Modelling Complex Systems

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Abstract

Empirical observations suggest that linear dynamics are not an adequate representation of ecological systems and that a realistic representation would require adoption of complex nonlinear dynamical systems with characteristics encountered in complex adaptive systems (CAS). Adequate modelling should include and combine, among others, strategic interactions among economic agents, nonconvexities induced by nonlinear feedbacks, separate spatial and temporal scales and modeling of spatiotemporal dynamics, and allowance of alternative time scales. Ignoring these characteristics might obscure very important features that we observe in reality such as bifurcations and irreversibilities or hysteresis. As a consequence, the design of policies that do not take CAS characteristics into account might lead to erroneous results and undesirable states of managed economic-ecological systems.

1 Introduction

Empirical observations suggest that linear dynamics are not an adequate representation of ecological systems and that a realistic representation would require adoption of complex nonlinear dynamical systems with characteristics encountered in complex adaptive systems (CAS), which can be defined as systems consisting of many interacting components in which macroscopic systems properties emerge from interactions among these components. These macroscopic properties may not be obvious from the properties of the individual components at the microscopic level. A complex system is adaptive if the macroscopic properties feed back and influence the interactions among systems components (Levin 1998). Economic, social and ecological systems are examples of CAS. Economic systems are comprised of individual agents that pursue own objectives and interact among themselves. These interactions lead to the emergence of macro behaviors that ultimately may feed back to influence the actions of individual agents, but typically on different time and spatial scales. The actions of individual agents and the emerging macroscopic outcomes may also be influenced by actions taken by regulatory institutions in their attempt to mitigate externalities associated with individual actions

CAS are characterized by three basic properties (Levin 1998): (i) diversity and individuality of components, (ii) localized interactions among those components, and (iii) an autonomous process that uses the outcomes of those interactions to select a subset of those components for replication. As a result CAS are dynamic nonlinear systems, evolving in time and space, which self organize from local interactions and are characterized by historical dependencies, complex dynamics, thresholds and multiple basins of attraction (Carpenter et al. 1999, Levin 1999b).

Economic systems such as CAS have been regarded as very similar to ecological systems (Levin 1999a). There are strong similarities between them, emerging from the fact that in both systems there is competition for limiting resources, but there is also a fundamental difference. In economic systems agents' behavior is *forward looking* since agents typically solve dynamic optimization problems by forming rational expectations, while in ecological systems the behavior of ecological agents is determined by solving optimization problems *backwards*, since evolution in these systems takes place in the context of Darwinian dynamics where adaptive evolutionary changes can be attributed to past mutations. This difference has a profound impact on economic policy design, since an efficient and successful regulatory framework should take into account the way in which the agents that are subjected to regulation form expectations about future.

Recent advances in environmental and resource economics emphasize the need for a realistic representation of the ecological system and stress the presence of thresholds, multiple steady states and hysteresis effects, which are empirically observable features in these ecosystems associated with CAS structures, as opposed to the traditional approach of simple linear dynamics. Good examples are lakes, grasslands and coral reef systems.[1] Agricultural economics, a field where economic agents interact with ecosystems, can be regarded as an area where modelling in the framework of CAS should increase our insights both with respect to the internal structure and the self organizing aspects of agricultural systems, and our capacity for designing of efficient policies.

Modeling of these systems will in broad terms include a dynamical system consisting of transition equations which describe the evolution of state variables char-

[1] See for example Scheffer (1997), Carpenter and Cottingham (1997), Crépin (2007), Crépin and Lindahl (2008).

acterizing the natural system such as biomass, water, pollutants, and state variables characterizing the economic systems such as physical capital, knowledge, R&D. The complex adaptive character of the system implies that: the dynamical system will be nonlinear and nonconvex to allow for nonlinear feedbacks and multiple basins of attraction, and will evolve in both the temporal and the spatial dimension to include spatiotemporal interactions; temporal scales of evolution could be different so that some state variables evolve in fast time and some in slow time; and the spatiotemporal dynamics will be affected by control variables which will be decided by interacting forward looking optimizing economic agents. Because of this optimization, the dynamical system describing the dynamics of the unified economic-ecological system can be regarded as a set of dynamic constraints which, along with other possible static constraints, is the relevant set of constraints for the economic problem of optimizing some criterion function, such as a utility or benefit function, of the economic agents, which is defined in the same spatiotemporal domain of the dynamic constraints.

Similar problems have been addressed recently both in the ecological and economic literature. For example, in the ecological literature Scheffer (1997), or Carpenter (2003) study nonconvex ecosystems, characterized by regime shifts, thresholds, hysteresis and irreversibilities using lake ecosystems as the reference ecosystem. Studies that examine semiarid savanna grazing systems from the same point of view can be found for example in Walker et al. (1981), Scholes and Walker (1993), Scholes and Archer (1997), Ludwig et al. (1997), while Mahon et al. (2008) discuss fisheries as CAS.

The economics of complex ecosystems have also been analyzed in recent years (e.g. Janssen 2002, Dasgupta and Mäler 2003). In particular management of ecosystems with non-convex positive feedbacks are examined by Brock and Starret (2003), while a similar problem is studied by Wagener (2003). Mäler et all (2003) and Kossioris et al. (2007) examine the management of a shallow lake which is regarded as a non-convex system with hysteresis, where individual forward looking optimizing agents strategically interact among themselves using open loop and nonlinear-closed loop (or feedback) strategies. This work can be regarded as close to the concept of managing a forward looking CAS, since individual agents interact among themselves and the underlying dynamics are characterized by thresholds, hysteresis, irreversibilities, and multiple basins of attraction. Another strand of this literature studies the spatial scale and analyzes interactions between agents at the temporal and the spatial scale of these systems, endogenous formation of spatial patterns and spatial regulation. These models originating from metapopulation fishery management models (e.g. Sanchirico and Wilen 1999), have moved to the analysis of continuous spatial dynamic processes (e.g. Wilen 2007, Smith et al. 2009). Brock and Xepapadeas (2008a, 2008b) have developed a version of Pontryagin's maximum principle for optimal control with spatial diffusion and study optimal spatial pattern formation and spatial regulation in unified ecological economic models.

In this context the purpose of the present paper is to describe the way in which a complex adaptive model that unifies economic and ecological systems can be developed and to present the mathematical tools that can be used to analyze it.

2 Strategic interactions and nonconvexities

The main approach for modeling strategic interactions among agents is to consider a setup where the utility or benefit function of each agent depends on the state variable x, that is we have stock dependent utility, and the evolution of the state variable depends on the controls u of all agents. This formulation is common in problems such as: common access resource harvesting problems where individual profits depend on resource stock through stock externalities and the evolution of the resource stock depends on harvesting by all agents; knowledge formation problems, where individual profits depend on total knowledge through positive knowledge externalities and the evolution of the total knowledge depends on R&D undertaken by all agents; global pollution problems where individual (country) utility depends on the stock of global pollutant through a damage function and the evolution of the global pollutant depends on emissions by all agents.

Let $U_i(x, u)$ be a standard utility or benefit function for agent $i = 1, ..., n$. Stock effects mean that $(\partial U_i/\partial x) \neq 0$ while $(\partial U_i/\partial u)$ measures the marginal control impact, which could be for example marginal profits, utility or costs. Let $\mathbf{u} = (u_1, ..., u_n)$ be the vector of controls undertaken by all agents. Then using single species logistic dynamics as an example, the optimization problem for the individual agent harvesting biomass x, is

$$\max_{\{u_i(t)\}} J_i = \int_0^{\infty} e^{-\rho t} U_i(x, u_i)\, dt \tag{1}$$
$$\text{subject to} \quad \dot{x} = sx\left(1 - \frac{rx}{s}\right) - \sum_{i=1}^{n} u_i \ , x(0) = x_0$$

where s is intrinsic growth rate and s/r is the environment's carrying capacity. Problem (1) is an infinite time horizon differential game with J_i being the payoff for agent i. A differential game can be defined as a situation of conflict where players choose strategies over time. For each player i, a control space exists whose elements are the control variables for each player. The strategy for each player is a function $u_i(t) = \theta_i(x(t), t)$. The set of strategies for player i is her/his strategy space. Crucial to the structure of the differential game is the specification of the information about the state of the game gained and recalled by each player at each point in time . There are a number of possible information structures for a differential game (Basar and Olsder 1982), only two of which are considered here.

i The differential game is said to have an *open-loop* informational structure if the players follow open-loop strategies: $u_i(t) = \theta_i(x_0, t)$.

ii The differential game is said to have a *closed-loop or feedback* informational structure if the players follow feedback strategies: $u_i(t) = \theta_i(x(t), t)$.

In the differential game (1), the open-loop Nash equilibrium is defined as the n-tuple of open-loop strategies $(u_1^*, ..., u_n^*)$ satisfying

$$J_i(u_1^*, ..., u_n^*) \geq J_i\left(u_1^*, .., u_{i-1}^*, u_i, u_{i+1}^*, ..., u_n^*\right). \tag{2}$$

A closed-loop (or feedback) Nash equilibrium is an n-tuple of closed-loop strategies that satisfies conditions (2) for every possible initial state x_0. The open-loop Nash equilibrium is not subgame perfect, while a closed-loop equilibrium is subgame perfect. By defining a cooperative solution as the problem (1) but with the objective replaced by $J = \int_0^{\infty} e^{-\rho t} \sum_{i=1}^{n} [U_i(x, u_i)]\, dt$, it is possible to study deviations between cooperative behavior and noncooperative behavior corresponding to open-loop or closed-loop Nash equilibrium.

Differential games are important analytical tools when it comes to the analysis of strategic interactions among players in a dynamic setup which is a fundamental ingredient of CAS. Differential games could become closer to CAS if non linear feedbacks are incorporated into the transition dynamics of the state variables. A commonly used way to model these nonlinear feedbacks is the Holling type-III functional response which is defined as

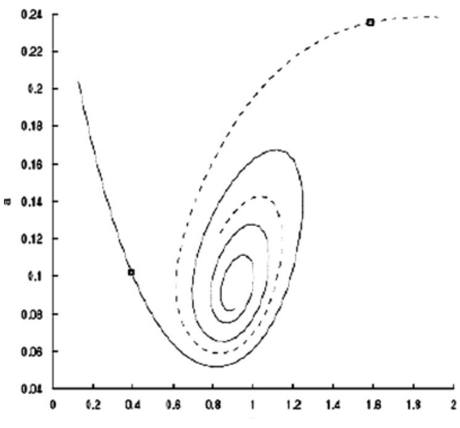
$$f(x) = \beta \frac{x^2}{\alpha^2 + x^2}. \tag{3}$$

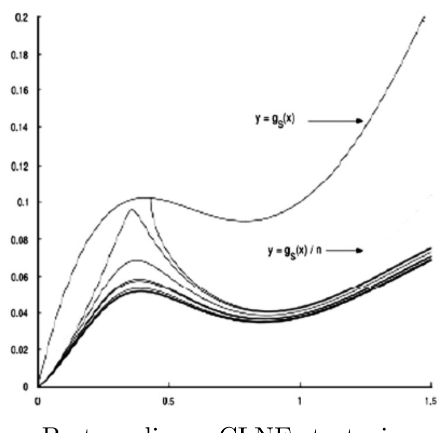
The introduction of (3) into (1) introduces nonconvexities and the possibility of multiple basins of attraction both in the cooperative and the noncooperative equilibrium. The functional form (3) was originally used by Ludwig, Jones, and Holling (1978) to understand the dynamics of the spruce budworm. Function (3), in a nondimesionalized form with $\alpha = \beta = 1$, has been extensively used recently to study, through the differential games framework, the economics of the so-called shallow lake problem (Mäler et al. 2003, Kossioris et al. 2008). The shallow lake problem, which incorporates strategic interactions among many agents and nonlinear dynamics leading to multiple steady states, hysteresis and irreversibilities, can be written as:

$$\max_{\{u_i(t)\}} J_i = \int_0^\infty e^{-\rho t} \left[U(a_i) - D(x)\right] dt \tag{4}$$

$$\text{subject to } \dot{x} = a_i + \sum_{j \neq i}^{n} a_j - bx + \frac{x^2}{1+x^2}, \; x(0) = x_0$$

where a_i is individual phosphorus loadings due to agricultural activities, x is accumulated phosphorus in the lake, $U(a_i)$ is benefits from agricultural activities and $D(x)$ is damages from eutrophication of the lake. This is a differential game with nonconvex dynamics for which analytic solutions cannot be obtained. Numerical solutions have shown that in the OLNE multiple steady states exist where saddle points are followed by unstable spirals with Skiba points. On the other hand the CLNE is characterized by equilibrium nonlinear closed-loop strategies which lead to inferior outcomes relative to the cooperative solution and the OLNE (see figures below).


Stable manifolds OLNE



Best nonlinear CLNE strategies

The deviations between cooperative and noncooperative solutions suggest that regulation is required. The problem of regulation becomes complicated due to hysteresis. Mäler et al. (2003) derived a fixed tax which in a decentralized regulated

OLNE moves the system to the cooperative steady state. Kossioris et al. (2009) present preliminary results from the far more difficult problem of deriving a fixed tax which steers a decentralized regulated CLNE with nonlinear strategies to the cooperative steady state. Brock and Xepapadeas (2004a) studied the problem of managing fisheries with interacting species, under nonconvex dynamics and hysteresis. They show that the cooperative solution leads to a unique steady state while open access equilibrium leads to multiple steady states with hysteresis effects. They also show that regulation of the hysteretic system in the sense of approaching the cooperative steady state depends on whether biomass dynamics move slower than economic dynamics. This result opens the issue of multiple time scales which will be addressed latter.

3 Spatiotemporal dynamics in CAS

3.1 Modelling spatial interactions in economics and ecology

Let $x(z,t)$ be a state variable denoting the concentration of a biological or economic variable at time $t \geq 0$ at the spatial point $z \in \mathcal{Z}$, where space is assumed to be one dimensional and modelled by a line segment. The classic approach for modelling spatial movements of this state variable is through diffusion. In this context diffusion is a process through which the microscopic irregular motion of an assemblance of particles such as cells, chemicals, animals, or resources results in a macroscopic regular motion of the group. This classical approach to diffusion implies that diffusion has local or short range effects. A measure of diffusion is the *diffusion coefficient*, or *diffusivity,* which measures how efficiently the particles move from high to low density. Let $F(x(z,t), u(z,t))$ be a density dependent source, or a growth function, for the state variable where $u(z,t)$ is a control variable such as harvesting by economic agents, and D_x be the diffusion coefficient. Then the basic diffusion equation describing the spatiotemporal evolution of the state variable is:[2]

$$\frac{\partial x(z,t)}{\partial t} = F(x(z,t), u(z,t)) + D_x \nabla^2 x(z,t), \quad \nabla^2 x(z,t) = \frac{\partial^2 x(z,t)}{\partial z^2}.^3 \tag{5}$$

If the source term represents logistic population growth net of harvesting $u(z,t)$ at spatial point z and time t, or $F(x) = x(z,t)(s - rx(z,t)) - u(z,t)$, then we obtain the *Fisher* equation:

$$\frac{\partial x(z,t)}{\partial t} = sx(z,t)\left(1 - \frac{rx(z,t)}{s}\right) - u(z,t) + D_x \nabla^2 x(z,t). \tag{6}$$

The Fisher equation can be generalized to several interacting species or activities. With two interacting species (x,y) which are both harvested and no cross diffusion, we obtain:

$$\frac{\partial x}{\partial t} = F_1(x,y) - u_x + D_x \nabla^2 x \tag{7}$$

$$\frac{\partial y}{\partial t} = F_2(x,y) - u_y + D_y \nabla^2 y. \tag{8}$$

System (7)-(8) is referred to as a *reaction-diffusion system* or as an *interacting population diffusion system.*[4] In more general diffusion models the diffusion coeffi-

[2] For details see Murray (2003).

[3] In general $\nabla^d y = \frac{\partial^d y}{\partial z^d}$, $d = 1,2$.

[4] Generalization to n species is straightforward.

cient could be density dependent or $D_x = D_x\left(x\left(z,t\right)\right).$

Modelling spatial movements through diffusion implies that diffusion is a *local* or *short range* effect. In many applications however it is necessary to model *nonlocal* or *long range* effects. This is done by using an integral equation formulation. In the presence of nonlocal effects the temporal change of the state variable at spatial point z depends on the influence of neighboring state variables in all other locations z'. Then the evolution equation analogous to (5) is:

$$\frac{\partial x\left(z,t\right)}{\partial t} = F\left(x\left(z,t\right),u\left(z,t\right)\right) + \int_{-\infty}^{\infty} w\left(z-z'\right)x\left(z',t\right)dz' \tag{9}$$

where $w\left(z-z'\right)$ is the *kernel function* which quantifies the effects of neighboring $x\left(z',t\right)$ on $x\left(z,t\right).$[5]

Local and nonlocal effects can be combined to produce models described by integrodifferential equations (e.g. Genieys et al. 2006) or

$$\frac{\partial x\left(z,t\right)}{\partial t} = F\left(x\left(z,t\right),u\left(z,t\right)\right) + D_x\nabla^2 x\left(z,t\right) + \int_{-\infty}^{\infty} w\left(z-z'\right)x\left(z',t\right)dz. \tag{10}$$

Nonlocal effects and the integral equation formulation are widely used in economics to model knowledge or productivity spillovers on the production function (e.g. Lucas 2001, Lucas and Rossi-Hansberg 2002) or to model long-range effects of knowledge accumulation (e.g. Quah 2002). Thus a constant return to scale production function with knowledge spillovers can be written as

$$Q(z) = e^{\gamma V(z)}L(z)^a K(z)^b X\left(z\right)^{1-a-b} \tag{11}$$

where Q is the output, L is the labor input, K is physical capital, X is land, and V is the productivity spillover which depends on how many workers are employed at all locations and represents a positive externality.

$$V(r) = \delta \int_{-\infty}^{\infty} e^{-\delta(z-z')^2}L(z')dz' \tag{12}$$

The function $e^{-\delta(z-z')^2}$ is the kernel. The production externality is a positive function of labor employed in all areas and is assumed to be linear and to decay exponentially at a rate δ with the distance between z and z'. The idea is that workers at a spatial point benefit from labor in nearby areas, and thus the distance between firms determines the production of ideas and the productivity of firms in a given region. A high δ indicates that only labor in nearby areas affects production positively. In terms of agglomeration economics, the production externality is a *centripetal* force, i.e. a force that promotes the spatial concentration of economic activity.

If we interpret K as the stock of knowledge, then in the context of R&D based growth the accumulation of knowledge depends on resources L_K, $0 < L_K < L$ devoted to the production of new knowledge,[6] and spatial spillovers or

$$\frac{\partial K\left(z,t\right)}{\partial t} = h\left(K\left(z,t\right),L_K\left(z,t\right)\right) + \int_{-\infty}^{\infty} w\left(z-z'\right)K\left(z',t\right)dz \tag{13}$$

where $h\left(K,L_K\right)$ is the R&D generating function.

[5] The spatial domain could be finite with appropriate boundary conditions.

[6] In this case L in (11) is replaced by labour used in output production L_Q, with $L_Q + L_K = L$.

Dynamical systems incorporating local and nonlocal effects capture interactions at both the temporal and the spatial scale and can be used as the foundations for modelling complex systems. There are however more features of complex adaptive systems which should be incorporated into this modelling framework. For the purpose of this paper we consider that the features to be incorporated should include non-convexities, different time scales, and strategic interactions among economic agents choosing controls u.[7]

3.2 Economic analysis and complex adaptive systems in temporal and spatial scales

To incorporate both the temporal and the spatial scale into economic problems, the control variables in transition equations like (5), (7)-(8), (9), (10) or (13) should be chosen by economic agents according to some behavioral assumption. It is assumed that an economic agent is located at each spatial point z. Each agent has a benefit function $U(\mathbf{x}(t,z), \mathbf{u}(t,z))$ defined over the state and the control variables, which is assumed to be increasing and strictly concave in the controls. In principle we can distinguish between three type of behavior.

3.2.1 Myopic economic agents

In this case each economic agent acts myopically both temporally and spatially and considers itself to be small in relation to the spatiotemporal evolution of the state variables. Thus it chooses controls to maximize an objective at each instant of time for the given spatial site, by treating the values of the state variables as exogenous parameters. Therefore agents ignore the impacts of their actions on other sites. However, these impacts emerge because of the diffusion of the state variables and this is the source of a diffusion induced spatial externality which exists along with the temporal externality

The private controls (or harvesting) can be defined in terms of two assumptions which are associated with the type of property rights prevailing in the spatial domain. If each agent owns enforceable property rights for her/his site, the optimal private controls are defined as:

$$u_{ij}^{0}(t,z) = \arg\max_{u_{ij}} U_i(\mathbf{x}(t,z), \mathbf{u}(t,z)) \;,\; i=1,...,n,\; j=1,...,m.$$

Assuming that the optimal private control is interior, $u_{ij}^{0}(t,z)$ is defined implicitly by the system of first-order conditions $\frac{\partial U(\mathbf{x}(t,z),\mathbf{u}(t,z))}{\partial u_{ij}} = 0$. Solution of the system of first-order conditions defines the private optimal controls as state dependent rules for given values of the state variables, thus,

$$u_{ij}^{0}(z,t) = h_{ij}^{0}(\mathbf{x}(t,z)) \;, j=1,...,m \text{ for all } i. \tag{14}$$

Under open access, controls are chosen so that rents are dissipated on each site, or $\hat{\mathbf{u}}(t,z) : U_i(\mathbf{x}(t,z), \hat{\mathbf{u}}(t,z)) = 0$ for all z. The open access controls are then determined in a feedback form as:

$$\hat{u}_{ij}(t,z) = \hat{h}_{ij}(\mathbf{x}(t,z)) \;, j=1,...,m \text{ for all } i. \tag{15}$$

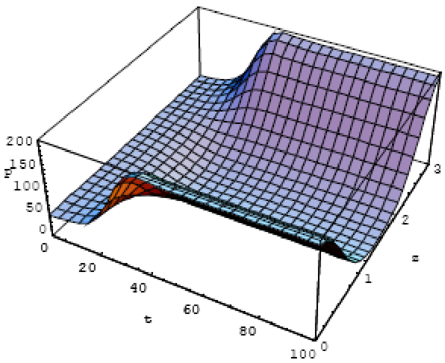
[7] Risk and uncertainty, and especially the distinction between traditional risk associated with risk aversion and unmeasurable uncertainty associated with multiple priors and uncertainty or ambiguity aversion, is another important feature of CAS. Its discussion is, however, beyond the purpose of the present paper.

Brock and Xepapadeas (2008b) study a semi arid grazing system with two state variables, plant biomass and underground water which evolve in time and space. The system is managed by economic agents located at each site, whose objective is maximization of private profit from cattle products. To obtain the cattle products the economic agents harvest plant biomass by exercising costly grazing or harvesting effort. In the Brock-Xepapadeas set up, individuals are myopic and disregard spatiotemporal dynamics. Substitution of myopic controls into transition equations like (7)-(8) produces a system of partial differential equations:

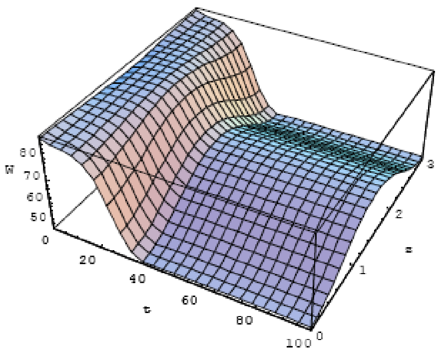
$$\frac{\partial x}{\partial t} = F_1\left(x, y, u^0\left(x, y\right)\right) + D_x \nabla^2 x \tag{16}$$

$$\frac{\partial y}{\partial t} = F_2\left(x, y, u^0\left(x, y\right)\right) + D_y \nabla^2 y. \tag{17}$$

The resulting system is a *reaction-diffusion*, *activator-inhibitor* system where spatial patterns emerge as a result of the celebrated Turing mechanism (Turing 1952) of diffusion induced instability. This means that the economic-ecological model leading to (16)-(17) can be used as a conceptual basis for explaining observed spatial pattern in semi arid grazing systems. Simulated spatiotemporal evolution for plant biomass and groundwater indicating the emergence and persistence over time of spatial patterns are shown below.



Plants biomass



Groundwater

3.2.2 Cooperative optimum

In the cooperative or social optimum a social planner is introduced whose objective is the maximization of discounted benefits over finite spatial domain, subject to the spatiotemporal constraints. By explicitly taking into account these constraints, the social planner internalizes spatial and temporal externalities which were not taken into account at the myopic private optimum. The problem of the social planner can be stated as:

$$\max_{\{\mathbf{u}(t,z)\}} \int_0^{\infty} \int_0^{L} e^{-\rho t} \left[U\left(\mathbf{x}\left(t, z\right), \mathbf{u}\left(t, z\right)\right)\right] dz dt \tag{18}$$

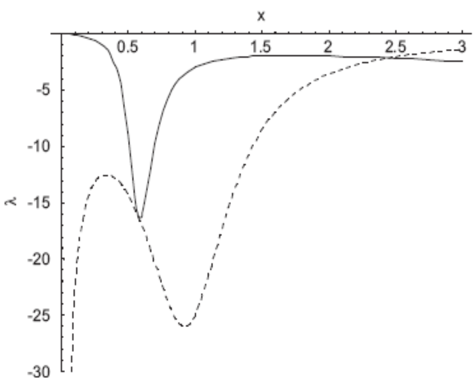
$$\text{subject to } \frac{\partial \mathbf{x}\left(z, t\right)}{\partial t} = \mathbf{F}\left(\mathbf{x}\left(z, t\right), \mathbf{u}\left(z, t\right)\right) + D\nabla^2 \mathbf{x}\left(z, t\right)$$

where D is a diagonal matrix of diffusion coefficients. Brock and Xepapadeas (2008a, 2008b) study this abstract problem by extending Pontryagin's maximum principle for the optimal control of systems governed by partial differential equations. They identify an optimal diffusion induced spatial instability mechanism, analogous to the Turing mechanism of diffusion induced spatial instability. This mechanism implies that it might be optimal to control the system in such a way that spatial patterns emerge. The Brock-Xepapadeas result can thus be regarded as an optimal agglomeration result applying to ecological economic systems.

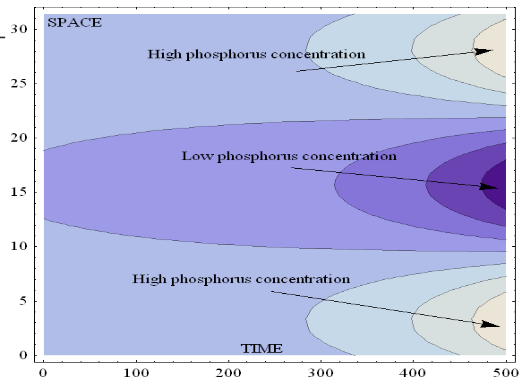
In this context Brock and Xepapadeas (2008a) study the cooperative solution for the shallow lake problem. The problem is:

$$\begin{aligned}
&\max_{a(t,z)} \int_0^\infty \int_0^L e^{-\rho t} \left[B\left(a\left(t,z\right)\right) - C\left(x\left(t,z\right)\right)\right] dz dt \\
&s.t.\ \frac{\partial x\left(z,t\right)}{\partial t} = a\left(t,z\right) - bx\left(t,z\right) + \frac{x^2\left(t,z\right)}{1+x^2\left(t,z\right)} + D\frac{\partial x^2\left(z,t\right)}{\partial z^2} \qquad (19)\\
&x\left(t,0\right) = x\left(t,L\right)
\end{aligned}$$

This problem describes a CAS where the spatial scale has been incorporated with nonconvex dynamics which induce hysteresis and multiple steady states. Numerical simulations using the generally accepted spatial forms and parametrization of the shallow lake problems show that spatial patterns emerge through the optimal diffusion induced spatial instability mechanism in the neighborhood of the oligotrophic steady state. The multiple steady state for the cooperative solution along with the emerging of the optimal spatial patterns for the concentration of phosphorus are shown below.



Oligotrophic, eutrophic steady states



Contours of phosphorus concentration

The analysis of CAS in the spatiotemporal domain under myopic noncooperative behavior and under cooperative behavior reveals the differences between the corresponding spatiotemporal paths for the state variables. These deviations suggest that regulation is necessary to steer noncooperative paths towards cooperative paths by internalizing spatiotemporal externalities. The problem of designing decentralized schemes in the form of taxes or quotas for these CAS is a difficult task as suggested by the attempts to regulate the OLNE in the shallow lake problem. Preliminary results suggest that for the specific systems under study instruments varying among spatial zones which partition the spatial domain, and among different time intervals, are feasible. This is a very promising area for further studies.[8]

[8]Instruments with spatiotemporal dimension have been studied by Goetz and Zilberman (2000), Sanchirico and Wilen (2005).

3.2.3 Strategic behavior, nonconvexities and spatiotemporal dynamics

The optimal control of spatiotemporal systems with nonconvexities has not, to the author's knowledge, advanced to the stage where strategic interactions among agents are introduced and myopic behavior is abandoned. This calls for the formulation of differential games in spatiotemporal domains. This is an area for further research, which hopefully might provide interesting new results. To obtain an idea about how such a model can be formulated one can adopt the shallow lake problem again. The problem of individual agent i, who has full property rights on site z, can be formulated as

$$\max_{a_i(t,z)} \int_0^\infty e^{-\rho t} \left[B\left(a_i\left(t,z\right)\right) - C\left(x\left(t,z\right)\right)\right] dzdt \ , i = 1, ...n \tag{20}$$

$$s.t. \ \frac{\partial x\left(z,t\right)}{\partial t} = a_i\left(t,z\right) + \sum_{l \neq i} a_l\left(t,z\right) - bx\left(t,z\right) + \frac{x^2\left(t,z\right)}{1+x^2\left(t,z\right)} + D\frac{\partial^2 x\left(z,t\right)}{\partial z^2}$$

$$x\left(t,0\right) = x\left(t,L\right).$$

In this set-up each individual acts as a planner for the site where he has full property rights and takes into account the impact of his own action on himself and on his own site, while ignoring his impact on others located at different spatial points. This means that each individual treats $\frac{\partial x^2(z,t)}{\partial z^2}$ as a fixed parameter x^e. However actions taken at site z affect state variable at site z' through spatial diffusion. Thus the forward looking individual does not fully internalize spatiotemporal externalities. This causes deviations between the noncooperative and the cooperative solution, but most likely deviations are not spatially homogenous so that correction of the externalities requires spatiotemporal regulation. To obtain a preliminary idea about the characterization of the solution, we assume open loop strategies, symmetry and set $B\left(a_i\right) = \ln a_i$, $C\left(x\right) = cx^2$. An open loop Nash equilibrium equilibrium is defined for $x^e = \frac{\partial^2 x(z,t)}{\partial z^2}$, and is characterized by the optimality conditions

$$\dot{\lambda} = \left[b + \rho - \frac{2x}{\left(1+x^2\right)^2} \right] \lambda + 2cx$$

$$\dot{x} = n\left(\frac{-1}{\lambda}\right) - bx + \frac{x^2}{1+x^2} + D\frac{\partial^2 x}{\partial z^2}$$

The nature of the deviations is revealed by a comparison with the optimality conditions for the cooperative equilibrium given below:

$$\dot{\lambda} = \left[b + \rho - \frac{2x}{\left(1+x^2\right)^2} \right] \lambda + 2ncx - D\frac{\partial^2 \lambda}{\partial z^2}$$

$$\dot{x} = n\left(\frac{-1}{\lambda}\right) - bx + \frac{x^2}{1+x^2} + D\frac{\partial^2 x}{\partial z^2}.$$

Closed loop information structure implies that $a_i = \theta\left(x\left(t,z\right)\right)$. Handling this problem requires solution of a nonlinear differential game with nonlinear closed loop strategies evolving in two dimensions. This suggests a challenging area for future research.

4 Economic-ecological systems evolving in different time scales

The interaction of fast and slow processes is an integral part of ecosystems analysis. As Simon Levin (2000, p. 499) points out:

> "Even the most basic notions of population dynamics involve recognition of multiple scales. Populations introduced into new areas typically grow exponentially on a fast time scale, before density dependence restricts growth. The intrinsic rate of natural increase characterizes the fast time scale dynamics, whereas the carrying capacity determines essential features on longer time scales."

In the analysis of coevolutionary process the conventional distinction implies a time scale separation between population dynamics which evolve in fast time scale, and evolution which takes place in slow time scale. In models of antagonistic coevolution of species the interaction of population (or biomass) dynamics and mutation (or trait) dynamics leads to the so-called "Red Queen cycles."[9]

In economics this time separation is not apparent, especially for purely economic variables, however when the economic system interacts with a system operating at different time scales, time separation might be important for management purposes. For example indications related to resistance development for genetically modified crops in agriculture, or to resistance development to antibiotics, suggest that the slow movement of mutation might be relevant and important in certain cases for analyzing the whole system and for developing sensible policies. Thus modelling fast-slow systems has been associated with issues such as biological resource management, water management, pest control (e.g. Brock and Xepapadeas 2004b, Grimsrud and Huffaker 2006, Huffaker and Hotchkiss 2006, Crépin 2007).

Formally dynamical systems evolving in a fast - slow time framework can be analyzed by using results from singular perturbation analysis (e.g. Wasow 1965, Fenichel 1979). In the context of a model with two interacting state variables which can be controlled by a vector of controls $\mathbf{u}$, a dynamical system in fast-slow time can be written as[10]

$$\begin{aligned} \varepsilon\dot{x} &= f\left(x, y, \mathbf{u}, \varepsilon\right) \\ \dot{y} &= g\left(x, y, \mathbf{u}, \varepsilon\right). \end{aligned} \tag{21}$$

where ε is a small positive parameter. Since $\dot{x}$ can be much larger than $\dot{y}$, x is the *fast variable* and y is the *slow variable.* Thus x could be the population of a species evolving in fast time and y could be a trait evolving in slow time. If $\varepsilon \to 0$ then system (21) is replaced by the algebraic-differential system

$$0 = f\left(x, y, \mathbf{u}, 0\right) \tag{22}$$

$$\dot{y} = g\left(x, y, \mathbf{u}, 0\right). \tag{23}$$

Assume that a smooth differentiable manifold, $x = m\left(y\right)$ exists which describes the solution of equation (22). Then $m\left(y\right)$ is called the *slow manifold* and the dy-

[9] A limit cycle or other non-point attractors in trait space dynamics are called "Red Queen" races because, for example, in predator-prey systems each is evolving its trait against the other and the traits are moving dynamically, unlike a fixed point. Red Queen cycles are observed in a slow time scale, since trait dynamics are assumed to evolve slowly, in contrast to the population, host - parasite, dynamics which are assumed to evolve fast.

[10] The presentation here is adapted from Berglund and Gentz (2003).

namics on it are described by the *reduced equation*

$$0 = g\left(m\left(y\right), y, \mathbf{u}, 0\right). \tag{24}$$

If we scale time, t, by $1/\varepsilon$ in order to define fast time $\tau = t/\varepsilon$, the system becomes in fast time

$$dx/d\tau = f\left(x, y, \mathbf{u}, \varepsilon\right) \tag{25}$$

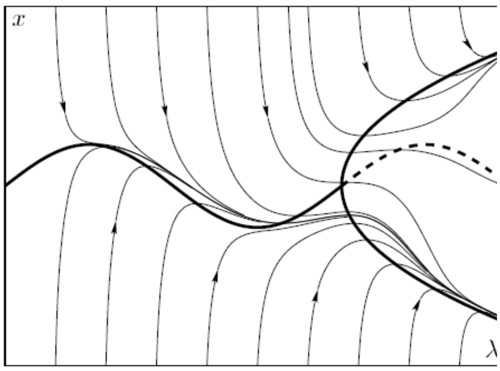
$$dy/d\tau = \varepsilon g\left(x, y, \mathbf{u}, \varepsilon\right). \tag{26}$$

By taking the limit $\varepsilon \to 0$ we obtain the *layer or associated system*,

$$dx/d\tau = f\left(x, y, \mathbf{u}, 0\right) \tag{27}$$

$$dy/d\tau = 0 \tag{28}$$

where the slow variable is treated as a fixed parameter. The slow manifold $x = m\left(y\right)$ consists of the steady states of (27) for fixed y. Fenichel (1979) developed the analysis of these systems in terms of invariant manifolds. From the point of view of CAS this type of modelling is important since the interaction of fast and slow processes is an integral part of the sudden large shifts that sometimes occur in ecosystems (Rinaldi and Scheffer 2000). Possible moments around a slow manifold are shown in the figure below, which depicts phenomena occurring as the slow variable approaches a supercritical pitchfork bifurcation point.



An attracting slow manifold

Incorporating optimal control and strategic interactions by choosing controls $\mathbf{u}$ according to economic criteria in systems characterized by fast-slow dynamics, and extending the few attempts recently undertaken along these lines is another area of promising new research for designing policies in different time scales.

5 Concluding remarks

This paper presents methodologies and tools for the modeling of CAS which are essential to the economic management of ecosystems. It is suggested that adequate modelling should include and combine among others, strategic interactions among economic agents, nonconvexities induced by nonlinear feedbacks, separate spatial and temporal scales and modeling of spatiotemporal dynamics, and allowance of alternative time scales. Undoubtedly issues like spatial scaling and uncertainty which were not tackled in this paper are also very important.

Modelling along these lines increases to a large extent the complexity of models and the cost of the analysis, reduces the possibilty of optaining general and analytic results, and increases the importance of numerical simulations in acquiring insights for the problems uder study. On the other hand however, empirical observations suggest that linear dynamics are not an adequate representation of ecological systems and that realism requires moving towards more complex nonlinear systems with many CAS characteristics. Ignoring these characteristics might obscure crucial features that we observe in reality, such as bifurcations and irreversibilities or hysteresis. As a consequence, the design of policies that do not take CAS characteristics into account might lead to erroneous results and undesirable states of managed economic-ecological systems.

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