TART CHERRY YIELD AND ECONOMIC RESPONSE TO ALTERNATIVE PLANTING DENSITIES

By

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ABSTRACT

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The study investigates the economic response of tart cherry yields to planting density using an unbalanced longitudinal yield data from tart cherry orchards in Northwest Michigan. The relationship between tart cherry yield and tree age is specified as a linear spline function and planting density interacts with tree age. A random effect method, treating block as random, is used to estimate the spline function. Stochastic simulation was used to estimate the mean and variance of the product of two random variables (price and yield), and the coefficient of variation was used as a measure of how much risk is involved in corn/soybeans production relative to tart cherries production. Estimates of the variance provided the discount factor (10%) and with yields predicted from the statistical model, relevant cost data and prices, a deterministic simulation was performed to determine the economically optimal planting density, using annualized net present value (ANPV) as the decision-making criterion. Results of the study show that at a discount rate of 10% and tart cherries priced at $0.30 per lb, planting 160 trees per acre is most profitable. A sensitivity analysis is carried out to determine the effect of variation in interest rates and tart cherry prices on the optimal planting density. Changing the discount rate to 12% or 15% or the price to $0.50/lb did not change the most profitable planting density.

DEDICATION

This work is dedicated to my son, Karsten and

To my nieces: Daniella, Sydney and Chelsey Ekane

for their love and fun in times of stress and desperation.

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Special thanks to my research advisor, Dr. Roy Black for his knowledge, time and other resources put together to see this work accomplished. It would not have been possible without him. Thanks to other members of my committee: Dr. Scott Swinton for helping me develop my knowledge base for this topic in the Production Economics class as well as the very useful comments on the final draft; Dr. Jeffery Andresen for his comments on the paper and to Dr. John Staatz for his guidance, encouragements and the funds provided to see this work accomplished. Many thanks to other lecturers:- Dr. Bob Myers in whose Introductory Econometric class I learnt some basis skills used in this study and to Dr. Eric Crawford whose Cost Benefit class enhanced my understanding of the economic model used in this study. Several colleagues also merit mention in their support of this work: Nicole Olynk, Nicole Mason, Joshua Ariga, Alda Tomo, Malika Chaudhuri, Tina Plerhoples, Kirimi Sindi, Adjao Ramzi and Mukumbi kudzai.

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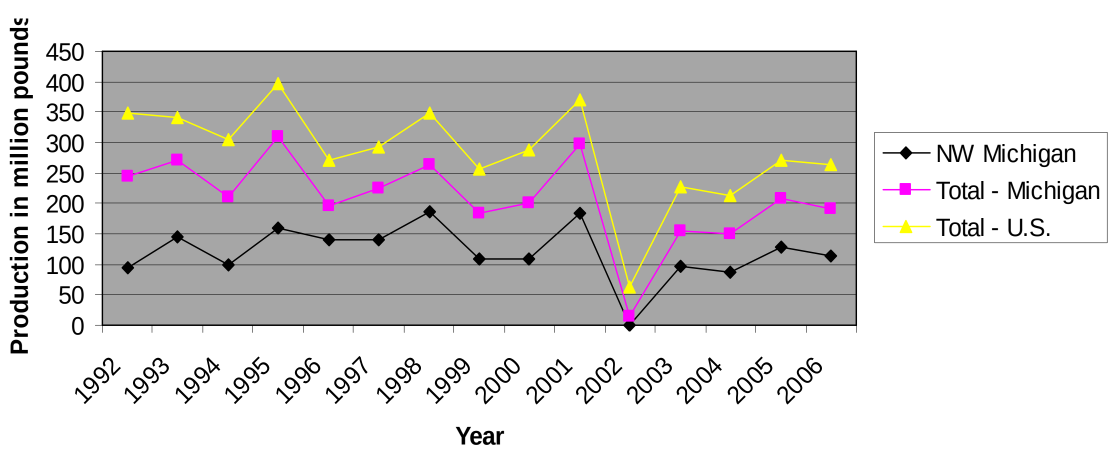
INTRODUCTION

Tart cherry is a perennial tree fruit produced in Michigan, Utah, Washington, New York and Wisconsin in the United States (U.S.) and widely consumed in different forms (frozen, fresh, and processed). The U.S. typically produces more than 200 million pounds of tart cherries each year[1] with significant year-to-year variability associated with weather conditions. In 2002, for example, the crop was severely damaged in Michigan by a non-inversion frost followed by an inversion frost resulting in zero or near zero yields. Much of the production is concentrated in Michigan (70-75%) and Northwest (NW) Michigan grows about 60 percent of Michigan's total. Figure 1 describes U.S., Michigan, and NW Michigan production from 1992-2006 and illustrates the trends and variability in production.

Figure 1. U.S., Michigan and North West Michigan Cherry Production. (1992-2006)

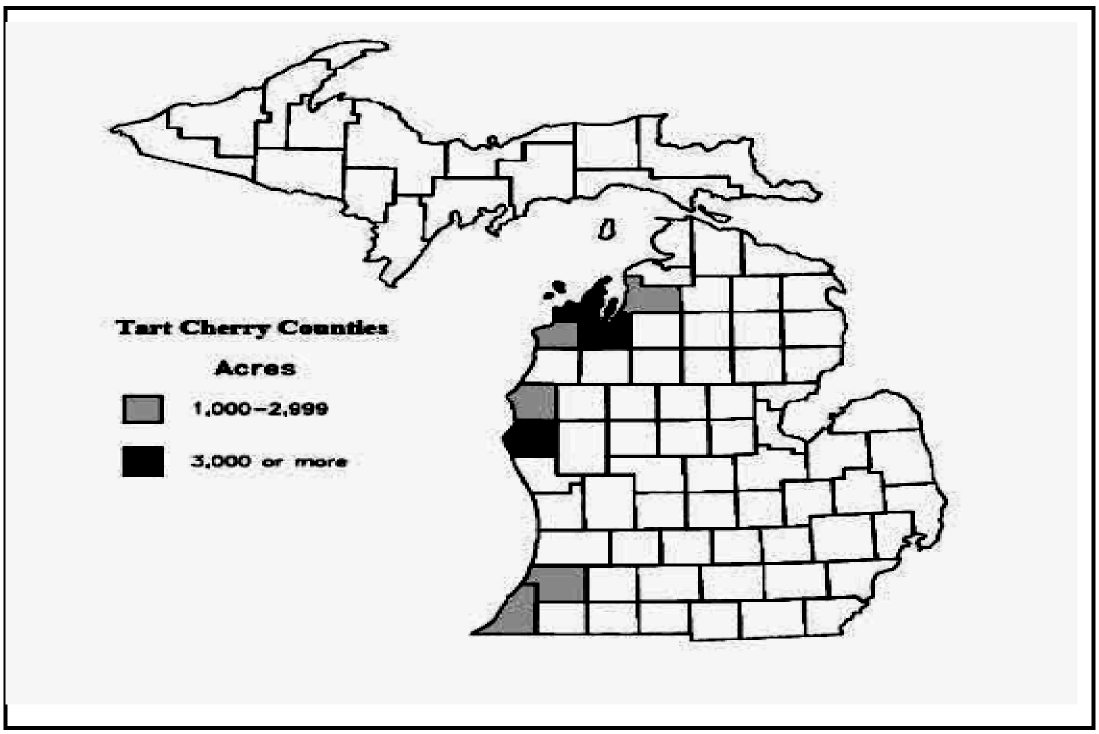


Source1: Pollack and Perez (2008)
Source 2: Cherry Industry Administrative Board (2009)

[1] http://www.cherryfestival.org/aboutus/history.php

Fluctuations in yield due to spring frost damage is observed to be common and the single most important weather related risk in Michigan tart cherry industry . However, tart cherries' proximity to Lake Michigan which bounds the west side of Michigan gives it a comparative advantage in tart cherry production (Figure 2). The lake provides a local warming effect during the night that helps to reduce the likelihood of severe spring freezes during critical stages of flower bud development. Site choice is an important determinant of the expected yield (in a probabilistic sense).

Figure 2 Michigan Tart Cherry Growing Areas.



Perennial crop production is distinguished from annual crop production by the long gestation period, time lag between initial input and first output, an extended period of output flowing from the initial investment decisions and eventually a gradual deterioration of the production capacity of the plant (French and Matthews, 1971). Tart cherry production involves farmers making complex design, replacement and annual management decisions. A common assumption in perennial crop studies is that the potential profitability of an orchard planting system is the most important factor in the decision a grower makes when planting a new orchard (Robinson et. al, 2007). Design decisions include site, acreage, cultivar/variety and planting density (number of trees planted per acre of land). Replacement is a strategic, longer reaching decision. Finally tactical management decisions include timing of the harvest, quantity of the fruits to harvest, pest and disease control and pruning. Pruning and disease control decisions have both current and future year impacts since they influence the long-run productivity of the cherry tree.

These decisions or choice of strategy are critical because 1) they involve high costs of reversal and influence earnings for the next 20 to 30 years and 2) are made in the context of risk. Risk involved in tart cherries can be described as revenue risk. Revenue risk could in turn be subdivided into risk associated with variations in tart cherry prices as well as production risk. This study focuses on production risk, how it affects yields and how this in turn affects revenues. Production risk is categorized further into trajectory risk (risk associated to the shape of the tart cherry yield response) as well as the variability in yield around a trajectory. These two categories of production risk have a different impact on new investment versus replacement decisions. For instance while the decision question of making investments in new sites can be treated as a random effect in a statistical model framework (since little can be said about the productivity of the site), replacement decisions can be treated as fixed effects because the productivity of the site is known at the time replacement decision is made. Further explanation on how the method used depends on the decision questions is provided under the section on theoretical framework underlying the study and methods.

RESEARCH PROBLEM/ KEY RESEARCH GAPS TO EXPLORE.

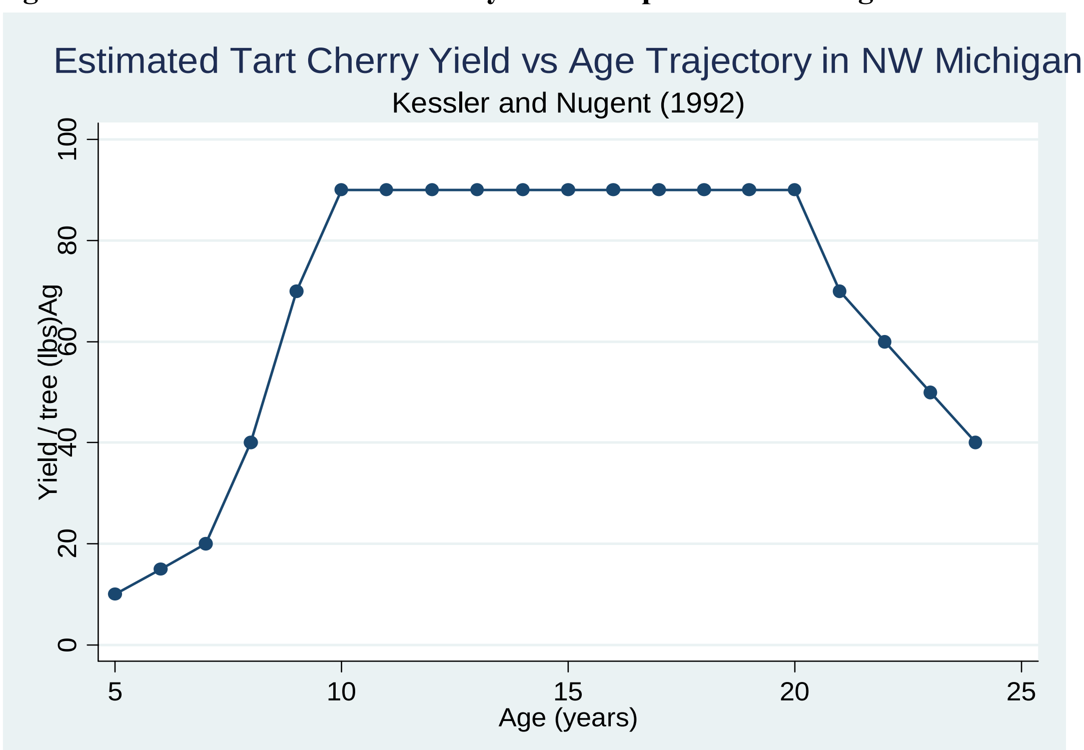
Numerous factors influence perennial crop yields, some of which can be controlled by the farmer either directly or indirectly (e.g., site, tree age, trees per acre, varietal choice, and pest and disease control) and others which are outside the influence of the farmer (weather and other stochastic factors). The yield-age trajectory and its response to planting density are crucial to a range of issues from farmer strategy to processor choices and to policy issues such as marketing orders. These relationships are a key element of supply response investigations which informs decisions at all levels of aggregation. For instance, a good proportion of existing literature highlights the significance of tree age in perennial crop supply response (French and Matthews, 1971; Rae and Carman, 1975; French et al.1995). In NW Michigan for instance, Michigan State University Extension educators are currently using the age-yield trajectory for tart cherries described in Table 1 in educational programming. The estimated tart cherry yield age trajectory based on data in Table 1 is represented in Figure 3.

Table 1 **Estimated Tart cherry yield Trajectory (NW Michigan)**

Age of tree	Yield/tree (lbs)
2-4	0
5	10
6	15
7	20
8	40
9	60-80
10-20	80-100
21-22	60-80
22-23	50-60
≥24	40

Source: C. Kessler and J. Nugent (1992), Michigan State University Cooperative and Extension Service, unpublished.

Figure 3 **Estimated Tart Cherry Yield Response to Tree Age.**



The number of plants in a crop community and the spatial distribution of the plants are also important determinants of yield. Wade and Douglas (1990) observe that plant density determines the number of individuals amongst which the limiting resources must be shared, whilst plant arrangement controls interception of light or retrieval of that resource. How planting density affects crop yields and how economic value responds to planting density has received attention from plant scientists, who have done their investigations using small plot experiments by varying plant density as a treatment (Springer and Gillen,2007; Seiter et al, 2004; Ngouajio et al , 2006 and Bednarz et al.,2006).

Through its impact on perennial crop yields, planting density can potentially influence the profitability of perennial crop production. Some work has been done to investigate the impact of variation in planting density on crop yields and hence on the profitability of perennial tree crop production; however, most of these are done on apples using yield estimates from field trials or replicated research plots. For instance, Robinson et al (2007), performs an economic comparison of five high density apple planting systems to determine which is the most economically profitable. The five planting systems were evaluated in field trials covering a wide range of densities and the yields for each system were composite averages derived from several replicated research plots.

No work has been done to investigate the impact of variation in planting density on the profitability of tart cherry production. This study builds on a statistical model to determine the impact of variation in number of trees per acre on the flow of tart cherry yields and the resulting impact on the profitability of tart cherry production. The study seeks to identify the appropriate statistical methods/procedures in determining the impact of variation in planting density on the tart cherry yield age trajectory and most importantly on the profitability of tart cherry production. Focus group discussions held by Dr. Black, J. R. (Department of Agricultural, Food and Resource Economics, Michigan State University) and James Nugent (Northwest Horticultural Research Station) with tart cherry farmers in NW Michigan established the need to re-evaluate the response of tart cherry yields to planting density (Black, J.R., personal Communication) . Tart cherry farmers need to know: 1) what happens to the trajectory of yield per acre as the planting density changes, 2) how the planting density affects the optimal economic life of a block and 3) which planting density gives the highest economic return measured by annualized net present value (ANPV). ANPV is used in contrast to NPV because it takes into account potential differences in lifespan (unequal rotation periods).

RESEARCH OBJECTIVES.

A systematic search in the literature revealed no studies on the impact of alternative planting densities on the trajectory of tart cherry yields. This research therefore seeks to make an important contribution to the existing literature on perennial tree crops by providing a road map for framing and defining the appropriate tools/statistical methods for determining the trajectory for perennial tree crop yield response. The study seeks to investigate how variations in planting density influence the trajectory of yields per acre over the lifetime of a tart cherry block and the corresponding effects on the profitability of tart cherry production as measured by the annualized net present value. Specific objectives of the study include:

1. To estimate the joint response of tart cherry yields to tree age and planting density using unbalanced, longitudinal data from tart cherry blocks under common management in NW Michigan.
2. To use information from the estimated tart cherry yield response model to simulate deterministically the impact of variations in planting density on the trajectory of yields, cash flows and profitability of production as measured by the ANPV.
3. To conduct sensitivity analyses to evaluate the impact of variations in tart cherry prices and interest rates on the optimal planting density and orchard economic life.
4. Make recommendations on the economically profitable planting density.

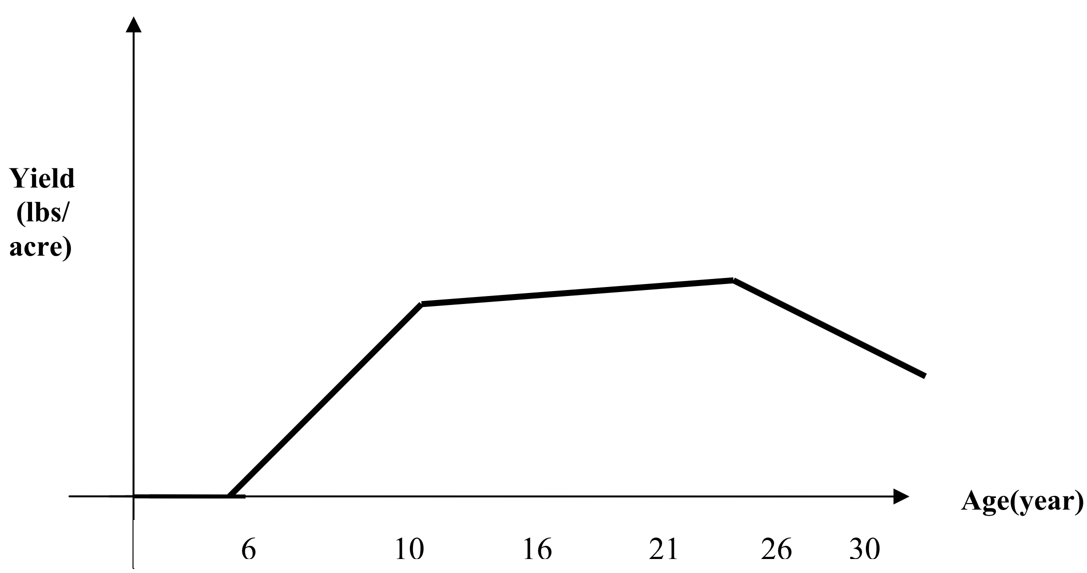
The results of this study are of interest to tart cherry farmers in NW Michigan and members of the tart cherry value chain.

THEORETICAL FRAMEWORK.

As mentioned earlier, tart cherry farmers are faced with critical decision-making questions. These decision questions may include new site, making new investments or replacements. Planting decisions are examples of investment decisions and they refer to all the possible options available to the farmer in varying the firm's future productive capacity through adjustments of tree stock. Given expected future prices and cost considerations, the farmer is assumed to make decisions about the desired age composition, the number of trees planted in a block and the choice of inputs in order to maximize present value over the lifetime of his investments. The normal life of a tart cherry tree is about 30 years. Hence, farmers who plant trees in year t are concerned about production over the period t + 5 to t + 30.

The hypothesized pattern in tart cherry production in NW Michigan is graphically illustrated in figure 4. Trees start bearing at about 4 years after planting; yields are low but increase slowly (stage 1). Stage 2 begins at about 5-6 years after planting when the yields begin to rise at an increasing rate and reach a peak at about 12 years. Then, starts stage 3 during which the yields maintain a steady rise to about 20 years. At stage 4, the last stage, yields gradually decline, as the trees get older.

Figure 4 Hypothesized pattern of tart cherry yield-Age trajectory. NW Michigan.



Cultivar, weather/climate or planting density could influence this hypothesized pattern. For instance a fast growing cultivar can start bearing fruits much earlier or severe damages due to spring frost when the tree is in stage 2 of its lifecycle can cause yields to decline to levels below stage one or even more. Higher plant densities would peak faster and give higher yields over a shorter period. Figures 5a and 5b illustrate two alternative trajectories for the tart cherry yield-age relationship. The trajectory is conditioned by site and planting density. Figure 5a illustrates possible differences in trajectory due to differences in site quality. Such a pattern presents enormous statistical estimation challenges as it involves capturing the differences in slopes as well as in the location of the knots[2] for trajectories A and B. With perfect information on site quality (site index) the effect of site on the trajectory can be investigated. However, this falls beyond the scope of this study which is to investigate the joint response of tree age and planting density on the trajectory of tart cherry yields.

The second factor that conditions the trajectory and therefore exposes the farmer to some trajectory risk is planting density. It is argued here that planting density influences the trajectory of yield per acre over the lifetime of the block[3] and hence the rotation[4] period. For instance, one hypothesis is that higher plant densities reach peak production sooner and decline faster. Therefore, an important decision facing tart cherry farmers is the number of trees to plant per-acre (planting density). The potential effect of planting

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[2] Knots refers to points on the trajectory of tart cherry yield-age relationship, where there is a significant change in the pattern of tart cherry yield response to age

[3] A block is a piece of land on which tart cherries are grown. Trees on a block share common characteristics such as variety, planting pattern, and exposure to weather/climatic conditions.

[4] A rotation period is the length of time from site prepartation to removal and subsequent replanting.

density on the tart cherry yield age trajectory is shown in figure 5b. A, B, C and D are alternative planting densities.

The trend in the trajectory could be either stochastic or a deterministic depending on whether the slope of the trajectory is drawn from some probability distribution that is unpredictable or predictable. For instance if the slope of the trajectory increases by some fixed amount on average but in any given planting density the trend deviates from the average by some unpredictable random amount, then the trajectory is said to exhibit a stochastic trend. The type of trend exhibited by the trajectory has implications for statistical modeling, hence the choice of random versus fixed effect. A fixed effect model is appropriate when it is assumed that the contribution of each block to our yields follows a deterministic (non-stochastic) trend that is predictable with yield increasing by some fixed amount over time, thus allowing us to estimate block specific effects. In contrast, the random effects approach is appropriate when the assumption is that the trajectory of tart cherry yields follows a stochastic trend.

Figure 5a Effect of Site Quality on Yield-Age Trajectory

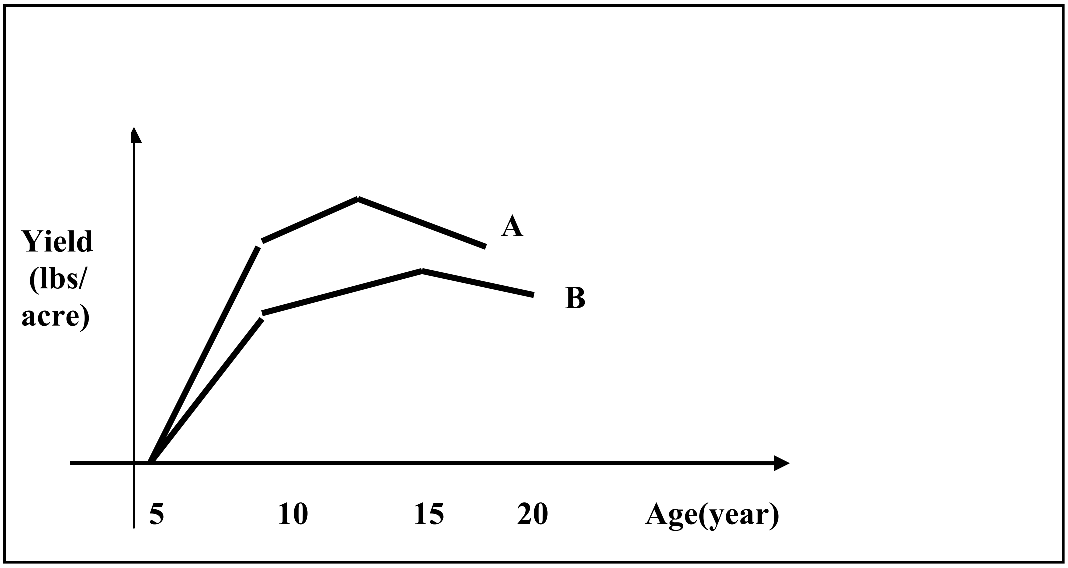
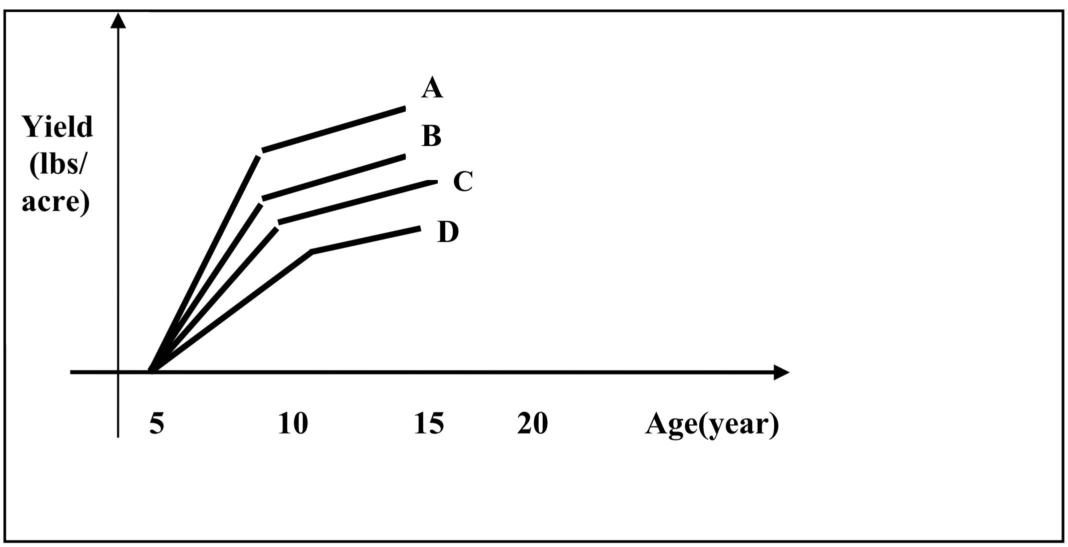


Figure 5b Effect of Planting Density on Yield-Age Trajectory.



As mentioned earlier, the observed data is longitudinal with multiple measurements of each individual block over time. There is considerable variation among blocks in the number of observations. The formulation of our problem should therefore rest on the structure of our data (description of data is provided under the section on data) and the assumptions we make about the probability distribution of multiple measurements in our data.

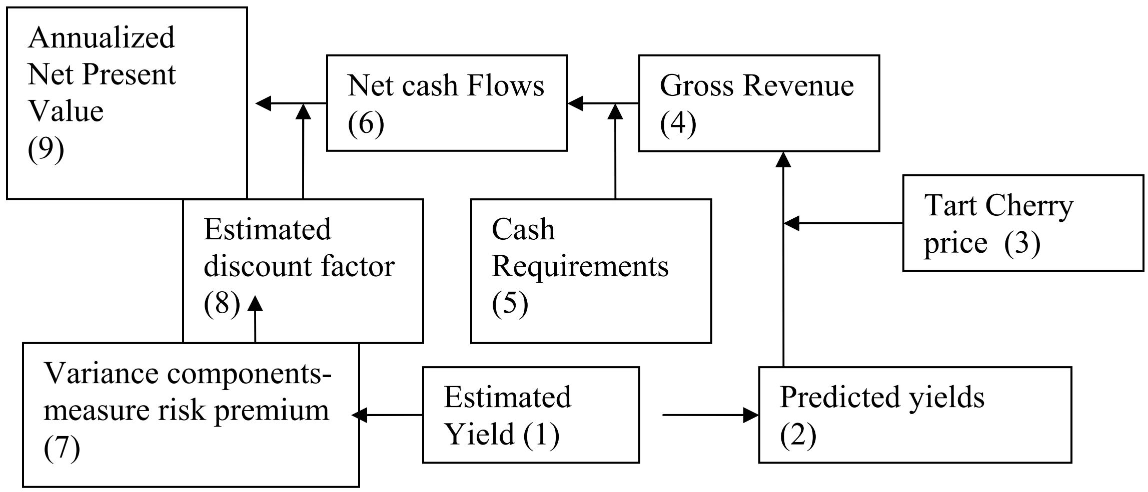
While the maintained hypothesis is that there is a piecewise linear relationship between yield per acre and tree age (Kessler and Nugent, 1992), no information exists either on the nature of the relationship between yield per acre and number of trees per acre or on the effect of trees per acre on the tart cherry yield-age trajectory. Thus in addition to the study contributing to existing literature by identifying the appropriate statistical tools necessary in modeling the impact of variation in planting density on the trajectory of yields and hence the profitability of tart cherries, the study also makes a contribution in the sense that it is the first study on tart cherries (to the best of my knowledge). For other perennial tree fruits such as apple, it has been argued that higher planting densities results in higher early yields and higher cumulative yields than lower planting densities (Robinson et al, 2007).

Generally, tart cherry production begins with costs, followed by annual benefits that continue over the full life of the trees until they have reached maturity. Variations in planting density are hypothesized to cause variations in the flow of benefits -by varying the pattern of yield over time. Variations in planting density also cause costs of production to vary over time. Goedegebure (1991, 1993) observe that higher planting density systems have greater investment costs and annual labor costs than low density systems. Robinson et al (2007) perform an economic comparison of five high density apple systems. They found that differences in establishment costs were largely related to tree density. In addition to increased orchard establishment costs (for instance cost incurred in buying trees), higher planting density might result in increased orchard maintenance cost in the earlier years and decrease cost in the later years due to shorter rotation periods. While this dual effect of planting density on revenue and costs is recognized, this study focuses on assessing the economic consequences of alternative planting densities taking into account variations in harvest cost and not maintenance cost. That is, planting density is allowed to affect net returns through its impact on yield per acre and the corresponding effect of yield on variable harvesting cost and on gross revenue. Under such considerations, an appropriate economic decision could be to find the most profitable planting density. That is, given expected future tart cherry prices, the planting density that maximizes potential tart cherry yields over time subject to cost constraints.

METHODS

The analysis consists of two major parts. The first part consists of a statistical estimation of the joint response of tart cherry yields to tree age and planting density and predicting values for yields over the lifetime of the block. The second part is the economic analysis and the economic choice criterion employed in the study is ANPV maximization. This part entails using estimates from the statistical model with relevant price and cost information to determine the profitability of production for different planting densities as measured by the ANPV. Figure 6 is a flow chart that outlines the major sections of the methods used in the analysis.

Figure 6 Flow chart to illustrate the method



DATA AND MODEL.

Economic Data

Price data used in this study are "Annual prices received for tart cherries", obtained from NASS,USDA-Quick Statistics[5]. Relevant cost data are from "Cost of Tart Cherry Production in Michigan" (Black et al, forthcoming). The document contains cost evaluations, developed through focus group discussions with cherry growers in each of the production regions in Michigan. The budget includes cash and labor costs per acre for large -scale cherry growers in the NW Michigan. For the non-bearing years major components of costs associated with establishing the orchard include; site preparation, planting, culturing and growing. Planting costs, incurred in year 1 is a function of trees

[5] http://www.nass.usda.gov/QuickStats/Create_Federal_All.jsp#top

planted, so for each of the planting densities considered in the study tree cost ($8.25) was multiplied by the number of trees planted per acre to obtain planting cost. For years 2, 3 and 4, there is still some variation in cost which causes slight differences in the total cost estimates for each of these years. Although the activity type is same for years 2, 3 and 4, the intensity of each activity varies, thus causing cost to vary. For instance pruning takes place in years 2, 3 and 4. However, in year 2, two hours of pruning, valued at $15.9 per acre is required, as opposed to three hours per acre ($47.9) and four hours per acre ($63.6) required for years 3 and 4 respectively. Other activities that vary in intensity and hence cost across years 2, 3 and 4 are mowing, pest control, and management and fertilizer material. See Table 2 for a breakdown of cost for the non-bearing years.

For the bearing ages, costs for each age beginning at 5 were calculated taking into account the fact that harvesting begins in year five. As previously mentioned, only variable components of the harvest cost were allowed to change with planting density. The following equation was used to calculate the cost for the bearing years:

$$\text{cost/acre} = (\gamma_0 + \frac{\gamma_1}{\text{yield/acre.}}) \times \text{yield/acre} + (0.0120 + 0.0055 + 0.005) \times \text{yield/acre..(1)}$$

where

$\gamma_0 = 0.0060$, and $\gamma_1 = 312$

The parameters in the cost equation were established from a budget system developed by J.R Black and J. Nugent in a focus group discussion with tart cherry growers in each of the production regions in Michigan. The parameter γ_0 was fixed irrespective of the rate of production while γ_1 depended on the rate of production. The cost equation was constructed to reflect decreasing cost per pound. Harvest costs had both fixed (equipments) and variable components. Fix components include; cost of 80 HP tractor/forklift, 85 HP tractor/forklift, double incliner shaker and skimmer. The variable costs that entered the calculations include; a) shipping charged at 0.0120 cent/pound, b) cooling pad operation charged at 0.00550 cents/pound and Tart Cherry Assessment[6] cost charged at 0.00500 cent/pound, which are multiplied by yield per acre in equation 1.

Table 2 Cost Data and Calculations.

Age	Activity	Cost ($) per Acre
0	Site Preparation and Fallow	900
1	Planting and Culturing	754+ (trees per acre) x $8.25
2	Growing	319
3	Growing	361
4	Growing	378
5	Growing+ other Operations,	456+ cost calculated using equation 1
6-25	Operations, harvest and management	Cost calculated using equation 1

Adapted from Black et al, forthcoming.

The Economic Model.

The objective of the economic model is to determine the planting density that maximizes ANPV. The economic model describes the costs and revenues associated with fruit

6 Assessment for advertising/ promotion of product.

production in an orchard system from planting to maturity. Predicted annual tart cherry yields from the statistical model and exogenously given prices are used to calculate the annual revenue from the system. The prices used in calculating the gross revenue are market prices averaged over a period of time. In principle, gross revenue should be yield multiplied by the effective price, which is the market price adjusted for quality (which is in turn a function of the age of the block). However, because of lack of data on fruit quality, the prices used in the analysis were not adjusted to reflect quality. The costs of establishing the orchard system as well as costs related to tree density and harvest enter the net cash flow calculation. Apart from the three harvesting costs listed above that were a function of yields per acre, all other costs were held constant across planting density.

To determine the profitability of tart cherry production under a specific planting density, the net cash flows are calculated using equation 2[7]. Net cash flow in year t is given by

$$NCF_t = P_t y_t - \sum_{i=1} (C_{0t} + C_{it}) \quad \text{.......................................(2)}$$

where NCF_t is the net cash flow at time period t, P_t is the average market price for tart cherry calculated from historical price data of tart cherry, y_t is tart cherry yield for a given tree age and planting density combination. Multiplying average prices and yield for a given age and planting density generates a stream of revenue over time. The cost of establishing the orchard system from bare ground (C_{ot}) as well as costs related to tree density and harvest, C_{it} enters the net cash flow calculations.

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[7] Equations 2, 3 and 4 are adapted from AEC 865 "Agricultural Cost Benefit Analysis" lecture notes.

Growing tart cherries is a capital investment due to the length of time that capital is committed to the orchard prior to them becoming productive assets; hence the discount rate and method used are important decisions. The discount factor should reflect time factor and risk involved in the venture, hence the discount factor to be used in the analysis should reflect the time and risk involved in the business. Ideally, when a firm finances using both debt and equity, the weighted average cost of capital (WACC) should be used to discount the project. The WACC is a weighting of a firm's cost of equity and its cost of debt (Ross et al., 2005).

The WACC is positively related to risk (that is, the WACC rises above the risk free rate as the level of risk involved in the business increases) .The WACC includes production risk and increases as the variability of after tax cash flows increase. Thus, one would expect the variability of after tax cash flows to be higher for a new site (site determines the degree of exposure to weather related risk - an important source of variation in tart cherry yields) than for the replacement decision when the productivity of the site is known. It is worth noting that this study performs a before tax analysis of risk while ignoring any income tax consequences

The WACC enables us to compare how much risk is involved in the tart cherry venture relative to other agricultural ventures like corn and soybean farms. Net cash flows are a function of revenue and cash outflows. While the cost data used in the analysis does not vary that much, variability in net returns are largely due to variability in cash flows, which are in turn triggered by variability in yield and tart cherry prices. Yield variability is measured by variance from our statistical model.

Given that cash flows from a tart cherry crop cycle extend over several years, any analysis of that cash flow should incorporate the time value of money. The Net Present Value (NPV) for a given mean planting density is calculated by discounting the net cash flows over time (revenue net of expenses for each year) summed over the lifetime of the investment) at the farm's opportunity cost of capital.. The net present value of the stream of annual profits obtained over the planning horizon t=1,…,T is defined as:

$$\text{NPV} = \sum_{t=1}^{T} \frac{NCF_t}{[1+\alpha]^t} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (3)$$

where:
NCF = annual net cash flows in period t (cash inflows minus cash outflows),
t = annual index,
α = the discount rate,
T = the length of the investment.

When comparing investments of different lengths, it is desirable to compare them in annuity form. To compare the relative profitability of different planting densities the net present value of the income stream must be annualized or converted to an average net return per year. This annualized value is obtained using:

$$\text{ANPV} = \text{NPV x } [r / 1 - (1 + r)^{-n}] \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (4)$$

Where r is the economic discount rate and n is the length of the rotation period.

The annualized net present value (ANPV) for a given planting density income stream can be interpreted as the average net return per acre per year over a particular planting density adjusted for the time value of money.

Annual yields are recalculated for different planting densities chosen arbitrarily and the economic model is simulated to determine the impact of variation in average planting density on annual tart cherry yields and consequently on the annualized net present value of the income stream. Such a deterministic simulation model is particularly useful means to evaluate the effects on yield and profitability of alternative planting density and other decisions under the direct control of the grower. The optimal planting density maximizes the ANPV of the income stream. If a planting density results in a decrease in the average annualized net income, it should not be considered even if a profit can be made from its harvest.

The length of the time horizon, *T*, also influences system ANPV. The time horizon is important in assessing orchard rotation strategies. Furthermore, an objective of this study is to determine the optimal economic life of the orchard block. That is, what is the marginal net revenue derived as a result of holding the block for an additional year? The optimal economic life is driven by fruit quality (as well as yields) and may vary with number of trees planted per acre. Discussions with some farmers from Northwest Michigan revealed that fruit quality is dropping off in instances where the blocks were being pulled. With no data on fruit quality, the effect of fruit quality on the optimal economic life cannot be determined. Moreover, different planting densities are hypothesized to result to different rotation periods, which ideally should be reflected in the computation of the ANPV, but data series was not long enough to test for this.

Tree Data.

The study uses a longitudinal dataset from a single farm with multiple tart cherry producing blocks, producing the Montmorency variety, under common management and located in Northwest Michigan. For confidentiality reasons, additional information on the tart cherry farm used for this study cannot be disclosed. Not all blocks were started under the same management. Some were started under the current management while others were acquired from other farmers. The planting density differs across blocks but is constant over time within a block. The blocks are spatially diversified resulting in variation in weather exposure and site quality. The blocks vary in year planted and therefore age and are measured at regular intervals (annually) over a period for yields. The oldest measurements for tart cherry yields were taken in 1979 and the youngest in 2003. Tree yields vary with age within a given block and across blocks measured at similar ages.

The number of measurement occasions varies across blocks thus resulting in variation in number of observation across blocks; hence we have an unbalanced data. See Table 3 for a general description of the dataset. Block number is just a number assigned by the researcher to identify each block. Trees per acre are the number of trees planted per acre. Beginning age observation is the age of the trees when the tree was first measured for yields. End age observation is tree age at last measurement and year planted is the year in which the block was planted. An examination of the age variable in the data reveals possibility of selection bias; very few blocks of trees are older than 25 years. The trees might have been pulled out because of the quality of the fruits. Data also contains information on number of acres per block, tree yields by block, and total production.

Table 3 Structure of the tart cherry (Montmorency) tree data under study.

Block number	Number of observation	Mean trees per acre	Beginning age observation	End Age observation	Year Planted
1	22	84.9	10	31	1969
2	25	112	6	30	1973
3	21	118	5	25	1975
4	4	107	23	26	1977
5	7	158	19	25	1978
6	21	116	4	24	1979
7	20	126	5	24	1979
8	22	172	3	24	1979
9	20	147	4	23	1980
10	19	135	5	23	1980
11	19	136	5	23	1980
12	18	140	5	22	1981
13	19	115	4	22	1981
14	3	176	19	21	1982
15	16	132	6	21	1982
16	14	151	5	18	1983
17	16	132	5	20	1983
18	16	156	5	20	1983
19	16	130	5	20	1983
20	16	138	4	19	1984
21	4	106	6	18	1985
22	16	105	3	18	1985
23	8	145	5	12	1987
24	10	132	5	14	1989
25	8	132	6	13	1990
26	8	136	6	13	1990
27	8	117	5	12	1991
28	3	110	7	9	1992
29	5	121	5	9	1992
30	8	123	4	11	1992
31	6	133	5	10	1993
32	5	116	5	9	1994
33	5	101	5	9	1994
34	5	116	4	8	1995
35	3	117	6	8	1995
36	1	134	5	5	1998

Statistical Model for the joint response of tart cherry yields to tree age and planting density.

The purpose of statistical model is to obtain statistical parameters/estimates that capture/describe the relationship between tart cherry yields, tree age and planting density. Two important issues are dealt with in this subsection: a) choosing a functional form that captures the nature of the relationship between tart cherry yields and tree age and planting density, and b) sampling and error structure. The choice of the functional form is driven by the underlying notion that perennial crop yields vary with variation in the age of the tree. From our hypothesized tart cherry yield age relationship, four different linear relationships exist between tree age and tart cherry yields for each stage of the lifecycle.

As earlier mentioned, the maintained hypothesis is that there is a piecewise linear relationship between tart cherry yields and tree age. One possibility is to fit 4 separate linear regressions (equations 5 to 8) for the four different stages.

(1) $y_{1jt} = \alpha_1 + \beta_1 X_{1jt}$ if age is < 5(stage 1)……………………………(5)

(2) $y_{2jt} = \alpha_2 + \beta_2 X_{2jt}$ if age≥ 5 and age<12(stage 2)………………….(6)

(3) $y_{3jt} = \alpha_3 + \beta_3 X_{3jt}$ if age ≥12 and age <23 (stage 3)……………….(7)

(4) $y_{4jt} = \alpha_4 + \beta_4 X_{4jt}$ if age ≥ 23 (stage 4)……………………….......(8)

Where the y's and x's are variables denoting tart cherry yields and age of tree respectively, for block j, and in each of the 4 stages.

However, fitting separate linear models for the 4 different stages of the hypothesized lifecycle will lead to a discontinuous function. We therefore need a mathematical function that enforces continuity between the stages in our hypothesized yield-age relationship described above. The model is therefore specified as a linear spline function that allows us to generate a piecewise continuous yield- age response, resulting in a smooth curve (Green, 2007). Generally, we do not expect much output within the first few years after planting because the fruit seeds planted have to grow into trees before they begin bearing any considerable amount of fruits. Although our data contains some yields for this stage, we ignore this stage in our analysis. Yield/acre for tree age less than 5 are significantly less than 5000 pounds per acre. Table 4 illustrates a breakdown of yield per acre by tree age.

Table 4 **Average yield per acre by tart cherry tree age and number of blocks measured.**

Tree age	Mean Yield (lbs /acre)	Number of blocks measured at tree age
1	0	
2	0	
3	548	2
4	1080	8
5	5970	25
6	5770	27
7	5850	27
8	10500	28
9	12000	27
10	11800	24
11	19700	23
12	14300	21
13	21000	20
14	21300	19
15	22500	19
16	23200	19
17	25700	15
18	16200	18
19	29500	15
20	23900	15
21	19400	12
22	44500	9
23	18800	8
24	47700	6
25	30100	4
26	5290	3
27	42100	2
28	17200	2
29	3360	1
30	15100	2
31	1980	1

Given the stylized pattern of tart cherry yield age trajectory and after ignoring stage 1 from the analysis, it is now possible to specify a spline equation for the remaining of the trajectory. Introducing 2 dummy variables to distinguish between the 3 stages left in the trajectory, we obtain the spline equation shown in equation 9.

$$y_{jt} = \alpha + \beta_0 X_{1jt} + \beta_1 d_1 (X_{2jt} - t_1^*) + \beta_2 d_2 (X_{3jt} - t_2^*) + \varepsilon_{jt} \text{ ----------- (9)}$$

Where d_1 and d_2 are dummy variables. d_1 takes on 1 if in stage 3 and 0 otherwise, and d_2 is takes on value 1 if in stage 4 and 0 otherwise. t_1^* and t_2^* are the hypothesized locations of the knots on the trajectory. In the case of tart cherry for instance, a knot is the point on the tart cherry yield-age trajectory where there is a remarkable change in the pattern of the relationship between tree age and tart cherry yields. It is the point that connects stage 2 to stage 3 and stage 3 to 4.

As shown in table 3, there are few observations with age above 25, this may be a selection bias issue because the trees were already pulled out. This therefore limits how much information we have to capture the trajectory in stage 4. Henceforth we limit our analysis to cases where age is greater than or equal to 5 and less than or equal to 25 years, thus enabling us to capture stages 2 and 3 of the hypothesized tart cherry yield age trajectory.

Furthermore, to reduce the effects of the incompleteness of the data, we eliminate blocks with less than 7 observations. Year 2002 was also dropped from the data because it was a really bad year thus resulting in extreme values for yields. Table 5 summarizes the resultant dataset.

Table 5 Blocks meeting statistical Estimation requirements.

Block number	Number of observation	Mean trees per acre	Beginning Age observation	End age observation	Year Planted
1	16	84.9	10	31	1969
2	20	112	6	30	1973
3	21	118	5	25	1975
6	19	116	4	24	1979
7	19	125	5	24	1979
8	19	172	3	24	1979
9	18	147	4	23	1980
10	18	135	5	23	1980
11	18	136	5	23	1980
12	17	140	5	22	1981
13	17	115	4	22	1981
15	15	132	6	21	1982
16	14	151	5	18	1983
17	15	132	5	20	1983
18	15	156	5	20	1983
19	15	130	5	20	1983
20	14	138	4	19	1984
22	13	105	3	18	1985
23	8	145	5	12	1987
24	9	132	5	14	1989
25	7	132	6	13	1990
26	7	136	6	13	1990
27	7	117	5	12	1991

The method used in estimating a spline function depends on whether or not the location of the knot is known with certainty. Generally, when the exact locations of the knots are known, dummy variables are introduced to distinguish between the different sections of the curve. However when the location of the knots is not known we use nonlinear least squares regression to estimate the location of the knots. In our case, the hypothesized location of the age that separates stage 2 from stage 3 was age=12. Even though non linear least squares gave tree age 13 as a likely location for the knot with a standard error of approximately 1.0, graphical examination of the data revealed that age 12 was most logical within 1 standard deviation. Hence we stick with age 12 as knot location between stage 2 and stage 3.

We now have an equation of the form

$$y_{jt} = \alpha + \beta_0 X_{1jt} + \beta_1 d_1 (X_{2jt} - 12) + \varepsilon_{jt} \quad \text{..} (10)$$

Where j denotes block, d_1 a dummy variable which takes on 1 if in stage 3(age>12) and 0 if in stage 2(age≤12), X_{1jt} and X_{2jt} are values for age (measured in years) in stage 2 and stage 3 respectively and finally β_0 and β_1 are parameters to be estimated.

Recalling that we are interested in the joint effect of age and planting density on tart cherry yields, we introduce planting density in equation 2. A quadratic relationship between yield per acre and planting density is hypothesized and is captured by the square terms and planting density is made to interact with age to allow yield to change its knots with respect to age and not with the planting density. The model therefore becomes

$$y_{jt} = \alpha + \beta_0 X_{1jt} Z_j + \beta_0 X_{1jt} Z_j^2 + \beta_1 d_1 X_{2t}^* Z_j + \beta_1 d_1 X_{2t}^* Z_j^2 + \varepsilon_{jt} \quad (11)$$

where $X_{2jt}^* = X_{2jt} - 12$ and

Z_j is the planting density

With a functional form for our model, the next issue to be addressed is what estimation method to use for our model. The formulation of our problem or the choice of method should be driven by the structure of our data and the assumptions we make about the probability distributions of the multiple measurements in our data.

Examining the data set reveals variation in crop yields between blocks (even when the blocks are of the same age- block effect) and variation across time (age effect). The block effect that implies that yields are random across blocks is investigated by running separate linear regressions for each block. The results in Table 5 illustrate much randomness in the slope and intercept coefficients across blocks. Three different estimations were performed for blocks with at least 7 observations with complete data set ($5 \leq age \leq 25$), pre-peak data ($5 \leq age \leq 12$), and post-peak data ($12 < age \leq 25$), respectively to understand variation in yields across blocks. Peak age is 12.

Table 6 Interblock variations in Yield response to tree age for pre-peak, post peak and complete data.

Complete Estimates (5<=Age<=25) $Y=\beta_0+\beta_1{*}Age+\beta_2{*}d{*}Age$					Pre Peak Estimation 5<=age<=12) $Y=\beta_0+\beta_1{*}Age$				Post Peak Estimation (12<age<=25) $Y=\beta_0+\beta_1{*}Age$			
Block no	**No. of observ ation**	β_0	β_1	β_2	**Block no.**	**No. of Observ-ations**	β_0	β_1	**Block no.**	**No. of Observ ations**	β_0	β_1
1	16	-17.4	1.80	-1.68	2	7	-5.04	0.80	1	13	5.85	-0.03
		(15.3)	(1.33)	(1.4)			(1.00)	(0.11)			(3.44)	(0.18)
			1.35	**1.19**				**7.39**				**-0.16**
2	20	-5.10	0.81	-0.55	3	8	-6.57	1.20	2	13	1.52	0.26
		(2.32)	(0.23)	(0.29)			**(1.40)**	(0.16)			**(2.73)**	(0.14)
			3.52	**-1.86**				**7.52**				**1.82**
3	21	-5.06	0.98	-0.86	6	8	0.81	0.20	3	13	3.15	0.22
		(2.51)	(0.26)	(0.35)			(1.29)	(0.15)			(3.60)	(0.19)
			3.82	**-2.48**				**1.37**				**1.19**
6	19	-0.49	0.39	-0.19	7	8	2.56	0.03	5	7	8.14	-0.15
		(2.20)	(0.23)	(0.32)			(1.76)	(0.20)			(14.02)	(0.63)
			1.70	**-0.59**				**0.16**				**-0.24**
7	19	0.21	0.37	-0.20	8	8	0.06	0.36	6	12	7.00	-0.10
		(2.36)	(0.24)	(0.35)			(1.76)	(0.20)			(4.24)	(0.23)
			1.51	**-0.57**				**1.82**				**-0.46**
8	19	-1.78	0.63	-0.49	9	8	-4.17	0.81	7	12	9.53	-0.22
		(1.96)	(0.20)	(0.29)			(1.97)	(0.22)			(4.04)	(0.21)
			3.08	**-1.69**				**2.61**				**-1.01**
9	18	-5.32	0.97	-0.91	10	8	-4.64	0.93	8	12	10.60	-0.23
		(3.31)	(0.34)	(0.59)			(2.47)	(0.28)			(3.87)	(0.21)
			2.81	**-1.79**				**3.31**				**-1.14**
10	18	-6.27	1.16	-1.16	11	8	-3.39	0.68	9	11	11.60	-0.30
		(3.15)	(0.33)	(0.48)			(1.80)	(0.20)			(6.65)	(0.36)
			3.53	**-2.39**				**3.31**				**-0.82**

Standard errors are in parenthesis below coefficients and t-values are highlighted

Table 6 Continues.

Complete Estimates (5<=Age<=25) $Y=\beta_0+\beta_1*Age+\beta_2*d*Age$					**Pre Peak Estimation 5<=age<=12) $Y=\beta_0+\beta_1*Age$**				**Post Peak Estimation (12<age<=25) $Y=\beta_0+\beta_{1*}Age$**			
Block no	**No. of observ ation**	**β_0**	**β_1**	**β_2**	**Block no.**	**No. of Observ-ations**	**β_0**	**β_1**	**Block no.**	**No. of Observ-ations**	**β_0**	**β_1**
	18	-3.87	0.75	-0.51	12	8	-4.19	0.77	10	11	14.94	-0.43
		(2.13)	(0.22)	(0.33)			(2.02)	(0.23)			(6.26)	(0.34)
11			**3.36**	**-1.56**				**3.37**				**-1.25**
12	17	-5.22	0.92	-0.49	13	8	-5.95	1.09	11	11	7.31	-0.08
		(2.17)	(0.23)	(0.35)			(2.51)	(0.28)			(5.08)	(0.28)
			4.05	**-1.41**				**3.82**				**-0.28**
13	17	-5.41	1.10	-1.02	15	7	-1.11	0.37	12	10	9.12	-0.10
		(2.35)	(0.25)	(0.38)			(1.78)	(0.19)			(6.71)	(0.38)
			4.11	**-2.7**				**(1.92)**				**-0.26**
15	15	-3.68	0.71	-0.56	16	8	-3.46	0.84	13	10	9.93	-0.23
		(2.15)	(0.22)	(0.33)			(2.18)	(0.25)			(5.60)	(0.32)
			3.23	**-1.71**				**3.4**				**-0.73**
16	14	-5.07	1.07	-1.35	17	8	-0.68	0.53	15	9	12.24	-0.41
		(2.51)	(0.27)	(0.53)			(1.09)	(0.12)			(4.55)	(0.26)
			3.94	**-2.54**				**4.33**				**-1.57**
17	15	-1.37	0.63	-0.68	18	8	-5.64	1.15	17	8	14.14	-0.53
		(1.79)	(0.19)	(0.33)			(1.49)	(0.17)			(6.90)	(0.41)
			3.33	**-2.07**				**6.76**				**-1.27**
18	15	-5.35	1.10	-1.27	19	8	-4.01	0.70	18	8	14.76	-0.52
		(2.40)	(0.26)	(0.44)			(2.76)	(0.31)			(8.89)	(0.53)
			4.32	**-2.89**				**2.25**				**-0.97**
19	15	-4.09	0.72	-0.75	20	8	-4.34	1.10	19	8	8.83	-0.30
		(3.03)	(0.32)	(0.56)			(2.43)	(0.28)			(8.17)	(0.49)
			2.22	**-1.35**				**3.99**				**-0.62)**

Standard errors are in parenthesis below coefficients and t-values are highlighted.

Table 6 Continues.

Complete Estimates (5<=Age<=25) $Y=\beta_0+\beta_1*Age+\beta_2*d*Age$					Pre Peak Estimation 5<=age<=12) $Y=\beta_0+\beta_1*Age$				Post Peak Estimation (12<age<=25) $Y=\beta_0+\beta_{1*}Age$			
Block no	**No. of observ -ation**	β_0	β_1	β_2	**Block no.**	**No. of Observ-ations**	β_0	β_1	**Block no.**	**No. of Observ-ations**	β_0	β_1
20	14	-3.50	0.98	-1.20	22	8	-4.29	0.89	20	7	14.42	-0.51
		(3.30)	(0.35)	(0.66)			(1.84)	(0.21)			**(13.16)**	(0.82)
			2.78	**-1.82**				**4.28**				**-0.63**
22	13	-4.28	0.89	-1.28	23	8	-2.22	0.71				
		(2.27)	(0.25)	(0.50)			(2.05)	(0.23)				
			3.63	**-2.55**				**3.06**				
23	8	-2.22	0.71	0.00	24	8	1.60	0.44				
		(2.05)	(0.23)	(0.00)			(4.15)	(0.47)				
			3.06	**-**				**0.94**				
	8	-1.94	0.78	-2.59	25	7	5.30	-0.08				
		(3.99)	(0.44)	(1.78)			(6.57)	(0.71)				
24			**1.78**	**-1.46**				**-0.12**				
25	7	-1.30	0.78	0.40	26	7	1.35	0.24				
		(6.27)	(0.72)	(4.61)			(5.34)	(0.58)				
			1.08	**0.09**				**0.41**				
26	7	-5.11	1.08	-2.10	27	8	3.17	0.19				
		(3.85)	(0.44)	(2.83)			(4.12)	(0.47)				
			2.43	**-0.74**				**0.41**				
27	7	0.41	0.62	0.00	30	7	1.83	0.13				
		(2.64)	(0.31)	(0.00)			(3.17)	(0.38)				
			1.99	**(-)**				**0.34**				

Standard errors are in parenthesis below coefficients and t-values are highlighted.

Possible causes of variation in tart cherry yield response across blocks could be site. The site at which a block is located determines degree of exposure to weather/climatic conditions such as freezes (wind freeze, inversion) and drought, and the extent of pollination.

An important characteristic of longitudinal data is within and between subject correlations. Using ordinary, least squares (OLS) methods to estimate this model by pooling the data would mean ignoring any within-block and between-block correlations. In the presence of such correlations, OLS could result in inefficient estimates (although could be consistent in large samples as estimates approach the unknown parameter value). Examples of models that are capable of handling the unobserved block-specific effect are the random and fixed effect models.

The fixed effects model is an appropriate specification if we are focusing on a specific set of N blocks, such that the inference is conditional on the set of blocks that are observed. A fixed effect model assumes the contribution of each block to our yields follows a deterministic (non-stochastic) trend that is predictable and it increases by some fixed amount over time. Thus a fixed effect model allows us to estimate block specific effects.

In contrast, the random effects model is an appropriate specification if we are drawing many blocks randomly from a large population. With the random effect approach, we assume that the trajectory of tart cherry yields follows a stochastic trend with the slope drawn from some probability distribution, which is unpredictable. A stochastic trend would imply that tart cherry yields increase by some fixed amount on average but in any given block the trend deviates from the average by some unpredictable random amount that can be modeled using a random effect model.

Thus, viewing blocks as random samples from a population of blocks and assuming that the probability distribution of the multiple measurements has the same form for each individual block, but that the parameters of that distribution varies over blocks, a random effect model is appropriate to estimate the joint response of tart cherry yields to age and planting density. A random effects model would explicitly account for the heterogeneity of blocks studied through a statistical parameter representing the inter-block variation and it allows us to characterize the probability distributions (pattern) for individual responses and change over time and to investigate the effects of covariates on these patterns.

The use of a random effects model enables us to draw statistical inferences from comparable but heterogeneous blocks beyond the particular values used in the study. Therefore, conceptualizing the selected blocks in the data as pieces randomly drawn from a larger universe of possible blocks, inferences can be made to a larger universe of blocks, farm and even location.

Random effect model specification:

Consider the following model;

$$y_{jt} = \alpha + \beta_0 X_{1jt} Z_j + \beta_0 X_{1jt} Z^2 + \beta_1 d_1 X_{2t}^* Z_j + \beta_1 d_1 X_{2t}^* Z_j^2 + \varepsilon_{jt} \text{..........} (12)$$

This specifies tart cherry yields as a joint function of age and planting density with some error. y_{jt} is the response of block j at time t, X and Z same as above and ε_{jt} is the residual. The residual captures variations in yields due to factors other than tree age and planting density, some of which can be treated as random across blocks (such as site). To account for within block dependence, the ε_{jt} is split into two components $\in_{jt}$ and δ_{jt} which account for deviation of y_{jt} from block j's mean (e.g. due to pest and disease) and random deviation of block j's mean yield from the overall mean (site is treated as random) respectively.

$$y_{jt} = \alpha + \beta_0 X_{1jt} Z_j + \beta_0 X_{1jt} Z_j^2 + \beta_1 d_1 X_{2t}^* Z_j + \beta_1 d_1 X_{2t}^* Z_j^{\ 2} + \delta_{jt} + \in_{jt} \quad \text{............(13)}$$

Where $\delta_j \approx N(0, \tau^2)$ and $\in_{jt} \approx N(0, \sigma_{jt}^2)$.

This is our estimation model.

In addition to the sampling error associated with each block, an assumption behind the random effects model is that the true yield effect in each time period is influenced by several factors, including tree characteristics (such as the age of the tree) , planting density and weather/climatic conditions for that year. The model stipulates that the true yield effects is given by all terms on the right hand side of equation 13 except $\in_{jt}$

Where α is the overall yield effect or average yields generated from a population of possible realizations of yields and δ_{jt} is the deviation of the *j*-th block's effect from α .

The variance of $\in_{jt}$, σ_{jt}^2, is the sampling variance reflecting within-block variance and the sample size of the study. The sampling variance, σ^2_{jt}, is usually unknown and is estimated from the data of the j-th observed block. The variance of δ_{jt}, τ^2, is the inter-block variance and represents both the degree to which true yield effects vary across time as well as the degree to which individual blocks give biased assessments of yield effects. With this formulation, the assumption is that the observed yield effects, $y_1, \ldots, y_n$, are realizations of independent random variables from a distribution with mean value μ and variances $\tau^2 + \sigma_1^2, \ldots, \tau^2 + \sigma_n^2$.

The variances reflect the two components of variance assigned to each observed effect: an inter-block variance τ^2, which reflects yield effects heterogeneity and an intra-block variance σ_{jt}^2, which reflects within-block sampling variance. An important question to answer after determining the variance components is whether or not the covariance structure of the yields is known. If the correlation structure is not known, then it is necessary to model the covariance between any two observations taken at arbitrary time points to predict a continuous yield curve. A random effects model provides an essential tool in estimating these variance components. The estimated model is tested for possible for heteroscedasticity.

RESULTS AND DISCUSSION

Random effect estimation was carried out on model (13). Results of the estimation are shown in table 6. Results reveals that while the linear (x_1z) and quadratic (x_1z^2) interaction between tree age and planting density when tree age is between 5 and 12 inclusive are significant at the 5 percent level, only the linear interaction (dx_2z) between tree age and planting density when tree age is greater than 12 and less than 25 was significant. The quadratic interaction (dx_2z^2) in this stage is insignificant at the 5 percent level. Regression results also show a between blocks (sigma_u in table 7) and within blocks (sigma_e in table 7) standard deviations of 0.53 and 2.05, respectively. That is, the between and within block variance is 0.28 and 4.20 respectively. There is much more variation in yield response within blocks than between blocks. As mentioned earlier, variation in tart cherry yield age trajectory could be caused by differences in planting density and/or site quality. Summary of the data used in this analysis (Table 5) supports some variation in planting density. Weather and pollination are large effects on yield. Offhand, it should not be surprising that the within block variance is larger than the between block variance because the within variance captures weather effects. A maintained hypothesis is that the within variance for poor sites is larger than that for excellent sites (quality of air drainage). However, we do not have enough information to test this. The proportion of the total variance in yields contributed by block level variance in yield response (rho in table 7) is 0.062 (6%).

Table 7 Results of the random effect regression of the joint response of tart cherry yields to tree age and planting density

Random-effects GLS regression		Number of observation:295		
Group variable: block no		Number of groups :18		
R^2 : within:0.5192		Observation per group: minimum:7		
R^2 between: 0.0188		average:16.4 maximum:21		
R^2 overall: 0.4874				
Random effects u_i ~ Gaussian		Wald chi2(4) : 292.47		
corr(u_i, X) : 0 (assumed)		Prob > chi2 : 0.0000		
Yield/acre	**Coefficient**	**Standard error**	**z**	**P>|z|**
x_1z	0095166	0.0014681	6.48	0.000
x_1z^2	-0.0000264	9.12e-06	-2.89	0.004
dx_2z	-0.0060399	0.0029646	-2.04	0.042
dx_2z^2	7.12e-06	0000207	0.34	0.732
_cons	-3.540716	6263516	-5.65	0.000
sigma_u :0.52985059				
sigma_e :2.0528581				
Rho : 0.06245687	(fraction of variance due to u_i)			

where:

yld_acre denotes yields in tons per acre.

x_1z denotes age x tree if age<= 12 years

x_1z^2 denotes age x planting density squared if age<= 12 years

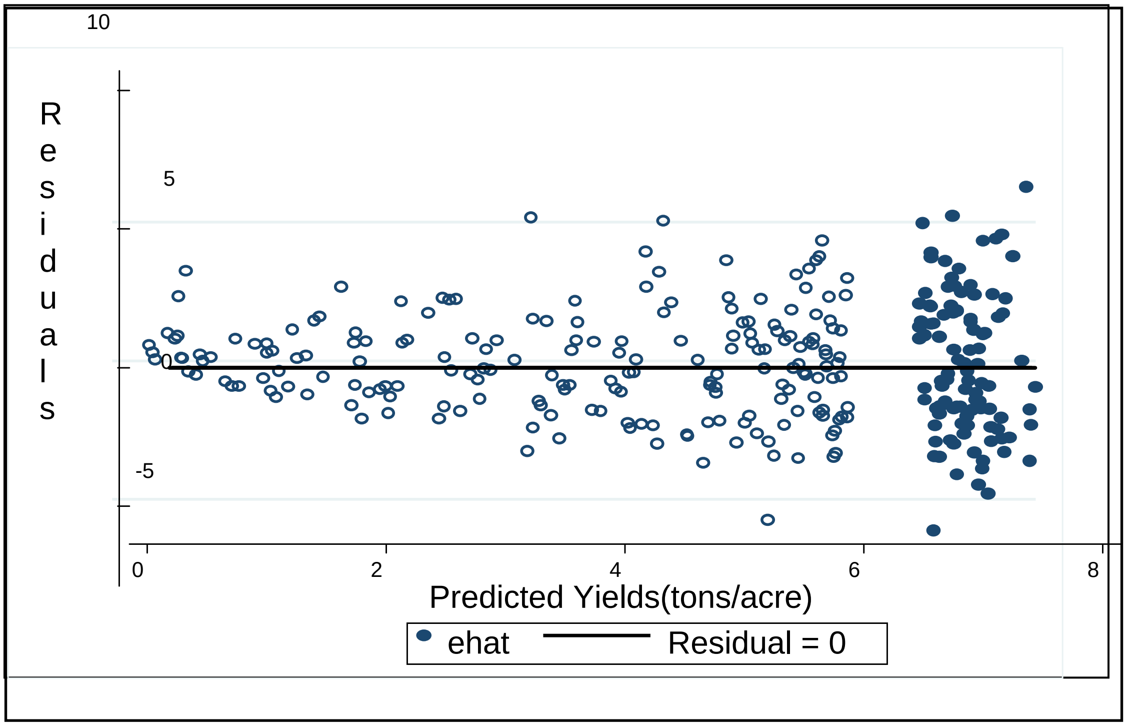
dx_2z denotes age x tree if age> 12 years <25 and

dx_1z^2 denotes age x planting density squared if age> 12< 25 years

The test for the statistical significance of regression coefficients reveal that while dx_2z and dx_2z^2 are not both individually significant, they are jointly significant with chi2 (2) equals 292.47 and probability greater than chi2 equals 0.00, which may be a result of multicollinearity amongst these two variables.

The estimated model was tested for heteroscedasticity by plotting the residuals calculated from the estimated model against predicted yields (Figure 7a), tree age (Figure 7b)as well as against trees per acre(figure 7c) . Figure 7c indicates that 84 and 176 trees per acre are the lowest and highest planting density respectively in the data. These points might cause high leverage on the estimates because including them in the analysis alters the estimated coefficients. While figures 7a, 7b and 7c do not suggest any evidence of functional form issues, the figures present evidence of non-constant variance. As observed by Greene (2003, pg 296), unbalanced panel data adds a layer of difficulty in the random effects model as it distorts the normal/ expected form of the variance-covariance matrix, resulting in group-wise heteroscedasticity, caused by unequal group sizes. Ideally, this problem could be dealt with by finding a transformation that would give a constant variance. However, this is problematic as the problem of non-constant variances seems to be more of an issue in the second part of the trajectory (age greater than 12) rather than in the first part (age less than or equal to 12). An alternative estimation method that could be taken up later to address this issue is the Generalized Least Squares (GLS) method in which the second stage is estimated to get weights used in estimation of the first stage. Another model that might be investigated would be a random coefficient model. As opposed to the random effects model which allows only variations in the intercepts across blocks, a random coefficients model allows variation in both the slope and the intercept terms across block.

Figure 7a Tart cherry yield residuals versus predicted yields



NB: ehat is the difference between the actual yield and the predicted yield

Figure 7b Tart cherry yield residuals against Tree Age

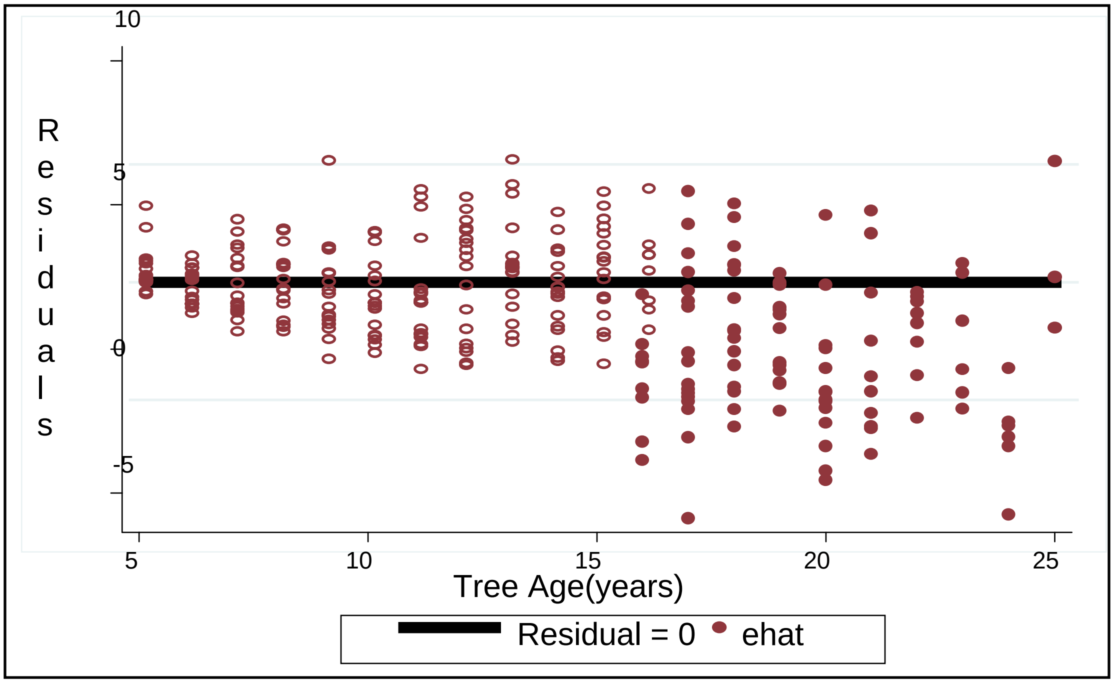
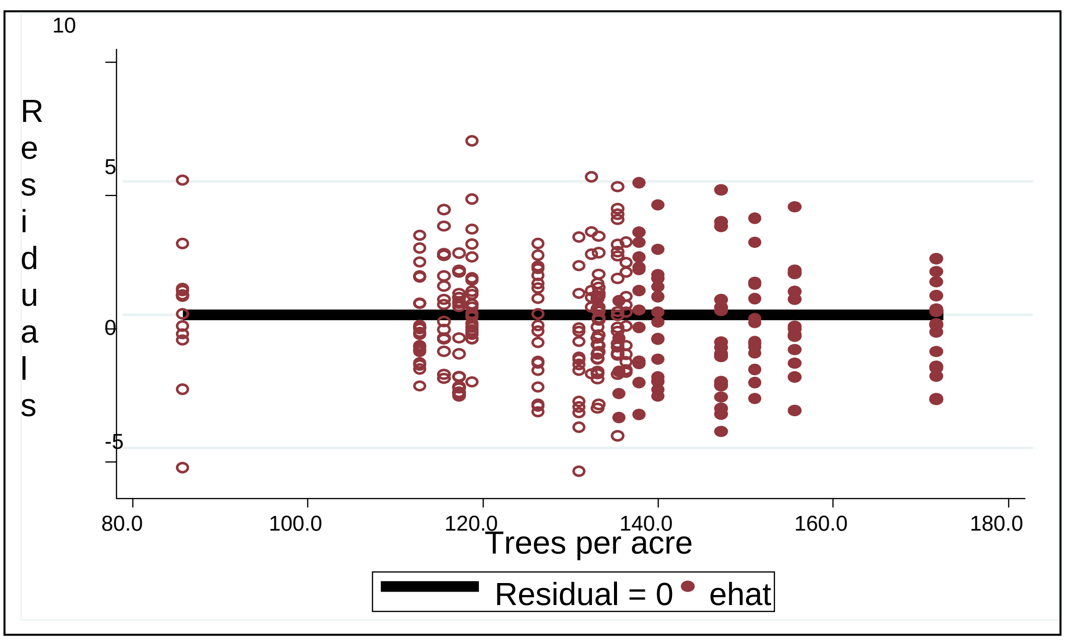
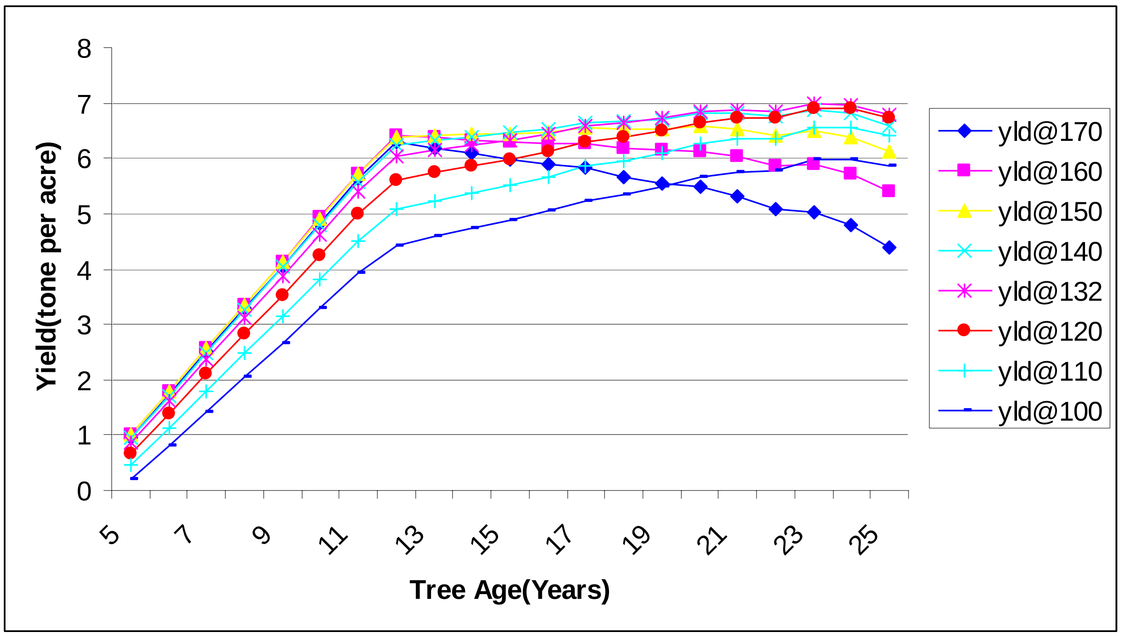


Figure 7c **Tart cherry yield residuals against Trees per Acre**



From the estimated random effect model for tart cherry yield joint response to tree age and number of trees planted per acre, yields were predicted for ages 5 to 25 and for average number of trees planted per acre for the final data used (approximately 132 trees/acre). Using coefficient estimates from the estimated model and the mean planting density as the baseline, values for tart cherry yields were calculated for different values of planting densities chosen arbitrarily. Figure 8 illustrates plots of values of yields against tree age, calculated for different planting densities.

Figure 8 Estimated Joint Response of Tart Cherry to Tree Age and Planting Density, NW Michigan.



From figure 5 above, it can be seen that at planting densities higher than 132 trees/acre (170, 160, 150 and 140 trees per acre), the slope of the trajectory in the interval 5 to 12 years is much higher than for planting densities 132, 120,110 and 100 respectively. Post peak age (12years), yields from the higher planting densities turn to decline faster than the lower planting densities, some of which continue to rise but at a much slower rate. This pattern supports the fact that higher planting densities will lead to richer yields during the early stages of the lifecycle but will decline even much faster resulting to differences in rotation periods.

With the predicted values for yields from 5 to 25 years and relevant cost information adjusted for yield in pounds per acre, the NPVs and ANPVs at various planting densities were calculated. The discount factor used in the calculations was obtained by comparing the variance of tart cherry relative to benchmark values for corn and soybeans, which are closely related ventures. Net cash flows from the tart cherry venture are the gross revenue per acre (product of Yield and price) minus the cost. Both yield and price are random variables; cost is treated as non random. The procedure used is to estimate the variation of revenue. Estimates of annual price standard deviation and the price-yield correlation were $250 per ton and -0.60 respectively (J.R. Black, personal communication). With estimates of yield variability from our statistical models (mean, standard deviation and coefficient of variation), stochastic simulations for tart cherry and corn were performed using @Risk stochastic simulation Excel add-in-version 5, and also using appropriate values for price and price-yield correlation coefficient in both cases. The coefficient of variation of revenue is used as a measure of risk. The coefficient of variation for corn/soybean is compared to that of tart cherry. The results of the simulations reveal that risk associated with corn/soybean venture is 67% of the risk involved in tart cherry venture. Mathematically, the rate used in corn/soybean venture divided by 0.67 gives an estimate of the rate to be used in the tart cherry venture.

Discount factor used in corn/soybean is usually 7% (see Pederson, 1998). Thus implying that an approximate discount factor to use for the tart cherry venture is 10 %(7 divided by 0.67). Discounting started at year zero.

It was mentioned earlier that the ANPV rather than the NPV was more appropriate as a criterion to compare the profitability of alternative planting densities due to the differences in rotation periods. Although this was a relevant argument, within the data used in the analysis (5-25years) yields never turn down so that the ANPV argument is no longer relevant but it is still calculated as it makes interpretation easier. Values for the NPV and ANPV for different planting densities using a 10% as the discount factor and price $0.30 per pound (computed by averaging Michigan Annual tart cherry prices from 1980-2003) annual are illustrated in Table 7. Figure 9 graphically illustrates the relationship between ANPV and planting density.

Table 8 Variation in predicted NPV with planting density.

Planting Density	NPV@10%	ANPV@10%
100	4560	500
110	6090	670
120	7230	800
132	8050	890
140	8320	920
150	8270	910
160	8500	940
170	7660	840

Tart cherry fruit price=$0.30/lb

Figure 9 Plot of ANPV @ 10% and $0.30/lb against planting density

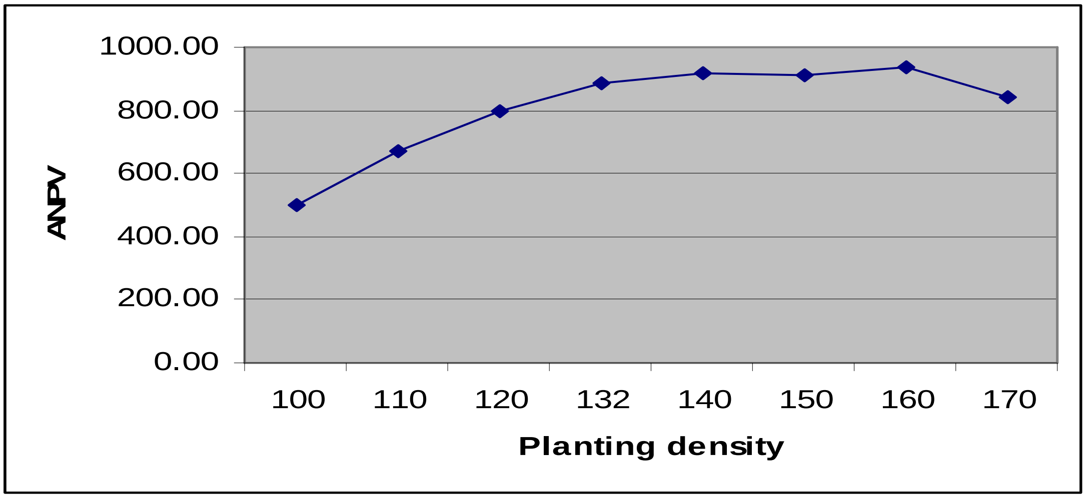


Figure 9 illustrates that as planting density increases from 100 trees per acre to 132 trees per acre, ANPV increase. Beyond 132 trees per acre, the ANPV continue to increase but at a rate slower than it was below 132 trees/acre. The ANPV reaches a peak at planting density of 160 trees per acre, beyond which ANPV declines.

A sensitivity analysis was conducted to evaluate the effect of altering tart cherry prices and discount rate respectively on profitability as measured by NPV and consequently on the choice of planting density. Recalculating the ANPV using 5%, 12% and 15% as the discount factor reveals some changes. As shown in Table 8, values for NPV and hence ANPV are sensitive to the discount rate used. Changing the discount factor from 10% to 12% and 15% respectively, still supports 160 trees per acre as the most profitable planting density. However, a decrease in the discount rate from 10% to 5% switches the most profitable planting density from 160 trees/acre to 140trees/acre. A high discount rate should reflect more risk in a project. A discount rate of 5% is not quite reasonable as it signifies that there is less risk involved in tart cherry relative to corn/soybeans. Further analysis of the impact of variation of tart cherry prices on NPV (Table 9) illustrates that at a 10% discount rate a reduction in tart cherry prices from 30 to $0.15/lb) causes significant declines in ANPV values while an increase in tart cherry price from $0.30/lb to $0.50/lb causes significant increases in ANPV. At $0.50/lb, 160tress per acre is still the most profitable planting density whereas at $0.15/lb, 160 tress/acre ties with 140 trees/acre. The values for NPV and ANPV shows that at a discount rate of 10%, 12% or 15%, irrespective of whether the price is $0.30/lb or $0.50/lb, planting density of 160 trees per acre is more profitable than the others.

Table 9 Effect of varying Discount Rate on NPV and ANPV.

Planting Density	**ANPV@5% & Price=$0.30/lb**	ANPV@10% & **Price=$0.30/lb**	**ANPV@12% & Price=$0.30/lb**	**ANPV@15% & Price=$0.30/lb**
100	950	500	340	120
110	1160	670	500	260
120	1310	800	610	360
132	1420	890	690	430
140	1450	920	720	460
150	1430	910	720	460
160	1430	940	750	490
170	1300	840	670	420

Table 10 Effect of varying Prices on NPV

Planting Density	**ANPV@10% & Price=15cents**	**ANPV@10% & Price=30cents**	**ANPV@10% & Price=50cents**
100	-95.0	500	1300
110	-24.0	670	1600
120	27.0	800	1820
132	63.0	890	1990
140	72.0	920	2040
150	65.0	910	2040
160	72.0	940	2090
170	26.0	840	1930

SUMMARY OF FINDINGS, CHALLENGES AND RECOMMENDATIONS

This study sought to investigate how variations in planting density influence the trajectory of yields per acre over the lifetime of a tart cherry block and the corresponding effects on the profitability of tart cherry production as measured by the ANPV. The study had four objectives. The first objective was to estimate the joint response of tart cherry yields to tree age and planting density. A statistical model for the joint response of tart cherry yield to tree age and planting density was developed and estimated. Results of the statistical model illustrate variation in the tart cherry yield-age trajectory with planting densities. It is found that pre-peak, higher planting densities give higher yields (steeper slope) than lower planting densities. However, post-peak, yields at higher planting densities decline much faster than those at lower planting densities.

The data used for this study had few observations in stage 4 of the hypothesized tart cherry yield age trajectory, which illustrates evidence of potential selection bias because the trees were already pulled out. Consequently it was not possible to capture the shape of the trajectory in stage 4. Nevertheless, comparing the tart cherry yield age trajectory estimated by Kessler and Nugent (figure 3) to that estimated here(figure 8) for consistency, we see that figure 8 reveals a much more linear structure for stage 1 of the lifecycle than figure 3. Looking at the residuals in figure 7b, there is no compelling evidence (residuals are randomly distributed around age), that yield response to tree age is not in fact linear. This means that the shape of the trajectory is more linear than illustrated by the maintained hypothesis by Kessler and Nugent (figure 3).

The second objective of the study was to use information from the estimated tart cherry yield response model to capture how variations in the trajectory of yield due to variations in planting density translates into variations in cash flows and profitability of production as measured by ANPV. Results of this economic analysis reveal that at a discount rate of 10% and tart cherry priced at 30cents a pound, it is most profitable to plant 160 trees per acre.

The third objective of this study was to conduct a sensitivity analyses to evaluate the impact of variations in tart cherry prices and interest rates on the optimal planting density and orchard economic life. The sensitivity analysis revealed that the optimum planting density did not change when price per pound was changed to 50cents or when the discount rate was changed to 12% or 15%. From the preceding analysis, it is seen that planting density has a potential influence on the trajectory of tart cherry yields over the life of an orchard. However, prevailing (and even expected) tart cherry prices as well as the discount factor which reflects the cost of capital are relevant in choosing the most profitable planting density. The impact of planting density on the economic life of the block (optimal rotation period) could not be investigated because the data series used in the analysis was not long enough to verify this effect. This problem was aggravated by the unbalanced nature of the data which led to the exclusion of blocks that were incomplete to capture the pattern.

The fourth and final objective of the study was to make recommendations on the economically profitable planting density. The results reveal that, for large scale farmers in Northwest Michigan, it is most profitable to plant 160 trees per acre when tart cherries are priced at 30 cents a pound and at a discount factor of 10%. Prevailing (and even expected) tart cherry prices and/or the discount factor are of course relevant in choosing the most profitable planting density.

Two challenges were encountered in the course of this study. There is a potential problem of sample selection bias. Little is known about how the blocks used in the analysis were selected. Another problem is the lack of data on fruit quality. This made it difficult to use the effective market prices (prices adjusted for fruit quality) in the calculation of gross revenue

The unbalanced nature of the data led to dropping of some data which if were complete would have been very useful in understanding more precisely the effect of planting density on the trajectory of tart cherry yields. Moreover, few observations, particularly on the low density end made it difficult to see clearly what happens to the yield-age trajectory at low planting density. Even more, yields never really turned down. This made it hard to see the shape of the trajectory in the later stages of the lifecycle as well as determine the life of the tree. Finally, site quality was identified to be an important factor that determines what statistical methods to use in the analysis. Variations in site quality can lead to variation in both the slope and the intercept of the tart cherry yield-age trajectory. What statistical method to use depends on whether or not we treat site as random or known. Lack of perfect information on site quality prevented a proper investigation of the extent to which site quality can influence tart cherry yield-age trajectory. As a result, the study treats site as given, thus allowing for the shape of the trajectory to be influenced by tree age and planting density. Yields at all planting densities are constrained to peak at 12 years and only the slope of the trajectory and not the intercept is allowed to change with planting density.

The preceding statistical analysis allows variation in slopes but constrains the origin of the tart cherry yield-age trajectory as planting density changes. As such, the method used in this study could be perceived as a constrained/specialized random coefficient model which goes beyond the standard random coefficient model (which allows variation in both the intercept and the slope coefficient), but was still variance components to useful in achieving part of the reason for the statistical estimation, which was to estimate the help in the economic model.

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