ISSN 1679-1614

PORTFOLIO SELECTION MODELS: COMPARATIVE ANALYSIS AND APPLICATIONS TO THE BRAZILIAN STOCK MARKET

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Abstract – This paper presents a comparison of three portfolio selection models, Mean-Variance (MV), Mean Absolute Deviation (MAD), and Minimax, as applied to the Brazilian Stock Market (BOVESPA). For this comparison, we used BOVESPA data from three different 12 month time periods: 1999 to 2000, 2001, and 2002 to 2003. Each model generated three optimal portfolios for each period, with performance determined by monthly returns over the period. In general, the accumulated returns from the Minimax modeled portfolios were superior to the BOVESPA's principal index, the IBOVESPA. The MV model was the least efficient for portfolio selection.

Key words: portfolio selection, stock market, Brazil.

1. Introduction

In his work *Portfolio Selection* (1952), H. Markowitz presented what is now known as modern portfolio theory. This theory is based on the Mean-Variance (MV) optimization model, which solves the portfolio problem by using two basic indicators: expected returns, represented by the mean return, and risk, measured by the return's variation.

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In spite of having attractive theoretical consistency, the MV model is not often used for portfolio selection (Konno and Yamazaki, 1991) because of difficulties related to its formulation and solution. The most relevant one is the covariance matrix dimension, which may result in a quadratic optimization problem that makes an optimal solution rather difficult to attain.

The MV model's drawbacks led researchers to develop alternative portfolio selection models. The Minimax (Young, 1998) and the Mean Absolute Deviation (MAD) portfolio selection models are now frequently employed in portfolio studies. Both models make use of linear equations, removing one of the MV model's major shortcomings and making them more suitable for practical use.

The main goal of this paper is to review the application and relative performances of the Minimax, the MAD, and the MV models for portfolio selection in the Brazilian stock market (BOVESPA). We contrasted the performance of model generated portfolios for three different time periods: from September 1999 to August 2000; from January to December 2001; and from February 2002 to January 2003. The first time period is typified by an up market, whereas the last two periods are dominated by down markets. Additionally, we evaluated the models' performance by using choice sets with different numbers of stocks available for investment. There are three choice sets: one comprised of 20 stocks that the models can choose among when making investments, another with 50 stocks, and another with 100 stocks. This procedure is added to meet the diverse needs of investors and may be useful as a guide in their choice of portfolio selection models under different economic environments.

2. Analytical Models

Portfolio theory (See Markowitz, 1952) is applied to the study of methods for selecting portfolios under risky conditions. Following this theory, a portfolio's expected return is estimated based on a probability distribution that takes into account the investor's utility function. A density function of the events is built, and its measure of central tendency gives the return from the assets; correspondingly, its standard deviation, which is the dispersion measure of the expected returns around the mean, is a convenient measure of asset risk. In the Mean-Variance (MV) model, the portfolio that minimizes the variance subject to the restriction of a given mean return is chosen as the optimum portfolio.

The MAD and Minimax models measure risk in an alternative way. The MAD model retains some of the theoretical characteristics of the MV model, and because of this, it is frequently used. The Minimax model is based on game theory and has also been employed in portfolio optimization studies.

As mentioned earlier, these last two models are expressed in linear forms and can be solved using linear programming techniques. This is an obvious advantage over the MV model with its quadratic form and need for quadratic programming. Use of either of the linear models considerably reduces the time needed to reach a solution, thereby making them more feasible for large-scale portfolio selection.

2.1. The MV model

According to Young (1998), the MV model can be described as:

$$\underset{w}{Min} \sum_{j=1}^{N} \sum_{k=1}^{N} w_j w_k s_{jk} \ , \tag{1}$$

subject to

$$\sum_{j=1}^{N} w_j \bar{y}_j \geq G \tag{2}$$

$$\sum_{j=1}^{N} w_j = \mathrm{W} \tag{3}$$

$$0 \leq w_j \leq u_j, \ \mathrm{j} = 1, \ldots, \mathrm{n} \text{ and } \mathrm{k} = 1, \ldots, \mathrm{N}. \tag{4}$$

where: $s_{jk} = 1/(\mathrm{T - N})\sum_{t=1}^{T}(y_{jt} - \bar{y}_{\mathrm{j}})(\mathrm{y}_{\mathrm{kt}} - \bar{y}_k)$, for a finite number of financial assets, N, at time T; y_{jt} denotes the return of the asset *j* at time *t*; $\bar{y}_j$ is the mean return of the asset *j*; y_{kt} is the return of the asset *k* at time *t*; $\overline{y_k}$ is the average return of the asset *k*; $\sum_{j=1}^{N} w_j \bar{y}_j$ is the portfolio average return*;* w_j e w_k are the portfolio allocations for the assets *j* e *k*, respectively; and u_j is the maximum budget share that can be invested in the asset *j*.

The MV portfolio selection model represents the portfolio with minimum variance (Eq. 1), subject to the restriction that the mean return of the portfolio overcomes a given level, G (Eq. 2), such that total allocations to the portfolio cannot exceed the total budget, W (Eq. 3).

The significant outcome of this analysis comes from the fact that as the correlation among the assets decreases, the benefits of the portfolio's diversification increases, that is, the risk level decreases for a given return rate. Thus, the lower the correlation among asset returns, the higher the risk diversification will be.

Note that many market agents do not consider the standard deviation of returns as a satisfactory portfolio risk measurement (Kroll *et al.*, 1984; Young, 1998). Also, as investors' perception of risk may not be symmetrical, the Markowitz model should be seen only as an approximation of the investor's optimization problem.

2.2. Minimax model

Young (1998) was the first to apply the Minimax model for portfolio selection. As mentioned before, this model is based on game theory. A game can have two or more players each one knowing the goals and the opponents' possible strategies (complete information games). If each player behaves rationally, then game theory asserts that a solution for every situation can be determined by assuming that the players seek to maximize their expected minimum returns–Maximin criterion – or, conversely, minimize their maximum expected losses – Minimax criterion. Situations that involve the agents' decision-making process under risky conditions have been very well represented and solved through game theory. Even though those situations usually involve only one agent, the Minimax model has shown to be suitable for solving those kinds of problems, as long as Nature is considered the other player and the player who makes the decisions protects himself from the worst possible outcome.

According to Young (1998), the formulation of the Minimax model applied to portfolio selection can be described as follows: For a finite number of financial assets, N, and horizon, T:

$$\bar{y}_j = \frac{1}{T}\sum_{t=1}^{T} y_{jt} \tag{5}$$

$$E_p = \sum w_j \bar{y}_j \tag{6}$$

$$y_{pt} = \sum_{j=1}^{N} w_j y_{it} \tag{7}$$

$$M_p = \min_t y_{pt} \tag{8}$$

where y_{jt} denotes the return of the money invested in the asset j at time t; $\bar{y}_j$ is the mean return from the asset j; w_j is the portfolio allocation to the asset j; y_{pt} is the portfolio return at time t; E_p is the average portfolio return *;* and M_p is the portfolio minimum return per time period. This formulation refers to the description of the Maximin portfolio selection method; however, the term Minimax will be used since it is more often mentioned in the specialized literature for this formulation. Nevertheless, it is necessary to highlight the fact that the formulation presented here is the Maximin criterion, which is not its dual formulation, Minimax.

The Minimax model attempts to obtain the maximum value of M_p (the portfolio minimum return per time period), such that E_p (portfolio mean return) exceeds a given level, G, and total portfolio allocations cannot exceed the total budget, W.

Based on the above definitions, the optimization problem can be described as:

$$\underset{M_p, w}{Max}\, M_p \tag{9}$$

subject to

$$\sum_{j=1}^{N} w_j y_{jt} - M_p \geq 0, \quad t = 1, \ldots, T \tag{10}$$

$$\sum_{j=1}^{N} w_j \overline{y_j} \geq G \tag{11}$$

$$\sum_{J=1}^{N} w_j \leq W \tag{12}$$

$$0 \leq w_j \leq u_j, \ \mathrm{j} = 1, \ldots, \mathrm{n}. \tag{13}$$

As noted earlier, the portfolio chosen by the model is the one that maximizes the minimum return over past observations (Maximin). Eq.10 assures that for every time period M_p will always be smaller than or equal to the portfolio return. Thus, M_p represents the portfolio's minimum return at the end of each time period and will be bounded from above by the minimum portfolio return. Since M_p is being maximized, the portfolio will take on the maximum value of the minimum returns (Maximin). According to Young (1998), this model presents logical advantages over other portfolio optimization models if asset prices are not normally distributed and similar results when they are.

2.3. Mean Absolute Deviation (MAD) model

Konno and Yamazaki (1991) were the first researchers to present the Mean Absolute Deviation (MAD) portfolio optimization model. The MAD model was designed to retain the advantages of the Mean-Variance (Markowitz's) model while removing some of its shortcomings, thereby making the MAD model more suitable for use by working brokers. The optimum MAD model portfolio is the one that minimizes the return's average absolute deviation subject to the restriction of a given mean return.

The MAD model was formulated with linear functions that are solved using linear programming techniques, thus avoiding the difficulties presented by quadratic programming. The model's construction process consists basically of creating a non-linear formulation that approximates a linear one. This non-linear form is the absolute deviation from the portfolio's mean return, described as follows:

$$\text{Absolute Deviation} = E\left[\sum_{j=1}^{N} Y_J w_j - E\left[\sum_{j=1}^{N} Y_j w_j\right]\right] \qquad (14)$$

where Y_j is a random variable that represents the return rate per time period for asset S_j, $j = 1, \ldots, \text{N}$. The other variables have already been defined.

Konno and Yamazaki considered that y_{jt} is the realization of the random variable Y_j during time period t (t = 1,..., T), which is consistent when using historical data to make future projections. They assumed that the expected value of a random variable can be approached by the average of the data set:

$$r_l = E(Y_j) = \sum_{t=1}^{T} y_{jt} / T \tag{15}$$

In this way, the Absolute Deviation can be approximated as

$$E\left[\sum_{j=1}^{N} Y_J w_j - E\left[\sum_{j=1}^{N} Y_j w_j\right]\right] = \frac{1}{T}\sum_{t=1}^{T}\left|\sum_{j=1}^{n}(y_{jt} - y_j)w_j\right|. \tag{16}$$

Denoting $a_{jt} = y_{jt} - y_j$, j $=1,\ldots,$N; t $= 1,\ldots,$ T, the problem still takes a non-linear form, that is

$$Mínimum = \frac{\sum_{t=1}^{T}\left|\sum_{j=1}^{n} a_{jt} w_j\right|}{T} \tag{17}$$

subject to

$$\sum_{j=1}^{n} w_j = W \tag{18}$$

$$0 \le w_j \le u_j, \text{ j } = 1,\ldots, \text{n.} \tag{19}$$

Denoting $\left|\sum_{j=1}^{n} a_{jt} w_j\right| = V_t$ (Eq. 17) and adding the restriction equations yields the following equivalent linear form:

$$Mínimum = \frac{\sum_{t=1}^{T} V_t}{T} \tag{20}$$

subject to

$$\sum_{j=1}^{n} \bar{y}_j w_j \geq G \tag{21}$$

$$\sum_{j=1}^{n} w_j = W \tag{22}$$

$$0 \leq w_j \leq u_j, \text{ j } = 1, \ldots, \text{n.}$$

All variables were previously defined.

The MAD model makes extensive calculations of the covariance matrix unnecessary, as opposed to the MV model. Due to its linear forms, entering data into the MAD model is much easier than entering data into the MV model, especially when new data are added, as the solution of a linear programming problem is considerably simpler than a solution derived through quadratic programming.

3. Data and Procedures

The initial step in the selection of the optimum portfolio of BOVESPA stocks during a selected month is the compilation of data from that market for the previous 12 months. In this study, transaction costs and taxes were ignored. The maximum budget share (u_j) that could be allocated to a single asset was set at 75%. There is no theoretical foundation to justify any restriction to the budget allocation share to a single asset. Nevertheless, many authors have suggested those constraints, among them Papahristodoulou (2003).

Using this method, the optimum portfolio for January 2001 is obtained by analyzing asset returns for BOVESPA stocks from January to December 2000; for February 2001, the asset returns for the previous 12 months are analyzed (February 2000 to January 2001). This procedure has been used by many authors (Kono and Yamazaki, 1991; Young, 1998; Papahristodoulou, 2003; and Sharpe, 1971). Initial data for selection of each month's optimal portfolio was obtained in this manner. The optimal budget allocation for each month was called a simulation. Portfolio monthly return was estimated by taking into account the ratio invested in each stock and the stocks' real profitability during that month.

Through use of the MV, MAD and Minimax models, optimal portfolios were generated for three time periods from a pre-selected universe of 20, 50 and 100 stocks. The three time periods were September 1999 to August 2000, January 2001 to December 2001, and February 2002 to January 2003. The stocks available for selection came from the January 2001 Brazil Index (IBX). This index is composed of 100 stocks and is the largest BOVESPA index. The first group contains the 20 most traded stocks in the IBX index, the second group is made up of the index's 50 most traded stocks, and the third group is the entire 100 IBX stocks. The IBX components were gathered from the BOVESPA Home Page (http://www.bovespa.com.br).

Each model ran 36 simulations for each period: 12 simulations with 20 stocks available for investment, 12 simulations with 50 stocks, and 12 simulations with 100 stocks. A total of 324 simulations were run (36*3*3) using Microsoft Excel. The Interbank Certificate of Deposit's (CDI) interest rate was assumed to be the risk free asset, and fluctuation of the 55 stocks comprising the IBOVESPA (BOVESPA Index—the market's principal index) represents market behavior. BOVESPA and IBOVESPA quotations were gathered from the Economática Software database for the period from September 1998 to January 2003. All the stock quotations were corrected for eventual payment of dividends and bonuses. The values were deflated by the Brazilian General Price Index (IGP-DI) calculated by the Getúlio Vargas Foundation. The monthly CDI quotations were collected from the Brazilian Institute for Social Economic Planning's (IPEA) web site (http://www.ipeadata.gov.br).

4. Empirical Results

Study results were divided into three groups according to time period: from September 1999 to August 2000; from January 2001 to December 2001; and from February 2002 to January 2003. We presented total portfolio results at the end of each 12 month period.

The models that sought to minimize variance (objective function) had some problems if their entire budget was not allotted to stock purchases. In almost all scenarios, if a partial budget investment was assumed, then the minimization models' investment became nil. In some sense, that should have been expected. Since the models do not contain any restriction on the minimum level of budget share invested and the objective function related to the variables that determine allotments is being minimized, then allotments become zero. To prevent this from happening, several authors have incorporated the restriction that the entire budget must be allocated (Konno and Yamazaki, 1991; Papahristodoulou, 2003). For that reason, allocation of the entire budget was assumed in our study.

By assuming its maximization form, the Minimax model did not present this type of problem. When the budget is allowed to be partially allocated, Minimax will attempt to allocate the largest possible value to achieve the optimal solution.

Besides analysis of risk and accumulated returns at the end of each period, each models' usefulness was also evaluated. Usefulness was a function of diversification and concentration. Allocation of the budget to a very few stocks can increase risk, not only the market risk but also credit risk. Credit risk or any form of risk other than the market risk was not considered in this study. A highly concentrated portfolio might have its risk underestimated. On the other hand, a high level of diversification causes greater transactional costs, which may overcome the benefits that come from using the model. Excessive concentration or diversification depreciates the models' results, and both must be taken into consideration when creating a portfolio.

4.1. Results of the 1st Period: From September 1999 to August 2000

As shown in Figure 1, all the portfolios generated from the 20 and 50 stock universes had accumulated returns at the end of the 1^{st} period superior to the IBOVESPA, which was in an up markets phase. Among the portfolios generated with 100 stocks, only the MAD model portfolio accumulated returns inferior to the IBOVESPA. Minimax stood out from the others as the only model to generate a portfolio with accumulated returns equal to or greater than 50% over the period, independent of the number of stocks that could be selected from.

The portfolio returns generated by all the models were always greater than the CDI accumulated rate. Hence, the variable rate investments (modeled portfolios) presented higher accumulated returns than the fixed rate investment represented by the CDI.

The Sharpe Index was used to clarify each investment portfolio's risk-return relationship (Figure 2). The Index correlates a portfolio's return above the risk-free rate of return with the standard deviation of the portfolio's returns. The Sharpe Index is given by the following expression: $S = (T - L)/Z$, where S is the Sharpe Index, T is the monthly mean return rate, L is the monthly mean return rate of the risk -

free stocks (CDI) and Z is the standard deviation of the monthly return rate over the time period considered. Greater portfolio efficiency is indicated by higher Sharpe Index values, as the ratio between excess return and the risk surrogate, the standard deviation of returns, increases.

Figure 2 shows the Sharpe Index for each generated portfolio at the end of the 1st period. The MAD model showed the best performance in this measure among the 20 stock universe portfolios, presenting the largest excess return/risk ratio. However, as the MAD model's universe of asset choices increased, that ratio began to decrease, suggesting a loss of efficiency. For the universe of 100 stocks, the MAD model was clearly the worst performing model based on the excess return/risk ratio. The MV model generated the least efficient 20 stock portfolio, but as the number of stocks available for investment increased to 50, the model's efficiency increased, reaching a Sharpe Index close but yet inferior to the Minimax model's. The Minimax model was the most efficient portfolio in the universes of 50 and 100 stocks.

4.2. Results of the 2nd period: from January 2001 to December 2001

All the generated portfolios had negative accumulated returns at the end of the down market 2nd period, making analysis of the return/risk ratio using the Sharpe Index impossible. The Sharpe Index presents ambiguous results if the returns are negative.

In this period, all simulated portfolios accumulated returns smaller than the CDI (Figure 3). Of the portfolios generated from the universes of 20 and 50 stocks, only Minimax portfolio provided greater returns than the IBOVESPA; however, these Minimax modeled portfolios did not generate solutions or allocate funds to stock assets except in January 2001. The absence of a solution was due to the simulated portfolio's non-positive mean return over practically all the period's months. The models did provide solutions for all months when allowed to select from the universe of 100 stocks. The Minimax portfolio presented the largest losses, although the accumulated return classification was ambiguous (Figure 3).

The MV model's portfolios showed increasing accumulated returns as the number of stocks that could be selected from increased, shifting from an accumulated return of about – 40% with 20 stocks to choose from, to about -1% with 100 stocks. This was the only model to generate a 100 stock portfolio with accumulated returns superior to the IBOVESPA. The MAD model generated portfolios that obtained returns close to but smaller than the IBOVESPA (Figure 3).

4.3. Results of the 3rd Period: from February 2002 to January 2003

At the end of the very down market 3rd period, all modeled portfolios except for the 50 stock MAD portfolio generated accumulated returns superior to the IBOVESPA, and these returns increased with the number of stocks available for investment (Figure 4). However, the Minimax portfolios were the only ones to present non-negative accumulated returns over the entire period.

The Minimax portfolio containing stocks selected from the 20 stock universe allocated no budget resources to equities, similar to that which occurred in the 2nd period. The Minimax portfolios generated from 50 and 100 stock universes did present solutions that allocated budget funds to equity purchases. As in the 2nd period, the negative returns presented by some modeled portfolios made use of the Sharpe Index impossible.

4.4. Aggregate Analysis of the results for the three periods

All portfolios showed positive accumulated returns in the 1st period (up market) that fell drastically in the 2nd period (down market). In the 3rd period's very down market, some MAD and Minimax modeled portfolios had return gains. This compared with losses for similar portfolios in the 2nd period. All the MV modeled portfolios and the IBOVESPA produced losses in both the 2^{nd} and 3^{rd} periods.

In the 1^{st} period, all portfolios achieved larger accumulated returns than the IBOVESPA; and in the 3^{rd} period, all but one portfolio (MAD 50) out-gained that Index. However, of all the portfolios modeled from equities in the 20 and 50 stock universes in the 2nd period, only the Minimax portfolios provided accumulated returns superior to the IBOVESPA, and its portfolios achieved the highest returns in the 3rd period. Of the three models, only Minimax generated portfolios with accumulated returns systematically superior to the market index.

Of the portfolios generated from the 20 stock universe only those generated by the Minimax model presented accumulated returns higher than the IBOVESPA during all three periods. The 50 stock universe Minimax portfolios showed behavior similar to its 20 stock universe portfolios, but the Minimax portfolio generated from the 100 stock universe behaved differently. In the 2^{nd} period that portfolio did not generate accumulated returns greater than the IBOVESPA.

Minimax provided larger returns than the other modeling techniques for the 1^{st} and 3^{rd} periods. Relative to the IBOVESPA, the MAD portfolios showed returns inferior to the IBOVESPA in the 1st period, nearly equal in the 2^{nd} period, and larger in the 3rd period. Only the portfolios generated by the MV model showed returns that were superior to the IBOVESPA and non-negative in all three periods.

4.5. Portfolio concentration and diversification

The optimum monthly portfolios generated by the MV model from a universe of 100 stocks was always less concentrated than those created using the Minimax and MAD models. Portfolios from the 20 and 50 stock universe were not analyzed since neither the Minimax nor the MP models reached solutions or generated interesting solutions. This made comparison of the portfolios impossible with regard to concentration and dispersion. These MV modeled portfolios always contained 29 or more stocks while the other models never generated a portfolio from the 100 share universe made up of more than 13 stocks, and the optimum portfolios chosen by Minimax and MAD models showed quite similar concentration levels. Another aspect that points the smaller concentration of the MV modeled portfolios is that the total value allocated to the MV model's 10 largest investments in the first two periods was considerably less than the value allocated to the top 10 stocks selected by the other models, and greater only in the last period, when this ranking reversed.

It was found that the MV modeled portfolios also contained the greatest number of stocks selected from the universe of 100 stocks that received less that 3% of the total budget over any complete period. The portfolio formed by the MV model held 448 stocks during the entire 1st period, 10 times more than held by the MAD (39 stocks) modeled portfolio, which held the second greatest number of stocks during that period. Over the three periods, the MV portfolios also presented a substantially greater budget partition level: at least 200 stocks that received less than 3% of the total period budget. In addition, the MV model provided the portfolio that had the greatest number of stocks that received less than 1% of the total investment budget. These results indicate that the portfolio dispersion using the MV model is considerably greater than from the other portfolio modeling techniques.

5. Concluding Remarks

In this paper, we reviewed the application and relative performances of the Minimax, MAD and MV models as tools for portfolio selection in the Brazilian stock market (BOVESPA) over three time periods: from September 1999 to August 2000; from January to December 2001; and from February 2002 to January 2003. The first time period is typified by up markets, whereas the last two periods are dominated by down markets that behaved differently.

In general, the portfolios generated by the models during the up market period presented a total return superior to that of a risk free control investment (certificate of deposit—CDI) regardless of the number of stocks invested in. During the down market periods, the model generated portfolios provided a total return inferior to the CDI, except for the Minimax modeled portfolios generated from a universe of 100 stocks in the February 2002 to January 2003 period. Results from this study's small sample indicate that use of any of the three models was more suitable during up markets.

The Minimax model generated portfolios with accumulated returns larger than the IBOVESPA (BOVESPA´s principal index) in 8 out of 9 scenarios, making it superior to the other modeling techniques considered. Moreover, the Minimax model presented a Sharpe Index value that was higher than the market index for the entire first period, the only period for which the Sharpe Index was appropriate, independent of the number of stocks invested in. The Minimax model is therefore considered the most appropriate of the models included in this study for the generation of market investment returns.

The Minimax modeled portfolios formed from the 50 and 100 stock universes presented the highest Sharpe Index, making it the most efficient among the models evaluated. The Minimax model also generated the most concentrated portfolio; however, this portfolio was only somewhat more concentrated than one generated by the MAD model.

The MV model presented practical problems that became more intractable as the number of stocks in the selection universe increased. These problems included a high level of apportionment and the computational inconvenience associated with solution of the quadratic form. Hence, use of the MV model as a large scale portfolio selection model can prove unfeasible. In this context, the Minimax model was found to be the most practical for investor use, due to its workability, efficiency, and its ability to create portfolios that generate generally greater returns than those provided by the MAD and the MV models.

As noted earlier, transaction costs were ignored in our evaluation, an omission at odds with the reality faced by investors. The models would be improved if these costs were included.

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Resumo – Neste artigo faz-se uma comparação de três modelos de seleção de portfólios, Média-Variância (MV), Desvio Absoluto Médio (MAD) e Minimax. Estes três modelos foram aplicados ao mercado de capitais brasileiro (BOVESPA). Para fazer a comparação, foram utilizados dados da BOVESPA referente a três períodos distintos de 12 meses: 1999 a 2000, 2001, e 2002 a 2003. Foram gerados três portfólios ótimos para cada modelo, um para cada período de tempo, tendo como medida de desempenho os retornos mensais acumuladas em cada período. De modo geral, os retornos acumulados fornecidos pelo modelo Minimax foram superiores ao principal índice da BOVESPA (IBOVESPA). O modelo MV foi o menos eficiente para seleção de portfólio quando comparado aos demais modelos.

Palavras-chave: seleção de portfólio, mercado de capitais, Brasil.

Appendix

Figure 1. Portfolio accumulated returns estimated by the models (Sept. 1999-Aug. 2000)

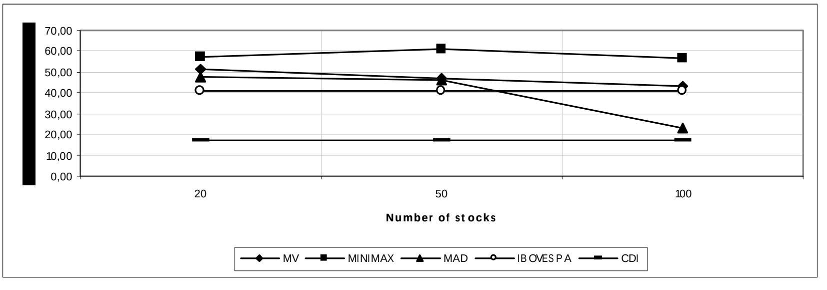

Figure 2. Portfolio efficiency measured by the Sharpe Index (Sept. 1999-Aug. 2000).

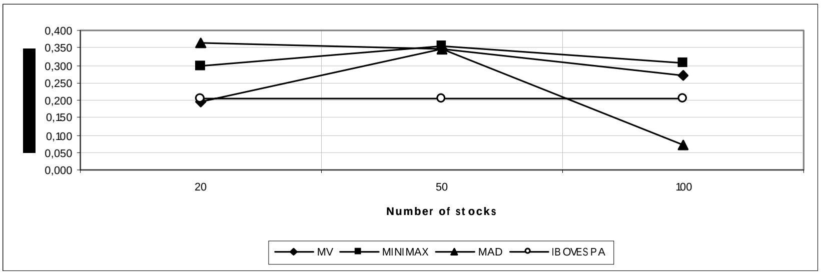

Figure 3. Portfolio accumulated returns estimated by the models (Jan. 2001-Dec. 2001)

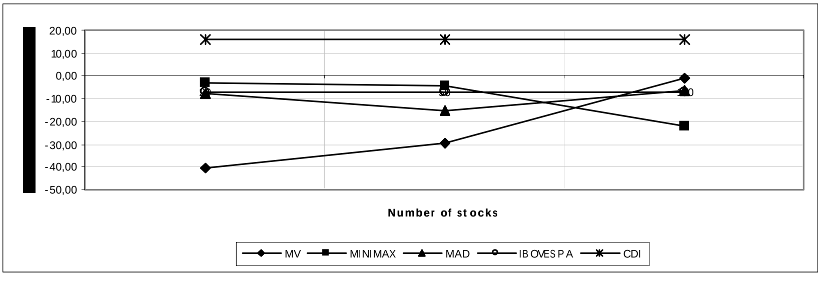

Figure 4. Portfolio accumulated returns estimated by the models (Feb. 2002-Jan. 2003)