	
Systematic Risk			
$P_t = \beta_{PD} \times X_t + \varepsilon_{PD,t}$	$P_t(L_t)$	PD (LGD) systematic risk	N(0,1)
$L_t = \beta_{LGD,t} \times X_t + \varepsilon_{PD,t}$	X_t	Systematic risk factor	N(0,1)
	$\beta_{PD}(\beta_{LGD})$	PD (LGD) sensitivity	---
	$\varepsilon_{PD,t}(\varepsilon_{LGD,t})$	PD (LGD) residual changes	N(0,-)
Idiosyncratic Risk			
$e^i_{PD,t} = \theta^i_{PD,t} \times x^i_t + \varepsilon^i_{PD,t}$	$e^i_{PD,t}(e^i_{LGD,t})$	Idiosyncratic PD (LGD) risk	N(0,1)
$e^i_{LGD,t} = \theta^i_{LGD,t} \times x^i_t + \varepsilon^i_{LGD,t}$	x^i_t	Idiosyncratic risk factor	N(0,1)
	$\theta^i_{PD,t}(\theta^i_{LGD,t})$	Idiosyncratic PD (LGD) risk sensitivity	
	$\varepsilon^i_{PD,t}(\varepsilon^i_{LGD,t})$	Residual changes	N(0,-)
Individual Risk			
$p^i_t = R_{PD} \times P_t + \sqrt{1-R^2_{PD}} \times e^i_{PD,t}$	$p^i_t(l^i_t)$	Individual PD (LGD) risk	N(0,1)
$l^i_t = R_{LGD} \times L_t + \sqrt{1-R^2_{LGD}} \times e^i_{LGD,t}$	$R_{PD}(R_{LGD})$	Systematic PD (LGD) correlation	
	$P_t(L_t)$	PD (LGD) systematic risk	N(0,1)

Description | Simulation | Portfolio Results | Portfolio Distribution

Figure 2. Model Description Window

Simulation Window

The simulation window enables the user to input all the model parameters (see Figure 3). The sensitivity and correlation parameters chosen by the user are reflected in the panel labeled “correlation parameters.” The user can either enter the parameter values directly into the spreadsheet, or can click the button directly above the “correlation parameters” panel.

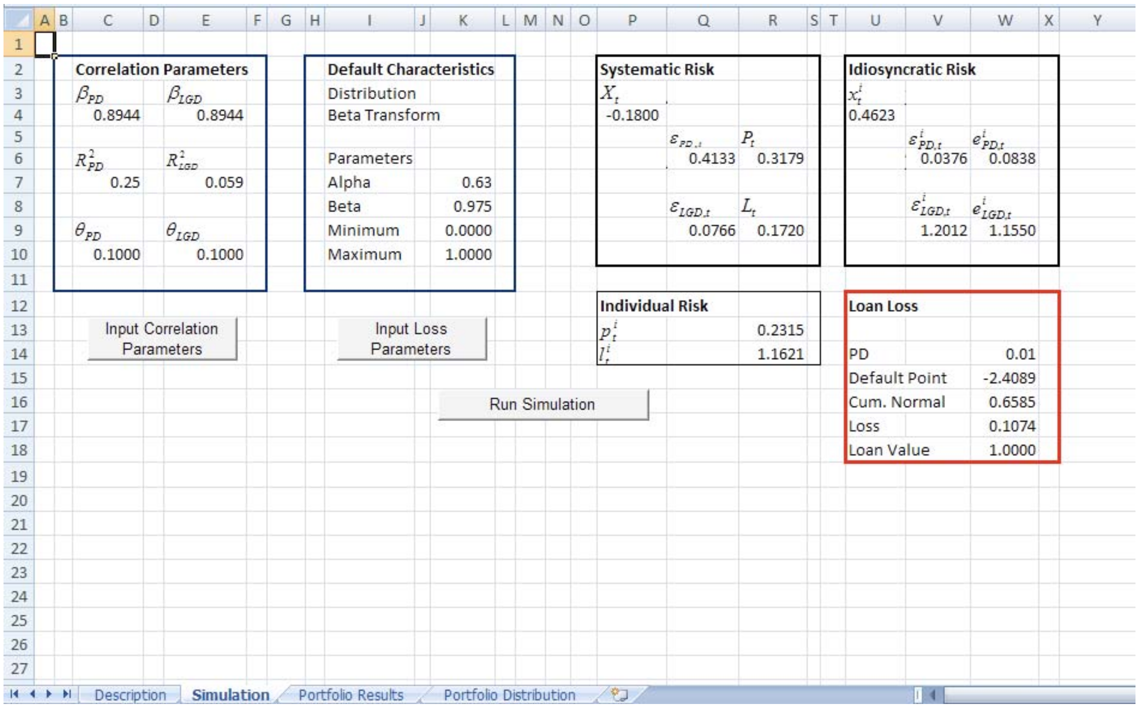

Figure 3: Simulation Window

Input Model Sensitivities

In Figure 4, the input model sensitivities dialog box allows the specification of the idiosyncratic and systematic sensitivities, as well as the pair wise PD and LGD correlations. A key assumption that we make is that the systematic and idiosyncratic sensitivities are equal (see Miu and Ozdemir). If a user would like to specify this assumption as such, he or she can conveniently enter a single value for each pair of systematic and idiosyncratic sensitivities. For example, if both the systematic sensitivities are equal to the square root of 0.8, we can enter the expression: “= SQRT(0.8)” into the dialog box labeled “systematic PD sensitivity = systematic LGD sensitivity = “.

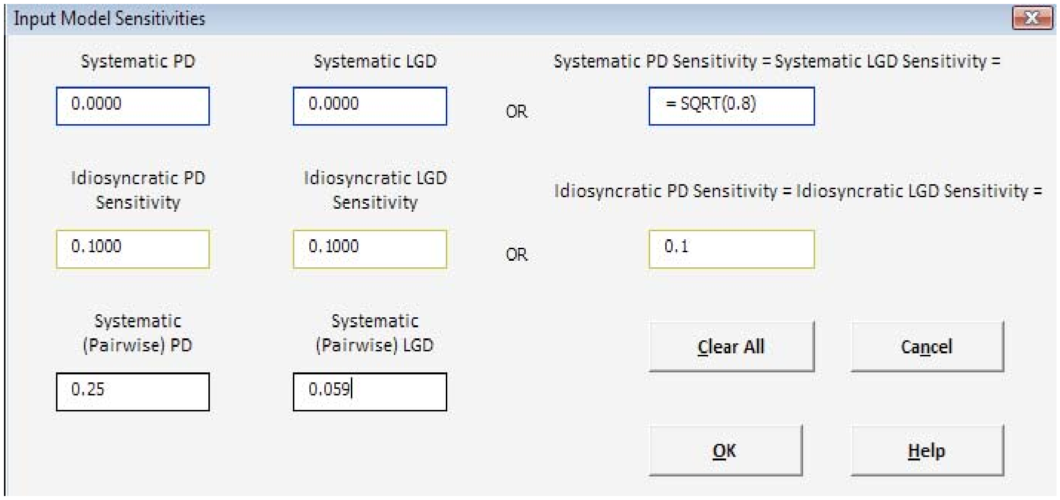

Figure 4. Input Model Sensitivities Dialog Box

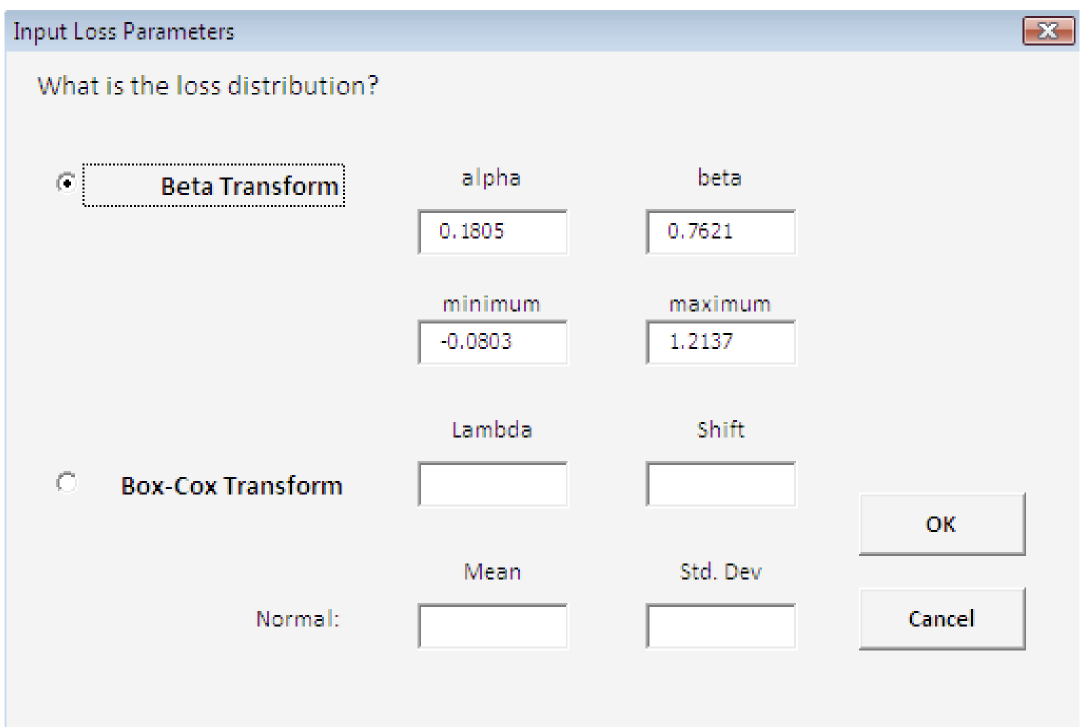

Figure 5. Input Loss Parameters Dialog Box

Input Loss Parameters

The Input Loss Parameters dialog box enables the user to input the specifications of the loss distributions used to translate the observed LGD rates into their standard-normally distributed equivalents (see Figure 5). The application provides the necessary functionality to translate the individual LGD risks l_t^i into their corresponding beta-distributed loss. Based on preliminary testing, we found that the Box-Cox[2] transformation may be a very useful and often-applicable transformation of the observed LGD values as well.

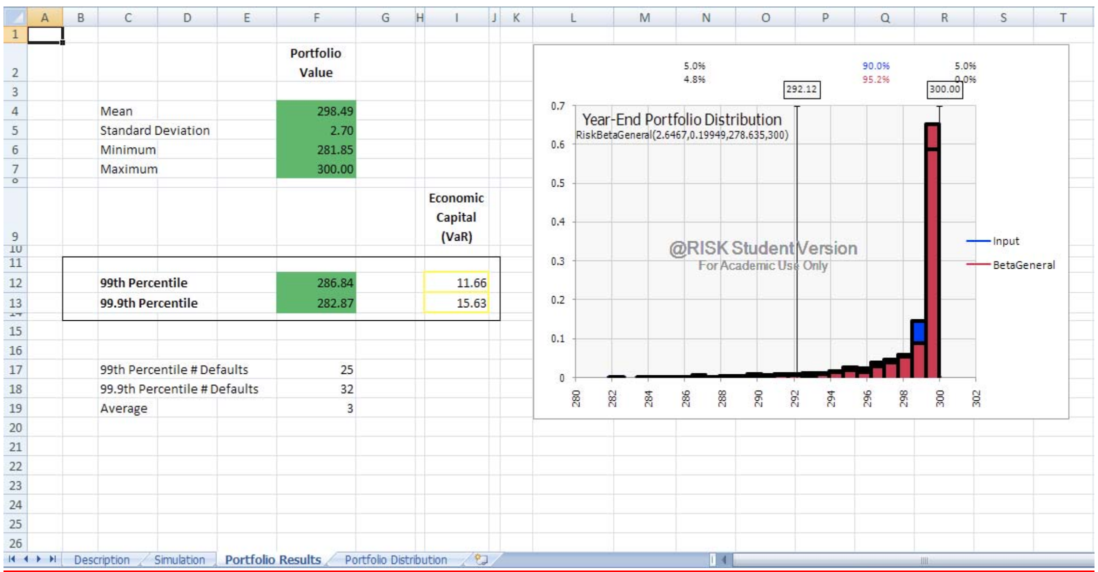

Figure 6. Results Window

Results Window

The Results Window gives a summary of the economic capital requirements of a given set of model parameters at the 99th and 99.9th percentiles (see Figure 6). Using @Risk's built-in distribution-fitting capabilities, we can further investigate and evaluate the input and output distributions both graphically and numerically.

While precisely reproducing Miu and Ozdemir's results would have been ideal in validating the model, limited computer processing and memory capacity necessitated that we minimize the number of simulations and iterations necessary to best approximate Miu and Ozdemir's simulation outcomes. We determined that 300

[2] The popular Box-Cox method attempts to transform the distribution of a continuous variable into an approximately standard normally-distributed range of values and is available in most statistical packages. We apply the computational form specified by Johnson and Wichern (2001).

simulations each consisting of 300 iterations was the most optimal combination of simulations and iterations that gave us comparable results while striking the right balance between the validity of the probability distributions being generated and the simulation running-time. Indeed, 300 iterations is a widely-used rule-of-thumb standard that allows a distribution to adequately approximate its true density[3].

Applying the Miu-Ozdemir Model to Agricultural Loans

We hypothesize that farm real estate and intermediate term loans may reasonably exhibit a significant level of systematic risk, which would make for a good case-study. Further, if it is reasonable to assume that the creditworthiness of agricultural borrowers depends on the value of their assets, we surmise that variations in agricultural land values are a plausible proxy for the systematic risk factor (X_t) which drives default and loss given default rates at the industry and lender levels. By extending the analysis to incorporate years in which land values were volatile (the "farm crisis" of the early 1980s) we can simulate the effects of an economic cycle on the agricultural lender's portfolio and economic capital (see Figure 7).

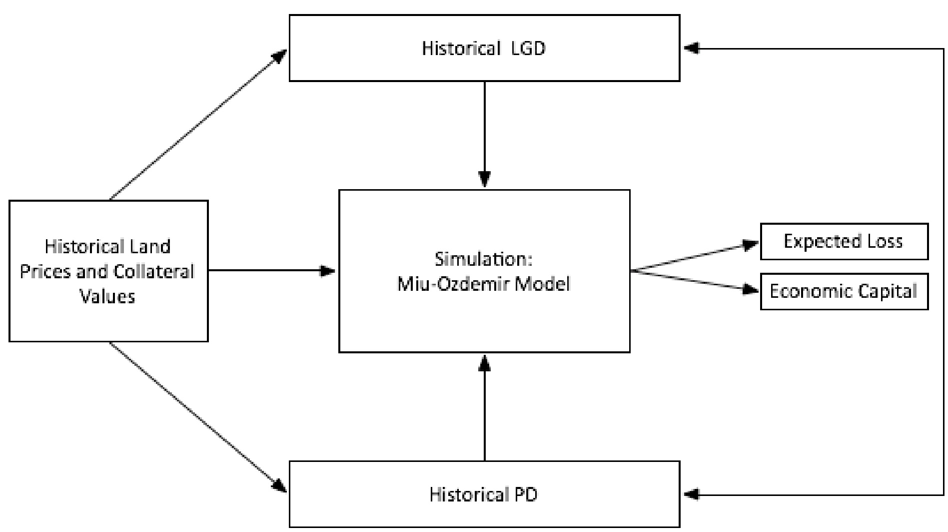

Figure 7. Agricultural Loan Portfolio Simulation

[3] We specify the "Latin Hypercube" random number generator as the sampling option. The Latin Hypercube method is a stratified-sampling algorithm especially conducive to the goal of attaining stabilization or "convergence" of a distribution much more efficiently than traditional sampling algorithms.

The model is calibrated to the agricultural lender's portfolio by identifying and measuring the key correlations in the model. These correlations are between; land values and PD, land values and LGD, and PD and LGD. We will use the Minnesota agricultural land values series provided by the United States Department of Agriculture (USDA, 2009) at the national and state levels during 1950-2008 (Appendix A).

The time-series of default and loss-given-default data provided by AgriBank spans a limited time horizon, 2002 through 2008, a period of time during which the agricultural industry was relatively prosperous and defaults were at historically low levels.

Due to insufficient data from the 1980s, reliable farm credit default and loss data is unavailable for the years corresponding to the farm financial crisis. Thus, we employ a credit officer survey to generate a proxy for the actual historical data. Our goal was to elicit expert estimates of the timing and the severity of the farm financial crisis with respect to this specific lender.

While there was some variation in their peak default estimates, the responding credit officers all agreed that 1986 was the year the Association's credit portfolio experienced the most stress. Further, responses indicated that the pre-farm crisis default rates were in the 0%-2% range. In contrast, the peak default rate ranged from 4.5% to 40% for real-estate loans, and 6% to 20% for intermediate term loans.

The survey responses were combined with actual historical Association level default data from the years 1999 through 2008. As a result, we had three sets of default data series, each consisting of 12 observations. 10 of these observations were from the actual 1999-2008 historical data, and 2 observations were based on the respondents' answers about the high default years in the early 1980s. Each observation consisted of the number of borrowers current at the start of the given year (n_t) and the number of these borrowers who defaulted by that year's end (k_t). Recall that the systematic PD risk as a function of our chosen systematic risk factor (land values), and it is defined by the linear equation $P_t = \beta_{PD} \times X_t + \varepsilon_{PD,t}$. Deriving the systematic PD sensitivity parameter, β_{PD}, then becomes a matter of estimating the linear regression of the time-series for systematic PD risk on the standard-normalized land values series.

Introducing Minnesota land values into the investigation of the systematic dependency of the credit risks sheds more light on the overall systematic dependency of the credit risks. A graphical display of -P_t, L_t and the Minnesota land values series helps us appreciate any underlying systematic trends more intuitively (see Figure 8).

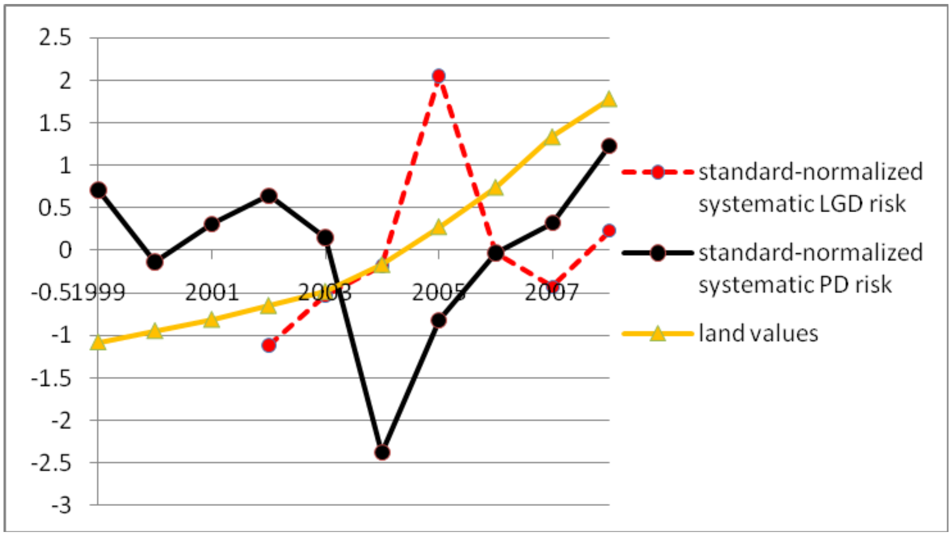

Figure 8. Systematic Credit Risks and Land Values, 1999-2008.

Due to the limited number of observations available and noise apparent in the data, we choose to apply the most statistically-significant systematic sensitivity estimate obtained via linear regression. We (more conservatively) use our regression sensitivity estimates of 0.98 and -0.98, for β_{PD} and β_{LGD} respectively, through the rest of our subsequent analyses. Before moving ahead, however, it is worth asking: are the coefficient estimates obtained by linear regression reasonable?

The business of agricultural lending is relatively sector-specific. That is, the value of an agricultural lender's credit portfolio may depend heavily on the overall financial and economic health of the agricultural industry. If covariant risk is a problem, it is because farms and/or agribusinesses are sensitive to a set of market-wide factors or forces that adversely affect many clients at once. Those factors may be more significant determinants of credit risk than the uncorrelated borrower-specific risks that may exist. In this paper we examine a predominantly secured subset of agricultural real-estate loans and intermediate-term loans. A sudden decline in the value of farm land may imply an unexpected loss of income to the farmer. If the farmer defaults on his/her debt, any recovery made by the lender upon possession of the collateral will likely be at a lower value because the market as a whole experiences stress, and the collateral is less valuable than when the loan was originated.

Thus, the value of farm land used as collateral to back these loans is an indicator of both the obligor's ability to repay debt *and* the resulting loss on an outstanding exposure if the borrower defaults. However, if this is true, the correlation parameters we have estimated ($R_{PD}^2 = 0.08$ and $R_{LGD}^2 = 0.9$) seem to be counter-intuitive: shouldn't we expect *both* systematic correlations to be reasonably high? In fact, an R_{PD}^2 value of about 8% is reasonable. We see that this value has an associated long-run probability of default of slightly less than 1%. This means that in the long run about 1 out of every 100 loans goes into default. This suggests that the agricultural industry is a relatively *stable* one, except for the farm financial crisis years in the 1980s. There are different *types* and sizes of farms and some farms are better able to continue to operate through a period of moderate-to-severe systematic shocks than others. Therefore, even though farms and farmland values may be negatively impacted at a specific point in time, all obligors are not likely to go into default simultaneously. However, loans of farms that actually *do* default at approximately the same time are subject to the same appraisal of their collateral, because this appraisal is a market-wide valuation of the worth of the sector-specific assets that are used to guarantee the loans.

Table 1. Calibrating the Idiosyncratic Sensitivities to Reported Default Frequencies

$\theta_2 = 0.1$	Reported Frequencies				Simulated Frequencies		
Year	%chg in land values	# Loans	# Loans Defaulting	Default Rate	# Loans	# Loans Defaulting	Default Rate
1982	-8.41%	200	3	1.5%	300	3	1%
1986	-22.72%	200	10.5	5.2%	300	14	4.6%

The agricultural sector, and the specific loan portfolio we are looking at, may be subject to a significant degree of market-wide risk. Yet, because farming is generally a stable industry, we expect that any individual borrower has a low-to-moderate borrower-specific risk of default. Thus, we will explore low levels of idiosyncratic sensitivity in the model application. We also assume that the idiosyncratic risks (for default and loss given default) are of equal magnitude ($\theta_{PD} = |\theta_{LGD}|$). For convenience we will refer to the equal idiosyncratic risk magnitudes as θ_2. We evaluate an appropriate level of

θ_2 that best approximates the default frequencies observed from the lender survey and the lender's historical data. These results are reported in Table 1.

As we conduct the simulation analysis we hold the systematic risk factor (X_t) fixed at the standard-normal value that represents the annual percentage change in land values at the peak and immediately before the farm crisis years. We simulate a distribution of 300 year-end portfolio values, each consisting of 300 borrowers for robustness. The model setup in @Risk generates the corresponding number of defaults for each year-end portfolio value. We compare the average of the number of defaults in all the portfolio-values generated to the default frequencies reported by the credit staff in 1982 and 1986. As expected, the low level of idiosyncratic sensitivity reported in Table 1 allows us to approximate the observed default frequencies almost exactly. The simulated default frequencies are reported next to the reported frequencies at the peak of the cycle (1986) and immediately before the cycle (1982). These two frequencies correspond relatively well.

Stress-Testing the Portfolio

The analysis is concluded by stress-testing the calibrated Miu-Ozdemir model by anticipating specific percentage changes for the chosen systematic risk factor (land values). These percentage changes are used to derive their standard-normally distributed equivalents (via the Box-Cox transformation, where necessary). Thus, we need to find appropriate interpolations for specific percentage declines within the range of the observed percentage changes. Specifically, we are interested in evaluating the economic capital requirement when land values decline by 5%, 10%, 15% and 22.72% (which was their highest year-on-year decline, reported in 1986). We seek to answer the question: "If the agricultural sector again experienced significant financial stress, and land values changed from their current values by the same proportionate shift as during the farm credit crisis, how would this impact the economic capital requirement of the intermediate-term and real-estate loan portfolios?"

We compare the "markup" to economic capital to the economic capital generated from the baseline scenario (which corresponds to an expected 6.3% increase in land values). We stress the portfolio relative using downward shifts in the land values series to capture deviations from the recent trend of expected gains.

If the most recent Minnesota land value estimate (in our case the 2008 value of $2970/acre) experiences similar percentage losses to those exhibited by land values through the farm crisis years, by how much should this particular agricultural lender mark up their economic capital estimates in order to compensate for a downturn in the economy? The annual percentage land value declines of interest are 0% (the baseline), 5%, 10%, 15%, and the historical 1986 decline of 22.72%. In order to interpolate the percentage changes not directly observed in the series, we use a simple linear regression to perform a fit of the observed values' standard-normal equivalents (via the Box-Cox method, where necessary) on the actual percentage changes. Of course, since one variable is merely the rescaling of another, the fit is perfect, allowing us to interpolate the standard-normal equivalents of the 0%, 5%, 10% and 15% declines.

We use the model parameters calibrated to the agricultural loan data to simulate a hypothetical loan portfolio of 300 borrowers experiencing a systematic risk that is centered about the standard-normal equivalents of the percentage declines. For each percentage decline, we evaluate the percentage markup that is required to increase the economic capital from the baseline level (0% change) to that of the relatively stressed level.

Table 2. Economic Capital Markup by Percentage Change in Land Values

% change	Average #Defaults (Default Rate)	99 Percentile Level		99.9 Percentile Level	
		Economic Capital	%Markup	Economic Capital	%Markup
+6.3%	3 (1%)	7.71	0%	8.09	0%
0%	4 (1.1%)	12.09	57%	21.59	167%
-5%	5 (1.7%)	22.01	185%	26.45	227%
-10%	8 (2.6%)	23.49	205%	38.86	380%
-15%	10 (3.3%)	29.75	286%	41.09	408%
-22.72%	16 (5.3%)	37.16	382%	44.52	450%

In Table 2 we summarize the results of the stress-test analysis. For example, if agricultural land values decline by 5%, the average number of defaults is expected to be equal to 5, which is equal to a default *rate* of 5/300 = 1.7%. Recall, we are simulating 300 borrowers each owing one dollar in a year's time.

Table 3. Economic Capital per Dollar of Exposure

At-Risk Percentile Level = 99%		
%change in land values	Economic Capital	% of exposure
+6.3%	7.71	2.6%
0%	12.09	4.0%
-5%	22.01	7.3%
-10%	23.49	7.8%
-15%	29.75	9.9%
-22.72%	37.16	12.4%

At-Risk Percentile Level = 99.9%		
%change in land values	Economic Capital	% of exposure
+6.3%	8.09	2.6%
0%	21.59	7.2%
5%	26.45	8.8%
10%	38.86	13.0%
15%	41.09	13.7%
-22.72%	44.52	14.5%

From the perspective of the lender, we can think of economic capital as the *proportion at risk* of one dollar over a specified time horizon at a given confidence level. For example, an anticipated decline of 15% in land values will result in 9.9% of every dollar in the portfolio being at risk over the coming year, at the 99% confidence level (see column 3 in Table 3).

In Table 2 we report the average (expected) number of defaults in the distribution of year-end portfolio values. This gives us the probability-of-default estimate, *PD*. For the expected (or baseline) increase in land values of 6.3%, we can write the equation

$$LGD = \frac{EL}{PD} = \frac{0.73}{300} \times \frac{300}{3}$$

Solving this equation for the implied LGD rate yields LGD = 0.24 or 24%. The actual observed average LGD of the loan portfolio is approximately 20%. Given the limited number of simulation cycles implemented here, we can see that the simulation estimates are comparable to what has been actually observed. This tells us that the baseline simulation is adequately calibrated to the lender’s actual experience, and suggests that that the estimates relative to the baseline, and the chosen model parameters, are quite reasonable.

Summary and Conclusions

We explore the Miu-Ozdemir model by incorporating farmland values as the systematic risk factor that drives credit defaults and loan losses. Because an agricultural credit portfolio is largely undiversified due to its dependence on the financial wellbeing of a single sector in the economy, we began with the hypothesis that agricultural loans would show a significant degree of systematic risk. This expectation proved to be true in the simulation model application. The real estate and intermediate-term loan categories exhibit a strong correlation between their systematic PD and LGD risks. This correlation represents a significant and positive systematic risk sensitivity to land values in the portfolio of the Association.

An advantage of the approach used here is that systematic sensitivities can be obtained without explicitly modeling the market-wide risk factor (Miu and Ozdemir, 2005). We have shown that at the same time, the Miu-Ozdemir model is sufficiently flexible to allow the explicit modeling of the chosen systematic risk driver. Given the limitations of the loan data in our study this is somewhat of a necessity.

Using historical and surveyed default rate data, we are able to calculate the sensitivity of systematic PD to land values. We find that the correlation of the systematic PD risk to changing land values is positive and reasonably strong. The pairwise LGD correlation estimate characterizes an exceptionally strong systematic relationship between the observed LGD values. This is reasonable due to the specific types of loans in the lender's portfolio. Close to 75% of these loans are well-secured or adequately-secured. The loan categories evaluated in this study, real-estate mortgages and intermediate-term loans, are traditionally secured by farm real estate as the dominant source of collateral Therefore, the portfolio exhibits strong dependence on the value of the collateral guaranteeing these loans.

Farm real estate values are sensitive to the economic performance of the agricultural sector overall. Although it is not an instantaneous relationship, when agricultural commodity prices fall the value of farmland also falls. Further, some of this collateral is industry-specific to the degree that it is difficult, if not impossible, to sell the collateral outside of the agricultural industry. For example, farming equipment and machinery has little or no application outside of the agricultural industry. Similarly, many agricultural buildings and structures have a single agricultural use (e.g., storage facilities and barns). Therefore, when the agricultural sector performs poorly, agricultural lenders may have to write off a greater degree of any exposures outstanding because the lenders cannot recoup 100% of the collateral that was secured when the loans were originated and the sector was performing more favorably. This explains the generally high correlation between the individual observations of loss given default, since the collateral across borrowers at any point in time experiences a similar proportionate change in value.

We conclude that the model explored in this paper provides a useful framework for empirical analysis. It gives credit risk practitioners a consistent way to account for the relationship between credit default rates and loss given default rates. The framework enables us to develop a simulation model which can serve as an effective credit risk management tool, either as a stand-alone application or as an aid to informing lender decisions which may be made in conjunction with other tools and methods.

Appendix A: Minnesota and Wisconsin Value of Land and Buildings (per acre)

(Source: U.S. Department of Agriculture)

	Minnesota	Wisconsin
1950	84	89
1951	98	99
1952	107	105
1953	109	107
1954	104	101
1955	109	101
1956	119	107
1957	129	116
1958	143	122
1959	152	131
1960	155	133
1961	150	137
1962	156	144
1963	158	143
1964	162	150
1965	167	155
1966	176	165
1967	188	182
1968	201	193
1969	216	213
1970	226	232
1971	231	255
1972	241	274
1973	269	328
1974	338	389
1975	429	434
1976	529	496
1977	672	598

	Minnesota	Wisconsin
1981	1281	1,152
1982	1272	1,144
1983	1165	1,113
1984	1131	1,104
1985	898	944
1986	694	836
1987	587	777
1988	700	826
1989	747	845
1990	810	801
1991	881	849
1992	884	865
1993	910	925
1994	914	968
1995	950	1,040
1996	1030	1130
1997	1090	1170
1998	1160	1240
1999	1240	1450
2000	1320	1700
2001	1400	1950
2002	1500	2150
2003	1600	2300
2004	1790	2470
2005	2060	2790
2006	2340	3100
2007	2700	3640
2008	2970	3850

Correlation between Minnesota and Wisconsin series: R-squared = 0.9846

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