RATIONALIZING TIME SERIES DIFFERENCES BETWEEN COW-CALF AND FEEDER RETURNS

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Abstract

This paper tries to justify the observation of different return patterns in the upstream and downstream sectors of US beef production. It builds a dynamic rational expectation model separating the cow-calf and feeding sector with the former sector being the residual claimer. The model shows that the cow-calf operation has positively autocorrelated return pattern while the feeding operation return only reflects random shock. Empirical study shows that 85.4% of the Ricardian rent is passed through to the upstream sector, and the downstream sector can only claim the unexpected return resulting from random shocks.
1 Introduction

Beef is the single largest sector within United States agricultural production, accounting for a fifth of farm market revenues. Unlike other animal products, only a small share of output is produced under vertically integrated arrangements. The industry divides between grass-based cow-calf operations during the first year of a beef animal’s life and grain-based feeder operations during the months preceding slaughter. Cow-calf sector, which produces calves that go into feedlots, is mostly pasture based. The cattle feeding sector purchases the feeder cattle from the open market, and use corn and other concentrates to finish animals for slaughter. The two sub-sectors also differ in regards to financial performance. Using data from the Livestock Marketing Information Center\textsuperscript{1}, figure 1 in appendix provides time series of returns for the two sub-sectors. Casual inspection suggests the former reflects strong positive autocorrelation in returns over time while the latter may be close to white noise. This difference can be viewed as the motivation for this paper. Our investigation of the link between the upstream and downstream of cattle industry will help understand the production decision mechanism. The results are important in explaining why the cattle feeding sector is relatively immune from demand and supply-side shocks whereas cow-calf operations are more exposed, a phenomenon well observed in the beef industry\textsuperscript{2}.

It is well known that beef production differs from other sectors because of long lags in production responses. The beef industry can not respond to a price signal quickly, but rather needs years of time to adjust the breeding stock. Producers make

\begin{itemize}
\item \textsuperscript{1}The Livestock Marketing Information Center (LMIC) is a institute providing economic analysis and market projections concerning the livestock industry since 1955. Return data used here include annual return data for cow-calf operation and feeding operation, ranging from 1975 to 2007.
\item \textsuperscript{2}USDA Economic Research Service has made great contribution to the understanding of different effects of feed cost on these two sectors. One good example is the research report by Stillman, Harley and Mathews in 2009, which indicates that the cow-calf operation is less affected by the current high feed cost.
\end{itemize}
decisions to expand or contract production before feed and product prices are known. Biological lags mean that animal products consumed today are based on production decisions made up to 2 years ago. Cow-calf operators make production decisions by choosing between calf sale for fattening and retention for breeding, that is a choice between consumption goods and capital goods.

In addition to this dynamic constraint, the cow-calf sector also differs from other production sector by the scarcity of suitable pasture for the cattle to graze on. The distribution of cow-calf operation region is illustrated in Figure 2. This map groups the cow-calf operation according to regions based on the survey of Agricultural Resource Management Study (ARMS). By this survey, cow-calf operators in the west and southern plains have significant cost advantages over operators in other regions due to the longer grazing season. The two regions account for 50 percent of the production of weaned calves. 3 The suitable pasture land for cow-calf operation is a scarce resource that can not be replicated.

By contrast, the feeding operation does not have such properties. It takes around 160-180 days to finish the fattening process, a much shorter time than the production of feeder cattle. Except for feeder cattle, the main cost for cattle feeding is corn and other feed grain, which can be purchased freely on commodity markets. The feeder sector allows free entry and exit. Based on this fact, we assume the cow-calf sector will obtain the Ricardian rent from beef sales, be it positive or negative. The Ricardian rent is passed to cow-calf sector through the price of feeder cattle. Feeder cattle prices are affected by prices paid for fed cattle which, in turn, are affected by consumer demand for beef as reflected in retail beef prices. At the time of the feeder cattle transaction, bid for cattle feeder will drive up the feeder cattle until there is zero expected economic profit. Since the futures market for live cattle is very mature,

3See USDA statistical bulletin report number 974-3.
market information is available to all the participants who utilize this finance tool to make hedging. With the assumption of full incidence pass through, we extend existing dynamic models of beef market equilibrium to rationalize the difference.

A substantial amount of progress have already been made in understanding cattle cycles. Javis (1974) was among the first to point out that a permanent increase of beef price might reduce the live cattle supply, and hence brought attention to how cattle investment decisions interact with biological production lags in the cattle cycle. Along the same line, Rucker, Burt and LaFrance (1984); Foster and Burt (1992); Rosen (1987) developed models to explain how the biological structure affect cattle supply by treating live cattle both as consumption and capital goods. Particularly, Rosen (1987) stripped away most of the details and focused on the exogenous shock's effect on the formation of cattle cycles. Rosen, Murphy and Scheinkman (RMS hereafter) (1994) extended this model to a more complete biological structure, and implied clear cyclical pattern for breeding stock and live cattle price.

Heterogenous expectation also attracted scholar's attention when trying to interpret the cattle cycle. Both Baak(1999) and Chavas (1999) tested for different forms of bounded expectations and estimated the weights of operators with these expectations. Baak's study found that approximately one-third of ranchers appear to have bounded rationality in the sense that they forecast future prices based solely on time series observations. Chavas found that less than one-fifth of cattle producers appear to behave consistently with full rational expectations. But as argued in Aadland (2001), despite of these empirical evidences, rationality in expectation formation is still mostly favored by economist seeking to explain the aggregate cattle stock behavior. The evidence on heterogenous expectations is not strong enough to reverse the conclusion made under rational expectation. So, in this study, we adhere to the rational expectation formation.
So far, most of the literature about cattle cycles does not separate the cow-calf and feeder sectors in the beef supply chain. One exception is the work by Aadland (2001), (2004), and (2005). Aadland distinguished the fed beef price from unfed beef price, and hence proposed two margin problems for cow-calf operators. Under this framework, producers will respond positively to relatively higher prices along one margin and will build up stocks along the other margin. Despite this segregation, the feeder sector was still ignored, and hence the interaction between the two sectors was not considered.

To investigate this interaction issue, our work employs the idea of Ricardian rent theory (RRT hereafter). In RRT, rent is defined as “that portion of the produce of the earth, which is paid to the landlord for the use of the original and indestructible powers of the soil” (Ricardo 1821, p. 67). Economic theory suggests that extra production profits resulting from high beef prices will ultimately accrue to the cow-calf operators because breeding stock as well as the suitable pasture land is the most limiting resource in beef production. However, the RRT is quite challenged in the recent study in farmland rent and price. Kirwan(2008) estimates that only 25 percent of the government subsidy will finally flow to the landlord. Du, Hennessy and Edward (2008) finds little support of RRT when examining the crop price increase effect on cropland rent. They attribute the failure of RRT to the lack of mobility for tenants and inertia in leasing contract re-negotiations. However, compared with tenants, feeder cattle are easier to transport and the feeder cattle market is quite liquid, which implies the failure of RRT reasons might not exist in the cattle industry.

The paper is organized as follows. Section 2 sets up the dynamic rational expectation model including two sectors, with an explicit form of the two sectors’ return derived. Section 3 tests the RRT using the live cattle futures price. Also in section 3, we have a formal test of the return’s pattern, and the implications from the model.
Section 4 concludes by summarizing the main findings of the paper and suggesting avenues for further research.

2 Theoretical Model

2.1 Background and Main Assumption

This paper clearly builds on the aforementioned work of Rosen (1987) and Aadland (2001). Before introducing the model setup, it is necessary to formally outline the environment being modeled. The separation of cattle life is a relatively recent phenomenon. Prior to the 1930s, feeding of high concentrate grains was rare and most cattle lives on the pasture or harvested forage for the whole life. Since then, the practice of finishing feeder cattle on grains has become commonplace and in more recent times (beginning in the 1960s), finishing has gravitated toward organized feedlots.

Within the first six months after the calf is born, there are few decisions to make. After weaning, a calf is typically six to ten months old. If it is male, the calf will most likely be castrated and sent to feeding lot later, with only a small portion left for breeding purpose. The problem for the female calves is complicated since it is a consumption good and also a capital good. Cow-calf operators need to decide whether to retain the female calf for addition to the breeding stock (capital good) or sell them for beef production (consumption good).

The calves for consumption will then be sold as feeder cattle in the open market. They will first go through the so called finishing process for four to six months. After this stage, the animal will reach the final stop, the feedlot, where they will be fed high-concentrate grains for approximately six months to be fattened for slaughter.
So, generally there are two main stages for a typical beef animal’s life. Roughly speaking, a beef cattle will grow up under two operations, cow-calf operation and feeding operation, with each one accounted for one year time. On the other branch, breeding cattle will be first bred when they are fifteen months old. The gestation period will last for nine months. So, it takes around two years for a calf to give birth to its offspring. Two years is also the age at which a meat animal is ready to be slaughtered.

A dynamic rational expectation model is set up to capture the essential components in the beef supply chain. The key to this model is the interaction between the two sectors through the pricing of feeder cattle. As discussed in the introduction, since breeding cattle sector faces dynamic constraint, and the suitable pasture land is inelastic as well, we will assume the cow-calf operation will obtain all the extra profit from beef production. With rational expectations of all the market participant, the feeder cattle price will be bid up when there is positive expectation concerning forward beef markets, and will be bid down when the forward beef markets are depressed.

The model is set in discrete time with decision intervals one year in length. The biology structure is assumed to be consistent with the reality. The cow-calf operators make decision when the calf is one year old. The calves reserved for retention will be added into the breeding stock while the feeder calves will be sold to the feedlot and enter the beef market in the following year. Because of separation of two sectors, there are two prices, feeder cattle price and beef price. To make the problem tractable, we assume the breeding cow has the same value before and after giving the first birth. This setup is different from Aadland (2001) which distinguished between fed and unfed beef price but ignores calf price. This simplification will not change the main conclusions of this paper if the fed and unfed beef is highly correlated, but will provide great convenience for the model setup.
For the market participants, we assume cow-calf operators to be forward-looking, rational agents that maximize a discounted expected future stream of profits subject to biological and market constraints. The feedlot operators have the same rational expectation as cow-calf operators, but they are take-it-or-leave-it participants that can freely enter and exit the market. We assume that operators in each type are identical and make decisions in competitive input and output markets.

Properties of market equilibrium are established by analyzing the activities of a representative cow-calf operator and a representative feedlot operator. Consistent with other animal cycle models, the present and future production possibilities are linked by a population dynamics constraint, which gives the trade off between current consumption and potential future consumption. The main difference from Rosen (1987) is that the cow-calf operators now have to make beef production decisions one year earlier by selling a fixed number of feeder calves to feedlots one year earlier.

The model is determined by a stochastic difference equation and the shocks come from three aspects. Two types of shocks originate on the supply side, the holding cost of breeding cattle, $h_t$, as well the finishing and marketing cost of feeder cattle, $m_t$. The other shock comes from the demand side, the income level shock $y_t$. As with Rosen (1987), we simplify the model setup by abstracting from sex and life-cycle aspects of herd management, assuming a homogeneous female population with a biologically determined constant birth rate.\footnote{As noted in Rosen (1987), p 548} Table 1 gives the connection between this model and that of Rosen (1987).
3 Dynamic constraints:

Consider a closed economy, the growth of breeding cattle stock $x_t$ is determined by two parts, the addition of total new born calves $gx_t$, and the deletion of sold calves $s_t$. So $gx_t - s_t$ is the net addition of calves, and they will grow to be the breeding stock in the next period. The feeder calves sold to feedlot will go through finishing and fattening, and end up in beef market in the next period. This evolution of cattle stock is shown in equation (1):

$$x_{t+1} = (1 + g)x_t - s_t$$  \hspace{1cm} (1)

with $x_0$ given and $s_t \geq 0, x_t \geq 0$ for all time points $t \in \{0, \infty\}$. We can solve equation (1) by forward substitution. From equation (1), we can get $x_t = \frac{s_t}{1+g} + \frac{x_{t+1}}{1+g}$. Then forward this result for one period, we can get $x_{t+1} = \frac{s_{t+1}}{1+g} + \frac{x_{t+2}}{1+g}$. Substitute $x_{t+1}$ into the expression of $x_t$, we can get $x_t = \frac{s_t}{1+g} + \frac{s_{t+1}}{(1+g)^2}$. Repeat this process, we can get the following complete intertemporal constraint:

$$x_t = \sum_{\tau=0}^{\infty} \frac{s_{t+\tau}}{(1 + g)^{\tau+1}}$$  \hspace{1cm} (2)

Also given the available information of period $t$, take expectation to both side of equation (2) implies:

$$x_t = E_t \sum_{\tau=0}^{\infty} (s_{t+\tau})/(1 + g)^{\tau+1}$$  \hspace{1cm} (3)

3.1 Market Equilibrium

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Market equilibrium is achieved through the dynamic decisions made by cow-calf operators as well as static decision of feedlot operators. Given the feeder cattle price and the rational expectation, the market supply and demand of feeder cattle must equate. Looking at the cow-calf operation side, cow-calf operator’s profit per animal is defined as

$$\pi_{t}^{cc} = q_{t}s_{t} - h_{t}x_{t+1}$$

(4)

where $q_{t}$ is the price of feeder cattle sold to feedlot, and the superscript cc stands for cow-calf. The cow-calf operator’s return per cattle can be written as:

$$R_{t}^{cc} = q_{t} - h_{t}$$

(5)

where $R$ is for returns and the superscript is for cow-calf. As assumed, cow-calf operators will make the reproduction decision, and they will get the Ricardian rent, which drives feedlot’s expected profit to be zero. At time point $t$, feedlot operator purchases $s_{t}$ feeder calves from cow-calf operators and sell them to the beef market in period $t+1$. The feedlot’s return per cattle can be written as:

$$R_{t}^{fd} = (p_{t+1} - m_{t+1})(1 + r) - q_{t}$$

(6)

where $p_{t+1}$ is the fed-cow price in the beef market, $m_{t+1}$ is the finishing and marketing cost, so $(p_{t+1} - m_{t+1})(1 + r)$ is the discounted revenue from one cow, while $q_{t}$ is the cost for purchasing a yearling. Ricardian rent theory implies that the time $t$ expected return is 0, so from equation (6), we can get:

$$q_{t}(1 + r) = E_{t}(p_{t+1} - m_{t+1}) \equiv p_{t+1}^{*} - m_{t+1}^{*}$$

(7)
where $E_t$ is the expectation operator at period $t$, and the notation $p^*_{t+1} = E_t(p_{t+1})$ is used to be consistent with Rosen (1987). At this stage, it’s necessary to sum the notation used for this model setup, as shown in table 2. Under this model setup, cow-calf operators will face a dynamic problem, and maximize the sum of discounted life-time profit:

$$V_t = E_t \sum_{\tau=0}^{\infty} \pi^\tau_{t+\tau}/(1+r)^\tau$$

(8)

The solution to this problem is characterized by the Euler Equation that make the cow-calf operators indifferent between holding and selling, that is:

$$q_t = \frac{1}{\beta} E_t(q_{t+1} - h_{t+1})$$

(9)

where $\beta = \frac{1+r}{1+g} < 1$

To get an analytical solution to this problem, we suppose the demand of beef follows a linear demand function:

$$b_t = \alpha - \gamma p_t + y_t$$

(10)

where in this demand function, $b_t$ is the demand for fed cow at period $t$, $p_t$ is the price for fed cow in beef market, $y_t$ is the demand shifter for fed cattle. As assumed, the supply of fed cow in period $t$ comes from the sold feeder cattle in period $t-1$, that is, $b_t = s_{t-1}$. So we can rewrite the beef demand function in terms of feeder cattle: $s_t = \alpha - \gamma p_{t+1} + y_{t+1}$. Take expectation given all the information at time $t$, we can get:

$$s_t = \alpha - \gamma p^*_{t+1} + y^*_{t+1}$$

(11)
where $p_{t+1}^* \equiv E_t(p_{t+1})$, $y_{t+1}^* \equiv E_t(y_{t+1})$ are the expected beef price and demand shifter in year $t+1$, which is consistent with Rosen (1987)’s notation. By assumption, the feedlot makes zero expected profit from finishing operation. Substitute equation (7) into equation (11), we can get the sold amount $s_t$ in terms of yearling’s price $q_t$:

$$s_t = \alpha + y_{t+1}^* - \gamma [q_t(1 + r) + m_{t+1}^*]$$  \hspace{1cm} (12)

Using the law of expectations, equation (12) can be rewritten as:

$$s_{t+\tau}^* = \alpha + y_{t+\tau+1}^* - \gamma [q_{t+\tau}^*(1 + r) + m_{t+\tau+1}^*]$$  \hspace{1cm} (13)

With the same approach, we can rewrite the Euler equation (9) as:

$$q_{t+\tau}^* = \beta (q_{t+\tau+1}^* + h_{t+\tau+1}^*)$$

where the price of q at the beginning is given. Solve this by forward substitution, $q_{t+\tau}^* = \beta q_{t+\tau-1}^* + \beta h_{t+\tau-1}^* = \beta^2 (q_{t+\tau-2}^* + h_{t+\tau-2}^*) + \beta h_{t+\tau-1}^*$. Repeating this process, we can get the expected future feeder cattle price in terms of current yearling price and expected holding cost. As shown in equation (14), it pins down the optimal path for cow-calf operator.

$$q_{t+\tau}^* = \beta^\tau q_t + \sum_{i=1}^{\tau} \beta^\tau h_{t+\tau-i}^*$$  \hspace{1cm} (14)

Collecting equations, the competitive market equilibrium is described by equation (14), (13) and the intertemporal budget constraint (3).
3.2 Solving the model

To illustrate the recursive property of the model, it’s convenient to express the variables in the deviation form. Equation (15) expresses the shocks in such a form, where the bar_expressions are "normal" values and $u_t^j$’s are deviation from normal.

$$y_t = \bar{y} + u_t^y, \ m_t = \bar{m} + u_t^m, \ h_t = \bar{h} + u_t^h$$  \hspace{1cm} (15)

Following this, define the capital notation as the deviations from the normal level:

$$X_t = x_t - \bar{x}, \ S_t = s_t - \bar{s}, \ Q_t = q_t - \bar{q}$$  \hspace{1cm} (16)

With this deviation form, we can rewrite equation (16) as follow:  

$$X_t = -\frac{\gamma(1+r)}{1+g-\beta}Q_t + \sum_{\tau=0}^{\infty} v_{t+\tau}^*/(1+g)^{\tau+1}$$  \hspace{1cm} (17)

$$Q_t = \frac{1+g-\beta}{\gamma(1+r)[-X_t + \sum_{\tau=0}^{\infty} v_{t+\tau}^*/(1+g)^{\tau+1}]}$$

$$S_t = (1+g-\beta)X_t - (1+g-\beta)\sum_{\tau=0}^{\infty} v_{t+\tau}^*/(1+g)^{\tau+1} + u_{t+1}^y - \gamma u_{t+1}^m$$

$$X_{t+1} = \beta X_t + (1+g-\beta)\sum_{\tau=0}^{\infty} v_{t+\tau}^*/(1+g)^{\tau+1} - u_{t+1}^y + \gamma u_{t+1}^m$$

where $v_{t+\tau}^* = u_{t+\tau+1}^y - \gamma u_{t+\tau+1}^m - \frac{(1+r)\gamma u_{t+\tau}^*}{1+g-\beta}$

Notice that, compare with Rosen (1987)\(^6\), the difference is that the effect of demand shifter $y$ and feedlot's feeding cost $m$ comes from time $t + 1$, while the effect of cow-calf operator's holding cost is the same. This difference comes from the as-

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\(^5\)Please refer to Appendix A

\(^6\)the counterpart can be found in the Appendix D.
sumption change that yearling’s price is determined by the expected beef price in next year. Assume that all the shocks evolves as serially correlated processes with parameter $\rho_j$:

\begin{align*}
    u_{t+1}^y &= \rho_y u_t^y + \varepsilon_{t+1}^y \\
    u_{t+1}^m &= \rho_m u_t^m + \varepsilon_{t+1}^m \\
    u_{t+1}^h &= \rho_h u_t^h + \varepsilon_{t+1}^h
\end{align*}

(18)

where the $\varepsilon$’s are pure noise. This implies $u_{t+\tau}^i = \rho_j^\tau u_t^i$, substitute into equation(17), we can get:

\begin{align*}
    Q_t &= \frac{1+g-\beta}{\gamma(1+r)} [-X_t + \frac{\rho_y}{1+g-\rho_y} u_t^y - \frac{\gamma \rho_m}{1+g-\rho_m} u_t^m] - \frac{\beta}{1+g-\rho_h} u_t^h \\
    S_t &= (1+g-\beta) X_t + \frac{\rho_y(\beta-\rho_y)}{1+g-\rho_y} u_t^y - \frac{\gamma \rho_m(\beta-\rho_m)}{1+g-\rho_m} u_t^m + \frac{\beta \gamma (1+r)}{1+g-\rho_h} u_t^h \\
    X_{t+1} &= \beta X_t - \frac{\rho_y(\beta-\rho_y)}{1+g-\rho_y} u_t^y + \frac{\gamma \rho_m(\beta-\rho_m)}{1+g-\rho_m} u_t^m - \frac{\beta \gamma (1+r)}{1+g-\rho_h} u_t^h
\end{align*}

(19) \hspace{1cm} (20) \hspace{1cm} (21)

Compare this solution with Rosen’s (1987), we can see the main difference is that the effect of shock $u_t^y$ and $u_t^m$ is weakened through multiplying by $\rho_j$, while the effect of shock $u_t^h$ is magnified through multiplying by $(1+r)$. The reason in the first change

7Please refer to Appendix B

8the counterpart can be found in the Appendix D
lies in the structure of $u_{t+1}^y$ and $u_{t+1}^m$. With these equations, we can back out the path for the profits which are of our interest.

### 3.3 Result

In this subsection, we will derive the main conclusions of the model. Under the assumption of Ricardian rent incidence on cow-calf operator, feedlot will earn a zero expected profit. It’s easy to see the realized return of feedlot is random, and there is no recursive property. Substituting equation (7) into equation (12), we can get

$$q_t = \frac{1}{1+r}(\frac{\alpha + u_{t+1}^y - s_t}{\gamma} - m_{t+1}^*).$$

Inserting this into the return function of feedlot (6), we can get:

$$R_{t}^{fd} = \frac{1}{1+r}(\frac{\alpha + y_{t+1} - s_t}{\gamma} - m_{t+1}) - \frac{1}{1+r}(\frac{\alpha + y_{t+1}^* - s_t}{\gamma} - m_{t+1}^*)$$

$$= \frac{\varepsilon_{t+1}^y/\gamma - \varepsilon_{t+1}^m}{1+r}$$

As both $\varepsilon_{t+1}^y$ and $\varepsilon_{t+1}^m$ are pure random variables, the profit for feedlot is also a random variable. Also, the expectation of this return is 0. This is the first conclusion of this paper:

**Proposition 1** The realized return of feedlot is random and only affected by the shock from demand shifter side and finishing feed cost.

Then look at the returns of cow-calf operator. Follow equation (5), we can rewrite the returns of cow-calf operators in deviation form:

$$R_{t}^{cc} = Q_t - u_t^h$$

$$= \frac{1 + g - \beta}{\gamma(1+r)}[-X_t + \frac{\rho_y}{1 + g - \rho_y} u_t^y - \frac{\gamma \rho_m}{1 + g - \rho_m} u_t^m] - \frac{1 + g + \beta - \rho_h}{1 + g - \rho_h} u_t^h$$

15
As $X_t$ has recursive property as shown in equation (21), the return of cow-calf operator’s profit also has recursive property. Also, notice that, if the correlation $\rho_j$ is small, which means the shock is temporary, the effect from demand shifter and finishing cost can be very small, while the effect from holding cost can’t be negligible.

To make this paint more clearly, we can solve the explicit form of cow-calf operator return.\(^9\). The recursive form for $R_t$ can be written as

$$R_{t+1}^{cc} = \beta R_t^{cc} +\lambda u_t^h + \Psi_{t+1}$$ \hspace{1cm} (24)

where $\lambda = \frac{(1+g+\beta-\rho_h)(\beta-\rho_h)}{1+g-\rho_h}$, $\Psi_{t+1} = \frac{1+g-\beta}{\gamma(1+r)} \frac{\rho_y}{1+g-\rho_y} \varepsilon_{t+1}^y - \frac{\gamma \rho_m}{1+g-\rho_m} \varepsilon_{t+1}^m$.

Equation (24) confirms our observations of returns of cow-calf operators, and this is our second proposition:

**Proposition 2** The return of cow-calf operation has first-order positive autocorrelation.

Examine this clear form of cow-calf operator’s return, we can get two corollaries:

**Corollary 3** The deviation level of beef demand shifter and finishing feed cost does not affect the cow-calf operator’s return in next period.

This is not a surprising result following previous assumptions. As the Euler equation (9) states, the price for selling the yearlings this period should equal the expected payoff from holding these yearlings and selling the new yearlings in next period’s market. Rewrite the Euler equation in the return form, we can get:

$$R_t^{cc} + u_t^h = \frac{1}{\beta} E_t( R_{t+1}^{cc} )$$ \hspace{1cm} (25)

\(^9\) Please refer to Appendix C.
So we can see that the demand shifter and finishing cost has dropped out from the optimal path for cow-calf operator’s return. The effect of two shocks on cow-calf operation only comes from the unexpected random term, as in $\Psi_{t+1}$. So, the cow-calf operation’s return is largely shielded from the price fluctuation of the two shocks.

This result comes from the rational expectation assumption. The three shocks have autocorrelation structure as defined in (18). So, at the time when feeder cattle is set the part of shocks $u_{t+1}$ coming from the correlation with current shock $\rho \mu_t$ has already been expected. This feeder cattle’s price, in turn, is reflected in time t’s return $R_{cc}^{t}$. So, current shock’s level $\mu_t$ will have no influence on the return structure when $R_t$ is also present.

**Corollary 4** The effect of holding cost on the return pattern can go either way, which is determined by the difference by $\beta - \rho_h$.

From the definition of $\lambda$ in equation (24), we can see the magnitude of $\lambda$ depends on the sign of $\beta - \rho_h$. If $\beta > \rho_h$, then $\lambda > 0$, and a high level of holding cost deviation from normal level in this period can bring a high level of cow-calf operation’s return. If $\beta < \rho_h$, $\lambda$ can be a small positive number or even negative, which means the high holding cost induces a low cow-calf operation’s return in next period. Particularly if $\beta = \rho_h$, $\lambda$ is degenerated to $\beta$, the net discount rate.

This result is also intuitive as can be seen from return form of Euler equation (25). If the holding cost is very high in the current period, it is optimal to get a higher return in next period in order to compensate this high cost. If the next period’s expected return is not high enough, cow-calf operators will sell more of feeder cattle at this time point, which will cut the supply capability in next period. This cut in supply in the next period will drive the expected return of cow-calf operation up in the next period. This compensation effect is captured by the net discount rate $\beta$. On
the other hand, if the autocorrelation of holding cost is large, a high holding cost this period means a good chance for high holding cost in the next period. So it is likely that the cow-calf operators realized returns will be small due to the high cost of feed or forage cost.

4 Empirical Work

The work in this section is in three folds. The first one is to test Ricardian rent theory, the base of this model. The second one tests the causal observation of different return pattern. The third one examines the implication from the theoretical work.

4.1 Ricardian Rent Theory Test

The model is largely based on the assumption that the Ricardian rent is passed to the Cow-calf operators through the price of feeder cattle. If Ricardian theory is correct, then the increase of market expectation for fattened cattle would bid up spot feeder cattle prices. The first data source is Chicago Mercantile Exchange (CME), which provides daily data of live cattle(lc) and feeder cattle(fc). The second data source is National Agricultural Statistics Service (NASS) under United States Department of Agriculture. Data found from NASS includes the monthly data of corn’s price, feed grain and hay index, Consumer Price Index. All the data are reported regularly in NASS’s monthly agricultural report. The time span of all the data goes from Jan 1979 to Feb. 2009. Altogether there are 362 samples for monthly data and 30 samples for annual data. Daily data are transformed to be monthly data by taking an arithmetic average. So, for the live cattle and feeder cattle futures, monthly data are used.

We need to further transform the available data to fit the purpose. Firstly of the test, we use the nearest maturing cattle feeder future price to substitute cattle
feeder’s spot price $P_{fd}^t$, ignoring the basis between the two prices. Secondly, assume the farmers could use live cattle’s futures price to lock in a certain price when the cattle is ready for slaughter. As reported by Iowa Beef Center, it typically takes 6 or 7 months for a calf to grow up to a steer. As the CME live cattle futures contracts are only settled in even months (like February, April and so on), we suppose in even month it takes 6 months for finishing while in odd months it takes 7 months so that when the fed cattle is mature there is a corresponding price. By this rule, we can get a series of live cattle future prices $F_{t,t+s}^{lc}$, the one matured when cattle are fattened. Here the upper script lc stands for live cattle while $s$ is the time needs to fatten cattle. Thirdly, we use the monthly corn price to represent the cost for feeding. As reported by USDA\textsuperscript{10}, 90 percent of feeding cost comes from corn. A preliminary test confirms that soybean price has no significant effect in feeder cattle’s price.

By the futures specifications of CME in 2009, the feeder cattle midpoint weight is 749.5 pounds while it is 1262.5 pounds for live cattle. So, we need to transform the "per animal expression" of RRT in equation (7) to the per pound expression. The corresponding regression is shown in as follow:

$$P_{fd}^t = \beta_0 + \beta_1 F_{t,t+1}^{lc} + \beta_2 P_{cn}^t + \varepsilon_t \quad \text{where } \varepsilon_t \sim N(0, \sigma^2) \quad (26)$$

Let the half year discount rate to 5%, and take into account the death rate of feeder calves as 1.2\textsuperscript{11}, then RRT implies $\beta_1 = \frac{1262.5}{(1+0.012)(1+0.05)*749.5} = 1.58$. The estimate of the coefficient $\beta_1$ will indicate the proportion of Ricardian rent passed to the cow-calf operator.

As summarized in Wang and Tomek (2007), the unit root is a common problem in the commodity price, especially the nominal price. Also, different unit root test

\textsuperscript{10}Please refer to the report by Stillman, Haley, and Mathews in 2009.

\textsuperscript{11}1.2% is the mean level of death rate in feeding process as reported by Food Link.
approaches do not agree in many cases. To establish that the regression is not spu-
rious, we subject the data to a detailed unit root test, and the results are listed in
table 3. Notice that, the hypothesis for ADF and P-P methods is the existence of
unit root while the hypothesis for KPSS is that the time series is stationary. The test
result shows there is strong evidence that the variables have unit root problem and
are not stationary time series. Despite of this news, we could do the cointegration
test to test whether the linear combination of these variables are stationary. Table 4
gives positive information with all the three variables having significant evidence of
cointegration. So, the three variables have inherent correlation, and the regression
result will not be spurious.

Using OLS to estimate (26) could give us a flavor of how the model works. Figure
3 presents the ACF and PACF of the OLS residual. From this ACF figure, we can
see there is strong seasonal effect with a seasonal lag of 12 months. This seasonal
effect is commonly observed in agricultural commodities, which is largely affected
by the weather and timing. The PACF figure suggests that the residual has strong
AR(1) correlation, as the PACF cuts off from lag 1. This result is consistent with
the theoretical work. Since the feeder cattle is the main revenue source for cow-
calf operations, the cow-calf return’s AR(1) structure implies the feeder cattle price
may also have AR(1) structure. To specify the model correctly, we need to remove
the seasonal and AR(1) correlation from the residual. A two step FGLS method is
employed to remove this correlation. In the first step, we run the SARMA model over
the OLS residual, with the SARMA structure shown as follow:

\[
(1 - \Phi_1 L^{12})(1 - \theta_1 L)\omega_t = \varepsilon_t
\]

With the coefficient estimates \(\widehat{\Phi}_1\) and \(\widehat{\theta}_1\), we can transform the regressors in equa-
tion (26). The new regressors \( y_t \) is defined as 
\[
y_t = (1 - \Phi_1 L^{12})(1 - \theta_1 L) x_t,
\]
where \( x_t \) is the original regressor. Then run OLS over this new regressors to get the estimate of \( \beta \). Referring to Greene(2007), there is no gain by iterating this process. As can be seen in table 5, the D-W test is close 2, implying that the residual has no correlation problem. Figure 4 compares OLS and FGLS residual diagnosis, it is clear that FGLS approach has removed the residual correlation problem from OLS estimation.

Table 5 lists the estimate for \( \beta \) using FGLS. Especially, the estimate for \( \beta_1 \) is 1.35, and the 5% level confidence interval for \( \beta_1 \) is \([1.25, 1.45]\). Compared with the ideal value of 1.58, the estimate value of 1.35 suggests that 85.4% of the increase of future live cattle’s price will transfer to the current feeder cattle. The confidence level for this estimate ranges from 78.1% to 90.6%. In another word, about 85.4% of the Ricardian rent is passed to the cow-calf operation. Compared with the 25% pass through ratio in the crop subsidy, 85.4% is significantly large. This indicates that the Ricardian rent incidence on cow-calf operators is a plausible assumption.

\section*{4.2 Different Return Patterns for Cow-calf And Feeding Sect-ors}

\textbf{Feedlot Return}

This subsection will seek to verify the casual observation from figure 1 that feedlot have random return while cow-calf operations have positive autocorrelation. This is also the main conclusion of our model.

The RRT implies that the feeding lot should have zero expected profit. If the market participants have rational expectation, the realized return should be consistent with this expectation, that is, the realized return has a zero mean. The t-test result in table 6 indicates that we can’t reject the zero mean hypothesis of feeding sector’s
return. So, the feeding sector does not earn a significant positive profit over the last thirty years.

If the market forms rational expectations, the realized return to feedlot should be pure white noise, without any correlation pattern. A complete correlogram can illustrate this test result well, which is summarized in table 7. For the lags up to 10, there is no autocorrelation or partial autocorrelation is significantly different from 0, and the corresponding p-values fail to reject the null hypothesis. So, we can say feeding sector’s return is just a series of random variable, without clue to show any correlation.

**Cow-calf Return**

As talked before, the cow-calf sector should have positively correlated returns. We can test this property by fitting an ARMA structure to the data. Before running any regression, we need to make sure this time series is stationary. A test for unit root in cow-calf return is listed in table 8. It shows that the unit root hypothesis is rejected at a 5% level for both of the two test methods. So, it is safe to run a regression over the undifferenced data.

As shown in the model, cow-calf sector’s return has first order positive autocorrelation. We will use several models to fit the data, and test the AR(1) coefficient respectively. Suppose the most general model has the following form, with $\beta_i$ as the AR(i) coefficient and $\gamma_i$ as the MA(i) coefficient:

$$R^1_t = \beta_0 c + \beta_1 R^1_{t-1} + \beta_2 R^1_{t-2} + \beta_3 R^1_{t-3} + \gamma_1 \varepsilon_{t-1} + \varepsilon_t$$

(27)

The test result is summarized in table 9 with several criterion listed to compare the performance of these models.

Firstly, notice that $\beta_1$ is the only parameter that is significant through different
models. This strongly suggests that the first order correlation is significant, which is consistent with the theoretical analysis. Secondly, both the AIC and S-C criterion shows that AR(2) model best fits the data. Also, the S-C criterion suggests that AR(1) model is the second best model to fit the data. But the D-W test indicates that the residual term of AR(1) model still has some correlation not explained by the model. Another approach to compare the performance of different model is to look at the AC and PAC graphs, which are listed in Figure 5. The pattern of AC and PAC also suggest that a higher order of autocorrelation term is preferable than the AR(1) model. In a word, the data suggest that cow-calf sector’s returns have strong first order correlation, but higher order correlation is still possible. And it helps to explain the data better.

So, this section’s empirical work verifies the casual observation about the different return pattern. It also provides strong support for the theoretical model.

4.3 Calibration and Test of Model Implication

This section will calibrate the parameters used in the model, which, in turn, will confirm the model setup.

Cattle Holding Cost Correlation

The data used to calibrate cow-calf sector’s holding cost correlation comes from USDA Economic research service (http://www.ers.usda.gov/Data). Among the listed cost items, only the total feeding cost is consistently surveyed from 1982 to 2007. So, we are going to use this total feeding cost as a substitute of the cattle holding cost.
Including time trend, we can get an AR(1) estimation as follow\textsuperscript{12}.

\[
h_t = 61.54 \pm 24.37^{* * * } + 0.45 h_{t-1} + 11.67 t + \varepsilon_t
\]

\textbf{Cattle Feeding Cost Correlation}

As we have talked before, the main grain used for feeding cattle is corn. We can use the historical corn price as a candidate to estimate cattle feeding cost correlation. The data used here come from USDA NASS agricultural price report, covering annual data from 1949 to 1999. Also including time trend, we can get an estimation as follow:

\[
f_t = 0.0094 \pm 0.0023^{* * * } + 0.48 f_{t-1} - 0.001 t + \varepsilon_t
\]

\textbf{Demand Shifter Correlation}

Different from the previous two cost variables, the demand shifter can not be observed directly. Instead, we will employ the FGLS approach to estimate the correlation of demand shifter. Follow equation (10), if the demand shifter \( y_t \) has AR(1) correlation structure, then the regression of equation (10) will have serial correlation problem. Using the two steps FGLS, we will run the OLS regression first, and then run the AR(1) auxiliary regression to the OLS residual in the last step. This auxiliary regression will have asymptotically efficient estimate of the demand shifter correlation \( \rho_y \), and there is no gain to iterate the two steps. So, we will use the estimator of the auxiliary regression as the estimate for \( \rho_y \).

The data we employ includes annual steer whole sale value and annual steer slaughter quantity, covering from 1970 to 2005. The data can also be found in USDA Economic research service. (http://www.ers.usda.gov/Data/). Run OLS to estimate

\textsuperscript{12}The numbers in parenthesis are the standard errors of estimates, this expression will be used for the rest of this section.
demand function, and then fit residuals into AR(1) model.

\[ b_t = \frac{208.2^{***}}{3.49} - \frac{0.21^{***}}{0.02} p_t + y_t \]

with \[ y_t = \frac{0.16^* y_{t-1}}{0.10} + \varepsilon_t \]

**Other Parameter**

Chavas (1999) estimates the expected birth rate for calf as 1, which means that the breeding cow will give birth to one calf. The mean death rate is 0.08, which is also reported in his work. Then, we can get a net birth rate to be \( g = 1 - 0.08 = 0.92 \). Take the annual discount rate \( r \) as constant 10%, we can get an estimate of \( \beta = \frac{1+0.1}{1+0.092} = 0.58 \).

In sum, the calibration for the parameters used in the model is listed in table 10.

**Test of Corollary 1**

The theoretical model shows that cow-calf operation return only relies on the maintain cost of breeding stock, but it is not directly related with the feeding cost or demand shifter. We collected corn’s price, which is the main feeding cost, to test this inference. Based on previous work, we will use both AR(1) and AR(2) model to test corn’s effect on the cow/calf sector’s return, which are presented in the two equations of (28) respectively.

\[
R_t^1 = \beta_1 R_{t-1}^1 + \alpha C_t + \varepsilon_t \\
R_t^2 = \beta_1 R_{t-1}^1 + \beta_2 R_{t-2}^1 + \alpha C_t + \varepsilon_t
\]

So, the hypothesis to test is:

\[ H_0 : \alpha = 0 \]

\[ H_1 : \text{otherwise} \]

The test result is summarized in table 11. It indicates that in both models, the
corn price effect is not significant. The AIC and S-C criterions are not better but worse off over the original models. Also, the corn’s price does not explain the residual term’s correlation in AR(1) model, which is reflected in the D-W test. So we can conclude that the corn price, which is a indicator of feeding cost, does not affect cow/calf sector’s return.

5 Conclusion

This paper seeks to explain the differences in return patterns of the upstream and downstream operators in the beef supply chain. Under the assumption that the Ricardian rent incidence is on the cow-calf operators, we set up a rational expectation dynamic model to investigate the interaction between cow-calf operators and feeding operators. The model shows that the cow-calf operators, who make production decision, will get positively correlated returns to maximize the whole life profit. With free entry and exit, the feeding operators can not affect the production decisions, and end up with pure random returns, which are only affected by random shocks. The model also suggests that feeding operation provides a cushion for cow-calf operators from the demand shifter and finishing feed cost. The empirical study shows that 85.4% of the Ricardian rent will go through to the upstream sector, giving strong support to the model’s validity. The key parameters are calibrated through real world data, which also adds credit to the model specification. We believe it is the first time in literature to explicitly discuss the relationship between the two sectors of beef industry under the dynamic rational expectation framework.

We have four remarks about future possible extensions to the present study. Firstly, the empirical study suggest that there is an AR(2) component in the cow-calf operator returns. This AR(2) structure can give rise to the cow-calf return cycles,
which can not be explained by the current theoretical work. This requires to extend the model to a more complete biology structure, such as that of RMS (1994). Secondly, in addition to the calf retention decisions, cow-calf operators also need to make cull decisions of breeding cow. But our model did not distinguish fed beef price from unfed beef price, so we can not analyze this double decisions problem explicitly. Aadland (2001) has shown there will be a different effect from the classic conclusion as in RMS (1994) when considering this price differences. Thirdly, compare the integrated industry and two-layer industry, we can find that the main difference is that the cow-calf operators have to make production decisions one year earlier. In the integrated industry, the operators can delay to make the feeding decisions until there is more clear information about market demand or feeding cost. So, one can investigate whether this real option value is significant to justify the integration of the two sectors. Fourthly, the beef industry was affected by exogenous shocks, for example, Oprah Winfrey’s comment about mad beef disease caused beef price plummet in 1996 although this effect disappeared quickly. By the setting of this model, such unexpected and uncorrelated shocks will affect the of down-stream sector profit but have little impact on the up-stream sector. Such case study on the different effects can be done in the future.

References


Tables

Table 1 Connection between this model and Rosen (1987)

<table>
<thead>
<tr>
<th></th>
<th>Rosen (1987)</th>
<th>This Paper</th>
</tr>
</thead>
<tbody>
<tr>
<td>biological structure</td>
<td>Not clearly specified</td>
<td>Yearling-Cow</td>
</tr>
<tr>
<td>market structure</td>
<td>one layer competitive market</td>
<td>up-stream and down-stream industry</td>
</tr>
<tr>
<td>slaughter at period t</td>
<td>made at period t</td>
<td>decision made at period t-1</td>
</tr>
<tr>
<td>shocks</td>
<td>coming from demand side,</td>
<td>as Rosen (1987)</td>
</tr>
<tr>
<td></td>
<td>holding cost and marketing cost</td>
<td></td>
</tr>
</tbody>
</table>

Table 2. Notation and Definition

\(x\) breeding stock

\(s\) the number of yearlings sold to feedlot

\(q\) price of yearling

\(p\) price of fed cow

\(m\) unit cost of finishing incurred by feedlot

\(y\) demand shifters

\(h\) unit holding cost of yearlings incurred by cow-calf operators

\(r\) the market rate of interest

\(\beta\) \((1+r)/(1+g)\), the net discount rate

\(R_{cc}\) cow-calf’s net return for operating cow-calf business

\(R_{fd}\) feeders net return for finishing fed cow

\(g\) net birth rate after accounting for natural deaths

\(\pi_{cc}^{cc}\) cow-calf’s net cash flow in period \(t\)

\(V_t\) capital value of operating the cow-calf business

\(E_t\) expectation operator, given all the information at period \(t\)

\(b\) the fed cow supplied in beef market

\(k_{t+r}^*\) short for \(E_t(k_{t+r})\)
Table 3 Unit root Test statistics for corn, live cattle and feeder cattle

<table>
<thead>
<tr>
<th></th>
<th>Single Mean Case</th>
<th>Trend Case</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>ADF</td>
<td>P-P</td>
</tr>
<tr>
<td>Live cattle future $F_{t,t+1}^{lc}$</td>
<td>-1.71</td>
<td>-1.98</td>
</tr>
<tr>
<td>Corn $P_t^{cn}$</td>
<td>-1.60</td>
<td>-1.80</td>
</tr>
<tr>
<td>Feeder cattle future $q_t$</td>
<td>-3.38**</td>
<td>-2.64*</td>
</tr>
</tbody>
</table>

Notice: the statistics for ADF, P-P and KPSS are $t$, adj-$t$ and adj-LM respectively.

* rejects the hypothesis at 10% level
** rejects the hypothesis at 5% level
*** rejects the hypothesis at 1% level

Table 4 Cointegration tests for corn, live cattle and feeder cattle

<table>
<thead>
<tr>
<th>No. of CE(s)</th>
<th>Trace Test</th>
<th>Maximum Eigenvalue Test</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Trace stat</td>
<td>Prob</td>
</tr>
<tr>
<td>None</td>
<td>55.6</td>
<td>0.000</td>
</tr>
<tr>
<td>At most 1</td>
<td>23.8</td>
<td>0.002</td>
</tr>
<tr>
<td>At most 2</td>
<td>4.2</td>
<td>0.039</td>
</tr>
</tbody>
</table>

Notice: the test result is obtained using Eviews 5
### Table 5  FGLS estimate of coefficients in regression (26)

<table>
<thead>
<tr>
<th>Coefficient</th>
<th>$\beta_0$</th>
<th>$\beta_1$</th>
<th>$\beta_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Estimate</td>
<td>-0.004</td>
<td>1.35***</td>
<td>-6.43***</td>
</tr>
<tr>
<td>Std. err</td>
<td>0.59</td>
<td>0.04</td>
<td>0.51</td>
</tr>
<tr>
<td>P value</td>
<td>0.993</td>
<td>0.000</td>
<td>0.000</td>
</tr>
</tbody>
</table>

**Adj $R^2$** 0.75  **D-W stat** 2.05

Notice: the test result is obtained using Eviews 5

### Table 6  Mean zero test result

<table>
<thead>
<tr>
<th>Sample mean</th>
<th>-10.01</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sample std.dev</td>
<td>60.52</td>
</tr>
<tr>
<td>t-statistic</td>
<td>-0.95</td>
</tr>
<tr>
<td>P-value</td>
<td>0.34</td>
</tr>
</tbody>
</table>
Table 7: Corrlogram of feeding sector’s return

<table>
<thead>
<tr>
<th>Lag</th>
<th>Autocorrelation</th>
<th>Partial Autocorrelation</th>
<th>P-values</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.039</td>
<td>0.039</td>
<td>0.812</td>
</tr>
<tr>
<td>2</td>
<td>0.021</td>
<td>0.019</td>
<td>0.964</td>
</tr>
<tr>
<td>3</td>
<td>0.065</td>
<td>0.064</td>
<td>0.971</td>
</tr>
<tr>
<td>4</td>
<td>-0.001</td>
<td>-0.006</td>
<td>0.994</td>
</tr>
<tr>
<td>5</td>
<td>0.030</td>
<td>0.028</td>
<td>0.998</td>
</tr>
<tr>
<td>6</td>
<td>0.116</td>
<td>0.110</td>
<td>0.991</td>
</tr>
<tr>
<td>7</td>
<td>0.029</td>
<td>0.021</td>
<td>0.996</td>
</tr>
<tr>
<td>8</td>
<td>0.059</td>
<td>0.051</td>
<td>0.998</td>
</tr>
<tr>
<td>9</td>
<td>0.079</td>
<td>0.063</td>
<td>0.998</td>
</tr>
<tr>
<td>10</td>
<td>0.139</td>
<td>0.134</td>
<td>0.993</td>
</tr>
</tbody>
</table>

Table 8: Unit root test for cow-calf return

<table>
<thead>
<tr>
<th>Test Method</th>
<th>ADF test</th>
<th>PP test</th>
</tr>
</thead>
<tbody>
<tr>
<td>Test Statistics</td>
<td>-2.187</td>
<td>-2.255</td>
</tr>
<tr>
<td>p value</td>
<td>0.029</td>
<td>0.025</td>
</tr>
<tr>
<td>1% Critical Value*</td>
<td>-2.637</td>
<td>-2.636</td>
</tr>
<tr>
<td>5% Critical Value</td>
<td>-1.952</td>
<td>-1.951</td>
</tr>
<tr>
<td>10% Critical Value</td>
<td>-1.621</td>
<td>-1.611</td>
</tr>
</tbody>
</table>
Table 9  Comparison performance of different models

<table>
<thead>
<tr>
<th></th>
<th>AR(1)</th>
<th>AR(2)</th>
<th>AR(3)</th>
<th>ARMA(1,1)</th>
<th>ARMA(2,1)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta_0$</td>
<td>5.24(7.98)</td>
<td>7.05(7.75)</td>
<td>7.81(8.21)</td>
<td>16.42(27.61)</td>
<td>18.71(19.59)</td>
</tr>
<tr>
<td>$\beta_1$</td>
<td>0.74**(0.11)</td>
<td>0.95**(0.17)</td>
<td>0.91**(0.19)</td>
<td>0.64**(0.17)</td>
<td>1.14*(0.43)</td>
</tr>
<tr>
<td>$\beta_2$</td>
<td>NA</td>
<td>-0.31(0.17)</td>
<td>-0.24(0.26)</td>
<td>NA</td>
<td>-0.46(0.33)</td>
</tr>
<tr>
<td>$\beta_3$</td>
<td>NA</td>
<td>NA</td>
<td>-0.07(0.20)</td>
<td>NA</td>
<td>NA</td>
</tr>
<tr>
<td>$\gamma_1$</td>
<td>NA</td>
<td>NA</td>
<td>NA</td>
<td>0.29(0.22)</td>
<td>-0.23(0.48)</td>
</tr>
<tr>
<td>F-test</td>
<td>40.18**</td>
<td>21.48**</td>
<td>11.89**</td>
<td>21.92**</td>
<td>14.03**</td>
</tr>
<tr>
<td>AIC</td>
<td>10.53</td>
<td>10.46</td>
<td>10.55</td>
<td>10.52</td>
<td>10.51</td>
</tr>
<tr>
<td>S-C</td>
<td>10.62</td>
<td>10.60</td>
<td>10.73</td>
<td>10.65</td>
<td>10.70</td>
</tr>
<tr>
<td>D-W</td>
<td>1.53</td>
<td>2.05</td>
<td>1.98</td>
<td>1.92</td>
<td>1.98</td>
</tr>
</tbody>
</table>

* significant at 10% level

** significant at 5% level

Table 10  Calibration of parameter value

<table>
<thead>
<tr>
<th>Parameter</th>
<th>$\rho_h$</th>
<th>$\rho_f$</th>
<th>$\rho_d$</th>
<th>$\beta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Estimate</td>
<td>0.45</td>
<td>0.48</td>
<td>0.16</td>
<td>0.58</td>
</tr>
</tbody>
</table>

Ste err    | 0.19     | 0.10     | 0.1      |
<table>
<thead>
<tr>
<th>Model</th>
<th>AR(1)</th>
<th>AR(2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha$</td>
<td>-25.223(23.62)</td>
<td>-11.60(25.88)</td>
</tr>
<tr>
<td>Result</td>
<td>Can’t reject</td>
<td>Can’t reject</td>
</tr>
<tr>
<td>AIC</td>
<td>10.53</td>
<td>10.52</td>
</tr>
<tr>
<td>S-C</td>
<td>10.62</td>
<td>10.70</td>
</tr>
<tr>
<td>D-W</td>
<td>1.53</td>
<td>2.02</td>
</tr>
</tbody>
</table>
Figures

Figure 1  Cow/Calf and Feeding Sector’s Return Series

The data used here is provided by LMIC. Both of the two returns are annual data.
Figure 2 Distribution of cow-calf operation regions in US

Figure 3 ACF and PACF for the OLS residual

Notice: the figure is generated using R 2.8.2
Figure 4 Comparison of OLS and FGLS residual diagnosis

OLS
Notice: The diagnosis is done using R 2.8.2

FGLS
Figure 5: Compare the theoretical and true ARMA structure.
Appendices

A Deviation form of X,S and Q

With this deviation form, equation (13) can be written as

\[
S^*_{t+\tau} = (\alpha - \alpha) + (y^*_{t+\tau} - \bar{y}) - \gamma[(q^*_{t+\tau} - \bar{q}) + (m^*_{t+\tau-1} - \bar{m})]
\]
\[
= u^y_{t+\tau+1} - \gamma[Q^*_{t+\tau}(1 + r) + u^m_{t+\tau+1}]
\]

The deviation form of (14) is \(Q^*_{t+\tau} = \beta^\tau Q_t + \sum_{i=1}^{\tau} \beta^\tau u^h_{t+\tau-i} \). Substitute this into the equation above, we can get:

\[
S^*_{t+\tau} = u^y_{t+\tau+1} - \gamma[(\beta^\tau Q_t + \sum_{i=1}^{\tau} \beta^\tau u^h_{t+\tau-i})(1 + r) + u^m_{t+\tau+1}]
\] (A.1)

Also rewrite the intertemporal budget constraint (3) in deviation form: \(X_t = \sum_{\tau=0}^{\infty} S^*_{t+\tau}/(1 + g)^{\tau+1} \). Then substitute equation (A.1) into this budget constraint, which implies:

\[
X_t = -\frac{\gamma(1 + r)}{1 + g - \beta} Q_t + \sum_{\tau=0}^{\infty} \frac{v^*_{t+\tau}}{(1 + g)^{\tau+1}}
\] (A.2)

where \(v^*_{t+\tau} = u^y_{t+\tau+1} - \gamma u^m_{t+\tau+1} - \frac{(1 + r)\gamma \beta u^*_t}{1 + g - \beta} \)

From this, we can solve for \(Q_t \) in terms of \(X_t \) and \(u^j_t \):
\[ Q_t = \frac{1 + g - \beta}{\gamma(1 + r)} \left[ -X_t + \sum_{\tau=0}^{\infty} v_{t+\tau}^*/(1 + g)^{\tau+1} \right] \]  

(A.3)

Then insert equation (A.3) into the deviation form of equation (12), we can express \( S_t \) in terms of \( X_t \) and \( u_t^* \):

\[
S_t = u_{t+1}^* - \gamma Q_t (1 + r) - \gamma v_{t+1}^*
\]

(A.4)

\[
= (1 + g - \beta) X_t - (1 + g - \beta) \sum_{\tau=0}^{\infty} v_{t+\tau}^*/(1 + g)^{\tau+1} + u_{t+1}^* - \gamma u_{t+1}^*
\]

Finally, substitute equation (29) into the deviation form of equation (1), we can get the cattle stock’s path:

\[
X_{t+1} = (1 + g) X_t - S_t
\]

(A.5)

\[
= \beta X_t + (1 + g - \beta) \sum_{\tau=0}^{\infty} v_{t+\tau}^*/(1 + g)^{\tau+1} - u_{t+1}^* + \gamma u_{t+1}^*
\]

**B Reduced form of X, S, and Q**

This appendix is used to show the derivation of X, S and Q.
\[
X_t = \sum_{\tau=0}^{\infty} (u_{t+\tau+1}^{y*} - \gamma (\beta^\tau Q_t + \sum_{i=1}^{\tau} \beta^\tau u_{t+\tau-i}^{h*})(1 + r) + u_{t+\tau+1}^{m*})/(1 + g)^{\tau+1} \\
= -\sum_{\tau=0}^{\infty} \left( \frac{\beta}{1 + g} \right)^\gamma (1 + r) Q_t + \\
\sum_{\tau=0}^{\infty} (u_{t+\tau+1}^{y*} - \gamma u_{t+\tau+1}^{m*} - \frac{(1 + r)\gamma u_{t+\tau+1}^{*}}{1 + g - \beta})/(1 + g)^{\tau+1} \\
= -\frac{1}{1 - \frac{\beta}{1 + g}} \frac{\gamma (1 + r)}{1 + g - \beta} Q_t + \sum_{\tau=0}^{\infty} v_{t+\tau}^*/(1 + g)^{\tau+1} \\
\text{where } v_{t+\tau}^* = u_{t+\tau+1}^{y*} - \gamma u_{t+\tau+1}^{m*} - \frac{(1 + r)\gamma u_{t+\tau+1}^{*}}{1 + g - \beta}
\]

\[
Q_t = \frac{1 + g - \beta}{\gamma (1 + r)}[-X_t + \sum_{\tau=0}^{\infty} (u_{t+\tau+1}^{y*} - \gamma u_{t+\tau+1}^{m*} - \frac{(1 + r)\gamma u_{t+\tau+1}^{h*}}{1 + g - \beta})/(1 + g)^{\tau+1}] \\
= \frac{1 + g - \beta}{\gamma (1 + r)}[-X_t + \sum_{\tau=0}^{\infty} (\frac{\rho_y}{1 + g})^{\tau+1} u_t^y - \gamma \sum_{\tau=0}^{\infty} (\frac{\rho_m}{1 + g})^{\tau+1} u_t^m - \frac{\beta}{\rho_h} \sum_{\tau=0}^{\infty} (\frac{\rho_h}{1 + g})^{\tau+1} u_t^h] - \\
\frac{\beta}{\rho_h} \frac{1}{1 - \frac{\rho_y}{1 + g}} \frac{\rho_y}{1 + g} u_t^h \\
= \frac{1 + g - \beta}{\gamma (1 + r)}[-X_t + \frac{\rho_y}{1 + g - \rho_y} u_t^y - \frac{\gamma \rho_m}{1 + g - \rho_m} u_t^m] - \frac{\beta}{1 + g - \rho_h} u_t^h
\]
C Reduced Recursive Form of Cow-Calf Return

In this appendix, a clear form of cow-calf return’s recursive form will be derived.

First, from equation (23), we can solve for $X_t$:

$$X_t = \frac{\rho_y u^y_t}{1 + g - \rho_y} + \gamma(1 + r)\frac{1 + g - \beta}{\gamma(1 + r)}[-X_t + \frac{\rho_y}{1 + g - \rho_y} u^y_t - \frac{\gamma \rho_m}{1 + g - \rho_m} u^m_t] \quad (B.3)$$

Then forward cow-calf operator’s return (23) for one period as follow:

$$X_{t+1} = (1 + g)X_t - [(1 + g - \beta)X_t + \frac{\rho_y(\beta - \rho_y)}{1 + g - \rho_y} u^y_t - \gamma \frac{\rho_m(\beta - \rho_m)}{1 + g - \rho_m} u^m_t] + \frac{\beta \gamma(1 + r)}{1 + g - \rho_h} u^h_t \quad (B.4)$$
\[
R_{t+1}^- = \frac{1 + g - \beta}{\gamma(1 + r)} [-X_{t+1} + \frac{\rho_y}{1 + g - \rho_y} u_{t+1}^y - \frac{\gamma \rho_m}{1 + g - \rho_m} u_{t+1}^m] (C.1)
\]

\[
= \frac{1 + g - \beta}{\gamma(1 + r)} [-X_{t+1} + \frac{\rho_y}{1 + g - \rho_y} (\rho_y u_t^y + \varepsilon_{t+1}^y) \\
- \frac{\gamma \rho_m}{1 + g - \rho_m} (\rho_m u_t^m + \varepsilon_{t+1}^m)] - \frac{(1 + g + \beta - \rho_h)}{1 + g - \rho_h} (\rho_h u_t^h + \varepsilon_{t+1}^h)
\]

\[
= \frac{1 + g - \beta}{\gamma(1 + r)} [-X_{t+1} + \frac{\rho_y^2}{1 + g - \rho_y} u_{t+1}^y - \frac{\gamma \rho_m^2}{1 + g - \rho_m} u_{t+1}^m] - \\
(1 + g + \beta - \rho_h) \rho_h u_{t+1}^h + \Psi_{t+1}
\]

where \( \Psi_{t+1} = \frac{1 + g - \beta}{\gamma(1 + r)} \frac{\rho_y}{1 + g - \rho_y} \varepsilon_{t+1}^y - \frac{\gamma \rho_m}{1 + g - \rho_m} \varepsilon_{t+1}^m \\
- \frac{(1 + g + \beta - \rho_h)}{1 + g - \rho_h} \varepsilon_{t+1}^h
\]

Then substitute equation (21) into equation (29), the returns for cow-calf operators can be recursively expressed as:
\[ R_{t+1}^r = \frac{1 + g - \beta}{\gamma (1 + r)} \left[ -\beta X_t \right. \left. + \frac{\rho_y (\beta - \rho_y)}{1 + g - \rho_y} u_t^y - \frac{\gamma \rho_m (\beta - \rho_m)}{1 + g - \rho_m} u_t^m + \frac{\beta \gamma (1 + r)}{1 + g - \rho_h} \right] \\
+ \frac{\rho_y^2}{1 + g - \rho_y} u_t^y - \frac{\gamma \rho_m^2}{1 + g - \rho_m} u_t^m - \frac{(1 + g + \beta - \rho_h) \rho_h u_t^h}{1 + g - \rho_h} + \Psi_{t+1} \]

\[ = \frac{1 + g - \beta}{\gamma (1 + r)} \left[ -\beta X_t \right. \left. + \frac{\rho_y \beta}{1 + g - \rho_y} u_t^y - \frac{\gamma \rho_m \beta}{1 + g - \rho_m} u_t^m \right] + \]

\[ \frac{\beta (1 + g - \beta) - (1 + g + \beta - \rho_h) \rho_h}{1 + g - \rho_h} u_t^h + \Psi_{t+1} \]

\[ = \frac{1 + g - \beta}{\gamma (1 + r)} \left[ -\beta \left( \frac{\gamma (1 + r)}{1 + g - \beta} R_t^r - \frac{\gamma (1 + r)}{1 + g - \beta} \frac{1 + g + \beta - \rho_h}{1 + g - \rho_h} u_t^h \right) \right] \\
+ \frac{\rho_y \beta}{1 + g - \rho_y} u_t^y - \frac{\gamma \rho_m \beta}{1 + g - \rho_m} u_t^m \]

\[ + \frac{\beta (1 + g - \beta) - (1 + g + \beta - \rho_h) \rho_h}{1 + g - \rho_h} u_t^h + \Psi_{t+1} \]

\[ = \beta R_t^r + \frac{(1 + g + \beta - \rho_h)}{(1 + g - \rho_h)} (\beta - \rho_h) + \frac{\beta (1 + g - \beta)}{1 + g - \rho_h} u_t^h + \Psi_{t+1} \]

In sum, the recursive form for \( R_t^r \) can be written as

\[ R_{t+1}^r = \beta R_t^r + \lambda u_t^h + \Psi_{t+1} \quad (C.3) \]

where \( \lambda = \frac{(1 + g + \beta - \rho_h)}{(1 + g - \rho_h)} (\beta - \rho_h) + \frac{\beta (1 + g - \beta)}{1 + g - \rho_h} \)

**D Counterpart in Rosen (1987)**

The counterpart of equation (17) in Rosen (1987) is summarized as:
\[ Q_t = \frac{1 + g - \beta}{\gamma} \left[ -X_t + \sum_{\tau=0}^{\infty} v_{t+\tau}^*/(1 + g)^{\tau+1} \right] \]
\[ S_t = (1 + g - \beta) \left[ X_t - \sum_{\tau=0}^{\infty} v_{t+\tau}^*/(1 + g)^{\tau+1} \right] + u_t^y - \gamma u_t^m \]
\[ X_{t+1} = \beta X_t + (1 + g - \beta) \sum_{\tau=0}^{\infty} v_{t+\tau}^*/(1 + g)^{\tau+1} - u_t^y + \gamma u_t^m \]

where \[ v_{t+\tau}^* = \frac{u_{t+\tau}^y - \gamma u_{t+\tau}^m - \frac{\gamma \beta u_{t+\tau}^h}{1 + g - \beta}}{1 + g - \beta} \]

The counterpart of equations (19)-(21) in Rosen (1987) is summarized as:

\[ Q_t = \frac{1 + g - \beta}{\gamma} \left[ -X_t + \frac{u_t^y}{1 + g - \rho_y} - \frac{\gamma u_t^m}{1 + g - \rho_m} \right] - \frac{\beta u_t^h}{1 + g - \rho_h} \]
\[ S_t = (1 + g - \beta)X_t + \frac{(\beta - \rho_y) u_t^y}{1 + g - \rho_y} - \gamma \frac{(\beta - \rho_m) u_t^m}{1 + g - \rho_m} + \frac{\beta \gamma u_t^h}{1 + g - \rho_h} \]
\[ X_{t+1} = \frac{\beta - \rho_y}{1 + g - \rho_y} u_t^y + \gamma \frac{\beta - \rho_m}{1 + g - \rho_m} u_t^m - \frac{\beta \gamma}{1 + g - \rho_h} u_t^h \]