Approximating optimal numerical solutions to bio-economic systems: How useful is simulation-optimization?

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Abstract

For applications in agricultural economics, complex ecological systems are often oversimplified to the extent that ecologists rarely consider model results valid. Recursive optimization of complex systems represents an alternative, but requires strong assumptions regarding time preference and uncertainty.
In this paper we explore the implications of merely approximating "true" optima of complex dynamic optimization problems using a technique called simulation-optimization. We develop a standard discrete renewable resource use problem and solve it numerically using both simulation-optimization and non-linear mathematical programming. We subsequently introduce non-linearity and uncertainty and graphically compare the performance of simulation-optimization vis-à-vis non-linear programming in predicting optimal control and state variable paths. On the basis of this comparison we discuss potential non-formal test procedures that could be used to assess simulation-optimization solutions of more complex problems that do not allow for such comparisons.
We find that simulation-optimization can be a useful exploratory optimization technique when standard numerical optimization approaches fail to find near optimal solutions. That said, modelers should be careful in designing management functions of simulation-optimization problems and test their functional forms for severe misspecifications.

1 Introduction

Finding optimal solutions to complex bio-economic systems is a key challenge for the quantitative analysis of natural resource policy and management problems in interdisciplinary agricultural research. Despite increasing interdisciplinarity in research design, applied numerical modelling approaches generally remain within disciplinary domains (Russell 1995, Vandermeulen and Van Huylenbroeck in press). This holds especially for the analysis of land use and vegetation change dynamics, such as the dynamic interaction of trees, grasses, and grazing animals in rangeland production systems. Resource economists favor economic optimization approaches that overly simplify bio-physical relationships - usually unacceptable to ecologists - whereas ecologists develop complex dynamic process simulation models that pose considerable challenges to standard optimization procedures or require assumptions that violate important aspects of economic theory (see Börner et al. (2007) for a discussion). This contributes to researchers favoring disciplinary modelling over integrating economic and ecological modelling systems. This paper illustrates a modelling approach that links optimization with simulation and discusses its potential to integrate across disciplines.

Optimization of dynamic systems usually requires solving systems of differential equations. Depending on the complexity of the system under study, an analytical optimal solution to the underlying system of differential equations may not exist. Analytical solutions to relatively simple and general problems provide important insights into biological and economic system dynamics. In the quest for analytical solutions to dynamic optimization problems, however, most analysts end up using numerical optimization techniques to approximate optimal solutions to real world problems that involve multiple state and control variables (Woodward 2000). For this purpose, dynamic problems are often reformulated as a constrained linear or non-linear optimization problem and optimized over the control variable(s) (Standiford and Howitt 1992).

Despite the rapid evolution in computing capacity and numerical optimization techniques, this procedure is and probably will long be limited by the 'curse of dimensionality' (Bellman 1961). As models become larger, computing time increases exponentially with the size of the planning horizon. Another important drawback of standard numerical optimization, if compared to the underlying analytical solution, is that it merely provides us with the optimal paths of state and control variables. This is useful information if the objective is to see whether theses paths are in accordance with specific policy targets or to test how alternative policies might influence these paths. Yet, it does not provide resource managers with rules of action, as the analytical solution would do (if it existed), by expressing the control variable in terms of time (i.e. optimal control), state variables (i.e. feedback control), or both (i.e. closed-loop control).

Moreover, in the face of complex and stochastic dynamics, optimal paths of state and control variables loose some of their informational value as they can only be calculated for a given sequence of exogenous driving factors, e.g. prices and/or environmental factors. Under these circumstances it is indeed desirable to describe optimal behavior by a set of functional relationships that tell us how control variables should be optimally adjusted according to the stochastic dynamics of the system.

Our specific interest lies in understanding the interactions of economic activities, such as livestock grazing, with vegetation dynamics in semi-arid savanna systems. These systems are highly non-linear, subject to stochastic environmental factors, such as rainfall and fires, and exhibit multiple-stable attractors (Higgins et al. 2000, 2007). Models that adequately describe such systems are inherently complex and usually not solvable by analytical procedures (Scheiter and Higgins 2007). To adequately analyze the evolution of dynamic equilibria and limit cycle behavior, savanna models have to be run for extended time horizons, which makes its numerical optimization using constrained non-linear programming technically infeasible.

Previous savanna modelling studies and also applications in water reservoir management have developed a workaround to overcome some of the obstacles described above (Janssen et al. 2004, Koutsoyiannis and Economou 2003). The approach has been termed simulation-optimization in the operations research literature. Quite generally, simulation-optimization refers to the application of optimization algorithms to find parameter combinations that optimize a performance measure of a simulation model (Fu 2001). The performance measure can potentially be any measure of interest, such as fitness indices in ecological applications, utility in economic applications, or a measure of how well the model reflects reality, if the objective is to calibrate model parameters to a given set of observations.

In this paper we analyze a very simple problem of renewable resource extraction that resembles the characteristics of livestock production in savannas. The objective of the analysis is:

1. to assess the degree of achievable approximation to the underlying optimal control path,
2. to provide guidance for the optimal specification of dynamic simulation-optimization models in natural resource applications,
3. and to propose procedures to validate simulation-optimization solutions in the absence of real world observations.

In addressing these objectives, we hope to gain insights with regard to the usefulness of simulation-optimization as tool to analyze natural resource management issues that cannot be addressed using standard economic optimization approaches.

Section 2 of the paper presents the renewable resource extraction problem in an optimal control form and compares it with a potential simulation-optimization specification. Section 3 evaluates the goodness of fit of the simulation-optimization approximation to the optimal numerical solution of the problem using alternative specifications of the simulation-optimization model under deterministic and stochastic conditions. Section 4 suggests simple procedures to validate simulation-optimization solutions to the model in the absence of information about the true optimum. Finally, we discuss our findings in section 5 and provide concluding remarks in section 6.

2 Optimal Control and Simulation-Optimization

We start out from a continuous optimal control problem with the objective to:

$$\max_{z_t} \int_0^T ((p - \frac{wz_t}{x_t})z_t)e^{\rho t} + F(x_T)e^{\rho T} dt \tag{1}$$

subject to

$$\begin{aligned} \dot{x} &= x_t(1+r) - z_t \\ x_t &> 0 \\ z_t &\geq 0 \\ x_0 &= x(0) \\ F(x_T) &= given \end{aligned} \tag{2}$$

where z_t is the control variable (say animal stock), t is time with T being the the final time period of the interval $[0, T]$ with a given length n, p is the price for a fixed fraction of z_t sold in every time step, $\frac{wz_t}{x_t}$ denotes stock dependent unit costs of keeping animals, x_t is the resource (grass biomass), ρ is the discount rate, and $F(x_T)$ is the terminal value of the resource. Equation (2) is the equation of motion with r being the rate of resource growth.

An analytical, yet non-trivial, solution to the problem can be obtained by formulating the current value Hamiltonian (equation (3)) and the necessary conditions for a maximum:

$$H(z_t, x_t, \lambda_t, t) = ((p - \frac{wz_t}{x_t})z_t) + F(x_T) + \lambda_t(x_t(1+r) - z_t) \tag{3}$$

$$\frac{\partial H}{\partial z_t} = p - \frac{2wz_t}{x_t} - \lambda_t = 0 \tag{4}$$

$$\dot{\lambda} = \rho\lambda_t - \frac{\partial H}{\partial x_t} = \lambda_t(\rho - r - 1) - \frac{wz_t^2}{x_t^2} \tag{5}$$

and equation (2). From (4) we get:

$$\lambda_t = p - \frac{2wz_t}{x_t} \tag{6}$$

Setting the derivative of (6) with respect to time equal to (5) yields two equivalent expressions for $\dot{\lambda}$. Substituting for λ_t in (5) and solving for $\dot{z}$ results in:

$$\dot{z} = \frac{wz_t(2x_t(\rho - r - 1) + 2\dot{x} + wx_t) + x_t^2 p(1 + r - \rho)}{2x_t w} \tag{7}$$

With (2) and (7) we have a system of differential equations that describes the optimal trajectories of state and control variables given the initial and terminal conditions. As a result we would obtain $z_t^* = z(x_t, t)$, a definition of what the optimal animal stock would be at each time step and at given levels of grass biomass.

Alternatively the problem can be converted into a constrained non-linear programming problem (cf. Standiford and Howitt (1992)) and solved for the optimal trajectories of z_t and x_t using a numerical optimization algorithm. This would make sense if p and r become random parameters or if we include more than one state and control variables, turning an analytical solution into a more tedious or even impossible exercise. It also makes sense in in this paper, because we aim at comparing two approaches of finding optimal numerical solutions to control problems.

We arrive at a simulation-optimization formulation of the same problem through substitution of z_t in equations (1 and 2) by a function of t and x_t such that:

$$z_t^{*1} = a + bx_t - ct \tag{8}$$

where a is a constant and b and c are slope parameters to be determined by numerical optimization. We call equation (8) a management function. The management function is much simpler and more intuitive than the function $z(x_t, t)$ we would obtain going through an analytical procedure. But, we restrict z_t to be linearly increasing in x_t and t, which makes the optimal numerical solution of the simulation-optimization problem inferior to the numerical solution of the equivalent constrained non-linear programming solution. The management function, however, has an additional advantage: In order to maximize equation (1) numerically using constrained non-linear programming we need to define a vector $\mathbf{z}$ of length $n + 1$, i.e. the number of parameters over which to optimize. For most numerical optimization algorithms this implies that computation time may increase exponentially with n. Substituting z_t by the right-hand-side of equation (8) reduces the optimization problem to the parameters a, b, and c and makes it rather independent of n.

In short, converting an optimal control problem into a simulation-optimization problem means postulating *ex ante* the functional relationship between control and state variables and/or time and optimizing over the parameters of this function. In principle any functional form can be used as long as its domain includes feasible trajectories of the control variable. However, model outcomes will depend on how well the chosen management function specification can approximate the true optimal control path.

3 How good is Simulation-Optimization

Here we evaluate the performance of the simulation-optimization specification of the dynamic optimization model presented in the previous section *vis-a-vis* the standard non-linear programming version of the model. Three issues appear to be relevant in this regard: First, the choice of functional form for the management function. Second, solutions in which the control trajectory lies on its boundaries. And third, the number of system drivers that exhibit variability and, hence, influence the optimal trajectory of the control variable. The parameter values used in the simulations are summarized in Appendix A.

3.1 Alternative Functional Forms

To assess the performance of simulation-optimization and the implications of using different functional forms for the management functions, we perform three optimization runs and compare them side by side. We first run the model using the non-linear programming formulation, which serves as the benchmark for comparison. We then specify two different functional forms for the management functions, namely equation (8) and:

$$z^{*2} = a + \frac{x^b}{\alpha^b + x^b} + ct \tag{9}$$

where a, α, b, and c are the parameters over which optimization takes place.

Figure 1 shows the optimal trajectories of profits, state and control variables and compares the two management function specifications with the benchmark. Both specifications of the simulation-optimization model achieve very close approximations to the non-linear programming solutions (see also Table 1). Comparing control and profit trajectories reveals that the management functions introduce a small bias at the beginning and towards the end of the planning horizon. The simulation-optimization specification using management function (9) fits the problem slightly better as it allows for the control variable to react more flexibly to changes in resource availability.

3.2 Control at boundaries

Solutions in which the optimal control switches between different potential states, say z_{min}, z_{max}, and z_t^* are called bang-bang solutions (Sonneborn and Vleck 1965). Our optimal control model does not exhibit bang-bang solutions, because the unit costs of extraction are not constant. It is, however, possible to arrive at solutions in which $z_t = 0$ for several time steps along the optimal control trajectory. Such solutions represent the same type of challenge as bang-bang solutions for simulation-optimization models that involve economic decision-making.

Neither of the two management functions (8) and (9) can adequately deal with such a trajectory. The reason is that these functions map the control variable as a function of resource availability, which, under certain parameter combinations, violates the first condition of maximum principle (equation 4). Namely, marginal benefits of resource use in t must be equal to the opportunity costs of resource use.

Figure 2 shows that $z_t = 0$ is optimal during initial time periods if the planning horizon increases from 50 to 150 time intervals. I.e., opportunity costs of resource use in initial time periods are higher than the marginal benefits of using the resource. Management function (8) poorly approximates this solution. Yet, a much better fit can be achieved through inclusion of the marginal benefit per unit of resource use in the management function, e.g. by adding an additional term to (8) such that:

$$z_t^{*3} = (a + bx_t - ct)(1 - \frac{MB^\beta}{(\alpha^\beta + MB^\beta)}) \tag{10}$$

where

$$MB = p - \frac{2w}{x_t} * \delta^t \tag{11}$$

with δ^t being the discount factor of the discrete version of the model for numerical optimization. The non-linear term in (10) now serves as an approximation of a step function that multiplies the linear term by one if MB reaches a threshold value and by zero otherwise. The right panel in Figure 2 shows that this management function specification achieves a much better approximation of the true resource and control trajectories than the original form. Note that for reasons of comparability we only use an approximated switching function to allow for both model specifications to be solved by the same derivative based optimization algorithm. Usually this involves the definition of feasible starting values to make sure that the algorithm does not get stuck in local optima. More complex applications, such as in Higgins et al. 2007, require the use of evolutionary types of algorithms, which are less demanding in terms of functional properties and starting values.

In this example with a longer planning horizon the approximation of the constrained non-linear programming solution is, however, not as close as in section (3.1) even if parameters are chosen such that optimal resource use is always positive (see Table 2 for a comparison). This suggests that the performance of simulation-optimization models will generally decrease *ceteris paribus* if the planning horizon increases.

3.3 Multiple Sources of Variance

Until now we have only looked at cases in which both resource growth and prices were constant over time. In more realistic settings, prices vary due to market forces and resource growth varies according to environmental conditions.

We now change our model described by equations (1 and 2) such that $p \rightarrow p_t$ and $r \rightarrow r_t$. For our purposes it is irrelevant whether price and resource growth trajectories are known or stochastic, because we are merely interested in finding out to what extent variability in parameters affects the degree of approximation to the non-linear programming solution by simulation-optimization. In order to keep the comparison as simple as possible we assume known price and resource growth trajectories.

If both prices and resource growth exhibit variability we need to modify management function (8), because it would not allow to adjust the control variable trajectory to price changes.

The most simple way to introduce the price in (8) is to include an additional term such that:

$$z_t^{*4} = a + bx_t + cp_t - kt \tag{12}$$

As Figure (3) demonstrates, this management function specification does not achieve a particularly satisfying result. One option to improve the result is to introduce a non-linear interaction term to allow for the price-resource relationship to influence the control trajectory (see Figure 3 in the center), such that:

$$z_t^{*5} = a + bx_t + cp_t + qx_tp_t - kt \tag{13}$$

An even better and more elegant way is to make use of the insights in section (3.2) and specify the management function as

$$z_t^{*6} = a + bMB + qx_tp_t - kt \tag{14}$$

which allows for a quite satisfying degree of approximation in the present case (see Figure 3 right panel).

4 Management function validation

The previous section has demonstrated that simulation-optimization can find satisfying approximations to standard optimal control problems of renewable resources without knowing the true analytical solution of the problem. Yet, the actual purpose of this exercise was to introduce simulation-optimization as an instrument to analyze problems that are difficult or impossible to solve using analytical procedures or constrained non-linear programming. Since it is not possible to improve a complex simulation-optimization model based on a comparison with its non-linear programming solution, it is necessary to develop validation procedures to make sure that an adequate management function has been found. As the true optimal trajectory is not known, such tests will always involve subjective judgments with regard to the plausibility of results. The procedures described below, however, can be useful in detecting severe misspecifications.

4.1 Objective values and the variables of the management function

A simple heuristic procedure is to compare the optimized objective function values of alternative management function specifications. If alternative functional forms provoke large positive changes in the optimal objective function value, the chosen form is clearly sub-optimal.

Often highly dynamic bio-physical relationships are represented in a simulation model that is coupled with an optimization module (cf. Higgins et al. 2006, Jannssen et al. 2002). In such models it can be tempting to manipulate optimized control paths by replacing highly variable state or stochastic exogenous variables in the management function by more slowly reacting variables, the dynamics of which are linked to the original more volatile variable. As result optimized control paths become smoother, which may appear intuitive, but is merely the result of imposing additional restrictions on the control path. Restricting control paths in this way is likely to introduce serious biases into the model that are similar to, for example, omitting cp_t in equation (12) if the underlying model includes stochastic prices.

In short, the management function specification should allow for maximum flexibility of the control trajectory with respect to all sources of variance in model parameters and the related optimal state variable trajectory. If the resulting control path reflects unexpected behavioral outcomes it is the objective function that needs to be reexamined. For example, opportunistic grazing strategies in rangeland models (Westoby et al. 1989) may only appear optimal, because price and weather risk or heard size adjustment costs are not properly taken into account (Börner et al. 2007).

4.2 Random testing of control trajectory sections

A more systematic way of validating the management function is to test whether the optimized control trajectory of a simulation-optimization model exhibits a clear bias in defined sections. This can be done by converting the simulation-optimization model to a non-linear programming model in which the control path is bound by the simulation-optimization solution. Randomly selected points of the control path can now be freely determined by an optimization algorithm (Figure 4). If optimization leads to the points being shifted away from the original solution, the management function is likely misspecified. Using the management function (8) we demonstrate such a random test for the example in section (3.2).

Note, however, that the direction of the shift at a given point can be misleading if the management function is severely misspecified. For example, a shift at a given point on the control path may simply compensate for suboptimal controls in later time steps irregardless of where the true optimal control at this point is located.

A related method is sensitivity analysis. Selected or random points on the control path that was determined through simulation-optimization can be subjected to sensitivity analyses observing changes in the objective function value (3). Large changes suggest poor approximation to the true optimum. Moreover, several points on the control path can be simultaneously manipulated through stochastic simulation, which allows to establish correlations between control variable values and the objective value. Such correlations can provide useful insights as shown in Figure 5. A sensitivity analysis was run on all control variable instances of the simulation-optimization model using management function 12using Monte Carlo sampling from normal distributions with $\mu = z_t^{*2}$and $\sigma = 0.5$. Figure 5 depicts the resulting correlations with the model objective function value and the pattern suggests that the control path is suboptimal. I.e. lower levels of z_t^{*2}would yield higher objective function values in the first quarter of the planning horizon, whereas higher resource extraction values would do so in the second and third quarter.

4.3 Challenges to validation

Complex ecological-economic systems may exhibit fundamental changes in system states once state variables reach critical thresholds (see Higgins et al. 2006). Such highly dynamic systems represent challenges to numeric optimization. Simulation-optimization solutions to such problems require testing for the robustness of model solutions, because sub-optimality means that critical thresholds may be missed or passed due to management function misspecifications. Again, sensitivity analysis can be a useful tool to detect instances of control variables that are extraordinarily strongly correlated with changes in the objective function value or critical system state variables. If changes in such control variable instances produce large changes in system state variables and positive changes in the objective function value, management function misspecifications are a likely root cause.

5 Discussion

A major caveat of using simulation-optimization to address natural resource management problems that cannot be solved by standard numerical optimization techniques is the inevitable lack of means to prove model validity. Ideally, models should be validated using real world observations. In some cases, field observations may be used as a benchmark to evaluate whether the model adequately reproduces field observations. Often, however, no data exists or available information merely allows validation of ecological or economic sub-models (cf. Börner et al. 2007). Nevertheless, by contrasting simulation-optimization solutions of a standard renewable resource management problem with its numeric optimum, we demonstrate that simulation-optimization can considerably well approximate the latter, even if optimal control paths exhibit high volatility. This is a particular purposeful feature if modelling is applied as an exploratory tool to understand the behavior of complex ecological-economic systems.

That said, how well simulation-optimization approximates true optimality depends critically on the specification of the management function used in the simulation-optimization framework. By restricting the control variable to a predefined functional relationship, modelers only allow for solutions that lie in the domain of this function. This introduces a bias, which is comparable to the bias introduced by specifying inadequate functional forms in other modelling applications (McKitrick 1998). Specifying appropriate management functions is therefore an inherently interdisciplinary task as it requires consistency with the theoretical underpinnings of ecological-economic system behavior. For example, if management functions omit price or time as arguments, control paths will not respond optimally to price fluctuations and discount rate induced changes in resource use intensity. Similarly, management functions need to be specified such as to respond to critical drivers of ecological dynamics, such as rainfall in terrestrial ecosystems.

Apart from validating model solutions through comparison with real world observations, simple procedures can be used to identify severe management function misspecifications in simulation-optimization models (Section 4). Well specified and optimized management functions come with a particular advantage over standard numerical optimization solutions as they can be used to derive simple management rules linking resource use to system state or critical thresholds of external system driving factors, such as prices and rainfall.

Simulation-optimization clearly requires concessions from both economic and ecological disciplines. Large ecological simulation models may be too computation intensive to be optimized irregardless of the approach chosen and even if optimization is possible the use of prespecified management functions means that control paths violate optimality conditions from an economic theory point of view. Our analysis, however, suggest that simulation-optimization is more than a trade-off between ecological authenticity and economic optimality. Even if error margins in more complex applications than the one presented here are inevitable, simulation-optimization opens up new avenues for economists and ecologists to work in an integrated manner.

Simulation-optimization allows economists to assess whether policy recommendations derived from standard constrained non-linear optimization models are valid under stochastic circumstances and related dynamic ecological feedback relationships. By the same token, ecologists may improve model-based recommendations for natural resource management by eliminating management strategies that are clearly economically suboptimal. Hence, through simulation-optimization we may be able to learn more about human-environment interaction in cases, where conventional approaches fail.

6 Conclusion

In this paper we analyze a simple renewable resource management problem under constant and variable price and resource growth and demonstrate that simulation-optimization can yield satisfactory approximations to standard numerical optimization. We illustrate the implications of management function misspecifications in simulation-optimization models and suggest simple procedures to identify severe misspecification.

Reformulating constrained numerical optimization problems as simulation-optimization systems allows solving more complex systems of differential equations including highly non-linear functional forms and non-continuous functions. Derivative-based optimization algorithms usually fail under such circumstances, such that a combination of simulation-optimization with evolutionary algorithms remains the only option to identify close-to-optimal management strategies.

As such, simulation-optimization bears potential to integrate across disciplines as it allows more balanced trade-offs between aspects of ecological and economic theory that appear relevant in improving the contribution of ecological-economic modeling to sustainable resource management.

References

R. Bellman. *Adaptive Control Processes: A Guided Tour.* Princeton University Press, 1961.

J. Börner, S. I. Higgins, J. Kantelhardt, and S. Scheiter. Rainfall or price variability: What determines rangeland management decisions? a simulation-optimization approach to South African savannas. *Agricultural Economics*, 37:189–200, 2007.

M. C. Fu. Simulation optimization. In Smith J.S. Medeiros D.J. Rohrer M.W. Peters, B.A., editor, *Proceedings of the 2001 Winter Simulation Conference, 9-12 December 2001, Arlington, VA, USA*. Available Online (10/26/2005): http://www.wintersim.org/prog01.htm, 2001.

S. I. Higgins, W. J. Bond, and W. S. W. Trollope. Fire, resprouting and variability: a recipe for grass-tree coexistence in savanna. *JOURNAL OF ECOLOGY*, 88(2):213–229, 2000.

Steven I. Higgins, Jochen Kantelhardt, Simon Scheiter, and Jan Boerner. Sustainable management of extensively managed savanna rangelands. *Ecological Economics*, 62(1):102–114, April 2007. URL `http://www.sciencedirect.com/science/article/B6VDY-4KJ0SSG-1/2/da1777fc1d81cb81685430a8beb4420e`.

M. A. Janssen, J. M. Anderies, and B. H. Walker. Robust strategies for managing rangelands with multiple stable attractors. *JOURNAL OF ENVIRONMENTAL ECONOMICS AND MANAGEMENT*, 47:140–162, 2004.

D. Koutsoyiannis and A. Economou. Evaluation of the parameterization-simulation-optimization approach for the control of reservoir systems. *WATER RESOURSES RESEARCH*, 39(6):1–17, 2003.

Ross R. McKitrick. The econometric critique of computable general equilibrium modeling: the role of functional forms. *Economic Modelling*, 15(4):543–573, October 1998. URL `http://www.sciencedirect.com/science/article/B6VB1-3TWY7WS-4/2/818f38c04a533f63dd490f4e909a5690`.

Clifford S. Russell. Are we lost in the vale of ignorance or on the mountain of principle? *Ecological Economics*, 14(2):91–99, August 1995. URL http://www.sciencedirect.com/science/article/B6VDY-3Y45V3R-2/2/5395936eba8a031a2440b6527cf8f75c.

Simon Scheiter and Steven I. Higgins. Partitioning of root and shoot competition and the stability of savannas. *The American Naturalist*, 170(4):587–601, 2007. doi: 10.1086/521317. URL http://www.journals.uchicago.edu/doi/abs/10.1086/521317.

L. Sonneborn and F. Van Vleck. *SIAM Journal on Control and Optimization*, 2:151–159, 1965.

R.B. Standiford and R.E. Howitt. Solving empirical bioeconomic models: A rangeland management application. *AMERICAN JOURNAL OF AGRICULTURAL ECONOMICS*, 74(2): 421–433, 1992.

V. Vandermeulen and G. Van Huylenbroeck. Designing trans-disciplinary research to support policy formulation for sustainable agricultural development. *Ecological Economics*, Ecological Economics:–, in press. URL http://www.sciencedirect.com/science/article/B6VDY-4SV1JW3-1/2/e6718ec4e8be1a3e1c96d8bca1c2cbe0.

Westoby, M., B. Walker, and I. Noy-Meir. Opportunistic management for rangelands not at equilibrium. *JOURNAL OF RANGE MANAGEMENT*, 42:266–274, 1989.

Richard T. Woodward. Sustainability as intergenerational fairness: Efficiency, uncertainty, and numerical methods. *American Journal of Agricultural Economics*, 82(3):581–593, 2000. URL http://dx.doi.org/10.1111/0002-9092.00048.

7 Tables and Figures

Table 1: Optimal solutions of optimizations in section 3.1

Measure	Non-linear programming	Simulation-optimization (linear)	Simulation-optimization (non-linear)
Objective function value	638.7397	638.7071	638.7171
Optimized variables	50	3	4
Parameter a	n.a.	-4.3064	3.098
Parameter b	n.a.	0.0604	0.7459
Parameter c	n.a.	0.077	-0.0496
Parameter α	n.a.	n.a.	1415.83

Table 2: Optimal solutions of optimizations in section 3.2

Measure	Non-linear programming	Simulation-optimization (linear)	Simulation-optimization (non-linear)
Objective function value	1169.3848	1156.0910	1168.2578
Optimized variables	150	3	5
Parameter a	n.a.	0.1942	-0.9837
Parameter b	n.a.	-0.0022	0.0096
Parameter c	n.a.	0.0209	0.0227
Parameter α	n.a.	n.a.	6.5331
Parameter β	n.a.	n.a.	17.8039

A Parameter Values used in Simulations

Simulations in section 3.1

Parameter	Description	Value
p	price	10
w	cost parameter	100
δ	discount rate	0.995
$F(x_T) = x_t ft$	terminal value of x	0.5
T	terminal time	50
r	resource growth rate	0.01
$x(0)$	initial resource level	100

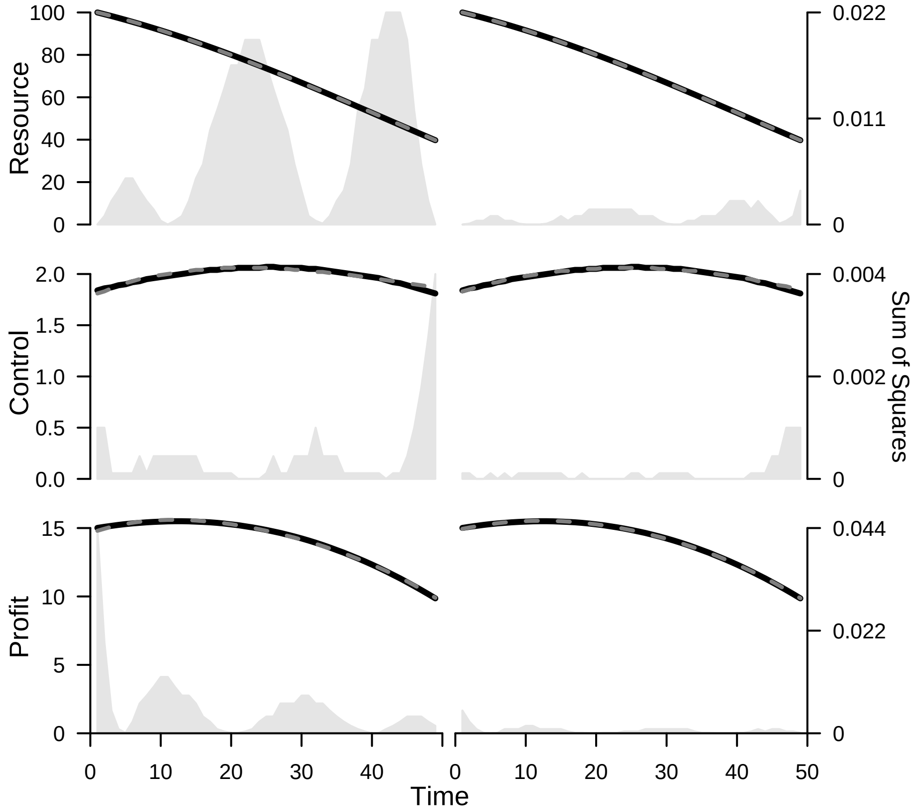

Figure 1: Optimal trajectories of state variable (upper panel), control variable (middle), and Profits (lower panel) for standard non-linear programming (black lines) and simulation-optimization (dashed grey lines). In the left panel a linear management function was used in for simulation-optimization, whereas a non-linear function was used in the right panel. The areas in light grey represent the squares of differences between the non-linear programming and the simulation-optimization solutions.

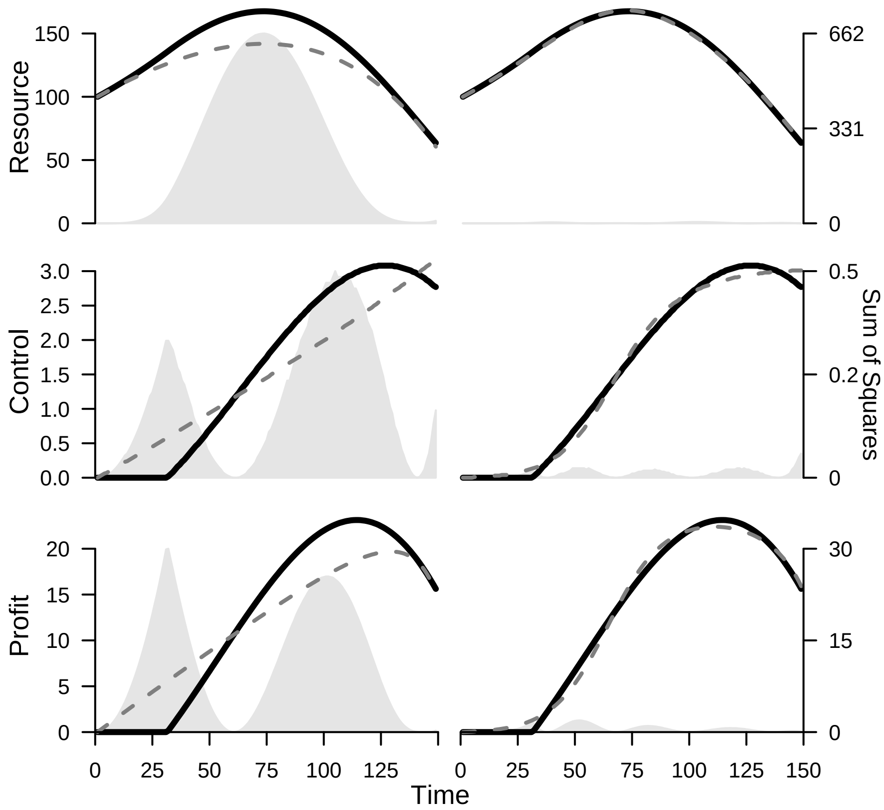

Figure 2: Optimal trajectories of state variable (upper panel), control variable (middle), and Profits (lower panel) for standard non-linear programming (black lines) and simulation-optimization (dashed grey lines) with an extended planning horizon. In the left panel a linear management function was used for simulation-optimization, which was multiplied with an approximated non-linear switching function in the right panel. The areas in light grey represent the squares of differences between the non-linear programming and the simulation-optimization solutions.

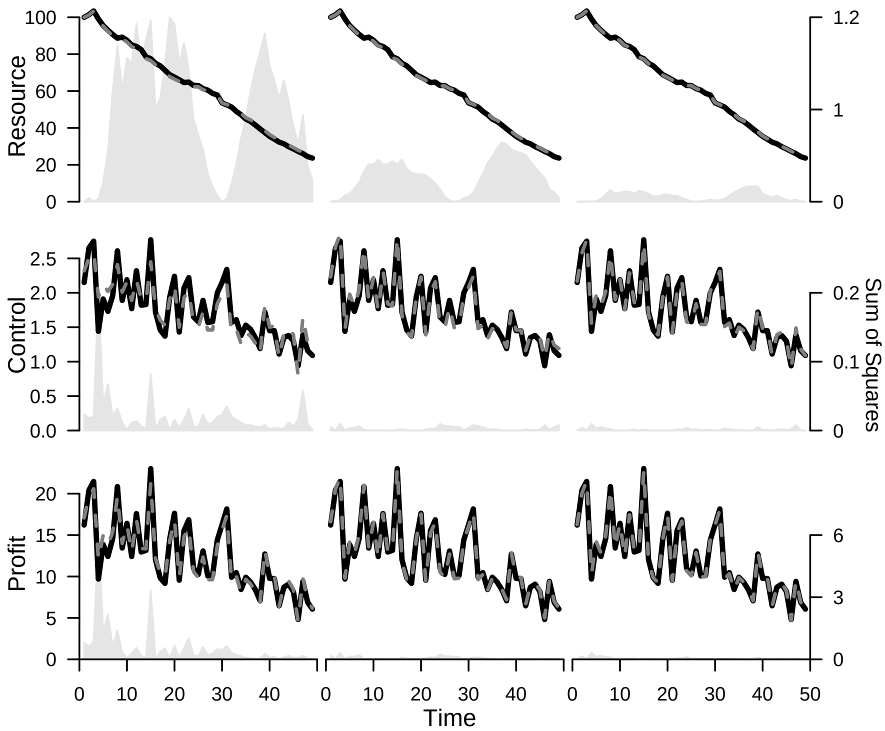

Figure 3: Optimal trajectories of state variable (upper panel), control variable (middle), and Profits (lower panel) for standard non-linear programming (black lines) and simulation-optimization (dashed grey lines) with an extended planning horizon. In the left panel, management function (8) was used for simulation-optimization. In the center panel a non-linear interaction term was used (equation 13). The right panel equation (13) was changed to equation (14). The areas in light grey represent the squares of differences between the non-linear programming and the simulation-optimization solutions.

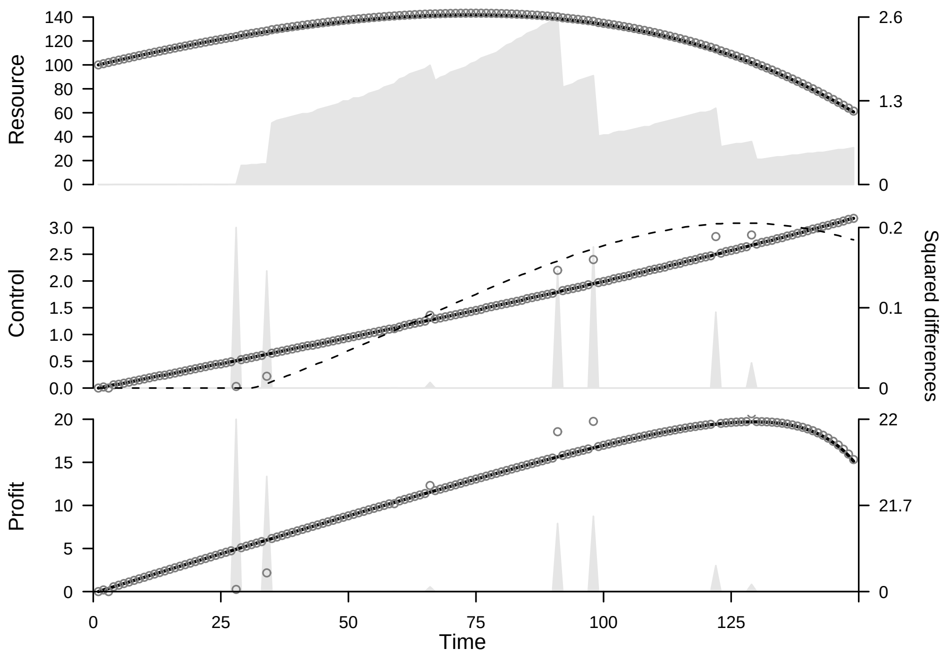

Figure 4: Optimal trajectories of state variable (upper panel), control variable (middle), and Profits (lower panel) for simulation-optimization (black lines). Dotted lines represent the same variable paths allowing for randomly selected points along the simulation-optimization solution to be determined freely in the optimization process. The areas in light grey represent the squares of differences between the the simulation-optimization solution and the dotted line.

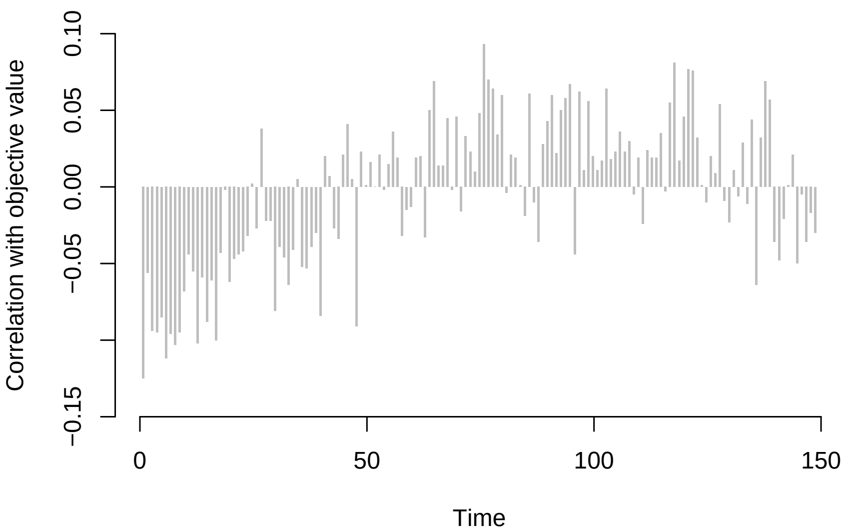

Figure 5: Sensitivity analysis: Correlations of control variable instances with objective value.