Implications of the Air Compliance Agreement for Livestock Producers

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Abstract:
Nutrient management and air emissions continue to be an area of increased management control on all livestock operations as environmental regulations become more stringent. Agricultural producers must consider uncertainty surrounding the timing of and potential increases of environmental policy controls, such as emission taxes, when making future investment decisions. An optimal control theory framework was applied to a dairy farm facing an uncertain increase in emission taxes at a known future date. The optimal investment path considering an uncertain tax increase was solved and compared to a full certainty case.

Keywords: air emissions, environmental regulations, irreversible investment, uncertainty

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Implications of the Air Compliance Agreement for Livestock Producers

Environmental regulation of livestock production continues to become more stringent as knowledge increases about production processes generating emissions and appropriate abatement technology required to control these emissions. In order to remain profitable agricultural producers must be cognizant of these changes and their implications on future emission-reducing technology investment decisions.

Increasing attention on regulating greenhouse gas emissions under the Clean Air Act from non-point pollution sources, such as the agricultural sector, has been evident with the introduction of the Environmental Protection Agency’s 2005 Air Compliance Agreement (Environmental Protection Agency, 2009). Livestock producers had an option to sign this agreement and pay a fee to EPA for past emissions that would be used to collect data on animal air emissions for various livestock enterprises and management practices. In return for signing the agreement, producers were not held liable for any emissions in the interim period prior to implementation of air emission policies --perhaps in 2011 or 2012--and agreed to abide by the decision. By declining to sign the agreement, those producers were left open to penalties for emissions during the interim period. This agreement foreshadows the reality that policy instruments in the form of carbon-equivalent air emission standards, taxes, or some combination thereof will be part of agricultural producer management decisions in the near future.

Air emissions from livestock production are a function of livestock species, animal housing, and manure storage and application methods. Previous studies reveal that livestock air emissions which vary greatly as a function of these factors (Koelsch and Stowell, 2009, USDA/ARS, 2008; Gay et al., 2003). For example, daily ammonia ($NH_3$)
air emissions for a dairy cow vary from 0.025-1.02 lbs/day (Air Quality, 2008; Koelsch and Stowell, 2009). The large variation in estimated air emissions makes it difficult for livestock producers to know if they are exceeding air emission limits.

Livestock producers can adopt abatement technologies to reduce animal emission levels. Abatement technologies exist for housing, storage, and manure application practices. Housing abatement technologies include bio-filtration system and urine-feces separation. Building long-term manure storage facilities with manure storage covers are examples of storage abatement technologies. Many times these abatement technologies involve irreversible investments. Therefore livestock producers must evaluate the trade-offs of various abatement technologies while considering the uncertainty surrounding future air emission regulations.

A farmer’s incentive to invest in emission-reducing technology is influenced by the environmental regulation chosen by the governmental agency. The policy instrument can be market based (emission taxes or tradeable pollution permits) or take on the form of a command-and-control policy (performance and technology standards). It has been argued that taxes encourage firms to invest in more efficient pollution abatement technologies than other market-based pollution control methods (Caswell, Lichtenberg, and Zilberman, 1990; Farzin and Kort, 2000; Millman and Prince, 1989; Tarui and Polasky, 2005). Indeed much of the discussion regarding the Air Compliance Agreement has focused on emission taxes as the likely policy instrument. The American Farm Bureau Foundation estimated yearly emission taxes for livestock operations at $175/dairy cow, $87.50/beef cow, and $21.87/hog. These emissions taxes were projected to apply to any agricultural operation with more than 25 dairy cows, 50 beef cattle, 200 hogs, or 500
acres of corn (Dairy Herd Management, 2009). If emission taxes are chosen as the environmental policy control instrument, these estimated animal levels indicate that almost all farms would face some form of an emission tax.

Past literature has evaluated the behavior of a firm subject to environmental regulation. Xepapadeas (1992) used an infinite duration dynamic game framework to develop incentive schemes for investment which accounted for the dynamics of non-point source pollution problems. Xepadadeas concluded that an increase in an emission tax always resulted in a larger stock of abatement capital for the firm compared to an emission standard. Comparing results from a dynamic to static incentive solution suggested that static incentives schemes were suboptimal in the long run.

Kort (1996) extended the theoretical framework of Xepapadeas (1992) assuming that abatement technology would be required to reduce pollution. He evaluated the effect of a pollution tax and marketable permits on firm investment in abatement technology. Emissions were generated as function of productive and non-productive capital stock. Productive capital stock was used in the production process whereas non-productive capital stock cleaned pollution. Using an optimal control theory model Kort determined that an increase in pollution tax does not necessarily result in lower pollution levels when increasing non-productive capital stock induces increased investment in productive capital stock. Kort also determined that in the long-run firm investment behavior was equivalent whether a pollution tax or marketable permit was imposed.

Hartl and Kort (1996) evaluated changes in the production process of a firm when an emissions tax was imposed, such as switching to cleaner inputs. Emissions were assumed to be generated through a production process defined by capital stock. Hartl and
Kort included an investment grant in the decision that was found to induce investment at a relatively earlier date.

Farzin and Kort (2000) extend Hartl and Kort’s model to consider two forms of uncertainty for the optimal investment policy of a firm facing environmental regulation: (1) an increase of unknown size in the future pollution tax rate and (2) an unknown timing of the tax increase. Secondly, Farzin and Kort assumed that emissions were a function of the production process as defined by a variable input rather than capital stock as specified in earlier models. This gave the firm the ability to decrease emissions by decreasing the variable input. Farzin and Kort determined that abatement investment rates were lower than the certainty case when uncertainty existed about the magnitude of the future tax increase. Uncertainty surrounding emission tax increase timing resulted in increased under-investment in abatement capital.

Investment in emission reducing technologies is a dynamic and potentially irreversible investment for many agricultural producers. Livestock producers are aware that an environmental policy instrument is scheduled to be imposed for air emissions December 31, 2011. However, uncertainty exists surrounding the stringency of new environmental regulations regarding air emissions for livestock operations. Producers must evaluate tradeoffs between investing in emission-reducing technology today versus waiting to invest at a later date when additional information regarding new emission reducing technologies could become available.

This analysis adapts the model of Farzin and Kort (2000) to evaluate the effect of an emission tax policy on farm investment in emission-reducing technology. This analysis differs from Farzin and Kort’s in three ways. First, the production process is
defined as a function of productive capital rather than a variable input, since animals are a form of capital on livestock operations. Second, in addition to a productive capital stock, non-productive capital stock is introduced in the model to reduce emissions. Therefore an emissions function is defined to address the interaction between productive and non-productive capital stock as outlined by Kort (1996), rather than a pollution function proportional to output level used by Farzin and Kort (2000). Finally, an empirical analysis is implemented which has been absent in previous analysis. Functional forms for production and emissions functions, and numeric values for price and tax parameters are defined to provide a tractable analysis which can be used in a policy context. The objective of this analysis is to determine the optimal investment path for (1) a certain emission tax at time $t=0$ and (2) an uncertain emission tax increase at time $T$ which considers the potential uncertainty regarding environmental regulation faced by agricultural producers.

The paper proceeds as follows: an analytical model of an emission tax that is known with certainty and does not change over time is presented for a dairy farm which is followed by an empirical analysis of the basic tax model for a known low and high tax rate. The second half of the paper presents an analytical analysis of an unknown tax rate at a known time $T$ with the empirical analysis followed by discussion and conclusions.

**Basic Emission Tax Model**

The basic model adapted from Farzin and Kort (2000) is an emission tax that is known with certainty and does not change over time. The basic model is applied to a
dairy farm but could be directly applied to other livestock enterprises. The results of this basic model are used as a benchmark to analyze uncertain tax policy in later sections.

Consider a risk-neutral perfectly competitive farm that has the opportunity to invest in two types of capital, productive and non-productive. The productive capital is a variable input, (dairy cows, (Ω)), used to produce a homogenous output (milk) according to a simple production process

\[ m = m(\Omega), \]

where \( m(0) = 0, \ m'(\Omega) > 0, \) and \( m''(\Omega) < 0. \) This production process generates emissions as a function of the input level of productive capital stock, cows. The second type of capital (K) used by the farm is non-productive, but reduces emissions. Examples of non-productive capital include animal housing, manure storage, and manure application methods used to minimize emissions (Gay et al., 2003; Koelsch and Stowell, 2009). The farm emissions function is given by

\[ A = A(\Omega, K), \]

where \( E \) is total emissions generated by productive and non-productive capital. The emissions function must satisfy the following conditions (Kort, 1996):

\[ \begin{align*}
(2a) & \quad A(\Omega, K) > 0 \ for \ all \ \Omega > 0 \ and \ \ K \geq 0, \\
(2b) & \quad A_\Omega(\Omega, K) > 0 \ and \ A_{\Omega K}(\Omega, K) > 0, \\
(2c) & \quad A_K(\Omega, K) < 0 \ for \ all \ \Omega > 0 \ and \ A_{KK}(\Omega, K) > 0 \ for \ all \ \Omega > 0 \\
(2d) & \quad A_{\Omega K}(\Omega, K) = A_{K \Omega}(\Omega, K) < 0.
\end{align*} \]

Condition (2a) implies that emissions are positive as long as cows are on the farm. Condition (2b) shows that emissions increase in a convex way with an increasing number of cows for a given level of emission-reducing capital. Diminishing returns for emission-
reducing technologies is shown with condition (2c) which states that emission output is smaller for larger amounts of emission-reducing technologies for a given level of cows (Ω). Condition (2d) implies that an increase in emissions due to one additional cow is smaller the larger the stock of emission-reducing capital. Therefore, emission-reducing technologies are more effective for reducing emissions than reducing the number of cows on the farm. The emissions function is not separable in Ω and K which implies that increased investments in emission-reducing capital stock is required to reduce emissions with some fixed level of productive capital, Ω.

The dairy herd is assumed to be a closed homogeneous female population. Following Rosen (1987) dairy herd population dynamics are equal to net births less the number of cows slaughtered, where the net birth rate is the difference between the birth rate and a constant natural death rate. Since cows produce emissions as a by-product of the production process, decreasing the number of cows through slaughter decreases the amount of emissions generated.

Emission abatement capital stock can be increased by making an investment, I, in emission-reducing technology. The total investment cost, \( C(I) \), is assumed to be a convex increasing function of the investment level such that,

\[
(3) \quad C(0) = 0,
\]

where \( C'(I) > 0 \) and \( C''(I) > 0 \). It is assumed that investment in emission abatement technology is irreversible such that \( I \geq 0 \). Without investment, \( I \), emission abatement capital stock is assumed to depreciate at a constant proportional rate of \( \delta \).

An emission tax, \( \tau > 0 \), is defined as the pollution tax per unit of emissions. The total farm emissions tax payment at any point in time is \( \tau A(cows, K) \).
The risk-neutral farm chooses the level of cows to slaughter, $S$, and emission-reducing technology investment, $I$, to maximize the present value of its cash flows over an infinite planning period,

$$
\max_{S,I} \int_0^\infty \left[ p_m m(\Omega) + p_s S - w\Omega - C(I) - \tau A(\Omega, K) \right] e^{-rt} dt,
$$

subject to

$$
\dot{\Omega} = \alpha \Omega - S,
$$

$$
\dot{K} = I - \delta K,
$$

$I \geq 0$.

where

- $\Omega$ = productive capital stock, variable input for milk production
- $m(\Omega)$ = milk production function
- $p_m$ = milk price
- $p_s$ = slaughter price
- $S$ = slaughter rate
- $w$ = input price of production (i.e., feed for cows)
- $K$ = non-productive capital stock
- $I$ = emission-reducing technology investment
- $A(\Omega, K)$ = emissions generated by cows and capital
- $\alpha$ = net birth rate
- $\tau$ = per unit emission tax
- $\delta$ = depreciation rate for non-productive capital stock, $K$
- $r$ = constant discount rate

The current value Hamiltonian for the optimal control problem is defined as,

$$
H = p_m m(\Omega) + p_s S - w\Omega - C(I) - \tau A(\Omega, K) - \lambda (\alpha \Omega - S) - \eta (I - \delta K)
$$

where $\lambda$ is the shadow price of the productive capital, cows, and $\eta$ is the shadow price of non-productive capital, emission reducing technology. The necessary conditions for the optimal policy are,

$$
\dot{\lambda} = p_s,
$$

$$
\eta = C'(I),
$$
Equation (6) shows the marginal impact of cow sales on the Hamiltonian where the shadow price of cows must equal the slaughter price along the optimal slaughter path. Equation (7) shows that the shadow price of the emission-reducing technology must equal its marginal cost along the optimal investment path. Equation (8) is the adjoint condition which states that an additional cow slaughtered is equal to the net revenue from that cow. Equation (9) is the adjoint condition which states that an additional unit of investment is equal to the savings on the emissions tax payment. The adjoint equations must hold at each point in time and can be expressed as “golden rule” equations (typically found in resource management literature) by taking the time derivative of the shadow price equation and setting it equal to the adjoint condition for $\lambda$ and $\eta$. Taking the time derivative of equation (6) and setting it equal to equation (8) and solving for, $r$, results in,

$$\dot{r} = \frac{p_m m'(\Omega)}{p_s} - \frac{w}{p_s} - \frac{\tau A_{11}(\Omega, K)}{p_s} + \alpha.$$  

Equation (10) equates the return from holding dairy cows (not slaughtering) to its opportunity cost, $r$. The first and second RHS terms are the marginal revenue and cost from keeping the dairy cow in the milking herd, respectively. The third term is the tax cost of investing in a larger dairy milk herd at the margin. The fourth term is the marginal impact of cows on reproduction.

Taking the time derivative of equation (7) and setting it equal to equation (9) and solving for $r$ results in,
Equation (11) equates the return from not investing in emission-reducing technology to its opportunity cost, \( r \). The first RHS term in the bracket is the capital gains to emission reducing technology less depreciation.\(^1\) The second RHS term is the marginal impact of taxes on investing in new emission-reducing technology.

From equation (6) we know that \( \lambda = p_s \), which results in a singular solution for the slaughter control variable since there are no control variables in equation (10). Therefore, we can solve for the number of cows in the herd as a function of capital, rather than \( S \). The number of cows in the dairy herd changes as the emission-reducing capital stock changes. We can solve for the \( \dot{I} = 0 \) and \( \dot{K} = 0 \) isoclines to analyze the phase diagram for the optimal investment path in the \((K,I)\)-plane rather than the \((K,S)\)-plane.

The \( \dot{K} = 0 \) isocline is a positively sloped straight line where \( I = \delta K \). The isocline for \( \dot{I} = 0 \) is defined by solving for \( \dot{I} \) in equation (11) such that,

\[
(12) \quad \dot{I} = \frac{(r + \delta)C'(I) + \tau A_K(\Omega, K)}{C''(I)}.
\]

From equation (12), a unique saddle point exists where \( \dot{I} = \dot{K} = \dot{\eta} = 0 \) and \( I = \delta K^* \) such that,

\[
(13) \quad -\tau A_K(\Omega^*, K^*) = (r + \delta)C''(\delta K^*).
\]

\(^1\) The first RHS term in equation (11) can also be represented as \( \frac{\dot{\eta}}{\eta} \) where \( \dot{\eta} = C'(I)\dot{I} \) and \( \eta = C'(I) \).
Numerical Example: Basic Emissions Tax Model

A numerical application of the basic emissions tax model was implemented to analyze the numerical phase plane diagram in the \((K,I)\)-plane rather than theoretical diagrams completed in previous analysis (Farzin and Kort, 2000). Data used to parameterize the model are provided in Table 1. Prices parameters used in the model were based on dairy industry average values. A 15 year milk price average for “All milk” in the U.S. resulted in a $14.32/cwt milk price (USDA-NASS, 2009). Slaughter price was valued at $650/cwt based on a five year average for dairy cow slaughter prices from USDA-NASS (2001-2006). Feed cost was assumed to be $4.68/cow/day for purchased, homegrown, and grazing feed which resulted in a yearly cost of $1,709/cow (ARMS-ERS, 2005).

The American Farm Bureau estimated a potential emissions tax for dairy of $175/cow. The emissions tax rate in the model is specified based on units of emissions. The per animal emissions tax rate is converted to pounds of ammonia \((NH_3)\) emissions assuming that a dairy cow produces 0.25 pounds of ammonia emissions on a daily basis or 91.25 pounds annually (USDA/ARS, 2009; Gay et al., 2003).\(^2\) The emission tax rate was calculated as $1.92/lb \(NH_3\) emissions.

The depreciation rate for capital, \(\delta\), was assumed to be 0.05 which assumes a 25 year useful life of the emission-reducing technology investment. A discount rate of 0.09 was used (Wolf et al., 2002).

A decreasing returns to scale milk production function was implemented to estimate milk production such that, \(m(\Omega) = 936\Omega^{1/2}\). It was assumed that the average

\(^2\) 0.25 pounds of ammonia emissions is the most common value used to estimate emissions and was used to adjust a per cow tax to per pound ammonia (NH3) emissions.
yearly milk yield was 22,000 pounds per cow. Annual yield was converted to hundredweight (cwt) basis to be in equivalent unit terms as milk price.

The functional form for the emissions production was specified using conditions (2a)-(2d) which resulted in $A(\Omega, K) = \Omega^{1/2} K^{-1/2}$. The negative coefficient on the non-productive capital, $K$, means that as the amount of emission-reducing capital stock increases through emission-reducing technology investment, farm level emissions decrease.

Total investment cost was assumed to be a convex increasing function of the investment level in the emission-reducing technology. This resulted in the following function form of, $C(I) = \frac{1}{2} I^2$ for investment cost.

**Table 1. Parameter description, values, and sources**

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Description</th>
<th>Value</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>$p_m$</td>
<td>Milk price</td>
<td>$14.32$/cwt</td>
<td>USDA-NASS (1990-2009)</td>
</tr>
<tr>
<td>$p_s$</td>
<td>Slaughter price</td>
<td>$650$/cow</td>
<td>USDA-NASS (2001-2006)</td>
</tr>
<tr>
<td>$w$</td>
<td>Feed cost</td>
<td>$1,709$/yr/cow ($4.68$/day/cow)</td>
<td>ARMS-ERS (2005)</td>
</tr>
<tr>
<td>$\tau$</td>
<td>Tax rate</td>
<td>$1.92$/emission unit ($175$/cow)</td>
<td>American Farm Bureau (2009)</td>
</tr>
<tr>
<td>$\delta$</td>
<td>Depreciation rate for non-productive capital</td>
<td>0.05</td>
<td>Assumption</td>
</tr>
<tr>
<td>$r$</td>
<td>Discount rate</td>
<td>0.09</td>
<td>Wolf et al. 2002</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>Net birth rate</td>
<td>1.8</td>
<td>Assumption</td>
</tr>
<tr>
<td>$m(\Omega)$</td>
<td>Milk production function</td>
<td>936 $cows^{1/2}$</td>
<td>Assumption</td>
</tr>
<tr>
<td>$A(\Omega, K)$</td>
<td>Air emissions production function</td>
<td>365 $cows^{1/2} K^{-1/2}$</td>
<td>Assumption</td>
</tr>
<tr>
<td>$C(I)$</td>
<td>Investment cost function</td>
<td>$0.5I^2$</td>
<td>Kort and Hartl (1996)</td>
</tr>
</tbody>
</table>
Using the assumptions in Table A, the numerical equivalents for equation (6)-(9) for the optimal control theory problem with a certain tax rate \( \tau = 1.92 \text{ lb ammonia} \) emissions are the following,

\begin{align*}
(14) \quad \lambda &= 650, \\
(16) \quad \eta &= I, \\
(17) \quad \dot{\lambda} &= -1.71\lambda - \frac{3318}{\Omega^{1/2}} + 1643 + \frac{3.84}{\Omega^{1/2}K^{1/2}}, \\
(18) \quad \dot{\eta} &= 0.19\eta - \frac{3.84\Omega^{1/2}}{K^{3/2}}.
\end{align*}

Taking the time derivative of (14) and setting it equal to (17) we can solve for \( \Omega \) in terms of emission-reducing capital since we have a singular solution for the slaughter control variable,

\begin{equation}
\Omega = \frac{1.42 \times 10^{-9}(36864 - 6.37 \times 10^7 K^{1/2})}{K}.
\end{equation}

Substituting equation (19) in (17) allows us to plot the isoclines for \( \dot{I} = 0 \) and \( \dot{K} = 0 \) for a tax rate \( \tau = 1.92 \text{ lb ammonia} \) as a numerical result for a singular solution with respect to the slaughter control variable (Figure 4a). The thin line in Figure 1 is the saddle path leading to the stable steady state equilibrium where \( I=9.20 \) and \( K=184 \). Along the saddle path investment falls as emission-reducing capital stock increases. This is a result of the functional form for the emissions production function including both cows (productive) and emission-reducing (non-productive) capital stock. The phase-plane diagram shows that a farm with a low level of emission-reducing capital stock (i.e., \( K=20 \)) will invest up to \( I=40 \) which will put them on the saddle path.
Along the optimal investment path (saddle path) the shadow price must always equal the marginal cost such that,

\[(20) \quad \int_{t}^{\infty} \tau A_{K}(\Omega, K)e^{-\left((r+\delta)(t-s)\right)} ds = C'(I(t)) = \eta\]

where the LHS expression of the equation is the reduction in emission tax payments resulting from an additional unit of emission-reducing technology investment at time $t$. When farm emission-reducing capital stock is relatively low (i.e., $K=20$) the optimal investment rate is high and decreases over time as it reaches the steady state at equilibrium at point A. A farm with a high level of emission-reducing capital stock ($K>200$) will have lower investment levels to reach the saddle path and the steady state equilibrium as compared to a farm with low initial capital stock levels.

![Figure 1. Optimal investment path for \( \tau = \$1.92 \)](image)
The tax rate influences investment decisions made by farmers. In the model we assumed \( \tau = \$1.92 \) per lb of ammonia emissions (\$175/cow) which is a high tax rate. For example a dairy farm with 100 cows would incur \$17,500 in emission taxes. If the milk price is \$14/cwt and a dairy cows produces 70 pounds of milk per day per cow, milk production revenue for approximately 18 days would be needed to pay the emission tax.

A second tax rate was included in the analysis to compare how investment decisions change with a lower tax rate. We assumed the lower tax rate was, \( \tau_L = \$0.48 \) per lb ammonia emission, which was 75\% lower than the tax rate reported by the American Farm Bureau (2009).

Figure 2 presents the phase-plane diagram for \( \tau_H = \$1.92 \) and \( \tau_L = \$0.48 /lb \) ammonia emissions. The \( \dot{K} = 0 \) isocline did not change with a new tax rate. \( \dot{I}_{\tau=0.48} = 0 \) shifted downward with the lower tax rate (purple line in Figure 4b). The steady state equilibrium for \( \tau_L = \$0.48 \) is represented by point B with \( I=5.29 \) and \( K=106 \). Increasing the tax rate by 75\% (\( \tau_L \) to \( \tau_H \)) increases the optimal investment rate by 43\% and emission reducing capital stock by 42\%. With the tax increase from \( \tau_L \) to \( \tau_H \) the productive capital stock, \textit{cows}, on the farm remained constant while the non-productive capital stock, \textit{K}, increased. Farmers will add emission-reducing technologies on their farms to decrease emission tax payments rather than decreasing their herd size.

Implicitly, the tax payment faced by the farm is \( \tau 365 \Omega^{1/2} K^{-1/2} \). Taking the derivative of the tax payment with respect to emission-reducing capital, \textit{K}, results in

\[-183\pi \Omega^{1/2} K^{-3/2} < 0 \] which shows that the emission tax payment decreases as emission-reducing capital stock increases.
An Uncertain Emissions Tax at a known Future Date

We now consider the case where the farm knows an emission tax will occur at a known future date, $T$, but the magnitude is uncertain. For this problem the farmer considers potential tax rates that may be imposed at time $T$, to adjust investment rates in emission-reducing technology from time $t=0$ to $t=T$. At time $T$, the actual tax rate is revealed and a jump in the investment rate may occur to adjust to the desired saddle path.

Before we solve the case with an uncertain tax increase, it is useful to analyze the case where the tax increase is known with certainty. Suppose a low tax rate, $\tau_L$, is imposed and at time $T$, the tax rate increases to $\tau_H$. The farmer solves the following problem,
\[
\max_{S, I} \int_0^T \left[ p_s m(\Omega) + p_s S - w \Omega - C(I) \tau_L A(\Omega, K) \right] e^{-\tau dt} d\tau + \int_r^\infty \left[ p_s m(\Omega) + p_s S - w \Omega - C(I) - \tau_H A(\Omega, K) \right] e^{-r \tau} d\tau \\
\text{s.t. } \dot{\Omega} = \alpha \Omega - S, \quad \Omega_0(0) \\
\dot{K} = I - \delta K, \quad K_0(0)
\]

Using the example presented in the previous section and Figure 2, the farmer adjusts investment rate to account for the high tax rate by time $T$. With an increase to $\tau_H$, the farmer’s investment rate will deviate away from the saddle path for $\dot{I}_{\tau} = 0$ and move towards the saddle path for $\dot{I}_{\tau_H} = 0$ as shown by the dashed line in Figure 3. The farmer’s new investment rate in emission-reducing technology changes such that at time $T$ when $\tau_H$ is imposed, the farmer is on the saddle path for $\tau_H$ and moving towards (or at) the steady state equilibrium (point A).
With an uncertain emission tax increase, the farmer is not certain how to adjust their investment rate by time $T$. The new problem can be set-up as a two-stage optimal control problem where in the second stage the farmer maximizes their expected present value of cash flows (Dosi and Maretto, 1990). In the first stage the farmer maximizes present value cash flows with the constraint that at that time $T$ the farm cash flows will be equal to the present value calculated in stage two. This leads to the farmer choosing the level of cows to slaughter, $S$, and emission-reducing technology investment, $I$, to maximize the present value of its cash flows over an infinite planning period such that,
We assume the farmer is risk neutral and the profit function is linear in taxes such that \( \pi E(\tau) = E(\pi(\tau)) \) which implies that the comparison of optimal investment paths with full certainty versus uncertainty depends on the tax rate at time \( T \) and expected tax rate after time \( T \). To solve the two stage optimal control problem we first solve the second term of equation (21),

\[
\max_{\tau} \int_0^\tau \left[ p_m(\Omega) + p_s S - w\Omega - C(I) - \tau A(\Omega, K) \right] e^{-\tau} dt
\]

\[
+ E \int_\tau^\infty \left[ p_m(\Omega) + p_s S - w\Omega - C(I) - \tau A(\Omega, K) \right] e^{-\tau} d\tau
\]

s.t \( \dot{\Omega} = \alpha \Omega - S, \quad \Omega_0(0) \)

\( \dot{K} = I - \delta K, \quad K_0(0) \)

Following this, the numerical solution to the second stage, \( e^{-\tau} \pi(K_T, \Omega_T, \tau_T) \), is included in the first stage problem,

\[
\max_{S, J} \int_0^\tau \left[ p_m(\Omega) + p_s S - w\Omega - C(I) - \tau A(\Omega, K) \right] e^{-\tau} dt
\]

\[
+ e^{-\tau} \pi(K_T, \Omega_T, \tau_T)
\]

s.t \( \dot{\Omega} = \alpha \Omega - S, \quad \Omega_0(0), \quad \dot{K} = I - \delta K, \quad K_0(0) \)
The necessary conditions for the optimal investment policy must include,

\[(24) \quad \dot{\lambda} = p_s = \frac{\partial \pi(K(\tau), \Omega(\tau), \tau)}{\partial \Omega} \]

\[(25) \quad \eta = C'(I) = \frac{\partial \pi(K(\tau), \Omega(\tau), \tau)}{\partial K} \].

If conditions (24) and (25) do not hold, the present value cash flows from stage 1 will not be equal to the present value calculated in stage two.

The numerical solution for the expected profit in the second stage is dependent on the tax rate. The optimal investment path was calculated for each tax rate (\(\tau_1, \tau_2, \text{ and } \tau_3\)) for a given level of emission-reducing capital stock. The expected profit is,

\[(26) \quad E(\pi|K_{0(i)}) = p(\tau_1)\pi(\tau_1) + p(\tau_2)\pi(\tau_2) + p(\tau_3)\pi(\tau_3) = e^{-\tau}\pi(K(\tau), \text{cows}(\tau), \tau).\]

The results of equation (26) are then included in the first stage regression to solve the optimal control theory problem for an uncertain tax increase at time \(T\).

**Numerical Example: An uncertain tax increase imposed at time \(t=T\)**

The two-stage optimal control theory model shows that the farm takes into account future tax rate increases to adjust (or not adjust) investment in emission-reducing capital stock even though the tax rate is not imposed until time \(T\). There are two cases that can be analyzed with the two-stage optimal control model. The first case is when a farmer is faced with a low tax rate from time \(t=0\) and knows a tax increase will be imposed at time \(T\). This two stage problem is equivalent to equation (23).

In the second case, an emission tax is not imposed in the first stage of the problem. Therefore, the adjustment to the optimal investment rate for the farm is dependent on the expected tax rate imposed at time \(T\) and its effect on the farmer profit.
function for time $T$ to infinity. The two-stage optimal control theory problem for case 2 is,

$$
\begin{align*}
(27) \quad \max_{s,d} & \int_0^T \left[ p_w m(\Omega) + p_s S - w\Omega - C(l) \right] e^{-\tau} dt + e^{-\tau} \pi(K(\tau_r), \Omega(\tau_r), \tau_r) \\
\text{s.t.} \quad & \dot{\Omega} = \alpha\Omega - S, \quad \Omega_0(0), \quad \dot{K} = I - \delta K, \quad K_0(0).
\end{align*}
$$

It may be hypothesized that without a tax imposed at time $t=0$, there exists little incentive to invest in emission-reducing technology. However, the possibility of future emissions taxes may create incentive for farmers investment rate up to time $t=T$.

A numerical application of an uncertain tax increase at time $T$ was implemented using the data and parameters presented in Table 4a for the case where the farmer does and does not have an initial emissions tax in the first stage of the problem. Three tax rates were assumed with equal probability to estimate the expected profit function for stage two at a given level of emission-reducing capital stock. The three tax rates were: $\tau_1 = \$0$ as the minimum tax, $\tau_2 = \$0.96$ as an average tax, and a maximum tax rate of $\tau_3 = \$1.92$. An equal probability was assigned to each tax rate where $P(\tau_1 = \$0) = 0.33$, $P(\tau_2 = \$0.96) = 0.33$, and $P(\tau_3 = \$1.92) = 0.33$. The expected profit function for a given level of emission reducing capital stock was estimated as,

$$
E(\pi|K_{0(i)}) = 0.33\pi(\tau_1 = 0|K_{0(i)}) + 0.33\pi(\tau_2 = 0.96|K_{0(i)}) + 0.33\pi(\tau_3 = 1.92|K_{0(i)}),
$$

and included in the first stage problem to solve for the optimal investment rate considering an uncertain tax increase at time $T$.

The optimal investment path for an uncertain tax increase at time $t=T$ with a low tax rate $\tau_L = \$0.48/lb$ ammonia emissions imposed from time $t=0$ to $t=T$ was solved and presented in Figure 4. First, if no tax rate increase was imposed, the farmer would
continue on the saddle path for $\dot{I}_{\tau}$ towards the equilibrium level of investment and capital for the low tax rate. However, the farmer knows that a tax increase will occur at time $t=T$, so they will deviate their investment away from the $\dot{I}_{\tau}$ and move towards $\dot{I}_{E(t)}$. The rate at which they change their investment path towards $\dot{I}_{E(t)}$ is determined by the transversality conditions holding (equation (24) and (25)).

For small levels of emission-reducing capital stock the farm stays on the investment path for the low tax rate, $\tau_L = \$0.48$, since at these points the present value cash flows in stage 1 are equal to stage 2. However at an emission-reducing capital stock level of $K=62$, it is optimal for the farmer to deviate away from the investment path for the low tax rate and move toward the investment for the expected tax rate. By time $T$, the farmer has adjusted the saddle path for the expected rate. If the actual tax rate is equivalent to the expected tax rate at time $T$, the farmer is already on the optimal investment path and will move towards the steady state equilibrium value of emission-reducing capital and investment. If the actual tax rate is higher than the expected rate, the farmer will have an immediate jump in their investment rate to the new saddle path for the actual tax rate. A realized tax rate at time $t=T$ which is less than the expected rate results in investment rates less than depreciation ($I < \delta K$) to reach the new steady state equilibrium.
Figure 4. Optimal investment rate for uncertain tax increase at $t=T$ with a low initial tax rate $τ_L = $0.48

In the second case, the expected future emissions tax at time $t=T$ is the first time a tax has been imposed. Farmers have not adjusted their investment rates as for the case when a tax rate was already imposed before the announcement of an increase. With the expected future tax, the farmer’s investment rate in emission-reducing capital stock increases up to time $t=T$ (Figure 5) for all levels of initial emission-reducing capital stock up the equilibrium for the expected tax rate. To reach the saddle paths for $\dot{τ}_L = 0$ the farmer must incur high investment rates for low levels of emission-reducing capital without knowing if an actual tax rate will be imposed since there is a positive probability that the tax rate could be zero at time $t=T$. Without prior investment adjustments,
\[ \dot{I}_{E(t)} = 0 \] farmers will need to increase investment rates based on the deviation saddle path, \( \dot{I}_{\tau=0} = 0 \) rather than moving directly to \( \dot{I}_{E(t)} = 0 \).

![Figure 5](image.png)

Comparing the change in investment rates for the two cases shows that when the uncertain tax increase is the first time a tax is imposed, adjustment in investment rates occurs for all levels of emission-reducing capital. When an initial low tax rate was imposed in stage 1, only farms with emission-reducing capital greater than \( K=62 \) adjusted their investment paths towards the expected tax rate. The difference in area between the saddle paths for the optimal investment rate between \( \dot{I}_{\tau=0} = 0 \) and \( \dot{I}_{E(t)} = 0 \) were smaller for lower levels of emission-reducing capital compared to the saddle path for \( \dot{I}_{\tau=0} = 0 \) and \( \dot{I}_{E(t)} = 0 \). Since the tax increase, of uncertain scale, in time \( t=T \) is the...
first introduction of an emissions tax, there are an increased number of possible outcomes for investment rates and greater uncertainty surrounding it compared to the case where there is a positive initial tax rate in the first stage, in which fewer possible outcomes exist.

Conclusions

A farmer’s incentive to invest in emission-reducing technology is influenced by the size and type of environmental policy instrument imposed. In this paper, we extended Farzin and Kort’s (2000) theoretical model to include a functional form for animal air emissions as a function of productive (cows) and non-productive (emission-reducing capital stock) capital stock. Further, numerical solutions for an introduction of a certain tax rate at time $t=0$ and an uncertain tax increase at time $t=T$ were estimated.

A certain tax rate imposed at time $t=0$ resulted in immediate increased investment in emission-reducing technology. An uncertain tax increase imposed at time $t=T$ for an initial low tax, $\tau_L = $0.48 lb/ ammonia emissions, demonstrated farmers adjusted their investment rates in emission reducing technologies towards the optimal investment level in anticipation of an expected tax increase imposed at time $t=T$. When farmers did not face an initial tax rate at time $t=0$ the amount of investment necessary to achieve the optimal path was increased compared to those farmers who had faced a positive initial tax rate. In other words, because farmers who had not faced any tax rate prior to time $t=T$ had not responded to potential tax increases, greater levels of investment were necessary approaching time $t=T$ relative to those facing positive initial tax rates.

The numerical solution for the optimal investment policy allows policy makers to evaluate how investment rates changed as the tax rate and uncertainty surrounding it
changed. This numerical model can be further extended to consider such items as emission standards and tradable permits rather than emission taxes as well as the inclusion of investment grants through cost-share programs to decrease investment costs associated with emission-reducing technologies.
References


