Journal of Cooperatives

Reprint of a Classic

The following article by Richard Phillips, then a new Iowa State Ph.D., was arguably the most thought-provoking piece on cooperatives published in the 1950s.

Economic Nature of the Cooperative Association

Richard Phillips

The cooperative association is an association of firms or households for business purposes—an economic institution through which economic activity is conducted in the pursuit of economic objectives. An understanding of economic concepts is basic to the sound organization and operation of cooperatives, and to public policies toward cooperative activity which are consistent with society's objectives. Much has been written about cooperative activity, but the literature is dominated by socio-reformistic, historical, and descriptive interpretations. The so-called basic "principles of cooperation" are referred to frequently and with considerable ardor. Seldom is their significance seriously questioned. Economic theorists have seldom addressed themselves specifically to the cooperative association. Where cooperative activity has been mentioned at all, the cooperative has been treated as a special kind of corporation, covered quite adequately by the general theory of the firm. As Aizsilnieks has suggested,[1] such a treatment led Clark[2] to generalizations regarding the effect of cooperatives on general welfare which are difficult to justify, at least until further analysis has been made.

This article attempts to develop, on the basis of the contemporary economic theory of the firm—but with adaptation to the cooperative structure—a realistic, workable, and reasonably complete theory of the economic nature of the cooperative association. This theoretical framework involves: (1) the economic structure of the cooperative association; (2) the economic relationships among the participating units; and (3) the conditions necessary for profit maximization in the cooperating firms.

The Jointly Operated Plant

When a group of individual firms form a cooperative association, they agree mutually to set up a plant and operate it jointly as an integral part of each of their individual firms (or households in the case of a consumer

cooperative).[3] The cooperative has no more economic life or purpose apart from that of the participating economic units than one of the individual plants of a large multi-plant firm. Instead the participating firms agree to function *coordinately* with respect to their joint activity. This agreement runs multi-laterally among the participating firms, rather than between each of them and the joint activity as such. These participating firms must function as an economic team in relation to their coordinated activity. They must forgo some of their individual sovereignty in favor of themselves as a team. It is technically correct to speak of the cooperative plant and of cooperating firms, but not of the cooperative firm.

As an economic institution, the cooperative association is not an organization of persons, either as laborers, as capital contributors, or as patrons, for that matter. It is an organization, as Emelianoff and Robotka both point out, of sovereign economic units—firms or households.[4] Although it is descriptively correct to say that a cooperative association is a business organization owned and controlled by its patrons and operated for their benefit as patrons, such a statement contributes nothing to the understanding of the economic structure of the cooperative. Capital, and even labor, may be involved in patronage; but patronage, per se, is not a productive resource capable of earning an economic return in the production of goods and services. Rather, the value of patronage to a given firm is reflected in the nature of the demand and supply functions faced by that firm. It is not possible "to organize an economic system from the standpoint and for the benefit of the . . . patron,"[5] as the controller of a given factor of production. It is possible for firms jointly to organize and operate a common plant with the idea of maximizing the economic returns to each of the individual associated firms.

The entrepreneurs of the associated firms each must allocate productive resources to their common plant, the same as a multi-plant firm must allocate resources to each of its plants. The associated firms in a cooperative cannot be in static partial equilibrium (under the profit maximization assumption) unless the marginal conditions for optimum resource allocation in each firm are met. This means that for each participating firm the marginal productivity of each resource allocated to the cooperative plant must be equal to the marginal productivity of that resource in the individual plants of that member firm. In addition, the marginal productivity of the last dollar must be equal in every use within each firm.[6]

In short, when two or more economic units cooperate with respect to some function or activity that is integrally related to their individual economic operations, the result is not a new firm; instead it is a common economic plant. The cooperative *association* consists of the sum of the multi-lateral agreements among the firms participating in the joint activity, in order that these firms may function coordinately through their common plant. The cooperative *activity* is an economic plant operated jointly as a part of these several firms. The cooperative, as such, has no entrepreneurial unit; its member units each have their entrepreneur. Except for the coordination necessary to enable a group of economic units to operate a common plant jointly, the member units function independently of one another in the pursuit of their individual economic careers. Each partici-

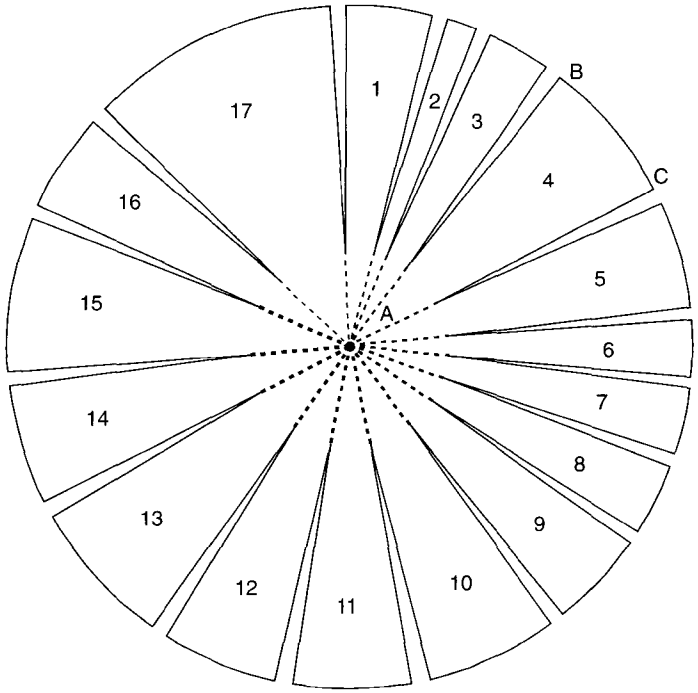
pating firm arrives at production equilibrium considering its separate activities plus its proportionate share of the joint activity as an integrated production unit.

This structure in the cooperative association is visualized in Figure 1. The triangular numbered sections represent the member firms. The small uninscribed circle at A denotes their joint plant. Notice that the complete firm in each case includes a part of the joint plant at the center (*i.e.*, BAC for firm 4). Observe that there is no connection between the firms except where they come together in the joint plant. Note also that the joint plant is indistinct except as a part of each of the member firms. If all firms were to withdraw (move outward from the center) there would be no cooperative plant left. If one firm were to withdraw (number 3 for example) a section of the plant would be removed, and its size correspondingly reduced.

Relationships Among Participating Firms

The participating firms usually do not share their joint plant equally. The proportion of the total joint plant included as a part of each of the participating firms is defined by the relative size of the production activities in the individual plant (or plants) of each firm with which the activities conducted through the joint plant are integrated (see Figure 1). This *proportionality* determines the manner in which the participating firms will share

Figure 1.—The Cooperative Structure



all inputs, including entrepreneurial inputs, and all outputs—all costs and benefits—of the joint plant. In order to achieve a static optimum allocation of resources *among* the participating firms, the entrepreneurial decisions, the bearing of uncertainties, the financial responsibility, the economic use, the costs, and the economic benefits in connection with the joint activity must be shared by the firms on the basis of this proportionality. Of course, only the participating firms will share in these economic functions which arise in their joint activity.

In other words, the firms which participate in a single department joint plant in each operating period will share all economic functions in connection with the joint plant on a proportional basis. From the standpoint of economic structure, voting in the cooperative association will not be shared on a per firm (one-firm one-vote) basis, but on a proportional basis. The economic use of the joint plant will be shared on a proportional basis. All costs, including costs of risks, will be shared on a proportional basis. Uncertainty bearing will not be shared on a per firm basis, but on a proportional basis. The financial responsibility (*i.e.*, either providing the actual capital, or paying the interest and providing the security required to obtain it) will be shared on a proportional basis. And the economic benefits, if any, of the joint activity will be shared by the participating firms on a proportional basis.

Given perfect knowledge and certainty, proportionality also applies over time and between departments. Under these assumptions, the firms jointly operating each department in the joint plant will participate in all of these economic functions proportionately as they share that department. Furthermore, all functions arising from the operation of the joint plant that are common to several departments will be shared by the groups of firms participating in each of the departments concerned. Such interdepartmental functions will be shared on a basis proportional to the fraction of the total operation of the joint plant represented by each department. Functions specific to a given department will be of concern only to the firms jointly operating that department.

What about the interfirm relationships extending over time which arise from participation in the joint activity? A given proportional sharing of the joint plant (such as that shown in Figure 1) for one operating period will not necessarily hold for other time periods. Proportionality must therefore be maintained between the groups of participating firms in each operating period as well as within each such group. This means that all functions which extend over time will be shared by the groups of firms operating the joint plant in each period on a basis proportional to the relative size of the plant in each period.[7] Functions which are specific to a given operating period will be participated in only by the group of firms sharing in the joint activity in this period.

Even when the assumptions of perfect knowledge and certainty are dropped, proportionality determines the manner in which the firms participating in a joint plant share the economic functions in the joint activity—including those which are interdepartmental and intertemporal. However, in this case planned proportionality— rather than actual realized proportionality—is the guide. Also, under these conditions, it may be economic

to share functions which are specific to a given department or time period interdepartmentally and intertemporally in order to reduce the expected dispersion around the most probable anticipated values of relevant variables facing each participating entrepreneur. How far the firms participating in the joint plant will go in this direction depends upon: (1) the anticipated uncertainties; (2) the expected reduction in this dispersion by interdepartmental and intertemporal pooling; (3) the percentage of owned equity in the participating firms; and (4) the indifference toward relative certainty and relative efficiency of the majority of the participating entrepreneurs. To the extent that specific functions are pooled, they also will be shared among the groups of participating firms involved on the basis of anticipated proportionality.

Profit Maximization in Participating Firms

Following the orthodox static assumptions, the cooperating firms individually seek to maximize their profits—considering that part of their operations in the jointly operated plant as well as their individual operations outside of it. Two conditions are necessary if this objective is to be accomplished. First, each participating firm must be in production equilibrium. Second, the joint plant must be of optimum size. Although these two criteria are related, it is possible to satisfy either without satisfying the other. The profits of the participating firms are not at their absolute maximum unless both are met.

The necessary production equilibrium conditions are the same as those for any firm. The participating firms must (1) achieve the best production combination, and (2) be of optimum size. The best combination of outputs will be produced at the least cost combination when the ratio of the *marginal expenditure for the input*[8] *to the product of the marginal physical productivity of the input*[9] times the *marginal return for the output produced*[10] is equal between all inputs in the production of all outputs within the firm. The optimum firm size will be defined at the output which equates the marginal revenue with the marginal cost of each product produced and for all products taken together. These conditions together mean that the marginal revenue productivity[11] must be equal to the marginal expenditure for each input in all alternative uses within the firm.

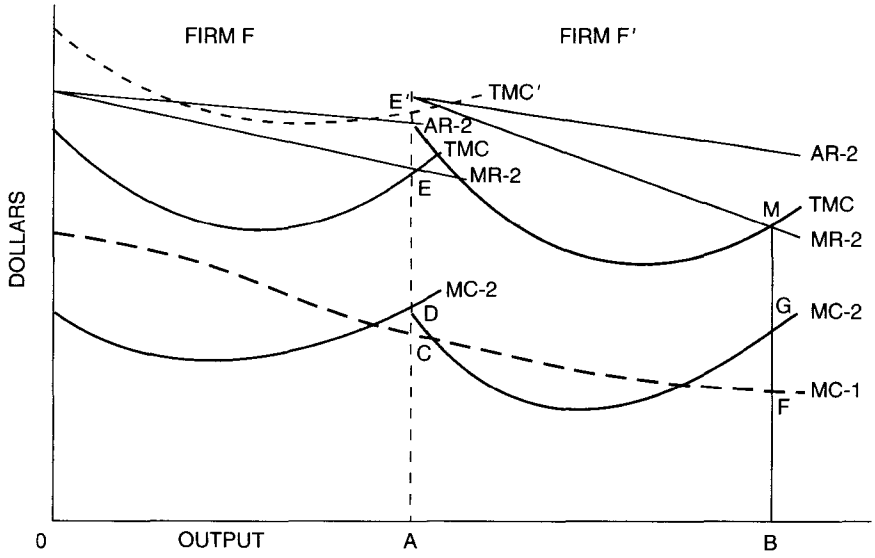
The participating firms are ordinarily vertically integrated in the sense that the output of the joint plant is the raw product input of the individual plants of the participating firms—or alternatively, the output of the individual plants of the participating firms is the raw product input of the joint plant.[12] As products move from one plant to another in a vertically integrated firm, no transfer of title takes place and no price is established for them. Such a firm is not interested in the profitability of any one of its plants considered alone. Rather it is interested in the profitability of the complete chain of integrated plants operating as a unit. The optimum output is determined on the basis of the complete operation. If measured in comparable units, the equilibrium output will be the same for each of its plants. The vertically integrated firm determines this optimum output by equating the sum of the marginal cost functions in all plants with the marginal revenue in the final plant from which the product is marketed.[13]

The cooperating firm equates the sum of the marginal cost in its individual plant or plants and the marginal cost in the joint plant with the marginal revenue facing the firm in the market where the product is sold. This criteria applies both to short run and to long run analyses.

The relevant segment of the marginal cost function in the joint plant to each participating entrepreneur is that beginning with the sum of the equilibrium outputs of all other participating entrepreneurs, rather than at zero output. Each entrepreneur must consider the additional costs in the joint plant resulting from his volume of output. The precise equilibrium output for any one participating firm cannot be determined unless the equilibrium output for all others is given; without this information the exact segment of the marginal cost function which is relevant cannot be determined.[14] Formal solution of the optimum output for each participating firm and in the joint plant, given the number of participating firms, is possible by simultaneous equations only. If the output product of the participating firms is marketed from their joint plant, everything which has been said about the relevant range of the joint plant marginal cost curve to each firm is true also of the relevant range of the marginal revenue curve.

The equilibrium position for two cooperating firms is illustrated in Figure 2. The two firms, F and F', operate plant 1 jointly. They each operate a second plant entirely independently of the other. The output of the joint plant is used entirely as an input for the two independent plants. MC-1 is the marginal cost function in the joint plant. MC-2 for firm F is the marginal cost in the second plant operated by this firm and MC-2 for firm F' is the marginal cost in its second plant. The output for the joint plant

Figure 2.—Participating Firm Equilibrium. Two Firm Plant



and the second plant of each firm is assumed to be in comparable units and measured on the same scale. Firm F′ produces at the output AB where the sum of the marginal cost in the joint plant and the marginal cost in its second plant equals the marginal revenue in the second plant. The relevant segment of the MC-1 curve to firm F′ is CF. The TMC curve is the sum of MC-2 for firm F′ and the segment of the MC-1 curve to the right of C. The sum of BF and BG is equal to BH. The equilibrium output for firm F′ assuming the best production combination, is AB, as defined by the intersection of TMC and MR-2, at H.

Firm F produces at the output OA where the sum of the marginal cost in its second plant and the marginal cost in the joint plant equals the marginal revenue in the second plant. The relevant segment of the MC-1 curve is that starting at the equilibrium of firm F′. In this diagram AB is equal to OA, so that the relevant segment of this curve to firm F is also CF. The TMC curve for firm F is the sum of its MC-2 curve and the MC-1 curve to the right of C. The sum of AD and BF is AE. The equilibrium output for firm F is defined by the intersection of TMC with MR-2 at E as OA units. If firm F operated plant 1 alone the total marginal cost to the firm would be TMC′ rather than TMC, since the relevant segment of MC-1 would be that starting from zero, and AC plus AD is equal to AE′. In such case the firm would not be able to cover costs, as contrasted to the positive profit margin when plant 1 is operated jointly with another firm.

Aside from variations due to the adjustment to optimum output in each firm, the size of the joint plant is determined by the number and size of the participating firms. One reason firms find it economic to operate a plant jointly, rather than carry out the same production processes individually, is a decreasing long run average cost curve over a considerable range for a part of the total economic function, or contemplated function, of the several firms. By pooling this part of their activities and functioning coordinately through a common plant, they are able, under these conditions, to increase their economic efficiency by approaching more nearly optimum size for this operation. Firms which jointly operate a common plant, once they have achieved the best production combination in their joint plant, are interested in the plant size which will maximize plant efficiency and make the greatest possible contribution to the profits of each participating firm. When the optimum size for the joint plant has been determined, this optimum is maintained over the long run by varying the number of participating firms. Plant size is reduced by failure to replace firms that retire from participation; it is expanded by encouraging new firms to participate.

How is the optimum size for the joint plant determined? The joint plant will be of optimum size when the long run average net return from the plant is at a maximum. This is true because the greatest contribution of the plant to the profit of each participating firm, regardless of the size of the firm, will be at that point.[15] Where the output of the joint plant is raw product or specific resource input for the individual plants of the participating firms, optimum size in the joint plant is defined by the minimum point in the long run average cost function in the joint plant. In this case, since they operate the joint plant to supply their individual plants, the participating firms face no revenue function from their joint plant.

Next, consider the case where the output of the individual plants of the participating firms is the raw product input of the joint plant. Here the optimum size for the joint plant is defined at the output which maximizes the vertical distance between the long run average revenue curve and the long run average cost curve in the joint plant. The joint plant long run average cost function in this case is net of the cost of raw product inputs supplied from the individual plants of the participating firms.

Typically, the cooperative activity consists of more than one department. Such a practice is economic when the production carried on by the participating firms makes it desirable and the long run per unit joint cost in the common plant decreases with volume. In such case, the participating firms will all maximize their profits by adding departments in the joint plant, each of optimum size, until a size is reached which minimizes the long run per unit joint cost.

The determination of the long run optimum size in the joint plant is distinct from the determination of the long or short run optimum size for each participating firm. Optimum long run joint plant size is achieved by varying the number of participating firms, each of long run optimum size. Both optimums must be reached if the profits of each participating firm are to be an absolute maximum.

Dynamic Consideration

Under dynamic conditions, the participating firm is interested in the maximum expected discounted value of the profit stream. But such a firm also is interested in the minimum expected variation in this stream over time. Each entrepreneur, in any given planning period, conceivably must achieve some optimum balance between these two objectives.

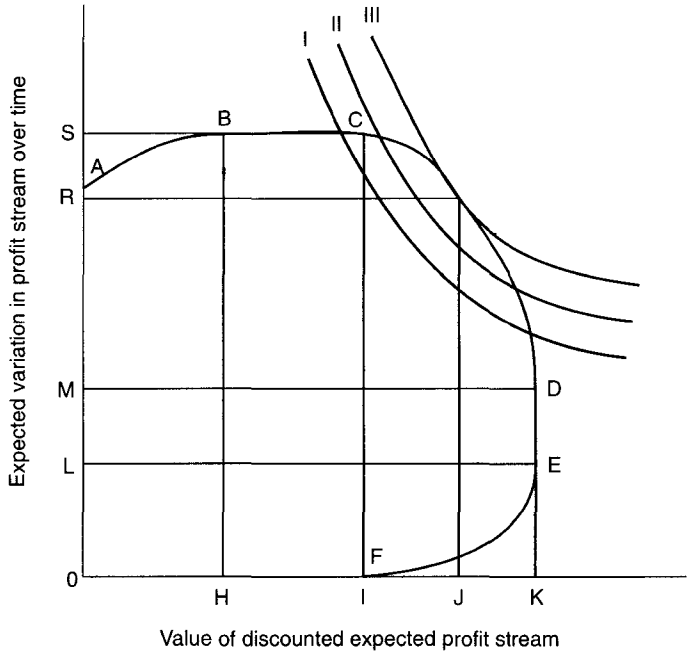
The necessary conditions for the maximum discounted expected profit stream for the firm in any period are similar to those under static conditions. The firm will produce the best combination of outputs at the least cost combination when the ratio of the discounted expected marginal expenditure to the discounted expected marginal revenue productivity is equal for all inputs in all alternative uses within the firm.[16] The optimum planned output in each period is determined by equating the expected discounted marginal cost function and the expected discounted marginal revenue function. The sufficient conditions must provide that any time the firm is thrown out of equilibrium, adjustments to meet again the necessary conditions will cause conversion to the efficiency equilibrium.[17]

In order to be in equilibrium under the objective of maximum stability of the profit stream over time, the participating firm must allocate resources and plan production so that the expected marginal gain in stability over time is equal to the expected marginal cost of the organization for stability. The ratio of the expected marginal gain in stability to the expected marginal expenditure of organizing for stability must be equal for all methods used by the firm to achieve profit stability over time. This equilibrium must also be stable in the sense that if the firm is thrown out of equilibrium, adjustments to meet again the necessary conditions will cause conversion to the stability equilibrium.

Given: (1) the production plan which maximizes the discounted value of the expected stream of profits, and (2) the production plan which minimizes the expected dispersion around the most probable profits over time for a participating firm, the task is resolved into the selection of the best compromise position between the two. The location of this compromise position for a given firm will depend upon: (1) the "technical" relationship between these two objectives; and (2) the indifference map of the entrepreneur defining the relative importance of the two objectives to him. If both the technical relationship and the indifference map are assumed to be definite and clearly defined and without discontinuities, then a unique optimum position will be described by the point of tangency of the technical relationship function and an indifference curve between these two objectives. The only rational segment of the technical relationship function is that part where the two objectives are competitive; the entrepreneur can improve his position by moving into this range regardless of the slope and shape of his indifference curves toward the two.

Such an optimum position is illustrated in Figure 3. In the diagram, efficiency is measured on the X axis, increasing to the right, and stability is measured on the Y axis, increasing upward. The distance OK represents optimum dynamic efficiency and OS represents optimum stability of profits over time. The technical relationship between efficiency and stability is represented by the curve ABCDEF. It illustrates the probable ranges of: (1) complementarity at very low efficiency, and at very unstable organi-

Figure 3.—Relationship Between Efficiency and Stability Objectives



zation; and (2) independence (BC and ED) at moderate efficiency and stability. The only segment representing rational combinations of anticipated efficiency and stability is CD, where the two objectives are competitive. The best position is defined by the point of tangency of the ABCDEF curve and indifference curve III at OJ efficiency and OR stability over time.

The conditions for optimum efficiency in the firm participating in a joint plant pointed out in the previous section are applicable under dynamic conditions if discounted anticipated variables, including allowances for uncertainties, are substituted for given ones and intertemporal relationships between variables are provided for.

What about the conditions for optimum stability in the participating firm? *First*, as long as it is economic, the anticipated conflict of interest among participating entrepreneurs must be minimized. This means an association of reasonably homogeneous, rather than heterogeneous, participating firms. Specification of the conditions of participation in the articles and bylaws of the association will also help to minimize this anticipated conflict of interest. It will also cause each entrepreneur to face less uncertainty as to the continued participation of the other firms.

Second, aside from this uncertainty as to the attitudes and actions of the other entrepreneurs, participation in the joint activity in itself tends to reduce the anticipated variability of the profits of each firm over time. Vertical integration, by reducing the number of markets of concern to the participating entrepreneur, makes the profits of his firm more stable under temporary fluctuations in market conditions. Also, because of the higher degree of coordination of the various processes, the vertically integrated operation will ordinarily be more adaptable to changing technical and economic conditions facing the firm.

The participating firms may gain further stability by diversifying horizontally and laterally, both in the joint plant and in their individual plants. Where there are several departments in the joint plant—all of which are integrated with the individual activities of the participating firms—anticipated stability is usually greater than would otherwise be the case. This is particularly true where counter-cyclic fluctuations are expected among the activities and outputs of the participating firms.

The opportunity to pool uncertainties within the group of participating entrepreneurs is another source of increased anticipated stability of profits over time to each firm. It often will be economic, because of the gain in anticipated stability, to pool uncertainties of the joint plant operation which are specific to a given department or time period between departments and over time. All such increases in the anticipated stability of profits to the participating firms over time result directly from the fact that they have carried out "concerted integration."[18]

Participation in the joint plant often reduces capital-rationing and other market imperfections facing the firm. Consider capital-rationing as an example. Participating firms usually are able to obtain credit as a group for the joint activity as such—even though some of them individually are unwilling or unable to obtain the use of any more capital. The structure provides machinery whereby participating firms with a high marginal productivity for capital may borrow from those with a low marginal productiv-

ity for capital. Firms are willing to borrow more capital because they jointly assume the anticipated new uncertainties arising from the additional borrowed capital and expanded production activities. The chief result of the reductions in capital-rationing and other market imperfections is that they provide for an increase in expected economic efficiency for each participating firm without a corresponding decrease in the expected profit variability over time, and vice versa.

Practical Considerations

The concept of the basic economic structure of the cooperative association, and the interfirm relationships resulting from the joint ownership and operation of a common plant, should provide an important working tool. The applied economist, the cooperative attorney, the cooperative accountant, the practical cooperative leader, and the legislator alike should find it useful. It should avoid the insurmountable difficulties and inconsistencies which arise when the cooperative is looked upon as a firm and its operations studied apart from the individual operations of the participating firms. The concept should shed new light on such things as: (1) the appropriate legal form for the cooperative association; (2) the cooperative income tax question; (3) the nature of "non-member" business in the cooperative; (4) the need for "grass-roots" control in the cooperative association; (5) the economic expansion rate for a given cooperative activity; and (6) economic and successful methods of financing cooperative activity.

For example, consider the cooperative income tax question. Whether the joint activity is conducted on a pooling basis, a purchase and sale basis, or on some other basis, it is operated for one purpose—to make their integrated business operations more successful. Theoretically and actually it is impossible in any short run period to determine the exact amount of net income which specifically arises from the operation of the joint plant. The net income is made in and accrues to the *participating firms as integrated production units.* Economically, there is only one accurate way in which the income arising from the joint activity can be taxed; that is to include it as a part of the total taxable income of each of the participating units. This is true whether such earnings are invested to expand the joint plant, whether they are invested to expand the individual plants of the participating firms, or whether some other disposal is made of them. It is true whether the participating units are corporate firms, partnerships, or individual proprietorships.

An understanding of the economic nature of cooperative business activity, as developed here, clearly reveals the absence of scientific basis for such statements as "in the cooperative association *people*, not capital, count," "factories are free to cooperative members," "cooperation is a superior way of doing business," "to the extent that the cooperative is different from other types of business activity, it is a *social* organization," and "in business, cooperation is the antithesis of competition." Such so called "cooperative principles" as "business with members at market prices" and "one-man one-vote" and "business on a strictly cash basis" are revealed as inaccurate or irrelevant. And, finally, the concept can contribute much

toward improved financial structures, improved accounting procedures, and more effective policy decisions within cooperative associations, and it can aid in the formulation of sound public policies as they relate to the cooperative activity.

Notes

1. Arnolds P. Aizsilnieks. Farmer Cooperatives and Economic Welfare—A Reply. *Journal of Farm Economics.* Vol. 34, August 1952, p. 400–403.

2. Eugene Clark. Farmer Cooperatives and Economic Welfare. *Journal of Farm Economics.* Vol. 34, February 1952. p. 35–51.

3. Cf. Edwin G. Nourse. *The Legal Status of Agricultural Cooperation.* The Macmillan Co. New York. 1927. p. 171, and Frank Robotka. Lego-Economic Implications of Cooperation. *In American Cooperation*; 1946. p. 522–533. American Institute of Cooperation. Washington, D.C. 1946.

4. Ivan V. Emelianoff. *Economic Theory of Cooperation.* Ivan V. Emelianoff. Washington, D.C. 1942. p. 249, and Robotka. *op. cit.* p. 526.

5. Nourse. *Op. cit.* p. 21.

6. Cf. Richard Phillips. *Economic Nature of the Cooperative Association.* Unpublished Ph.D. Thesis. Iowa State College Library. Ames. 1952. p. 149–246.

7. Provided the relationships affect each operating period on a basis proportional to the size of the plant operation in each period.

8. Defined as the increment of the firm's total cost for inputs resulting from an additional unit of this input.

9. Defined as the net addition to the physical outputs of the firm resulting from the use of an additional unit of this input.

10. Defined as the increment of the firm's total revenue for output resulting from the sale of the added amount of this output plus the sale of the net change in amount of other outputs.

11. Defined as the product of marginal physical productivity and marginal return.

12. The exception is the plant operated jointly by two or more firms to procure specific capital resources, such as farm machinery for agricultural firms. In this case each participating entrepreneur will equate the marginal cost in the joint plant with the marginal revenue productivity of the resource in his individual plant. Such participating firms are integrated in the sense that several stages in the production process are brought under one entrepreneurial control. The more narrow definition of vertical integration used in the text is necessary, however, for analytical purposes.

13. Cf. Werner Zvi Hirsch. The Economics of Integration in Agricultural Marketing. Unpublished Ph.D. Thesis. Univ. of California Library, Berkeley. 1950. p. 97–144. Hirsch did not deal with this general case, although it follows logically from his analysis. He dealt instead with the following conditions respectively: (1) no charges (costs) in any plant except that farthest from the consumer, (2) only fixed charges (costs) in all plants except that farthest from the consumer, (3) per unit charges (costs) at a constant rate in all plants except that farthest from the consumer, (4)

where fixed payments are made outside the firm for services, and (5) where a fixed per unit payment is made outside the integrated firm for services.

14. Of course it is not necessary to make this determination if the marginal cost function is horizontal over the relevant range.

15. Cf. Richard Phillips. Sharing Risks and Uncertainties in Farmer Cooperatives. Unpublished M.S. Thesis. Iowa State College Library. Ames. 1949. p. 72–84.

16. Provided the ME and MRP functions are so defined as to include intertemporal relationships. The same input or output in different time periods is considered as separate inputs and outputs. The anticipations are adjusted for uncertainty premiums and discounts. A constant discount rate is assumed in each planning period. Cf. the following:

J.R. Hicks. *Value and Capital.* 2nd. ed. The Clarendon Press. Oxford. 1946. p. 192–226.

Sune Carlson. *A Study on the Pure Theory of Production.* Stockholm Economic Studies. No. 9. P. S. King and Son, Ltd. London. 1939. p. 103–126.

Oscar Lange, *Price Flexibility and Employment.* Cowles Commission for Research in Economics, Monograph No. 8. The Principia Press, Inc. 1944. p. 20–44.

Albert Gaylord Hart. *Anticipations, Uncertainty and Dynamic Planning,* Univ. of Chicago Press. Chicago. 1940.

17. Cf. Paul Anthony Samuelson. *Foundations of Economic Analysis.* Harvard University Press. Cambridge. 1948. p. 257–349.

18. See D.H. Robertson. *The Control of Industry.* Harcourt, Brace, and Co. New York. 1923. p. 49.

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