The non-permanence of optimal soil carbon sequestration

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Abstract:
Carbon sequestration in agricultural soils is considered as an option of greenhouse gas mitigation in many countries. But, the economic potential is limited by the dynamic process of saturation and the opportunity cost of land use change. In addition, this article shows that permanence cannot, in general, be achieved in the strict sense of maintaining the soil carbon stock on an increased equilibrium level. Rather, a cyclical pattern with periodical release of sequestered carbon can be economically optimal from both the farmers’ and societal point of view.

Keywords: Agriculture, Climate policy, Carbon sequestration, Land use change, Economic analysis.

JEL classification: Q15, Q24, Q54.

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1 Introduction

The use of biological sinks is regarded as an appealing option of greenhouse gas (GHG) mitigation in many countries. It is explicitly considered in the Kyoto protocol as an alternative measure besides the reduction of energy-intensive activities and investment in less carbon-intensive technologies. In order to make use of this climate policy option, countries can declare pieces of land as selected for carbon sequestration and balance the net amount of carbon that is fixed in the corresponding soils and forest biomass against their national GHG emissions.

The scientific basis for the assessment of sequestration activities is compiled in a special report of the IPCC (Watson et al., 2000) which provides a comprehensive state-of-the-art examination of the global carbon cycle and the scientific and technical implications of carbon sequestration.\(^1\) It shows that the amount of carbon stored globally in soils is much larger than the global carbon stock in vegetation,\(^2\) and that “changes in soil carbon stocks are at least as important for carbon budgets as changes in vegetation carbon stocks” (Watson et al., 2000: 26). Accordingly, attention must not only be given to afforestation, forest management and agro-forestry, but also to the various options of soil carbon sequestration. The latter can be used to partly reverse negative effects of cultivation and partly recover past losses of soil organic carbon (SOC) from agricultural land. Thus, soil carbon sequestration can enhance agricultural productivity in the long run and by this way particularly contribute to sustainable development in less developed countries (Robert, 2001; Lipper and Cavatassi, 2004).

Various authors emphasise that farmers may benefit from providing sequestration services to private markets or government programs (Sandor and Skees, 1999; Marland et al., 2001a; McCarl and Schneider, 2001; Antle and Diagana, 2003; Young, 2003; Lehtonen et al., 2006). In addition, society might enjoy multiple side-benefits of soil carbon sequestration, such as improved water quality, biodiversity and landscape

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1 To get a concise insight in the role of agriculture in the global carbon cycle, see also Lal (2004a, b).

2 The ratio between the carbon stock in the soil and in vegetation ranges from 1:1 in tropical forests to 5:1 in boreal forests, and it is much higher for grasslands, wetlands and croplands (Watson et al., 2000: 4, 31).
amenities (McCarl and Schneider, 2001; Feng et al., 2004, 2007). Finally, different economic studies reveal that soil carbon sequestration through conservation tillage and land use change, respectively, can constitute a cost-effective option in a nation’s GHG mitigation portfolio with some considerable potential (e.g., Antle et al., 2001, 2003, 2007; McCarl and Schneider, 2000, 2001; Pautsch et al., 2001). However, one important feature of soil carbon sequestration is largely neglected in existing studies. This is the non-linearity and saturation of the dynamic process of soil carbon accumulation and its impact on the net present value of a sequestration program. Related problems are the threat of future carbon releases and the question of permanence.

The aim of this article is to investigate from an economic perspective the climate policy option of carbon sequestration in agricultural soils with special consideration of the dynamics and permanence issues. In particular, our focus is on the consequences for contract and policy design that follow from the non-linearity and saturation process. The remainder of the paper is organised as follows. Section 2 provides a brief overview of the prospects of soil carbon sequestration. Section 3 is devoted to an introduction of the economic allocation problem, incentive schemes and the cost of permanent sequestration. This is further developed in Section 4 with special consideration of the non-linearity in the sequestration process. Finally, Section 5 concludes with a general evaluation of soil carbon sequestration from an economic perspective.

2 Prospects of soil carbon sequestration

In contrast to carbon fixation by afforestation and improved forest management, the idea of carbon sequestration in agricultural soils only appeared in the economics literature in recent years. Following numerous scientific assessments, these studies have been motivated by the need to evaluate the feasibility and competitiveness of soil carbon sequestration from an economic perspective. They investigate the role which economic incentives could play in inducing farmers to adopt practices that would increase the amount of carbon in the soil. Such incentives could be given through direct government payments or private markets. In either case, contracts between buyers

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3 See Watson et al. (2000) for an overview.
(emitters of GHGs) and sellers (farmers) of carbon sequestration services would specify the payment mechanisms and other terms for either a government program or carbon markets (Antle and McCarl, 2002).

Candidate measures of soil carbon sequestration include conservation tillage, ley-arable farming, partial elimination of bare fallow, conversion of cropland to permanent grassland, and restoration of wetlands and organic soils (Watson et al., 2000, ch. 4; Lal, 2004). Related flows and stocks of SOC are not uniform across geographical regions, but vary with soil types and climatic conditions. Correspondingly, the suitability of sequestration measures can be different from one location to another. For instance, conservation tillage is seen as the primary means for increasing soil carbon in Iowa (Pautsch et al., 2001), while conversion of cropland into permanent grass and elimination of bare fallow through continuous cropping are considered adequate for Montana (Antle et al., 2001). In the latter case, site-specific marginal sequestration costs range from 12 to 140 US$/t C for continuous cropping and from 50 to 500 US$/t C for conversion of cropland into permanent grassland.

The difference in the cost of these two sequestration activities is primarily a consequence of differences in policy design, rather than due to an effective difference in marginal costs of sequestration. Under the continuous cropping scenario, farmers are assumed in the analysis of Antle et al. (2001) to receive payments on a per hectare basis only for fields switched to continuous cropping, whereas all cropland and pasture is assumed eligible for payments under the permanent grass scenario. In principle, this differentiation in eligibility has the same effect as the distinction between payments to all adopters of conservation tillage and payments to new adopters only that is made in the study of Pautsch et al. (2001). Their estimates of average sequestration costs in Iowa are much higher than those of Antle et al. for Montana. To a certain extent, this higher cost is a consequence of the initial adoption rate of conservation tillage, which was already above 60 percent in the reference year 1992.

With a more comprehensive approach, using the integrated assessment model ASMGHG for the US agriculture and forestry sector, McCarl and Schneider (2001) show that the economic potential of carbon sequestration in agricultural soils largely exceeds the potential of abating methane and nitrous oxide emissions from US
agriculture. For carbon prices below 100 US$/t C, it also exceeds the potential of carbon sequestration through afforestation. Compared to cost estimates for non-agricultural compliance with a Kyoto-like target that averaged between 44 and 89 US$/t C, with a maximum estimate of 227 US$/t C (cf. McCarl and Schneider, 2001; Antle and McCarl, 2002), these figures illustrate the competitiveness of agricultural soil C sequestration with other measures of GHG mitigation in the USA. Together with more recent studies of Antle et al. (2007) for the Upper Mississippi Basin and Feng et al. (2007) for the Central United States, these results indicate a large potential of soil carbon sequestration in the US that could be realised with rather moderate carbon prices.

In contrast, Manley et al. (2005) conclude in a meta-analysis across 52 studies that “in most places creating carbon offsets by changing tillage practices is simply not cost-effective.” Their results show a range of sequestration costs from a low of 1.94 US$/t C to well over 300 US$/t C, depending on region, crop grown and other factors. Thus, from an economic point of view, one cannot draw a general conclusion in favour or against soil carbon sequestration. Rather, situation factors must be taken into account. This is also supported by results from different European studies that reveal substantial differences in the cost-effectiveness of soil carbon sequestration in different countries.

Thus, the use of soil carbon sequestration as a GHG mitigation measure must be carefully evaluated from both scientific and economic perspectives. This particularly requires adequate consideration of the relevant geographical and political circumstances, and of the dynamic patterns of the sequestration processes. The latter restrict the sequestration potential of GHG mitigation for both biophysical and economic reasons.

First, depending on the type of activity, the removal of atmospheric CO₂ through sequestration may be offset by enhanced emissions of nitrous oxide (N₂O) and methane (CH₄). Increasing emissions of CH₄ must particularly be expected following the restoration of cultivated peatlands, while the net GHG effect of conservation tillage

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4 The various options and costs of GHG mitigation from agriculture have also been investigated in different European countries with the use of integrated assessment models, but with less emphasis on the examination of particular sequestration measures and without explicit assessment of marginal sequestration costs (e.g., De Cara and Jayet, 2000; Hartmann et al., 2008; Lehtonen et al., 2006).
might be reduced or even completely offset by additional N$_2$O emissions due to higher residue returns and denitrification rates (Leifeld et al., 2003: 90; Li et al., 2005). In contrast, grassland extensification may be a promising option for reducing N$_2$O emissions, while any extension of permanent grassland may induce CH$_4$ emissions if the additional grass is fed to ruminants (Lehmann and Hediger, 2004). Thus, to comprehensively evaluate carbon sequestration potentials and agricultural GHG mitigation strategies it is important to take the various interdependencies of crop and livestock management into account (McCarl and Schneider, 2001; Hediger, 2006).

Second, since carbon sequestration is a dynamic process of carbon accumulation in soils and biomass, special attention must be given to the issue of saturation and permanence. As emphasised in the IPCC special report, the rate of soil carbon sequestration following a particular change in land use or management practice cannot be sustained indefinitely. Rather, the temporal pattern of soil carbon accumulation can be represented as a non-linear process which describes the shift to a new equilibrium carbon stock above the initial level $S_0$ that existed prior to adoption of a particular sequestration activity.

![Figure 1. The sequestration curve and average sequestration rate](Adapted from Watson et al. (2000: 201))
In general, the rate of carbon gain following application of a given land use or management practice will decrease over time. Rates of change in the soil carbon stock observed during initial periods after adoption of sequestration activities are usually higher than the average rates that are generally reported by scientists (cf. Watson et al., 2000), and decrease toward zero when saturation is achieved (West and Six, 2007). This process is illustrated in Figure 1 with the solid line representing the idealised accumulation curve, while the dashed line represents the simplified linear time path that implies the average rate of carbon sequestration $q$.

Third, the issue of permanence is related to the fact that sequestered carbon in the soil constitutes a potential source and may be released again in the future if farmers would return to original land use and management practices (IPCC, 2000; Marland et al., 2001a, b; Antle and McCarl, 2002; Feng et al., 2002; Vercammen, 2002; Antle and Diagana, 2003; Thomassin, 2003). For the design of an economically efficient policy, it is therefore essential to account for potential future releases of sequestered carbon and the opportunity cost of maintaining carbon in the soil. Correspondingly, the time horizon in economic analyses and contract design must be extended beyond the point of saturation. Moreover, the non-linearity of the carbon accumulation process must adequately be taken into account. This is investigated in the remainder of this paper, starting with a general representation of the allocation and incentive problem and a first investigation into the cost of permanence using a simplified linear model.

### 3 Economic incentives for soil carbon sequestration

As mentioned above, the economic incentive problem can be formalised as a contract between buyers and sellers of carbon sequestration services. Such contracts are associated with two classes of costs: on-farm opportunity costs of sequestration and transaction costs (contracting and monitoring costs). The former depend on the opportunity cost of changing land use or management practices, divided by the rate of

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5 See also Cacho et al. (2003) that provide a comprehensive analysis and comparison of carbon-accounting methods in the context of carbon sequestration through reforestation. Further recommended reading on issues related to the accounting problem includes Murray et al. (2007) and Reilly and Asadoorian (2007).
soil carbon accumulation on a given piece of land. This cost is relevant for a farmer’s decision to adopt a particular land use or sequestration technique. The second class of costs consists of negotiation costs that are similar for different contract types, and monitoring costs that are presumably higher for per-ton contracts than per-hectare contracts.

3.1 The farmers’ land allocation problem

Following Antle et al. (2001, 2003) and Antle and McCarl (2002), a farmer’s decision problem can be formalised in terms of an economic allocation problem that maximises for each site the net present value of expected returns from a set of available production activities, using either a per-hectare or a per-ton payment scheme for carbon sequestration.

3.1.1 Per-hectare payments

For a contract with duration $T$ years and constant annual per-hectare payment $g^{is}$ received for switching from management practice $i$ to the sequestration activity $s$, a farmer’s decision problem is represented by

$$\max_x \left( \sum_{t=1}^T (1 + r)^{-t} \left[ \pi^i_t + g^{is} - I^i_t \right] x + \sum_{t=1}^T (1 + r)^{-t} \pi^s_t [1 - x] \right)$$

(1)

with $r$ denoting his interest rate, and $\pi^i_t$ and $\pi^s_t$ the respective per-hectare profits of the two activities at time $t$. Moreover, $x$ is the decision variable which takes the value $x = 1$ for adoption of the sequestration activity or $x = 0$ for non-adoption. $I^i_t$ represents the fixed cost per hectare of changing from system $i$ to system $s$.

To give a farmer an incentive to adopt the sequestration technique $s$, the annual per-hectare payment must exceed the annuity value of the farmer’s overall period opportunity cost. In the special case with constant expected returns over time ($\pi^i_t \equiv \bar{\pi}^i$ and $\pi^s_t \equiv \bar{\pi}^s$) and without fixed cost of changing practices, the condition for entering into a contract can be simplified to $\bar{\pi}^s + g^{is} > \bar{\pi}^i$ for all $t$. Rearranging, this inequality becomes

$$g^{is} > \bar{\pi}^i - \bar{\pi}^s$$

(2)
In this case, the farmer will benefit from the contract if his short-term opportunity cost of sequestration is less than the contract payment per period. This is a rule of thumb which only compares the variable cost of land use change per hectare and per year with the annual per-hectare compensation payment. It is used in similar form by Antle et al. (2001, 2003) and Pautsch et al. (2001) for the seek of tractability in empirical studies. Moreover, it can be taken as first step of an in-depth economic evaluation that uses less restrictive assumptions and takes a longer time horizon into account.

### 3.1.2 Per-ton payments

In case of a contract with payments per ton of carbon sequestered, the following equality holds for the carbon price $p$:

$$ p = g^{s} / q^{s} $$

(3)

Here, $q^{s}$ denotes the average annual increment of soil carbon that can be realised with sequestration technique $s$ until the level of saturation of soil carbon is achieved. Apparently, this implies that the contract duration cannot exceed saturation time, and therefore particularly involves the prospective problem of permanence. The condition for participation in the program is

$$ p > \left( \frac{\pi^{i} - \pi^{s}}{q^{s}} \right) / q^{s} = \frac{c^{i} / q^{s}}{q^{s}} $$

(4)

In other words, the farmer has an incentive to accept the contract when the price per ton of carbon is greater than the short-term farm opportunity cost per ton of carbon sequestered, given the above assumptions.

### 3.2 The cost of permanent sequestration

With the payment schemes considered so far, farmers would have no incentive to maintaining the sequestered carbon in the soil beyond the end of the contract. Rather, permanent sequestration would require a contract offered to farmers which would establish continuous responsibility for sequestered carbon and provide adequate incentives. In principle, this could be realised through a rental contract with credits assigned when carbon is sequestered and debits accruing when carbon is emitted.

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6 We still assume constant expected returns, and the absence of fixed costs for switching from system $i$ to system $s$. 
(Marland et al., 2001b), or through an extension of the above per-hectare contract with compensation for the variable incremental cost granted as long as the sequestration practice is applied.

To formally analyse the latter case, we extend the above analysis from a fixed to infinite time horizon. We assume again constant expected returns, but restrict our analysis to one single sequestration technology (land use or management practice). Correspondingly, we can omit the time index and the indices for different practices. Using a simplified notation from equation (2), the farmer’s short-term per-hectare opportunity cost of carbon sequestration is defined \( c = \pi^i - \pi^s \). This is the difference between the expected returns per hectare between the two practices. The fixed cost for changing practice is again denoted by \( I \), and \( r \) represents the farmer’s interest rate.

Assuming a constant annual per-hectare payment \( g_x \) provided for application of a given sequestration practice, the net present value of a contract for permanent sequestration is

\[
NPV_{(PST)}(r, \hat{T}) = \sum_{t=1}^{\hat{T}} \left[ \left( \frac{1}{1+r} \right)^t g_x (1+r)^{-t} - c \left( \frac{1}{1+r} \right)^t - I = \frac{g_x - c}{r} - I \right]
\]

The contract would be acceptable to a farmer if this value is positive. This formally requires a payment stream that compensates at any time the incremental cost of cultivation plus the annualised capital cost: \( g_x \geq c + rI \).

The same outcome could theoretically be achieved with a payment for each additional ton of carbon sequestered. This would be directly compatible with a carbon tax or a system of tradable carbon permits. In this case, the contract would commit the farmer to permanent sequestration, but only provide payments until saturation of soil carbon is achieved at time \( \hat{T} \). Assuming a constant payment \( p \) per ton of carbon, a constant rate of sequestration \( q > 0 \) for the sequestration period, and no further addition to the stock of soil carbon afterwards, the net present value of this contract is

\[
NPV_{(PS)}(r, \hat{T}) = \sum_{t=1}^{\hat{T}} pq (1+r)^{-t} - \sum_{t=1}^{\hat{T}} c (1+r)^{-t} - I = pq \left[ rD(r, \hat{T}) - \frac{c}{r} - I \right]
\]

with

\[
D(r, \hat{T}) = \sum_{t=1}^{\hat{T}} (1+r)^{-t} = \frac{1 - (1+r)^{-\hat{T}}}{r}
\]
To compensate the farmer for the total cost of sequestration, \( (c/r) + I \), the annual payment \( pq \) granted for any addition to the soil carbon stock must be higher than the minimum annual per-hectare payment required in a contract with limited duration:

\[
pq \geq \frac{c + rI}{rD(r, T)} = \frac{c + rI}{1 - (1 + r)^{-T}} > c + rI
\]

Thus, in the short term, the seller of the contract faces higher cost in case of per-ton payments than in the case with per-hectare payments. However, the net present value of total payments is in principle the same for both payment schemes:

\[
\sum_{i=0}^{\hat{T}} pq(1+r)^{-i} = pqD(r, \hat{T}) \geq \frac{c + rI}{r} \equiv \frac{g_{\text{min}}}{r} = \sum_{i=0}^{\hat{T}} g_{\text{min}}(1+r)^{-i}
\]

with \( g_{\text{min}} \) denoting the minimum rate of payment required under the per-hectare scheme:

\[
g_{\text{min}} = \arg \min \{g_\ast \} = c + rI.
\]

Hence, permanent sequestration can be achieved with either a contract that pays a farmer on a per-hectare or per-ton basis. It would, however, require higher annual payments and imply higher costs to the sellers of contracts than in the restricted static case presented in Subsection 3.1, where the challenge of permanence is not formally addressed. Moreover, our analysis has been restricted so far to the linear case that only considers the average rate of sequestration. Yet, to effectively assess the economic potential of carbon sequestration in agricultural soils and investigate the economic efficiency of alternative policies or contracts for soil carbon sequestration, one should further take into account the non-linearity and saturation of the sequestration process.

### 4 Economic efficiency and the dynamics of sequestration

Investigating the role of spatial heterogeneity, Antle et al. (2003) show that, for some agro-ecozones, the marginal cost of soil carbon sequestered under contracts with per-hectare payments can be as much as five times higher than the marginal cost in case of contracts that provide payments per ton of carbon sequestered. On this basis, they conclude that contracting parties could afford to bear a significant cost (monitoring cost) to implement the per-ton contract in more spatially heterogeneous regions and still achieve a lower total cost per ton of soil carbon sequestered than would be possible with a per-hectare contract. This confirms the relative inefficiency of per-hectare payments
for carbon sequestration in the presence of spatial heterogeneity, and provides an efficiency-based argument for the use of per-ton contracts. The latter would be directly compatible with tradable carbon permits and other efficiency-oriented government programs to mitigating greenhouse gas emissions. Therefore, we only consider per-ton payments in our further analysis of economic efficiency and dynamics of soil carbon sequestration. In principle, this requires a comparison of different GHG mitigation options with respect to their marginal cost per ton of carbon equivalent. Moreover, the analysis must be based on consideration of effective rates of sequestration, rather than average rates. Using the latter would not in general lead to an economically efficient outcome.

Since soil carbon accumulation is a non-linear process of transition between two equilibria, effective rates of soil carbon sequestration are not constant over time. As drawn in Figure 1, sequestration rates are usually higher at initial stages of the sequestration process and decline with soil carbon accumulation over time. Formally, this process can be represented for a given site as follows:

\[ \dot{S}_t = dS_t / dt = \phi(S_t) \geq 0 \]

with \( \phi(S_0) = \phi_{\text{max}} \), \( \lim_{S \to \hat{S}} \phi(S) = 0 \), \( \phi' < 0 \), \( \phi'' \leq 0 \) and \( S_0 \leq S_t \leq \hat{S} \) (10)

where \( S_t \) denotes the current soil carbon stock at time \( t \), \( S_0 \) the initial equilibrium stock at time \( t = 0 \), \( \hat{S} \) the level of saturation that would be achieved at saturation time \( \hat{T} \), and \( \phi(S_t) \) the effective sequestration rate at time \( t \).

The decline of the sequestration rate with the stock of soil carbon is also crucial from an economic efficiency point of view. The instantaneous payment which a farmer should receive for sequestering carbon under this regard is state and time dependent:7

\[ g_t = p_t \phi(S_t) \]  

(11)

7 This corresponds to the equality in equation (3), but assumes a state-dependent sequestration rate \( \phi(S_t) \) rather than the constant rate \( q \).
Hence, the incentive to sustain the sequestration process and permanently maintain carbon in the soil may diminish with the accumulation process. To further examine this issue, we first analyse the concave problem without a penalty for future carbon releases. Then, we introduce a charge for carbon releases and furthermore examine the effect of an increasing carbon price.

4.1 Optimal sequestration without penalty for releasing carbon

To investigate the impact of declining rates of soil carbon accumulation upon a farmer’s optimal choice, we first consider a contract with flexible terminal time, which grants him a fixed and constant price \( p \) per ton of effectively sequestered carbon but not charges him for the subsequent release. In analogy to Section 3, we furthermore assume constant expected returns over time and constant marginal cost of sequestration per hectare \( c \). But, as an extension of the above model, we consider the non-linear sequestration function of equation (10). Consequently, the net present value of the contract to an individual farmer is:

\[
V_0^x = \int_0^{T^*} e^{-\tau} \left[p \phi(S_T) - c\right] d\tau - I
\]

In this case, the farmer’s decision is not only about the adoption of a particular sequestration practice, but also about the optimal time horizon \( T^* \). Formally, the latter is determined by

\[
\frac{dV_0^x}{dT^*} = e^{-\tau^*} \left[p \phi(S_{T^*}) - c\right] = 0
\]

with \( S_{T^*} = S(T^*) \).

Hence, at the optimal terminal time, the rate of sequestration (the slope of the sequestration curve in Figure 1) must be strictly positive:

\[
\phi(S_{T^*}) = \frac{c}{p} > 0
\]

Thus, it would be optimal from a farmer’s perspective to terminate the contract at time \( T^* < \hat{T} \), this is before saturation of the soil carbon stock is achieved (\( S_{T^*} < \hat{S} \)). In other
words, farmers might be given an incentive to stop sequestration before the saturation point is reached.

This conclusion does not fundamentally change, if the carbon price is not constant. Rather, if it increases over time at a rate $\alpha > 0$, such that the instantaneous price is $p_t = p e^{\alpha t}$, the net present value of the contract to an individual farmer is:

$$\tilde{V}_0^x = \int_0^{\tilde{T}_1^x} p e^{-(r-\alpha)s} \phi(S_t) dt - \int_0^{\tilde{T}_1^x} c e^{-n} dt - I$$

(15)

and condition (14) changes to

$$0 < \phi(\tilde{S}_T^x) = \frac{c}{p} e^{-x \tilde{T}_1^x} = \frac{c}{p} e^{-\alpha \tilde{T}_1^x} < \frac{c}{p} = \phi(S_T^x)$$

(16)

with $\tilde{T}_1^x$ and $\tilde{S}_T^x$ denoting the optimal terminal time and terminal stock in this modified case.

From equation (16) we get $\tilde{S}_T^x > S_T^x$ and $\tilde{T}_1^x > T_1^x$ for $\alpha > 0$, and vice versa for $\alpha < 0$. Thus, the optimal terminal point to the farmer is postponed to higher levels of soil carbon for increasing carbon prices ($\alpha > 0$), while the economic sequestration period would be curtailed and the maximum stock of soil carbon be lowered in the case with declining carbon prices ($\alpha < 0$) but without a charge for released amounts of carbon.

In short, our analysis of the case without for the release of sequestered carbon shows that it would be optimal from a farmer’s perspective to terminate the contract before the saturation point is achieved and return to the original land use and management practice. However, this is not efficient from a social point of view, since economic efficiency also requires charging farmers for the release of carbon, rather than solely paying (subsidising) them for the absorption of carbon dioxide from the atmosphere and sequestering carbon in their soils.

4.2 Optimal sequestration if future carbon release is charged

If farmers would be charged a penalty for the release of sequestered carbon, then the above result changes. To investigate this effect, we first assume a contract paying a farmer a fixed price $p$ per ton of carbon sequestered through conservation tillage, for instance, and charging him the same price per ton $C$ for any future release of soil carbon.
that results from ploughing after a sequestration period of duration $T_0 \leq \hat{T}$. In this case, the net present value of the contract for the first “sequestration-tillage rotation” is

$$V_0^* = \int_0^{T_0} e^{-r t} \left[ p \phi(S_t) - c \right] dt - I - e^{-r(T_0+1)} pX_{T_0}$$

(17)

with $X_{T_0}$ denoting the amount of carbon released from the soil due one-time tillage at the end of the sequestration period.

Through the tillage of the soil and release of carbon, the farmer brings himself in the position to enter a new sequestration contract at time $T_0+1$ for which the same logic applies as above. Accordingly, a rational farmer would be advised to maximise not only the net present value $V_0^*$ of the first sequestration-tillage sequence, but also to take into account the net present value of future sequestration-tillage sequences $n = 1, \ldots, \infty$:

$$V_n^* = \int_0^{T_n} e^{-r t} \left[ p \phi(S_t) - c \right] dt - e^{-r(T_n+1)} pX_{T_n}$$

(18)

with $T_n$ denoting the optimal duration of the $n$th sequestration contract and $X_{T_n}$ the amount of carbon that would be released in case of tillage at the end of the $n$th sequence.

Crucial for determining the optimal duration of a sequestration contract are the values of $p$, $c$, $I$ and $r$ but also the sequestration function $\phi(S)$ and the carbon-release rate $X_{T_n}$. If the latter would be zero, we could simply apply the case presented in the previous subsection. However, the occasional tillage of agricultural soils may result in a partial loss of the previously sequestered carbon. Quincke et al. (2007), for instance, report on different results found in the literature ranging from complete loss of the sequestered carbon due to a one-time inversion tillage operation (e.g. Stockfish et al., 1999) to the observation that tillage did not cause significant losses of total or labile soil organic carbon.
4.2.1 Complete loss of the sequestered carbon

For analytical purposes and mathematical tractability, we start with the simplifying assumption that the sequestered carbon will be completely released due to one-time tillage at the end of each sequestration contract:8

\[ X_{T^*} = \int_{0}^{T^*} \phi(S_t) dt = S^*_T - S_0 \quad \text{with} \quad S^*_T = S(T^*) \]  \hspace{1cm} (19)

In this stylised case, each new contract has the same length \( T^* \) and the same value \( V^* = V_n^* = V_0^* - I \) at the beginning of each contract period. Thus, the net present value of the infinite sequence with sequestration-tillage rotation, assuming a constant carbon price \( p \), is:

\[ NPV = V^* + e^{-r(T^*+1)} V^* + e^{-r(2(T^*+1))} V^* + e^{-r(3(T^*+1))} V^* + e^{-r(4(T^*+1))} V^* + ... - I \]  \hspace{1cm} (20)

Following simple transformation, this results in

\[ NPV = \frac{V^*}{1 - e^{-r(T^*+1)}} - I \]  \hspace{1cm} (21)

Accordingly, the farmer could accept the contract if the \( NPV > 0 \), which is the case only if the value of a single rotation \( V^* \) exceeds the fixed cost \( I \) by the factor \( (1 - e^{-r(T^*+1)}) \). Yet, the value \( V^* \) is not exogenous. Rather, it is the result of the economic optimisation process which is to determine the optimal length of the sequestration contract and the optimal tillage time \( T^* \). The corresponding first-order optimality condition is:

\[ \frac{dNPV}{dT^*} = \frac{dV^*}{dT^*} \left( \frac{1}{1 - e^{-r(T^*+1)}} \right) - \frac{V^* e^{-r(T^*+1)}}{(1 - e^{-r(T^*+1)})} = 0 \]  \hspace{1cm} (22)

which results in

\[ \frac{dV^*}{dT^*} = \frac{V^* e^{-r(T^*+1)}}{1 - e^{-r(T^*+1)}} \]  \hspace{1cm} (23)

Using

8 This assumption will be relaxed in the next subsection.
\[
\frac{dV^*}{dT^*} = e^{-\tau^*} \left[ p \phi(S^*_T) - c \right] - e^{-r(T^*+1)} p \frac{dS^*_T}{dT} + re^{-r(T^*+1)} p \left[ S^*_T - S_0 \right] \\
= e^{-\tau^*} \left[ p \phi(S^*_T) - c - e^{-r} p \phi(S^*_T) + re^{-r} p \left[ S^*_T - S_0 \right] \right]
\]

(24)

it furthermore follows for the simplified case represented in equation (19):

\[
(1 - e^{-r}) p \phi(S^*_T) - c + re^{-r} p \left[ S^*_T - S_0 \right] = \frac{re^{-r}V^*}{1 - e^{-r(T^*+1)}}
\]

(25)

and consequently

\[
e^{-r} (1 - e^{-r}) p \phi(S^*_T) - c + p \left[ S^*_T - S_0 \right] = \frac{V^*}{1 - e^{-r(T^*+1)}}
\]

(26)

Optimal tillage time is given if the farmer’s capitalised instantaneous opportunity cost of tillage equalizes the total value of the contract, net of the initial investment cost. At this instance in time, the farmer would not only have to pay the charge \( p[S^*_T - S_0] \) for releasing carbon, but should also take into account the total value of foregone net revenue from sequestration due to postponing this event by one more period.9

Furthermore, the optimal rate of sequestration at terminal time \( T^* \) is

\[
\phi(S^*_T) = \frac{c + re^{-r} \left[ \frac{V^*}{1 - e^{-r(T^*+1)}} - p \left[ S^*_T - S_0 \right] \right]}{(1 - e^{-r}) p}
\]

(27)

Taking into consideration that the rate of sequestration is declining with the stock of soil carbon \( S \),10 it would be optimal for a farmer to sequester carbon in the soil as long as the current rate of sequestration \( \phi(S) \) exceeds the optimal rate \( \phi(S^*_T) \), that is, as long as \( \phi(S) > \phi(S^*_T) \). In contrast, it would be optimal to interrupt sequestration if equation (27) is satisfied and if the net present value, \( e^{-r} V^* / (1 - e^{-r(T^*+1)}) \), of all the subsequent sequestration-tillage sequences exceeds the penalty for releasing carbon, \( e^{-r} p \left[ S^*_T - S_0 \right] \). In this case, it is optimal for the farmer to till the soil and, one period later, to restart the cyclical process with sequestration and tillage, as illustrated in Figure 2.

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9 The latter is represented by the first term on the LHS in equation (26).

10 Cf. equation (10).
Compared to the situation without penalty for releasing carbon, the optimal tillage time $T^*$ is earlier than in the case without penalty for releasing carbon (Section 4.1); i.e. $T^* < T^x$. This is due to the fact that because of $c/[1 - (1 - e^{-r})p] > c/p$ and $V^*/(1 - e^{-r(T^*+1)}) - p[S^*_T - S_0] > 0$ we have $\phi(S^*_T) > \phi(S^*_T) = c/p$ and thus $S^*_T < S^*_T$.\(^{11}\)

4.2.2 Partial loss of the sequestered carbon

Qualitatively, the above result remains the same if we relax the simplifying assumption of the complete loss of the sequestered carbon due to one-time tillage. Instead, we assume that only a fraction $0 < \beta < 1$ is effectively lost to the atmosphere. As a consequence, the penalty for releasing carbon is lower than in the previous case, such that equation (17) changes to

$$V_0^{**} = \int_0^{\tau_0^*} e^{-rt} [p\phi(S_t) - c] dt - I - e^{-r(t_0^*+1)} p\beta \int_0^{\tau_0^*} \phi(S_t) dt$$  \hspace{1cm} (28)

---

\(^{11}\) Cf. equation (14).
with $T_0^{**}$ denoting the optimal terminal time of the first rotation in the case with $\beta < 1$. Moreover, equation (20) is replaced by

$$NPV_0 = V_0^{**} + e^{-r(T_0^{**}+1)}NPV_1$$

(29)

with

$$NPV_1 = V_1^{**} + \sum_{n=2}^{\infty} e^{-r(T_n^{**}+1)}V_n^{**}$$

(30)

The latter denotes the net present value of the infinite sequestration-tillage rotation at time $T_0^{**}+1$, and $T_n^{**}$ and $V_n^{**}$ the optimal length and value of the $n$th rotation. The value of this rotation (contract), with $X_{T_n}$ denoting the amount of carbon released in case of one-time tillage, is

$$V_n^{**} = \int_{0}^{T_n^{**}} e^{-rt} \left[ p\phi(S_t) - c \right] dt - e^{-r(T_n^{**}+1)}pX_{T_n}$$

(31)

Hence, the optimal duration of the first rotation is determined by the first-order condition

$$\frac{dNPV_0}{dT_0^{**}} = \frac{dV_0^{**}}{dT_0^{**}} - re^{-r(T_0^{**}+1)}NPV_1 + e^{-r(T_0^{**}+1)} \frac{dNPV_1}{dT_0^{**}} = 0$$

(32)

Using equation (28) and $S_{T_0}^{**} = S(T_0^{**})$, this involves

$$\frac{V_0^{**}}{dT_0^{**}} = e^{-rT_0^{**}} \left[ p\phi(S_{T_0}^{**}) - c \right] + re^{-r(T_0^{**}+1)}p\beta \int_{0}^{T_0^{**}} \phi(S_t) dt - e^{-r(T_0^{**}+1)}p\beta p\phi(S_{T_0}^{**})$$

$$= e^{-rT_0^{**}} \left[ (1 - e^{-r}\beta)p\phi(S_{T_0}^{**}) - c + re^{-r}p\beta \int_{0}^{T_0^{**}} \phi(S_t) dt \right]$$

(33)

while according to equation (31), $dNPV_1 / dT_0^{**} = 0$.

Thus, equation (32) can be replaced by

$$(1 - e^{-r}\beta)p\phi(S_{T_0}^{**}) - c + re^{-r}p\beta \int_{0}^{T_0^{**}} \phi(S_t) dt - re^{-r}NPV_1 = 0$$

(34)
Accordingly, the optimal rate of soil carbon sequestration at the terminal time of the first rotation period is given by

$$
\phi(S_{T_0}^{*}) = \frac{c + r e^{-\gamma} \left[ NPV_1 - p \beta \int_0^{\tau_0} \phi(S_i)dt \right]}{(1 - e^{-\gamma})p} \quad (35)
$$

Depending on the different price and parameter values, this rate can be larger or smaller than the optimal rate $\phi(S_0^*)$ in the previous case, where we assumed complete release of the sequestered carbon after tillage; i.e. $\beta = 1$. In other words, it is an empirical question whether the sequestration period is made longer or shorter and whether the maximum stock of soil carbon is larger or smaller for lower values of $\beta$. Moreover, the optimal rotation period can no longer be assumed constant if $\beta < 1$, such as illustrated in Figure 3 for the case with a constant carbon price $p$.

Since $\beta < 1$ signifies that only some of the previously sequestered carbon would be released under the plough, for every rotation, the initial stock of soil carbon for re-contracting must be higher than it was at the beginning of the preceding sequestration

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**Figure 3.** Optimal sequestration-tillage rotation when only a fraction of the sequestered carbon is released due to one-time tillage at the end of each rotation.
contract. Hence, the optimal sequestration time must become shorter from one rotation to the next. According to equation (35), it is indirectly determined for the \( n \)th rotation by the optimal sequestration rate at the end of this rotation in the general form

\[
\phi(S^*_{tn}) = \frac{c + re^{-\tau} \left[ NPV_{n+1} - p\beta \int_0^{t_n} \phi(S_t)dt \right]}{(1 - e^{-\tau})p}
\]  

(36)

Given the fact that the optimal values for the \( n \)th rotation depend on the NPV of all the subsequent rotations, we get an infinite sequence of recursive dynamic equations. Thus, to numerically solve this problem suitable algorithms and adequate empirical data about the sequestration process and release rate will be required.

Since it would only be rational for a farmer to follow a sequestration-tillage rotation if the net present value of all the future rotations exceeds the instantaneous penalty for releasing carbon in case of tilling the soil, we have

\[
\phi(S^*_{tn}) > \frac{c}{p} = \phi(S^*_t)
\]  

(37)

Thus, the optimal rate of carbon sequestration at terminal time is larger under a contract that charges the farmer for releasing carbon than under a contract that does not involve any penalty. Accordingly, charging farmers for the release of carbon results in a lower stock of soil carbon before tillage and a shorter duration of an optimal contract.

4.2.3 The effect of charging farmers for the release of sequestered carbon

Altogether, we can remark that, even if a farmer would be charged for the release of sequestered soil carbon, it can be optimal to accept a contract that pays him for the additional amounts of soil carbon accumulated in course of time. However, the optimal duration of a sequestration interval – that is the time period until the first interruption of the sequestration process by tilling the soil, for instance – is shorter than in the case that only paid farmers for sequestration but would not charge them for the carbon releases. This follows from the fact that the expression on the RHS in equations (27) and (36) are both larger than the relative sequestration price \( c/p \), which determined the optimal sequestration level in the previous case without penalty for releasing carbon.
Consequently, the optimal stock of soil carbon at the end of the sequestration phase is smaller if farmers are charged for the release of carbon than under a contract that only subsidises farmers for sequestration. Moreover, a contract that compensates farmers for carbon sequestration and charges them for subsequent release provides an incentive to restart sequestration at the end of each cycle, such a visualised in Figures 2 and 3. Thus, an optimal sequestration contract can be designed with a flexible terminal time that is implicitly determined by optimal tillage time. Subsequently a new contract can start. Paying and charging farmers according to the absorption or emission of carbon, respectively, such a contract is in principle compatible with carbon trading schemes that determine market prices or carbon tax schemes that use intertemporal efficiency prices.

4.3 Optimal sequestration with an increasing carbon price

Since carbon accumulation and mitigation are a dynamic processes that imply gradual changes of relative scarcities over time, the carbon price will hardly remain constant (Falk and Mendelsohn, 1993; Sohngen and Mendelsohn, 2003; Veld and Plantinga, 2005). Rather, to ensure intertemporal efficiency, the carbon price must increase over time at a rate that can be referred to as “carbon discount rate”, an extension of the social utility discount rate which also accounts for the rates of technical progress and disappearance of atmospheric GHGs (Nordhaus, 1982).

To cope with this issue, we extend our analysis and let the carbon price exponentially increase over time. For simplicity, we assume that the rate of increase is equal to the discount rate \( r \), and set \( p_t = pe^{rt} \). The fraction of sequestered carbon that would be instantaneously released through one-time tillage is again denoted by \( \beta \), with \( 0 < \beta \leq 1 \).

Accordingly, the net present value of the first “sequestration-tillage cycle” changes to

\[
\tilde{V}_0^* = \int_0^{\infty} e^{-rt} \left[ p_t \phi(S_t) - c \right] dt - e^{-rT} \int_0^{\infty} p_{t+1} \phi(S_t) dt
\]

\[
= p \int_0^{\infty} \phi(S_t) dt - c \frac{1 - e^{-rT}}{r} - p \beta \int_0^{\infty} \phi(S_t) dt
\]

\[
= (1 - \beta) p \left[ S_0^* - S_0 \right] - c \frac{1 - e^{-rT}}{r}
\]

This value does not remain the same for each rotation. Rather, due to the rising carbon price, it increases over time. Assuming here for simplicity that the rotation lengths
remain unchanged (i.e., $\tilde{T}^* = \tilde{T}^*_n$ for all $n$), which is the case for $\beta = 1$, the value of the $n+1^{st}$ sequestration cycle starting at $n(\tilde{T}^* + 1)$ writes as follows:

$$
\tilde{V}_n^* = p e^{n(\tilde{T}^* + 1)} [\tilde{S}_{n+1}^* - S_0] - c \frac{1 - e^{-r \tilde{T}^*}}{r} > \tilde{V}_0^*
$$

(39)

With a rising carbon price and constant sequestration cost, it becomes increasingly beneficial to a farmer if he accepts the contract, even if he is charged for the release of carbon at the end of the rotation. However, this does not imply that a farmer must initially accept the sequestration contract. Rather, he may reasonably wait until the net present value of the first effective rotation exceeds the initial cost. In other words, he might reject any contract until the net payoff in the first sequestration phase covers the initial cost.12

For convenience, we define the point in time where the first accepted contract starts by $t = 0$, and the corresponding price by $p_0 = p$. Thus, the entry condition in the first period is $N \tilde{P} V_0 = \tilde{V}_0^* - I + \exp[-r(\tilde{T}^* + 1)]N \tilde{P} V_1 \geq 0$, with

$$
N \tilde{P} V_1 = \sum_{n=1}^{\infty} e^{-r(s-1)(\tilde{T}^* + 1)} \tilde{V}_n^*
$$

(40)

Moreover, the optimal length of the first sequestration phase is determined by

$$
\frac{dN \tilde{P} V_0}{d\tilde{T}^*} = \frac{d\tilde{V}_n^*}{d\tilde{T}^*} - r e^{-r(\tilde{T}^* + 1)} N \tilde{P} V_1
$$

$$
= p \phi(\tilde{S}_{T_0}^*) - c e^{-r \tilde{T}^*} - r e^{-r(\tilde{T}^* + 1)} N \tilde{P} V_1 = 0
$$

(41)

Rearranging this equation, we get the optimal rate of sequestration at terminal time $\tilde{T}^*$ of the first rotation:

$$
\phi(\tilde{S}_{T_0}^*) = \frac{c + r e^{-r} N \tilde{P} V_1}{p} e^{-r \tilde{T}^*}
$$

(42)

12 Notice that the farmer would not accept the contract if the carbon price increases at a rate above his discount rate: $\alpha > r$. As shown in the Appendix A, this would strictly result in a negative net present value of the contract.
Given equation (39) and thus

\[ p e^{r n(T_{n+1})} \phi(S_{n+1}^*) - c e^{-r T^*} - r e^{-r(T_{n+1})} N \tilde{P} V_{n+1} = 0 \]  

(43)

The optimal rate at the end of the \( n+1 \)st rotation is smaller than in the first rotation:  

\[ \phi(S_{n+1}^*) = \frac{c + re^{-r} N \tilde{P} V_t}{pe^{r n(T_{n+1})}} e^{-r T^*} \]

\[ = e^{-r(T_{n+1})} \phi(S_{n+1}^*) < \phi(S_{n+1}^*) \]  

(44)

Accordingly, under the above assumptions, the optimal terminal stock \( S_{n+1}^* \) increases from rotation to rotation, such that a continuous increase of the carbon price over time may bring about a gradual approach of the saturation level. However, given the non-linearity of the sequestration curve, the saturation level \( \hat{S} \) will never be achieved nor will the sequestered carbon be permanently kept in the soil as the optimal solution. Rather, a cyclical pattern with an extended sequestration period and one-time tillage at the end of the contract proves to be optimal, once the value of the contract in the initial phase exceeds the initial cost. Afterwards, it will be optimal for a farmer to repeatedly accept a new contract that pays him for sequestration and implies a charge for carbon release at the end of each rotation.

Altogether, a continuous increase of the carbon price, such as suggested from economic theory (cf. Nordhaus, 1982), does not result in permanent sequestration of carbon in the soil, if incentive payments are required to initiate land use or practice change. But, it makes soil carbon sequestration increasingly attractive to farmers and gradually extends the length of the sequestration period and the maximum stock of soil carbon. However, it does not eliminate the incentive for periodical tillage and release of some fraction of the sequestered carbon.

5 Conclusion

The use of biological sinks for GHG mitigation is limited by natural and economic forces. These include the non-linearity and saturation of the dynamic process of carbon

13 The results in Appendix B show that this result is he most likely to hold also fort he more general case with \( \beta < 1 \).
accumulation in the soil, which implies changes in relative costs and benefits of carbon offsets and embraces the problem of permanence, that is, the maintenance of the sequestered carbon in the soil. Like the sequestration process, permanence cannot in general be achieved without adequate incentives, such as payments that induce farmers to alter their land use and management decisions in order to increase the carbon content in their soils. Thus, the problem of saturation and permanence is not only relevant from a scientific and political point of view but especially from an economic perspective.

First, carbon sequestration is a problem of cost sharing and efficiency. An incentive scheme with permanent payment to maintain saturated carbon sinks over time would be extremely costly to society. Furthermore, economic efficiency requires that subsidies or compensation payments granted for sequestering additional amounts of carbon must be stopped at the latest when the level of saturation is reached. This would give farmers an instantaneous inducement to change behaviour and reverse the sequestration effect again, as soon as the payments are stopped. As a consequence, a major part of the sequestered carbon would be released into the atmosphere. In other words, permanence can hardly be achieved in the strict sense that the stock of carbon in the soil is permanently maintained at an increased equilibrium level. Rather, the theoretical analysis in this article proves that a cyclical behaviour with an infinite rotation of sequestration and periodical tillage might be economically efficient, both from a farmer’s and societal point of view.14

Second, carbon sequestration is a non-linear process with declining rates of soil carbon accumulation. An incentive scheme that aims at achieving economic efficiency therefore should grant payments according to these effective rates. As a consequence, a farmer’s decision is not only about the adoption of a particular sequestration practice, but also about the optimal terminal time of the individual sequestration program (contract). With positive marginal cost of sequestration, optimal timing always implies a positive rate of sequestration and a termination of the sequestration before the saturation level is reached, irrespective on whether farmers are charged for the release of

14 Notice that there are also scientific reasons for cyclical behaviour. It may, for instance, be necessary to periodically plough no-till soils to redistribute surface accumulations of phosphorus throughout the root zone (Sharpley et al., 1994).
sequestered carbon or not. But, when farmers are charged for the release of carbon, it would be optimal for them to till the soil at the optimal terminal time, pay the charge for the released amount of carbon, and then enter a new sequestration phase where they can receive sequestration payments again.

Third, to ensure intertemporal efficiency, the carbon price must increase over time at a rate which is equal to the “social carbon discount rate”. In this case, it could be optimal for a farmer to postpone the acceptance of a sequestration contract until the moment where the net present value of the first sequestration period exceeds the initial cost. Afterwards, a cyclical pattern with sequestration and tillage will be optimal for him, even if he is charged for the release of carbon. Moreover, the increasing carbon price induces an extension of the duration of the sequestration period from one rotation to the next. However, for efficiency reasons, the maximum stock of carbon in the soil remains always below the saturation level. This must be taken into consideration when assessing sequestration potentials in agricultural soils and designing policies to provide adequate incentives to farmers in order to alter their land use and management practices with the intention to foster GHG mitigation through soil carbon sequestration.

Altogether, it must be emphasised that, in general, permanence of soil carbon sequestration can only be achieved in a weaker sense that soil carbon is accumulated to an economically optimal level and, following perturbations (periodical release of sequestered carbon), achieved again at the end of each cycle of rotation, but not maintained intact over time. The optimal level that is periodically attained with a particular sequestration practice is below the maximum level of saturation, but increasing over time if the carbon price increases at a rate which is not above the farmers’ discount rate. Thus, the economic sequestration potential is restricted by the competing uses of agricultural land and due to the non-linearity and saturation of the carbon accumulation process in the soil. This must, in particular, be taken into consideration when designing policy schemes that aim at inducing an economically optimal portfolio of GHG mitigation measures.
References


Appendix A

If we shall assume that the carbon price increases over time at a rate $\alpha > r$ then the equivalent of equation (38) is

$$
\tilde{V}_0^* = p \int_0^{\tilde{T}_0^*} e^{(\alpha-r)t} \phi(S_t) dt - c \frac{1-e^{-r\tilde{T}_0^*}}{r} - pe^{(\alpha-r\tilde{T}_0^*+1)} \beta \int_0^{\tilde{T}_0^*} \phi(S_t) dt
$$

$$
= p \cdot (1-\beta) \int_0^{\tilde{T}_0^*} e^{(\alpha-r)t} - e^{(\alpha-r\tilde{T}_0^*+1)} \phi(S_t) dt - c \frac{1-e^{-r\tilde{T}_0^*}}{r}
$$

(A1)

Since $\alpha > r$, it is $e^{(\alpha-r)t} - e^{(\alpha-r\tilde{T}_0^*+1)} < 0$ for all $t \in [0, \tilde{T}_0^*]$. Consequently, we get $\tilde{V}_0^* < 0$, saying that the net present value of the sequestration program in equation (A1) is negative.

In contrast, if we assume $\alpha < r$,

$$
\tilde{V}_0^* = p \int_0^{\tilde{T}_0^*} e^{-(r-\alpha)t} \phi(S_t) dt - c \frac{1-e^{-r\tilde{T}_0^*}}{r} - pe^{-(r-\alpha(\tilde{T}_0^*+1))} \beta \int_0^{\tilde{T}_0^*} \phi(S_t) dt
$$

$$
= p \cdot (1-\beta) \int_0^{\tilde{T}_0^*} e^{-(r-\alpha)t} - e^{-(r-\alpha(\tilde{T}_0^*+1))} \phi(S_t) dt - c \frac{1-e^{-r\tilde{T}_0^*}}{r}
$$

(A2)

can either be positive or negative, depending on the relative prices and parameter values.
Appendix B

If we relax the assumption of a fixed duration of the different sequestration periods, equation (39) changes to

\[ \tilde{V}_1^* = (1 - \beta)pe^{r(\tilde{T}_0^1)}\left[\tilde{S}_{T_1} - S_1\right] - e^{1 - e^{-\tilde{r}_0}} \]  

(B1)

As a consequence, \( d\tilde{N}\tilde{P}V_1 / d\tilde{V}_0^* > 0 \), such that the first-order condition for the first rotation is

\[ \frac{d\tilde{N}\tilde{P}V_0}{d\tilde{T}_0^*} = \frac{d\tilde{V}_0^*}{d\tilde{T}_0^*} - re^{-r(\tilde{T}_0^1)}\tilde{N}\tilde{P}V_1 + e^{-r(\tilde{T}_0^1)}\frac{d\tilde{N}\tilde{P}V_1}{d\tilde{T}_0^*} = 0 \]  

(B2)

Accordingly, we get

\[ \frac{d\tilde{N}\tilde{P}V_0}{d\tilde{T}_0^*} = \frac{d\tilde{V}_0^*}{d\tilde{T}_0^*} - re^{-r(\tilde{T}_0^1)}\tilde{N}\tilde{P}V_1 + e^{-r(\tilde{T}_0^1)}\frac{d\tilde{N}\tilde{P}V_1}{d\tilde{T}_0^*} \\
= (1 - \beta)p\phi(\tilde{S}_{T_0}) - ce^{-r\tilde{T}_0} - re^{-r(\tilde{T}_0^1)}\tilde{N}\tilde{P}V_1 + e^{-r(\tilde{T}_0^1)}\frac{d\tilde{N}\tilde{P}V_1}{d\tilde{T}_0^*} = 0 \]  

(B3)

Accordingly, we get

\[ \phi(\tilde{S}_{T_0}) = \frac{c + e^{-r}\tilde{N}\tilde{P}V_1 - e^{-r}\frac{d\tilde{N}\tilde{P}V_1}{d\tilde{T}_0^*}}{(1 - \beta)p} e^{-r\tilde{T}_0} \]  

(B4)

and

\[ \phi(\tilde{S}_{T_1}) = \frac{c + e^{-r}\tilde{N}\tilde{P}V_2 - e^{-r}\frac{d\tilde{N}\tilde{P}V_2}{d\tilde{T}_0^*}}{(1 - \beta)p e^{r(\tilde{T}_0^1)}} e^{-r\tilde{T}_0} < \phi(\tilde{S}_{T_0}) \]  

(B5)

Assuming that the costs and benefits of postponing tillage by one time unit remains the same from one rotation to the next, the optimal rate of carbon sequestration at the terminal time of each rotation increases over time and the maximum stock of soil carbon gradually approaches the level of saturation. However, there is always an incentive for periodical tillage, such that permanence in the strict sense cannot be achieved.