Food safety and nutritional quality – Firms’ strategies and Public intervention

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Abstract: The aim of our paper is to determine the conditions under which firms tend to offer the best nutritional quality of food products, and the public regulation required to obtain this in a context where diet and nutritional status plays an important part in maintaining health and preventing disease, and with increasing pressure for public intervention on food quality in developed countries. To this end, we develop a duopoly model where products can be horizontally (variety) and vertically (quality) differentiated. We analyze the perfect Nash equilibriums in a two period competition game where in the first stage, the firms decide simultaneously on the variety and the quality of the product to be sold and in the second stage, firms set prices. The model firstly highlights that in the absence of the public intervention, the spontaneous choice of the firms will not lead to the desired level of nutritional quality. An imposition of a minimum quality threshold is also necessary. Secondly, without assistance from authorities, firm that engages in the “upmarket” may have some difficulties such as the loss of customers due to a high price associated with high quality, a loss of traditional taste (for example, products containing less sugar), or a high cost of innovation (to induce the loss of customers) which casts doubt on the wisdom of taking such an approach. To correct this, the subvention and tax policies are suggested.

Key words: Nutritional product, health cost, public regulation, nutrition information, quality threshold, vertical and horizontal differentiation.

JEL: L13, L15, L51, Q18
Introduction

Until recently, we recognize the relationship between diet and health through major nutrient deficiency diseases. Evidence of this relationship was based on clinical tables accompanying famines and food shortages or food lack of nutrient which have occurred in the human history. Lack of protein was responsible for kwashiorkor, lack of vitamin C for scurvy, vitamin D for rickets, and iron for anemia...

In recent decades, in France as in all other industrialized countries, a situation of abundance has developed (except during the two world wars), which induce the disappearance of major deficiency diseases present in Europe and North America in the earlier twentieth century, and still widespread in developing countries today. Thus, the most obvious aspects of the relationship between diet and health have apparently faded in industrialized countries while they are still dramatically present in many other regions of the world.

The public health issues arising from the relationship between nutrition and health to which developed countries like France, USA etc must face in the 21st century have another nature: the quality of foods and ingredients is no longer enough, they must move into an era of nutritional quality because food (and nutritional status) participates actively in the determinism of their most prevalent diseases today. These include obesity, heart disease, hypertension, stroke, type 2 diabetes, cancers, osteoporosis, anaemia, dental caries... For example, Doll & Peto (1981) shown that 30-40% of cancers in men and 60% of cancers in women were due to diet and nutritional status. Many other researches emphasize that the consumption of some food can reduce the risks of these diseases, etc.

In terms of public health these diseases cause serious problems: they may cover an important fraction of the population and have serious consequences to the human, social and economic plan.

In France, obesity affects 11.3% of the population in 2003, representing an increase of 17% per year. At that rate, it should reach the U.S. obesity rate by 2020 (with 30,6% of adults), which will impose an annual cost of 14 billion euros only for the health insurance. Regarding cardiovascular diseases and cancers, these two main causes of death in France account each one for approximately 30% of all deaths and for health insurance schemes the two leading causes of admission to long-term treatment with 4 millions persons per year, representing a cost of 34 billion euros in 2006. Regarding the US, these three diseases account for approximately two-thirds of all deaths and about $700 billion in direct and indirect economic costs each year. In Europe 15, 75-130 billion euros are spent each year only for obesity.

Consequently, the public authorities are increasingly mobilised for the nutritional question domestically and internationally. From an economic point of view, the question posed is the choice of priority interventions to reduce, at the lowest possible social cost, the extent of these diseases.
This involves the evaluation of conditions for the effectiveness of public authorities’ possible tools on the basis of an overall assessment taking into account all economic dimensions related to the nutritional issue.

The authorities in charge of public health issues have indeed various action levers. The first ones, through governmental plans such as PNNS (National program nutrition and health) in France, aim to play on demand and to guide consumer behaviour, making them aware of the dangers of certain consumption practices such as the excessive consumption of fat, salt or sugar... In other words, public authorities act on the dietary choices made by consumers. Theses actions can be information campaigns on diet and nutritional status’s role in the health; nutritional education; relayed actions at the local level, and incentives for physical activities etc. On the contrary, the second ones are focused on the offer side through the modification on the relative price structure or the improvement of nutritional characteristics of food products sold to final consumers. In other words, decision-makers act on the choices of nutritional characteristics to supply the market made by firms.

All first levers’ tools such as the advertising, the labeling or the information… are totally legitimate even if we should discuss more about their modalities and their costs. In this light, the legitimacy of the public intervention on demand is not controversial but on offer one is, notably when it invokes the responsibility of the individuals. However, economic analysis offers arguments justifying the need to take simultaneously both of these actions levers because the only intervention on consumer behaviour or the only action on offer may not be sufficient to “impose” a healthy and balanced diet. For example, an enhancement of education and information on the role of nutritional quality in health can make consumer pay more attention to their choice during their purchase but without intervention on offer these positive effects may be cancelled by the advertising or the differential pricing structure.

However, the definition and the implementation of an integrated policy taking into account the various dimensions induced by the nutritional issue is based today on an insufficient evaluation of the effects given by different action levers and their interactions especially by the second ones - actions on offer. The purpose of this paper is therefore to try to move in this direction by analyzing possible tools of public intervention on offer and identify some issues to provide a coherent framework in order to guide the development of an integrated policy. To this end, we build a duopoly model where two firms are competing to provide the different nutritional products to consumers in the final market. The originality of our model is that it takes into account the double differentiation: in variety or horizontal (locational) differentiation, consumers differ in their most preferred variety. In quality (or vertical) differentiation all consumers desire more of the quality feature but they differ in the willingness to pay for improvements in quality. Or, the literature on models with double differentiation is quite little. Furthermore, in our model, a possibility of
arbitrage at firms’ level will be introduced, which permit to enrich the vertical and horizontal analysis. The model will be analyzed as a sequential game where firms choose simultaneously the location on variety and the quality of their product to be sold to consumer in the final market in the first stage and set prices in the second stage.

The model examines the role played by three types of intervention: minimum quality threshold, campaign information, and taxation. We identify the effects of this regulation on the strategy of firms and shows how certain regulations that may seem conducive to the supply of high nutritional quality may have adverse effects when the firms’ strategic reaction is considered. The model highlights that in the absence of public intervention, the spontaneous choice of the firms will not lead to the desired level of quality. Hence, it may appear that the public intervention is necessary: Firstly, an imposition of a minimum quality threshold may decrease the health cost. Secondly, intervention by the authorities in the form of campaigns aiming to inform, educate and guide young people, as suggested by several papers on the food demand will not induce the desired effect. Thirdly, without assistance from authorities, firms that engage in the “upmarket” may have some difficulties such as the loss of customers due to the high price or the loss of traditional taste (for example, products containing less sugar), which casts doubt on the wisdom of taking such an approach. To correct this, this work concludes that the authorities should use tax policies such as the “tax on product” which affects profits through a reduction in profits per unit of output, with a higher nutritional quality of product offered being taxed at a lower rate than poorer quality products.

The paper is organized as follows: Section 1 sets up the model; section 2 establishes the game; section 3 presents the main results by showing the price equilibrium and the quality and variety equilibrium; section 4 discusses the results and concludes.
I. Section 1: Model without relationship between the taste and the nutritional quality

1. The model

In reality, many products are differentiated in two dimensions. First, products can be classified as horizontally differentiated, a case where consumers have different preference orderings over product characteristics. In other words, consumers differ in their most preferred variety. Thus, this is sometimes called spatial differentiation (see Hotelling 1929; Salop 1979; Chamberlin 1933; Dixit & Stiglitz 1977). Alternatively, when consumers agree over the preference ordering of products, the market is said to be vertically differentiated. The best example of vertical differentiation occurs when products differ in terms of quality. With this form of differentiation, all consumers have the greatest preference for the product of highest quality. However, in this dimension, they differ in the willingness to pay for improvements in quality (see Musa & Rosen 1978; Gabszewicz & Thisse 1979).

In the case of nutritional products, they are differentiated in variety by the taste (chocolate/honey, salty/sweet; crunchy/soft...) and also in quality by their nutritional score (level of sugar, salt, fat, fibre, calcium, vitamins and minerals, complex carbohydrates, etc.).

In the model developed in this paper each differentiated product is defined by one feature of variety and one feature of quality. Variety is indexed by the parameter $y$ where a higher value of $y$ indicates a newer variety and quality by the parameter $s$ where a higher value of $s$ indicates a higher level of quality. Therefore, a product is defined as a pair ($y$, $s$). This method of double differentiation facilitates the study of the effects of quality variations on such central questions as the over-abundance of varieties and the under-supply of quantities; specifically, this allows the exploration of the basic substitutability of variety for quality and the extent to which strategic interactions among firms define the variety-quality mix that the market provides.

Regarding the firms, we assume that two firms (indexed by 1 and 2) compete in the market, and each firm produces a single product, defined as brands 1 and 2. Each firm can choose its location anywhere between the two extremes of a straight line running from 0 to 1 for variety: $y_i \in [0,1]$ and from $s$ to $\bar{s}$ for quality: $s_i \in [s, \bar{s}]$; $s \geq 0, \bar{s} \leq 1$. We call $y_1$ and $y_2$ the respective variety; $s_1$ and $s_2$ the respective quality chosen by the firms 1 and 2. Product supplied by the firm 1 is characterized thus by $(y_1, s_1)$ and product supplied by the firm 1 by $(y_2, s_2)$. To simplify, we assume that $y_1 = 0, y_2 = 1$ and $s_1 \geq s_2$.

Firms’ strategic variables are price, variety, and product quality.

Cost of production
Moreover, the firm is assumed to have production costs increasing in variety and quality, which means that the newer variety, the higher production costs. And it is the same for the quality:

\[ C_i = \lambda (1 + s_i)(1 + y_i) \]

with \( \lambda > 0 \), \( C(y_i, s_i) \geq 0 \) \( \forall (y_i, s_i) \in [0,1] \times [s, \bar{s}] \); \( \frac{\partial C_i}{\partial y_i} \geq 0 \) \( \forall y_i \); and \( \frac{\partial C_i}{\partial s_i} \geq 0 \) \( \forall s_i \).

However, note that the production of a product having a “low” variety, i.e. near to traditional taste, but a good nutritional quality may be less expensive than the production of the one having a new taste even if its quality is lower.

**Consumers**

We suppose that consumers defined by two parameters, \( x \) and \( \theta \), are uniformly distributed over the square \([0,1] \times [\theta, \bar{\theta}]\).

*First*, \( x \) is interpreted as its most preferred variety corresponding to its preferred taste in the case of nutritional product. This parameter is assumed to lie on the interval \([0,1]\). A consumer located at the point \( x \) on the horizontal line must pay a quadratic cost (or “transport cost”) such as \((x - y_i)^2\) due to the distance from the variety of the product supplied \( y_i \) to his taste type \( x \), which can also be considered as its disutility for not having its favorite product. Therefore, the utility given by the purchase of a product supplied by the firm \( i \) is: \(-(x - y_i)^2\), which means that consumer prefers to buy a product that is located near to its "home" (its taste) in order to reduce the “transport cost”. Its utility is maximal when the consumer is located in the same place as the firm.

*Second*, \( \theta \) denotes the relative intensity of preference of each consumer for quality, and it lies in the interval \([\theta, \bar{\theta}]\). It implies that increases in the level of quality of good \( s \) are valued equally by consumers of the same intensity of desire for quality \( \theta \), irrespective of their varietal preference. The higher \( \theta \), the more consumer gives weight to the quality. We may consider well informed, rich, or diseased persons as consumers of the type high \( \theta \) and young people or low-income class as consumers of low \( \theta \).

A consumer \( \theta \) on this dimension who buys a product \( i \) having the quality \( s_i \) will obtain a utility increasing in \( \theta \) and \( s \), which means that the higher \( \theta \), the higher satisfaction given by the quality \( s \). \( \theta \) may also represent its willingness to pay for the quality \( s_i \).

Moreover, suppose that consumers obtain a unit utility \( u \) when they buy a product \( i \) regardless of its variety and quality. \( u \) is assumed to be high enough to ensure that all consumers choose a product. Therefore, a consumer of type \((x, \theta)\) derives the following utility from purchasing a product of variety \( y_i \) and quality \( s_i \) supplied by the firm \( i \) at the price \( p_i \):
We can observe here the difference between two types of preferences: the first term of the above expression \( \theta s_i \) represents a ranking of qualities that is identical to all consumers. Indeed, if \( s_1 > s_2 \), we have \( \theta s_1 > \theta s_2 \), which means that at the same price, all consumers prefer the quality \( s_1 \) to the quality \( s_2 \). Meanwhile, the identical point of view of consumer is introduced by the second term \((x-y_i)^2\): Two firms located in the abscissa at positions \( y_1 \) and \( y_2 \) will not be classified in the same way by all consumers.

**Consumers’ demands**

Given the above assumptions, we can derive the set of consumers who are just indifferent between purchasing the product supplied by the firm 1 and the one by the firm 2 as following.

The consumer who buys the product of the firm 1 has a utility given by:

\[
U(s_i, y_i, x, \theta) = u - p_i + \theta s_i - (x - y_i)^2
\]

Its utility when it buys the product of the firm 2 is given by:

\[
U(s_2, y_2, x, \theta) = u + \theta s_2 - (x - y_2)^2 - p_2
\]

Hence, the set of indifferent consumers is located in a position such that

\[
\frac{\theta}{\theta - \theta} = \frac{\theta}{\theta - \theta} = \frac{\theta}{\theta - \theta}
\]

\[*\]

Posing \( \Delta s = s_1 - s_2 \); \( \Delta y = y_2 - y_1 \); \( \Delta y^2 = y_2^2 - y_1^2 \) et \( \Delta p = p_1 - p_2 \), the equation (1) becomes:

\[
\theta^*(x) = \frac{2x\Delta y - \Delta y^2 + \Delta p}{\Delta s}
\]

\( \theta^* \) is thus a linear function decreasing in \( x \), which means that consumers who appreciate quality prefer buy the products supplied by the firm 2.

In this light, the consumers will buy the products supplied by the firm 1 when:

\[
U(s_1, y_1, x, \theta) > U(s_2, y_2, x, \theta)
\]

i.e.

\[
\theta > \theta^*(x) = \frac{2x\Delta y - \Delta y^2 + \Delta p}{\Delta s}
\]

On the contrary, they will buy the products supplied by the firm 2 when:

\[
U(s_1, y_1, x, \theta) < U(s_2, y_2, x, \theta)
\]

i.e.

\[
\theta < \theta^*(x) = \frac{2x\Delta y - \Delta y^2 + \Delta p}{\Delta s}
\]

The product’s aggregate demand can be described as the measure of a subset of \([0,1] x [\theta, \bar{\theta}]\). It is calculated as following:
**Case 1:** The $\theta^*(x)$-line crosses the upper side and the lower side of the square: $\theta^*(0) \leq \bar{\theta}$ and $\theta^*(1) \geq \bar{\theta}$.

We have the following conditions:

\[
\begin{align*}
\theta^*(0) &= \frac{2.0 \Delta y - \Delta y^2 + \Delta p}{\Delta s} \leq \bar{\theta} \\
\theta^*(1) &= \frac{2.1 \Delta y - \Delta y^2 + \Delta p}{\Delta s} \geq \bar{\theta} \\
\Rightarrow \Delta y^2 - 2\Delta y + \bar{\theta}\Delta s &\leq \Delta p \leq \Delta y^2 + \bar{\theta}\Delta s
\end{align*}
\]

Hence, this case is possible only if $(\bar{\theta} - \theta)\Delta s \leq 2\Delta y$

Consumers in the trapeze $[\theta, \bar{\theta}, 0, x_0, x_1]$ decide to buy the product offered by the firm 1 while consumers belonging to the trapeze $[x_i, x_1, 1, \theta, \bar{\theta}]$ choose the one offered by the firm 2.

![Fig 1: $\theta^*(0) \leq \theta$ et $\theta^*(1) \geq \bar{\theta}$](image)

The demand of the firms 1 and 2 is given by:

\[
\begin{align*}
D_2 &= \int_{x_0}^{x_1} \theta^*(x) dx + (\bar{\theta} - \theta)(1 - x_1) \\
D_1 &= (\bar{\theta} - \theta) - D_2
\end{align*}
\]

Where

\[
\begin{align*}
\theta^*(x_0) &= \frac{2x_0 \Delta y - \Delta y^2 + \Delta p}{\Delta s} \Rightarrow 2x_0 \Delta y - \Delta y^2 + \Delta p = \bar{\theta} \Rightarrow x_0 = \frac{\bar{\theta}\Delta s + \Delta y^2 - \Delta p}{2\Delta y} \\
\theta^*(x_0) &= \bar{\theta}
\end{align*}
\]
\[
\begin{align*}
\theta^*(x) &= \frac{2x\Delta y - \Delta y^2 + \Delta p}{\Delta s} \\
\theta^*(x_i) &= \overline{\theta}
\end{align*}
\]

Thus, we have:

\[
D_2 = (\overline{\theta} - \theta) \frac{4\Delta y - (\overline{\theta} + \theta)\Delta s - 2\Delta y^2 + 2\Delta p}{4\Delta y}
\]

and

\[
D_1 = \overline{\theta} - \theta - D_2 = (\overline{\theta} - \theta) \frac{(\overline{\theta} + \theta)\Delta s + 2\Delta y^2 - 2\Delta p}{4\Delta y}
\]

**Case 2:** The \( \theta^*(x) \)-line crosses the left side and the right side of the square: \( \theta^*(0) > \underline{\theta} \) and \( \theta^*(1) < \overline{\theta} \).

We have the following conditions:

\[
\begin{align*}
\theta(0)^* &= \frac{2.0\Delta y - \Delta y^2 + \Delta p}{\Delta s} > \theta \\
\theta(1)^* &= \frac{2.1\Delta y - \Delta y^2 + \Delta p}{\Delta s} < \overline{\theta}
\end{align*}
\]

\[
\Rightarrow \Delta y^2 + \theta\Delta s \leq \Delta p \leq \Delta y^2 - 2\Delta y + \overline{\theta}\Delta s
\]

Hence, this case is possible only if \( (\overline{\theta} - \theta)\Delta s \geq 2\Delta y \)

Consumers in the trapeze \([\overline{\theta}, \theta^*(0), \theta^*(1), 1]\) decide to buy the product offered by the firm 1 while consumers belonging to the trapeze \([\theta^*(0), \underline{\theta}, 1, \theta^*(1)]\) choose the one offered by the firm 2.

![Diagram](image)

**Fig 2:** \( \theta^*(0) > \underline{\theta} \) and \( \theta^*(1) < \overline{\theta} \)

The demand of the firms 1 and 2 is given by:
\[
\begin{aligned}
D_2 &= \int_0^1 \theta^*(x)dx \\
D_1 &= 1 - D_2 
\end{aligned}
\]

Where

\[
\begin{aligned}
\theta^*(0) &= \frac{-\Delta y^2 + \Delta p}{\Delta s} \\
\theta^*(1) &= \frac{2\Delta y - \Delta y^2 + \Delta p}{\Delta s}
\end{aligned}
\]

Thus, we have:

\[
D_2 = \int_0^1 \theta^*(x)dx = \frac{\Delta y - \Delta y^2 + \Delta p}{\Delta s} - \theta
\]

And \( D_1 = \bar{\theta} - \theta - D_2 = \frac{\Delta s - \Delta y + \Delta y^2 - \Delta p}{\Delta s} + \bar{\theta} \)

**Case 3**: The \( \theta^*(x) \)- line crosses the left hand side and the upper side of the square: \( \theta^*(0) > \theta \) and \( \theta^*(1) > \bar{\theta} \).

We have the following conditions:

\[
\begin{aligned}
\theta^*(0) &= \frac{2.0 \Delta y - \Delta y^2 + \Delta p}{\Delta s} > \theta \\
\theta^*(1) &= \frac{2.1 \Delta y - \Delta y^2 + \Delta p}{\Delta s} > \bar{\theta} \\
\Rightarrow \Delta p &> \Delta y^2 + \text{Max} \left\{ \frac{\theta s, \bar{\theta} s}{\Delta y} - 2 \Delta y \right\}
\end{aligned}
\]

Moreover, we must have \( \theta^*(0) \leq \bar{\theta} \) in order to have \( D_1 \) non negative. This condition is thus given by:

\[
\theta^*(0) = \frac{-\Delta y^2 + \Delta p}{\Delta s} \leq \bar{\theta} \Rightarrow \Delta p \leq \bar{\theta} \Delta s + \Delta y^2
\]

Hence, the case 3 is possible only if \( \Delta y^2 + \text{Max} \left\{ \frac{\theta s, \bar{\theta} s}{\Delta y} - 2 \Delta y \right\} < \Delta p \leq \bar{\theta} \Delta s + \Delta y^2 \)

Consumers in the rectangle \([\bar{\theta}, \theta^*(0), x_i]\) decide to buy the product offered by the firm 1 while consumers belonging to the pentagon \([\theta^*(0), \bar{\theta}, 0, 1, x_i]\) choose the one offered by the firm 2.
The demand of the firms 1 and 2 is given by:

\[
\begin{align*}
D_2 &= \int_{0}^{\theta_2} \theta^*(x)dx + (1 - x) \\
D_1 &= 1 - D_2 
\end{align*}
\]

Where

\[
\begin{align*}
\theta^*(0) &= \frac{-\Delta y^2 + \Delta p}{\Delta s} \\
x_i &= \frac{\bar{d}s + \Delta y^2 - \Delta p}{2\Delta y}
\end{align*}
\]

Thus, we have:

\[
D_2 = \int_{0}^{\theta_2} \theta^*(x)dx + (1 - x_f) = \bar{\theta} - \theta - \left(\frac{\bar{d}s + \Delta y^2 - \Delta p}{4\Delta y\Delta s}\right)^2 \\
\text{And } D_1 &= \frac{(\bar{d}s + \Delta y^2 - \Delta p)(\bar{d}s + \Delta y^2 - \Delta p)}{4\Delta y\Delta s} = \frac{(\bar{d}s + \Delta y^2 - \Delta p)^2}{4\Delta y\Delta s}
\]

**Case 4:** The $\theta^*(x)$- line crosses the lower side and the right side of the square: $\theta^*(0) < \bar{\theta}$ and $\theta^*(1) > \bar{\theta}$.

We have the following conditions:

\[
\begin{align*}
\theta^*(0) &= \frac{2.0.\Delta y - \Delta y^2 + \Delta p}{\Delta s} < \bar{\theta} \\
\theta^*(1) &= \frac{2.1.\Delta y - \Delta y^2 + \Delta p}{\Delta s} < \bar{\theta} \\
\Rightarrow \Delta p &< \Delta y^2 + Min\{\bar{d}s, \bar{d}s - 2\Delta y\}
\end{align*}
\]
Moreover, we must have $\theta^*(1) \geq \bar{\theta}$ in order to have $D_2$ non negative. This condition is thus given by:

$$\theta^*(1) = \frac{2\Delta y - \Delta y^2 + \Delta p}{\Delta s} \geq \bar{\theta} \Rightarrow \Delta p \geq \Delta y^2 - 2\Delta y + \theta \Delta s$$

Hence, the case 4 is possible only if $\Delta y^2 - 2\Delta y + \theta \Delta s \leq \Delta p < \Delta y^2 + Min\{\theta \Delta s, \bar{\theta} \Delta s - 2\Delta y\}$

Consumers in the pentagon $\left[\bar{\theta}, \bar{\theta}, 0, x_0, \theta^*(1)\right]$ decide to buy the product offered by the firm 1 while consumers belonging to the rectangle $[x_0, 1, \theta^*(1)]$ choose the one offered by the firm 2.

**Fig 4:** $\theta^*(0) < \bar{\theta}$ and $\theta^*(1) > \bar{\theta}$

The demand of the firms 1 and 2 is given by:

$$\begin{align*}
D_2 &= \int_{x_0}^{1} \theta^*(x)dx \\
D_1 &= 1 - D_2
\end{align*}$$

Where

$$\begin{align*}
x_0 &= \frac{\theta \Delta s + \Delta y^2 - \Delta p}{2\Delta y} \\
\theta^*(1) &= \frac{2\Delta y - \Delta y^2 + \Delta p}{\Delta s}
\end{align*}$$

Thus, we have:

$$D_2 = \int_{x_0}^{1} \theta^*(x)dx = \frac{(2\Delta y - \Delta y^2 + \Delta p - \theta \Delta s)^2}{4\Delta y \Delta s}$$

And $D_1 = \bar{\theta} - \bar{\theta} - D_2 = \bar{\theta} - \bar{\theta} - \frac{(2\Delta y - \Delta y^2 + \Delta p - \theta \Delta s)^2}{4\Delta y \Delta s}$
Public health surplus:
Assume that the public health surplus is increasing in $s_i$ and $D_i$, its function is given by:

$$ PHS = \sum s_i D_i $$

The higher the quality offered by firms and the higher public health surplus

Profit of the firms
Taking into the cost of production and the demands on the market, we can calculate the profits of the firms as following: $\Pi_i = (p_i - C_i)D_i$.

The model proves analytically that the demand of the firm 2 is a continuous function in $p_2$. It is convex if $p_2^a \leq p_2 \leq p_2^b$, linear (decreasing) if $p_2^b < p_2 \leq p_2^c$, and concave if $p_2^c < p_2 \leq p_2^d$.

Regarding the profit of this firm, it is continuous too in $p_2$.

2. The Game
To identify the interactions between stakeholder decisions and the impact on the final market, we consider the following two-stage game:

Stage 1: The firms choose the location (variety) and quality of their products sold to the consumer on the final market.

Stage 2: The firms choose the price of the products they wants to sell on the final market by anticipating the consumers’ demand.

The game is solved by backward induction so as to achieve perfect Nash equilibriums in sub-games.

At the stage 2: the firms chooses the price to sell their products by anticipating demand on the final market. This must maximize their profit. It is therefore obtained by:

$$ p_i^* (s_i, y_i) = \arg \max_{p_i} \Pi_i $$

The resolution of the conditions of the first-order system ($\frac{\partial \Pi_i}{\partial p_i} = 0$) represents the sole sub-game equilibrium.

At the stage 1: After determining the products’ selling price, firms choose their variety and quality. These must maximize their profit. These are therefore obtained by:

$$ y_i^*, s_i^* = \arg \max_{y_i, s_i} \Pi_i $$

The resolution of the conditions of the first-order system ($\frac{\partial \Pi_i}{\partial y_i} = 0$ and $\frac{\partial \Pi_i}{\partial s_i} = 0$) represents the sole sub-game equilibrium.
3. Equilibrium

3.1 Equilibrium Prices

**Proposition 1:** The results highlight that there exists the price equilibrium for any location pair \((y_1, s_1)\) and \((y_2, s_2)\).

**3.1.1. \((\bar{\theta} - \bar{q})\Delta s < 2\Delta y\)**

The prices equilibriums are characterized by the following system:

\[
\frac{\partial \Pi_1}{\partial p_1} = 0 \quad \text{and} \quad \frac{\partial \Pi_2}{\partial p_2} = 0.
\]

These are also firms’ reaction functions.

\[
p_a^i = p_i - \bar{\theta}\Delta s - \Delta y^2
\]

\[
p_b^i = p_i - \Delta y^2 - \bar{\theta}\Delta s
\]

\[
p_c^i = p_i - \Delta y^2 - \bar{\theta}\Delta s + 2\Delta y
\]

\[
p_d^i = p_i - \Delta y^2 + 2\Delta y - \bar{\theta}\Delta s
\]

✓ If \(p_a^i \leq p_2 \leq p_b^i\) \textit{D1 is convex and D2 is concave}

We derive the equilibrium in prices:

\[
p_1^* = \frac{1}{8} \left[ \Delta y^2 + 7C_1 + C_2 + \bar{\theta}\Delta s + A \right]
\]

\[
p_2^* = \frac{1}{8} \left[ -5\Delta y^2 + 5C_1 + 3C_2 - 5\bar{\theta}\Delta s + 3A \right]
\]

Where \(A = \sqrt{16(\bar{\theta} - \bar{q})\Delta s\Delta y + (\Delta y^2 - C_1 + C_2 + \bar{\theta}\Delta s)^2}\).

The condition is given by:

\(p_a^i \leq p_2 \leq p_b^i\) \quad i.e. \quad -\Delta y^2 + C_1 - C_2 - \bar{\theta}\Delta s \leq A \leq -\Delta y^2 + C_1 - C_2 + 3\bar{\theta}\Delta s - 4\bar{\theta}\Delta s

The demands and profit function are given by:

\[
D_1^* = \frac{(\Delta y^2 - C_1 + C_2 + \bar{\theta}\Delta s + A)^2}{64\Delta s\Delta y}
\]

\[
D_2^* = (\bar{\theta} - \bar{q}) - D_1 = (\bar{\theta} - \bar{q}) - \frac{(\Delta y^2 - C_1 + C_2 + \bar{\theta}\Delta s + A)^2}{64\Delta s\Delta y}
\]
\[
\Pi_1^* = \frac{(\Delta y^2 - C_1 + C_2 + \bar{\theta} \Delta s + A)^3}{512 \Delta s \Delta y}
\]

\[
\Pi_2^* = \frac{1}{8} \left[-5 \Delta y^2 + 5 C_1 - 5 C_2 - 5 \bar{\theta} \Delta s + 3 A \right] \times \left[ (\bar{\theta} - \theta) - \frac{(\Delta y^2 - C_1 + C_2 + \bar{\theta} \Delta s + A)^2}{64 \Delta s \Delta y} \right]
\]

✓ If \( p^b_2 < p_2 \leq p^c_2 \) : \textbf{D1 and D2 are linear}

The price equilibrium is given by:

\[
p_1^* = \frac{1}{6} \left[ 2 \Delta y^2 + 4 C_1 + 2 C_2 + 4(\bar{\theta} - \theta) \Delta y + (\bar{\theta} + \theta) \Delta s \right]
\]

\[
p_2^* = \frac{1}{6} \left[ -2 \Delta y^2 + 2 C_1 + 4 C_2 + 8(\bar{\theta} - \theta) \Delta y - (\bar{\theta} + \theta) \Delta s \right]
\]

The conditions is given by:

\[
p^b_2 < p_2 \leq p^c_2
\]

\textit{i.e.} \( \Delta y^2 - 6 \Delta y + 2(\bar{\theta} - \theta) \Delta y + 2 \bar{\theta} \Delta s - \theta \Delta s < C_1 - C_2 \leq \Delta y^2 + 2(\bar{\theta} - \theta) \Delta y - \bar{\theta} \Delta s + 2 \theta \Delta s \)

The demand and profit functions are:

✓ \( D_1^* = \frac{2 \Delta y^2 - 2 C_1 + 2 C_2 + 4(\bar{\theta} - \theta) \Delta y + (\bar{\theta} + \theta) \Delta s}{12 \Delta y} \)

✓ \( D_2^* = -\frac{2 \Delta y^2 - 2 C_1 + 2 C_2 - 8(\bar{\theta} - \theta) \Delta y + (\bar{\theta} + \theta) \Delta s}{12 \Delta y} \)

✓ \( \Pi_1^* = \frac{\left[ 2 \Delta y^2 - 2 C_1 + 2 C_2 + 4(\bar{\theta} - \theta) \Delta y + (\bar{\theta} + \theta) \Delta s \right]^2}{72 \Delta y} \)

✓ \( \Pi_2^* = \frac{\left[ -2 \Delta y^2 + 2 C_1 - 2 C_2 + 8(\bar{\theta} - \theta) \Delta y - (\bar{\theta} + \theta) \Delta s \right]^2}{72 \Delta y} \)

✓ If \( p^b_2 < p_2 \leq p^d_2 \) : \textbf{D1 is concave and D2 is convex}

The price equilibrium is:

\[
p_1^* = \frac{1}{8} \left[ 5 \Delta y^2 + 3 C_1 + 5 C_2 - 10 \Delta y + 5 \bar{\theta} \Delta s + 3 B \right]
\]

\[
p_2^* = \frac{1}{8} \left[ -\Delta y^2 + C_1 + 7 C_2 + 2 \Delta y - \theta \Delta s + B \right]
\]
where \( B = \sqrt{16(\bar{\theta} - \theta)\Delta s\Delta y + (\Delta y^2 - C_1 + C_2 - 2\Delta y + \Delta s)^2} \)

The conditions is given by:

\[ p_1^e < p_2 \leq p_2^d \]

i.e. \( \Delta y^2 - C_1 + C_2 - 2\Delta y + \Delta s < B \leq \Delta y^2 - C_1 + C_2 - 2\Delta y - 3\Delta s + 4\Delta s \)

The demand and profit functions are:

\[ D_1^* = \bar{\theta} - \theta - \left( \frac{(\Delta y^2 - C_1 + C_2 - 2\Delta y + \Delta s - B)^2}{64\Delta s\Delta y} \right) \]

\[ D_2^* = \left( \frac{(\Delta y^2 - C_1 + C_2 - 2\Delta y + \Delta s - B)^2}{64\Delta s\Delta y} \right) \]

\[ \Pi_1^* = \frac{1}{8} \left[ 5\Delta y^2 - 5C_1 + 5C_2 - 10\Delta y + 5\Delta s + 3B \right] \left[ \bar{\theta} - \theta - \left( \frac{(\Delta y^2 - C_1 + C_2 - 2\Delta y + \Delta s - B)^2}{64\Delta s\Delta y} \right) \right] \]

\[ \Pi_2^* = \left( \frac{(-\Delta y^2 + C_1 - C_2 + 2\Delta y - \Delta s + B)^3}{512\Delta s\Delta y} \right) \]

The price equilibrium will be found in the same manner for the case \((\bar{\theta} - \theta)\Delta s > 2\Delta y\)

3.2. Variety and quality equilibrium

**Proposition 2:** With the cost function \( C_i = \lambda(1+s_i)(1+y_i) \); \( \lambda > 0 \), for all \( s \) and \( \bar{s} \in (0,1) \), there exists \( \lambda(\bar{s}, \bar{s}, \bar{\theta}, \bar{\theta}) > 0 \) and \( \bar{\lambda}(\bar{s}, \bar{s}, \bar{\theta}, \bar{\theta}) > 0 \) such as

- If \( \lambda < \lambda(\bar{s}, \bar{s}, \bar{\theta}, \bar{\theta}) \) a perfect duopoly equilibrium will be given by: firm 1 \((y_1^*, s_1^*) = (0, \bar{s})\); Firm 2 \((y_2^*, s_2^*) = (1, \bar{s})\)
- If \( \lambda(\bar{s}, \bar{s}, \bar{\theta}, \bar{\theta}) \leq \lambda \leq \bar{\lambda}(\bar{s}, \bar{s}, \bar{\theta}, \bar{\theta}) \) a perfect duopoly equilibrium will be given by: firm 1 \((y_1^*, s_1^*) = (0, \bar{s})\); Firm 2 \((y_2^*, s_2^*) = (1, s)\)
- And if \( \lambda > \bar{\lambda}(\bar{s}, \bar{s}, \bar{\theta}, \bar{\theta}) \) a perfect duopoly equilibrium will be given by: firm 1 \((y_1^*, s_1^*) = (0, s)\); Firm 2 \((y_2^*, s_2^*) = (1, s)\)
Proof:

- When \((y_1^*, s_1^*) = (0, \bar{s})\), \(\frac{\partial \Pi_2}{\partial s_2} > 0 \forall s_2 \) if \(\lambda < \bar{\lambda}(\bar{s}, \bar{s}, \bar{\theta}, \bar{\theta})\)

\[ \text{and } \frac{\partial \Pi_2}{\partial s_2} < 0 \forall s_2 \) if \(\bar{\lambda}(\bar{s}, \bar{s}, \bar{\theta}, \bar{\theta}) \leq \lambda \leq \bar{\lambda}(\bar{s}, \bar{s}, \bar{\theta}, \bar{\theta})\)

and vice versa.

- When \((y_1^*, s_1^*) = (0, \bar{s})\), \(\frac{\partial \Pi_2}{\partial s_2} < 0 \forall s_2 \) if \(\lambda > \bar{\lambda}(\bar{s}, \bar{s}, \bar{\theta}, \bar{\theta})\)

and vice versa.

The optimal position of the firms is shown in the following figure.

![Diagram of firms' optimal position](image)

**Fig 5:** Firms' optimal position

When \(\lambda\) is low, the optimal location of firms is \(s_1=1 \& s_2=1\) and when \(\lambda\) is high, this one is \(s_1=s_2=0\).

**Corollary 1:** There exists the maximum differentiation equilibrium on both of 2 dimensions.

The locations of the firms 1 and 2 in the interval \([\lambda_1, \lambda_2]\) correspond to the strategy of maximum differentiation in both variety and quality: for example, if the one decides to offer the standard product ie product with a "traditional" taste (close to the consumer’s one) but having a low quality (for example, products that are high in salt, added sugar or fat) in order to save its production cost, the other will tend to choose the supply of a new variety and a high nutritional quality. For example, firm 1 supplies the traditional yoghurt, firm 2 the calcium-rich and low-fat product in different fruit flavours, etc.

Nevertheless, in the absence of public intervention, the simulations highlight that in this same interval, the firm 1 prefers \(s_1=1 \; ; \; s_2=0\) while the firm 2 prefers \(s_1=0 \; ; \; s_2=1\), which may induce a
conflict of interest between firms. By consequent, we risk having the worst case scenario: \( s_1 = s_2 = 0 \) (or the best one: \( s_1 = s_2 = 1 \)), which is an out-of-equilibrium.

In short, this result means that in the absence of public intervention the firms’ spontaneous choice may lead to the worst quality level and then the lowest optimal social welfare. Hence, it may appear that the public intervention is necessary.

**Corollary 2:** Public health surplus depends on the segment size.

The PHS depend on the segment size, which is characterized by \( \Delta \theta \). In fact, if the latter is small, firms tend to offer their best quality because of the intensity of the concurrency. If it is medium, only one firm supplies the product of good quality. And if it is big, both of firm prefers the worst quality.

For example, the size of the fast food Mac Donald segment is big, the quality offered is very bad: extremely fat products… Meanwhile, the size of the new fast food pasta segment in France is small, the quality supplied is clearly better: products with less fat Industrial, etc.

**II. Section 2: Model where there exists a relationship between the taste and the nutritional quality.**

The framework in the section 1 is retained in this section but we suppose that there exists a relationship between the taste and the nutritional quality of the product: An increase of the firms’ quality may have bad impacts on the consumer’s taste (light product with less sweet taste for example…).

To simplify we assume that the location of the firm 2 is fixed: \( y_2^* = 1; s_2^* = 0 \), which means that the firm 2 offers a bad quality of food. The firm 1 may thus make a trade-off of offering between a product of good quality that serves consumers who are sensitive to the quality and don’t care so much about the taste and a product of low quality that satisfies the taste of more consumers.
If the firm 1 increases its quality, its consumer must pay a higher transport cost (for its taste) than the initial situation where there is no relationship between the taste and quality of food. Suppose that this extra-cost is given by:

\[ e(m, s_1) = ms_1 \]

\[ s_1 = 0 \rightarrow e(0) = 0 \]
\[ s_1 = 1 \rightarrow e(1) = m \]

where \( m \) is the state of innovation of the firm 1.

\( e \) is increasing in \( s_1 \) et \( m \) : the higher \( s_1 \) et \( m \), the higher \( e \).

A firm 1’s consumer located at the point \( x \) on the horizontal line must pay a “transport cost” such as

\[ (x - y_1 - ms_1)^2 \]

The new utility function is given by thus:

\[ U(s_i, y_i, x, \theta) = u - p_i + \theta s_i - (x - y_i - ms_i)^2 \]

The following figure describes the new firms’ demand:

\[ \text{Fig : New demand of firms} \]

To control the loss of consumers, firm 1 sets up an investment in the manner that the higher investment and the lower the loss of consumers.

The function of this investment is given by:

\[ I(m, s_1) = a(1-m)^2s_1 \]

The firms’ profit is given by:

\[ \Pi_1 = (p_1 - C1)^* D1 - I1 \]
\[ \Pi_2 = (p_2 - C2)^* D2 \]
**Proposition 3:** The firm 1 prefers an average effort of innovation

**Proof:**

There exists an \( m(s1) \) such as 

\[
\frac{\partial \Pi}{\partial s_1} = 0 \quad \forall \ s_1
\]

The following figure describes the optimal level of effort of innovation of the firm 1

![Graph showing the optimal level of effort of innovation](image)

**Fig:** The optimal level of effort of innovation of the firm 1

In fact, when the level of effort of innovation of the firm 1 is low, its demand is low too. By consequent, its profit decreases even if a low level of effort induces a low cost of investment. On the contrary, when the effort is high, its demand may be high too but its investment is also too high, which induce a loss of profit. An average effort is the optimal choice of the firm 1.

The result of this section emphasizes also that the higher quality \( s_1 \) and the higher the optimal level of effort of innovation, which means that in the case of an imposition of a quality \( s^* \) minimum by the public authorities, the higher \( s^* \) and the lower effort. The question arises is thus how the public authorities should intervenes to increase this latter. We try to respond à this question in our future paper where the firm that sets up an effort of innovation in order to offer a product of good quality will obtain a subvention while the firm that offers a bad quality will be taxed.
Conclusion

The aim of our paper is to determine the conditions under which firms tend to offer the best nutritional quality of food products, and the public regulation required to obtain this in a context where diet and nutritional status plays an important part in maintaining health and preventing disease, and with increasing pressure for public intervention on food quality in developed countries. To this end, we develop a duopoly model where products can be horizontally (variety) and vertically (quality) differentiated. We analyze the perfect Nash equilibriums in a two period competition game where in the first stage, the firms decide simultaneously on the variety and the quality of the product to be sold and in the second stage, firms set prices. The model examines the role played by three types of public intervention: minimum quality threshold, campaign information, and taxation. We identify the effects of this regulation on the strategy of firms and shows how certain regulations that may seem conducive to the supply of products of high nutritional quality may have adverse effects when the strategic reaction of firms is considered. The model firstly highlights that in the absence of the public intervention, the spontaneous choice of the firms will not lead to the desired level of nutritional quality. Hence, it may appear that the imposition of a minimum quality threshold is necessary. Secondly, without assistance from authorities, firm that engages in the “upmarket” may have some difficulties such as the loss of customers due to a high price associated with high quality, a loss of traditional taste (for example, products containing less sugar), or a high cost of innovation (to induce the loss of customers) which casts doubt on the wisdom of taking such an approach. To correct this, the subvention-tax policies are suggested.

The methodological contribution of this work is important because there does not exist, to the best of our knowledge, a theoretical model which analyses the competitive offer and industrial organization of nutrition firms endogenously. This relative scarcity of theoretical approaches is to be contrasted with the statistical and econometric work on nutrition products.
References