Modeling Recreational Amenities in an Urban Setting: Location, Congestion, and Substitution Effects

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In this article, we introduce a recreational amenity—a greenbelt park—into a simple urban economic model. For multiple possible park placements, we solve for the associated equilibrium urban structure, including the equilibrium rent gradient, city boundary, total number of park visits, the overall utility level, and total vehicle miles traveled. We examine how these change with alternative park placement sites. We then show how two modifications of the basic model—allowing congestion at the site to affect site quality, and introducing the possibility of a substitute site at the city's periphery—affect our conclusions about how greenbelt location influences urban structure.

Key Words: urban structure, greenbelt, congestion

The recent proliferation of open space acquisition programs in metropolitan areas has rekindled interest in the question of how the location of urban amenities affects urban structure and welfare. While early papers employed monocentric city models to assess the effects of generic urban amenities on residential density, city size, and welfare (Diamond and Tolley 1982, Kanemoto 1980), recent papers have modeled urban open space amenities more explicitly by allowing the amenity to take up real space on the landscape. For example, Lee and Fujita (1997) examine the efficiency of alternative greenbelt locations. Wu (2001) and Plantinga and Wu (2003) focus on exploring the relationship between open space provision and urban structure, illustrating the implications of alternative sizes, configurations, and locations of an open space amenity.

In all of these papers, the benefits of living near an urban park are modeled as being derived from proximity: benefits are highest adjacent to the amenity, and decline gradually with distance from it. If the quality of the open space is considered, the benefit function is adjusted to reflect quality levels, with higher quality amenities conferring a higher level of benefits. The bid-rent function, and the corresponding residential density function, emerge in an equilibrium in which households, when selecting a residential location, consider commuting costs to the central business district, the size of their residential lot, and their distance from the amenity. These papers show that the amenity attracts residential development nearby and consequently drives up neighboring rental rates.

While mere proximity may confer benefits, many open space programs are supported explicitly because they provide recreational opportunities. The potential for public access into new parkland is an important component leading to the passage of bonding referenda that fund park acquisition. Recent papers (Kline and Wichelns 1998, Duke and Aull-Hyde 2002) measure the motivations for preserving open space, and public access rates as one of the most important factors.

This article considers the implications of alternative location choices for an urban greenbelt, where the recreational function of the park is explicitly modeled: benefits from the greenbelt are derived from visits taken by urban residents. The focus on recreation allows us to accomplish sev-

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eral goals. First, we mirror actual behavior by incorporating the use value of open space into a model of this type: urban parks are used for recreational purposes, and residents are likely to consider access to recreation sites when making residential location choices (as noted by McConnell 1990, Parsons 1991, Colwell, Dehring, and Turnbull 2002, Smith, Van Houtven, and Pattanayak 2002). Second, because we model visits, we are able to analyze the effects of congestion in the greenbelt: as more residents visit the park, congestion increases and the quality of the park (and therefore the contribution of visits to residents' utility) falls. Third, we introduce the possibility of a substitute recreation site—in particular, the open space that lies beyond the periphery of the urban area. This allows us to explore the implications of substitutability and differential quality levels on the welfare effects of alternative greenbelt locations. For example, the periphery may act as an escape valve, which has a feedback effect on the quality of the interior park through the congestion variable. Our interest in recreational trips is shared by Kovacs and Larson (2007), who also consider recreational benefits in their examination of how the location of open space affects urban structure. They use a different model specification to focus on amenity and recreational benefits but do not allow for the feedback of visits on site quality.

When we model congestion, we solve for the utility level that arises in equilibrium where the planner has a choice only about the location of the park. This is not the social optimum, as any congestion externality remains uncorrected: we are modeling the typical situation where a planner does not have the ability to enact a corrective mechanism such as a congestion tax.[1] However, we do show how the effect of the congestion externality on residential utility can be mitigated by choosing the site of the park appropriately.

Our primary focus is on the effects of urban park placement on residential utility, but we also report on other urban structure variables of interest. For example, what is the effect of greenbelt location on city size and residential density? Does establishment of an urban park encourage increased density, and does this effect depend on the location of the park and the existence of attractive open space beyond the periphery? Also, since we model trips, we can assess the impact of an open space amenity on the total number of car trips and vehicle miles traveled. Because urban park location will have an effect on residential settlement patterns, commuting patterns will also change—with implications for the number of commuting miles traveled. A recreation site of this type adds both trips and vehicle miles traveled, and these changes may be of concern as they relate to air pollution or traffic congestion. We explore the sensitivity of these various effects to the location of the park, the presence of the congestion externality within the park, and the existence of attractive open space beyond the city limits.[2]

The rest of this article proceeds as follows. First, we develop a simple closed-city model in which urban residents derive utility from housing, a composite good, and visits to two recreation sites: the interior urban greenbelt and the open space at the city's periphery. The utility specification we use ensures that a fixed budget share is spent on recreation visits, but consumers allocate that budget share between the two sites according to the substitution parameter, the quality levels of the two sites, and the relative costs of visits. Second, we provide some intuition into the mechanisms by which park location affects residential utility. Third, we present our results. In each case, for multiple possible greenbelt placements, we use numerical optimization to solve for the associated equilibrium urban structure, including the equilibrium rent gradient, city boundary, total number of greenbelt visits, the overall utility

[1] Thanks are due to an anonymous reviewer for pointing out this distinction.

[2] City size, air pollution, and traffic congestion may affect utility, though in the interest of simplicity we do not model these effects explicitly. It is not obvious whether or not city size would intrinsically affect utility, apart from its effects through the variables we do model (e.g., commuting costs, housing consumption). Air pollution would be a public bad, uniformly affecting all residents. A utility increment that reflects the relationship between automobile trips and air pollution, along with the relationship between air pollution and utility, could be subtracted from the city's equilibrium utility level. This would likely change the optimal park location. Traffic congestion affects residents at different locations differently. Including traffic congestion in the utility function would alter the equilibrium in fundamental ways, as residents subject to heavy traffic congestion would have to be compensated in other ways (e.g., by lower housing prices) to achieve the same utility levels as other residents. This, in turn, would affect traffic patterns. It would be possible to introduce traffic congestion and solve for the associated equilibrium, but we choose not to do so here.

level, and total vehicle miles traveled. We then examine how these change with alternative greenbelt placement sites.

The Model

We use a monocentric urban model in the tradition of Alonso (1964), using Solow's (1973) indirect utility approach. We consider the case of a closed city, where urban population is fixed.[3] As in the standard model, utility is derived from housing and a consumption good. In addition, we introduce into the utility function a new category of spatially explicit goods—recreation amenities that residents must travel to in order to consume. Two possible recreation sites are considered: a circular, donut-shaped amenity, or greenbelt, located at distance x_1 from the city center, and the peripheral open space boundary located at the city fringe x_2.[4]

The utility function is a hybrid that combines features of the Cobb-Douglas and the CES utility function.[5] Utility is assumed to be a function of a consumption good (c), housing (h), and visits (w_1, w_2) to the two recreation sites:

$$(1)\, U(c,h,w_1,w_2) = \alpha \ln h + \beta \ln c + \frac{\gamma}{\rho} \ln[\mu_1 w_1^{\rho} + \mu_2 w_2^{\rho}]\,.$$

This formulation allows residents to substitute between trips to the recreation sites, according to a number of factors that determine the desirability of one site over another. One of these factors is the relative qualities of the different sites, as represented by μ_1 and μ_2. Another is the degree of substitutability, as determined by ρ: if $\rho = 1$, the two destinations are perfectly substitutable; if $\rho = 0$, the two goods have a unitary elasticity of substitution. This formulation also allows us to consider the possibility that the peripheral site is unavailable. If the number of trips taken to the second site is equal to zero, this utility function reduces to a Cobb-Douglas with γ as the preference parameter for trips to the greenbelt and

$$\mu_1^{\frac{\gamma}{\rho}}$$

as the constant term. In this case, the parameter ρ is not a substitution parameter: it affects only the utility contribution of the greenbelt, not the optimal number of trips taken.

In this standard model, it is assumed that all individuals commute to a central business district (CBD) to work, and pay commuting costs that increase with distance to the urban center. Each individual receives a fixed wage, but total income differs from wage due to the way in which we model land ownership. There are two extremes in the treatment of land ownership in these models (Kanemoto 1980). One is to assume that one or more absentee landlords own all the land and collect all the rents. If one of the items of interest is how residents' utility responds to the establishment of a greenbelt, this is a complicating assumption because when rents leave the city, one must account for the effect of the open space on the welfare of the absentee landlord. The alternative we use is the case of public ownership; we assume the existence of a city government that pays for all the land for the city and the greenbelt at the agricultural rental rate, rents it out to the city residents, then redistributes the net proceeds as a "social dividend" evenly among the population. This assumption keeps rents, and all welfare effects associated with moving the greenbelt, within the city.

Total income therefore comprises both wage income (Y) and the social dividend (SD) received. Employment transportation costs are paid out of this amount; for our purposes, the income that remains is referred to as disposable income, Y_d. Because each resident receives an equal wage and an equal dividend, the farther the individual lives from the city, the smaller his or her disposable income.

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[3] In a closed city model, urban migration is not permitted. This is appropriate in the short run, or when the attractiveness of the amenity is not sufficient to overcome barriers to migration. A closed city model also permits changing utility levels in response to changes in park location. For these reasons, we confined ourselves to a closed city model.

[4] Several authors [starting with Plantinga and Wu (2003)] embed environmental amenities into a city with Cartesian coordinates. This allows them to have amenities and cities with various shapes. Our focus is on the location of the park relative to the urban core, so a one-dimensional model serves our purposes adequately. Embedding this type of model into a Cartesian plane would give us more interesting shapes and would likely amplify our results: placing a park farther from the central city would make it less accessible to residents of the urban core as well as residents on the other side of town.

[5] This specification is similar to the specification used in Cavailhes et al. (2004), who use a hybrid Cobb-Douglas-CES model to consider substitution among different kinds of green amenities.

With the price of the consumption good normalized to 1, the budget constraint is written as

$$(2)\ Y_d = Y + SD - jtx = r(x)h + c + [m + td_{x_1}(x)]w_1 + [m + td_{x_2}(x)]w_2,$$

where t is a per-unit travel cost, d_{s_1} represents distance from the residence to the greenbelt, and d_{x_2} represents distance from the residence to the open space boundary. Two costs are associated with amenity consumption: a fixed access cost and a variable travel cost that varies according to residential location. Commuters must take j trips per year to the central city to work, but choose the number of visits to each recreational amenity. It is assumed that the park occupies a fixed area a, which at any given distance to the CBD is associated with a depth $b(x_1, a)$.[6] Visitors to the park travel to whichever park border [x_1 or $x_1 + b(x_1, a)$] is closer. Recreation trips have both a fixed cost component (m) and a travel cost component. In this model, Y, j, t, m, a, α, β, and γ are all exogenous.

Inserting the optimal quantities of c, h, w_1, and w_2 into the utility function yields an indirect utility function expressing the maximum achievable utility for a set of commodity prices and income. In equilibrium, individuals are distributed on the landscape so that their utility is equalized. The rent function is found by setting the indirect utility equal to this constant (endogenous) utility level $\bar{v}$ and solving for rent. As in Solow (1973), we use two additional equations to close the system. The first sets the rent level at the extensive margin equal to the agricultural rent generated by the land at the city's periphery, and the second equates supply and demand for housing for a given population size (see Appendix).

We examine four cases. In the first two cases, the only recreational amenity that provides utility is the greenbelt, located at x_1. What distinguishes the first two cases is the way we treat quality: in the first case, the quality of trips to the greenbelt is fixed ($\mu_1 = 1$, $\mu_2 = 0$) (Case 1), and in the second, the quality of greenbelt trips is a function of park congestion, which arises from total trips to the park, W_1, [$\mu_1 = f(W_1)$, $\mu_2 = 0$] (Case 2). W_1 is defined as the sum of all trips that individuals take,

$$\textstyle\sum_i w_{1i}\ ,$$

where i indexes residents. We define the congestion variable as a linear function of aggregate number of trips to the park and park area, a:

$$(3)\qquad \mu_1 = 1 - \frac{W_1}{\theta a}.$$

The congestion variable represents the quality of a park experience, and it influences the amount of utility that is received from trips to the park. *Ceteris paribus*, as aggregate number of trips increase, or park area decreases, the quality of the park experience, and the utility derived from trips, declines. The parameter θ represents the strength of the effect of congestion on utility.

In the next two cases, two recreation areas are available; residents can travel to either the park or the peripheral open space to derive their recreation utility. We again consider the two ways of modeling quality: first, the qualities of the greenbelt and of the peripheral open space are fixed and equal ($\mu_1 = 1$, $\mu_2 = 1$) (Case 3), and second, the quality of the greenbelt is sensitive to congestion, though the quality of trips to the peripheral open space is still fixed and equal to one [$\mu_1 = f(W_1)$, $\mu_2 = 1$] (Case 4). In Cases 2, 3, and 4, we consider two values for the parameter ρ. Table 1 summarizes this taxonomy.

Characterizing Equilibrium with Changing Greenbelt Location

While all residents enjoy the same level of utility, $\bar{v}$, the elements that compose that utility differ across the landscape according to how far the resident is from the CBD. Therefore, while changes in park location lead to the same change in utility for all residents, the sources of that utility change vary with the distance from the urban center.

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[6] The function

$$b(x_1, a) = \sqrt{a/\pi - x_1^2} - x_1$$

is derived from the difference in the area of two circles, one with radius $x_1 + b$ and one with radius x_1.

Table 1. Taxonomy of Cases

	Single Site	Two Sites: Interior Site Plus Periphery
Constant quality	Case 1	Case 3a: $\rho = 0.50$ moderate substitutability between sites
		Case 3b: $\rho = 0.99$ high substitutability between sites
Congestion-sensitive quality	Case 2a: $\rho = 0.50$ high weight of quality on utility	Case 4a: $\rho = 0.50$ moderate substitutability between sites
	Case 2b: $\rho = 0.99$ low weight of quality on utility	Case 4b: $\rho = 0.99$ high substitutability between sites

Further, the number of residents at any distance from the central city will vary with park location as citizens redistribute themselves across the landscape to equalize utility. Generally, changes in park location affect utility through changes in the consumption of housing, the composite good, visits to the park, and visits to the periphery. These changes occur due to income, price, and quality changes. While some of these intermediate effects affect everyone in the landscape uniformly (e.g., the social dividend, the quality of the park), others vary with location on the landscape (e.g., housing prices, distance to the park, distance to the periphery), and others work through a redistribution of residents on the landscape (e.g., commuting costs).

Disposable income affects the consumption of all goods in the utility function, and it adjusts with changes in park location. The location of the park affects the social dividend through the aggregate amount of rent collected from, and redistributed to, city residents. All residents receive an equal share of this aggregate rent. Commuting costs (jtx) at any given location are not affected by park location, but park location will affect the distribution of residents on the landscape and consequently the distribution of the amount spent on commuting. As city size changes, the range of costs incurred for commuting also changes. For example, a resident located at the outermost edge of the city will have a longer commute and lower disposable income if the city expands due to a change in park location.

Changes in utility are due to changes in the consumption of each of the goods in the utility function, and these changes occur due to changes in income and prices that arise from changes in park location. It is straightforward to derive analytical expressions for parts of this relationship, specifically the changes in utility that occur with changes in the social dividend, commuting costs, quality levels, and price changes for a resident at any given distance from the central business district. However, tracing how those variables change as park location changes is a task that can be done only numerically as it involves solving equations (A6)–(A8) in the Appendix simultaneously for each greenbelt location. Therefore, the overall changes in utility from changes in park location are derived using numerical methods. We use a simulation model programmed in Mathematica (Wolfram Research 2004) to solve the model, with the greenbelt placed at various distances from the central business district. The parameter values we use for generating simulation results are presented in Table 2.[7]

The simulation model we developed incorporates a number of feedback loops through iterating several variables to convergence. One of these variables is the social dividend, as this is derived from aggregate rents, which in turn are determined in part by the income of residents of which the social dividend is a component. A second variable is the number of trips when congestion affects the quality of the park (Cases 2 and 4): individual trips sum up to total trips, which affect park quality through μ_1, which in turn affects the optimal choice of individual trips. The algorithm iterates until there is no difference between the

[7] Many of these parameter values are taken from Wu (2001), including the population size, household income, and the rough magnitude of the housing and composite good utility parameters. The agricultural rental value was reduced to be more in line with actual annual agricultural rental rates. The area of the interior park is 40 square miles. Other values (round trip per mile costs, number of commuting trips per year) were chosen as roughly realistic for a typical U.S. citizen.

Table 2. Parameter Values Used in Simulation Model

Parameter	Meaning	Value
α	housing utility parameter	0.3
β	consumption good utility parameter	0.67
γ	recreational trips utility parameter	0.03
t	cost (round trip) per mile (dollars)	0.5
j	number of trips to the CBD per year	225
m	per visit cost of recreational trip (dollars)	5
Y	per capita income (dollars)	30,000
N	population	200,000
r_A	annualized agricultural rent (dollars per square mile)	640,000
θ	congestion parameter	0.000001
a	area of interior greenbelt park (square miles)	40

sum of optimally chosen individual trips and the aggregate number of trips in the congestion function. A third variable that is iterated to convergence is the size of the city when trips to the periphery enter the utility function: the number of trips taken to the periphery enters the utility level of residents, and this affects the location of the rent gradient and the location where urban rents equal agricultural rents. This location feeds back into the number of trips taken by residents. Details of the simulation model are available upon request.

Results

Case 1: Without Congestion, No Substitute Recreational Amenity

In the classic urban models, the rent gradient is found to decrease monotonically with distance to the CBD. Because inhabitants of the outer city rings are spending more money on commuting, they have less disposable income to spend on consumer goods. The price of the composite good is invariant with distance to the CBD, so rent is the only factor price that can equilibrate in order to equalize utility among all city residents. If outer-ring inhabitants are to receive the same utility as inner-city residents, they must be able to consume a greater amount of housing at a reduced price, yielding a declining rent gradient.

Introduction of the greenbelt disturbs the rent gradient by providing a location-specific source of benefits to residents at a specified distance to the CBD. Due to travel cost differences, residents who live closer to the park enjoy a relatively lower price of recreation than do those who live farther from the park. The equilibrium distribution of residents in the city will therefore reflect a trade-off between the benefits derived from living close to the CBD, which are reflected in a higher disposable income, and the benefits derived from living close to the park, which are reflected in a lower effective cost of greenbelt visits. In order for an equilibrium to occur, one would expect the benefits derived from living close to the park to be capitalized into the rents that must be paid for residential proximity. Thus, people who live closer to amenities earn benefits from the decreased effective costs of recreation visits, but in order for utility to equalize across the city, these benefits must be offset by increased residential rental rates.

In an open city where migration is allowed, the benefits would be fully capitalized into rents, so that the increase in rents can be used as a measure of the benefits derived from the park (McConnell 1990). In an open city model, by definition, landscape utility cannot be increased through provision of the recreation amenity; population will continue to flow into the city, driving up rents and altering city structure, until utility has been returned to its exogenously given level. In a closed city model such as this one, the increase in rents may not fully capture the benefits derived from the park; the benefits from the park provision are split between residents and people who receive rent, or landlords. In our model, we have removed the landlords in order that the benefits associated with park provision are fully reflected in residents' utility levels. Certain positions of the park confer greater benefits than others because of the interaction that exists among park location, size of the city, density of development, and resident accessibility to both park and CBD.

The results of our model are consistent with the expected rent capitalization effect: park location within the city is characterized by a local peak in rental rates. See Figure 1 for rent profiles for three different park locations. Because this is a closed city model, utility also responds to park placement. Our results confirm the importance of park accessibility in determining the utility-maximizing park location; Figure 2 illustrates the relationship between park location and landscape-level utility. For our choice of parameter values, optimal placement of the park in Case 1 puts it at $x_1^* = 19.6$; at this park location, the city's fringe occurs at $x_2 = 27.6$. Table 3 summarizes our results regarding preferred greenbelt placement.

For intuition, consider a resident who lives at the city's edge ($x = x_2$). The price of housing for the outermost resident is unaffected by the location of the park, so park placement changes two variables of interest to this resident—income and the cost of park visits. Simulations show that, for this set of parameters, changes in the social dividend and commuting costs caused by changes in park location x_1 affect income in roughly the same way: until about $x_1 = 9.5$, income increases as the park location is pushed out because the social dividend increases and the city boundary decreases (decreasing commuting costs for the resident at the city's edge). In this range, pushing out the park location both decreases the distance that the outermost resident has to travel to get to the park (the distance between x_2 and $x_1 + b$) and increases income. The outermost resident, whose utility level reflects overall landscape utility level, will clearly be better off. Beyond $x_1 = 9.5$, increases in park location reduce disposable income through both reducing the social dividend and increasing commuting costs. This leads to a reduction in the consumption of housing and the composite good. Utility still rises, however, as more visits are taken to the recreational area due to the fall in the cost of access, even though the increase in visits from a lower price is dampened by falling income. At $x_1 = 19.6$, the loss in income outweighs the benefits of decreased cost of access to the park and utility begins to fall.

For all residents besides the outermost resident, a change in park location affects housing prices. For example, inner-city residents will experience higher costs of travel to the recreation site—and will therefore take fewer trips—as the park is pushed out from the CBD, but these central residents simultaneously benefit from the increased housing consumption that comes with lower central city rents. This increase in housing consumption outweighs the loss in utility arising from the drop in disposable income from a reduced social dividend and from the increased cost of access to the park in the range between $x_1 = 9.5$ and $x_1 = 19.6$.

The park's impact on city structure variables such as city size and density pattern is also mediated through the rent function. According to our utility specification, each household's total expenditure on housing is independent of price. Therefore, if rents near the CBD drop and income is held constant, per household housing consumption must increase. However, the social dividend component of income is sensitive to park location. The social dividend increases as the park is moved outward from 0 to 9.5, but falls thereafter. Thus, the income effect of moving the park from 0 to 9.5 accentuates the tendency toward increasing housing consumption at the urban core. Residents near the CBD may consume a larger amount of housing, i.e., settle less densely, as the park location is pushed farther from the center of the city. People close to the park, where rent reaches a local peak, will settle more densely.

The changing rent structure also causes the urban area to expand or contract in response to greenbelt placement at different points. Greenbelts located at some distance from the urban core were shown to shrink the city size relative to its equilibrium size without the park. Surprisingly, however, the location of the greenbelt that minimizes city size is not, as one might expect, at the CBD. In fact, over some range, placing the park

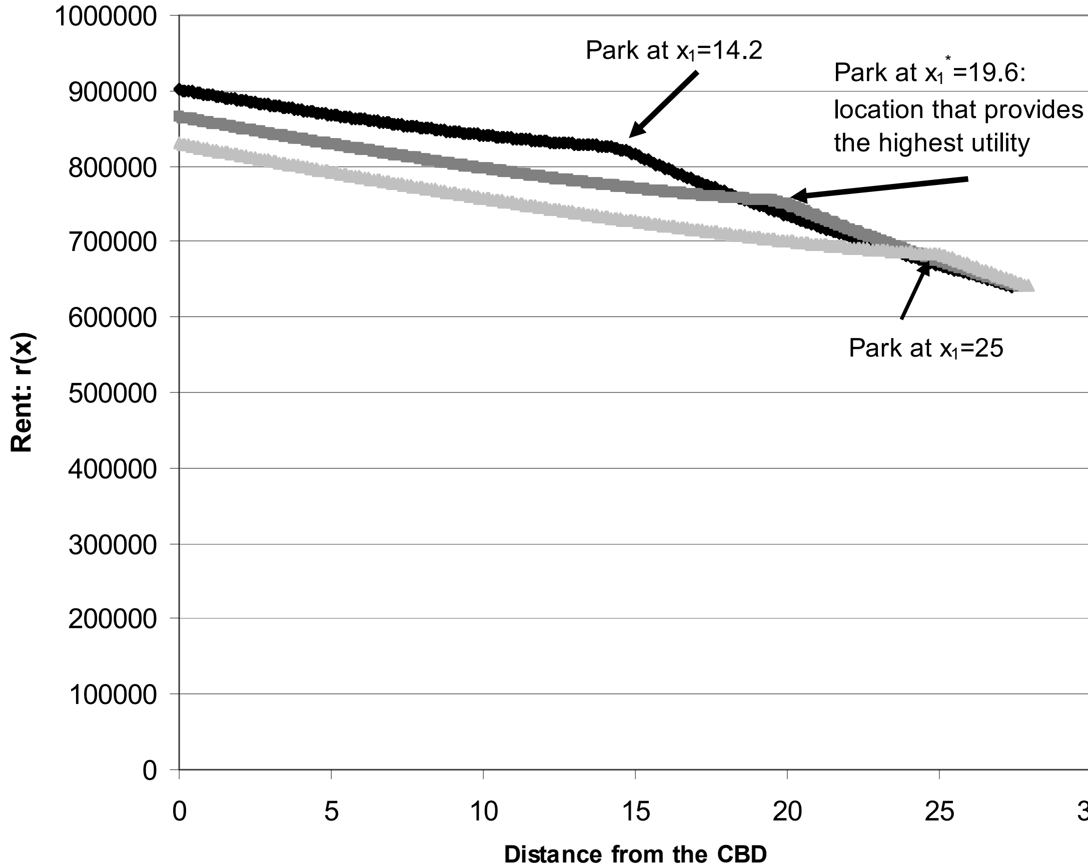

Figure 1. Rent Gradients for Three Different Park Locations for Case 1: No Greenbelt Congestion and No Substitute Recreation Amenity

farther away from the CBD actually serves to contract the city due to increased accessibility for peripheral residents. In this case, the city shrinks as the greenbelt is placed further from the CBD until $x_1 = 9.5$, beyond which the urban area expands. The urban area with a park, however, rarely exceeds the equilibrium size of a parkless city ($x_2 = 28.09$). Only when the park is located beyond $x_1 = 26$ does the combination of altered residential density patterns and development restrictions on greenbelt land lead to a larger urban area than without a park. Table 4 summarizes results on city structure.

Our final consideration in looking at the urban impact of park placement is its effect on vehicle use, including total vehicle miles driven (Table 5). To the extent that individual utility increases with access to both the CBD and the park, one might expect to observe a certain amount of congruence between decreasing vehicle miles and increasing utility. However, the relationship is complicated by the fact that a park increasingly accessible to one group of city residents becomes less accessible to another. The overall effect is that total vehicle miles driven (including miles traveled while taking both recreation trips and trips to the CBD) initially decreases as the park is pushed outward from the CBD, making it more accessible to the majority of city residents. However, this trend eventually reverses (here, at $x_1 = 17.7$), with total vehicle miles traveled beginning to climb as the park gains additional distance from the CBD. Peripheral residents continue to enjoy increased accessibility, and to increase their trips to the park, but central-city residents must travel increasing distances to reach it.

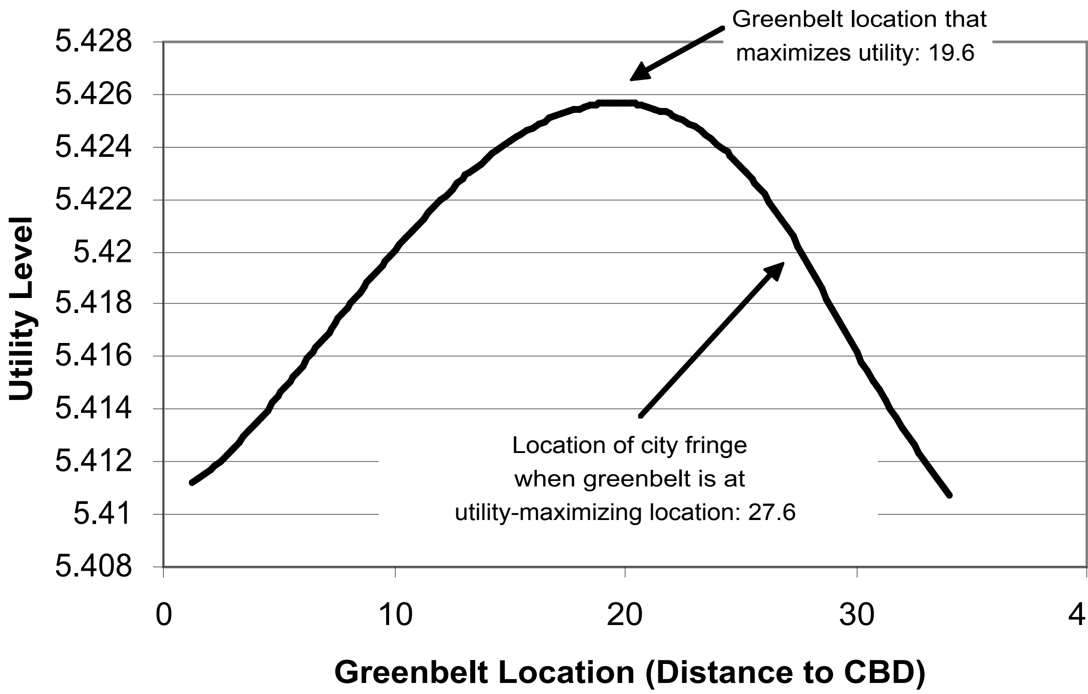

Figure 2. Relationship Between Greenbelt Park Location and Landscape-Wide Utility Level for Case 1: No Greenbelt Congestion and No Substitute Recreation Amenity

Table 3. Characterization of the Location of the Park That Maximizes Residential Utility

	Utility-maximizing greenbelt placement	Location of city fringe, x_2	Vehicle miles traveled (millions of miles)	Total trips to the park (million trips)
Case 1	19.6	27.64	1,839	23.1
Case 2a: ρ = 0.50	36.3	27.93	2,062	12.6
Case 2b: ρ = 0.99	28.7	28.10	1,792	18.4
Case 3a: ρ = 0.50	18.7	27.98	1,888	12.8
Case 3b: ρ = 0.99	15.8	28.06	1,824	15.8
Case 4a: ρ = 0.50	18.3	28.08	1,906	9.7
Case 4b: ρ = 0.99	10.5	28.20	1,863	9.9

Note: In Cases 1 and 3, these locations represent the social optimum. In Cases 2 and 4, these are second-best results due to the congestion externality.

Although increases in total vehicle miles driven are often cited as sprawl statistics, perhaps even more relevant to the problem of urban air quality is the effect of park location on total number of automobile trips. Because a major component of air pollution comes from cold starting the car, total amount of automobile-related air pollution may be more closely related to number of trips (and length of time on the road) than to length of trips in miles (Heimlich and Anderson 2001). Our analysis found a significantly different relationship between greenbelt location and total trips, as

Table 4. City Structure Results

	Location of Greenbelt That Minimizes City Size, x_2	Minimum City Size, x_2^{min}
Case 1	9.6	27.4
Case 2a: ρ = 0.50		
Case 2b: ρ = 0.99		
Case 3a: ρ = 0.50	13.1	27.9
Case 3b: ρ = 0.99	13.3	28.1
Case 4a: ρ = 0.50	11.8	28.0
Case 4b: ρ = 0.99	10.3	28.2

Table 5. Vehicle Miles Traveled and Trips Results

	Location of greenbelt that minimizes vehicle miles traveled	Minimum vehicle miles traveled (millions of miles)	Greenbelt location that maximizes total trips to the greenbelt	Maximum total trips to the greenbelt (million trips)
Case 1	17.7	1,836	19.7	23.1
Case 2a: ρ = 0.50				
Case 2b: ρ = 0.99				
Case 3a: ρ = 0.50	16.6	1,886	19.2	12.8
Case 3b: ρ = 0.99	15.8	1,824	26.3	19.2
Case 4a: ρ = 0.50	16.2	1,905	18.8	9.7
Case 4b: ρ = 0.99	14.2	1,853	17.7	12.7

compared to the relationship between park location and total vehicle miles. The optimal greenbelt location corresponds roughly to the location that maximizes total trips ($x_1 = 19.7$), though the total vehicle miles traveled is close to its minimum here. A less accessible greenbelt park (i.e., one located at either the CBD or the extreme urban periphery) may result in an increase in total vehicle miles traveled, but those miles represent fewer trips overall and therefore may be associated with relatively less automobile-related air pollution.

Case 2: With Congestion, No Substitute Recreational Amenity

The parameter ρ scales the strength of the quality weight variable μ_1, with a lower ρ corresponding to a higher importance of quality in the utility function. We consider both a high weight (ρ = 0.50, Case 2a) and a moderate weight (ρ = 0.99, Case 2b) scenario. Due to the utility specification used, and because the introduction of congestion does not affect relative prices in any way, decisions made regarding trips and housing/good consumption will be unaffected by the congestion variable.[8] The only impact of the variable lies in its effect on the utility derived from park trips,

[8] Utility without visits to the periphery is

$$U(c,h,w_1,w_2) = \alpha \ln h + \beta \ln c + \frac{\gamma}{\rho} \ln[\mu_1 w_1^{\rho}] .$$

Taking the exponent, we get

$$U(c,h,w_1,w_2) = h^{\alpha} c^{\beta} (\mu_1 w_1^{\rho})^{\frac{\gamma}{\rho}} = h^{\alpha} c^{\beta} \mu_1^{\frac{\gamma}{\rho}} w_1^{\gamma} .$$

Congestion affects μ_1, which does not appear in the marginal conditions and so does not affect consumption. It does, however, affect the utility level.

and therefore on the landscape-level utility derived from different park placements.

The introduction of congestion dramatically changes the results. There are now two opposing forces affecting preferred park placement: the desire for accessibility, which affects the travel cost that residents face for park use, and the role of congestion, which limits the park quality when too many residents visit. Increased access increases congestion as well; utility maximization requires a trade-off between the two. The park position considered optimal in Case 1 (x_1 = 19.6) is no longer preferred; this position in fact is close to the global minimum for utility on this landscape (see Figure 3). Instead, landscape utility can be increased by moving the park either inward toward the CBD or further outward toward (and beyond) the periphery, in both cases thereby decreasing the number of trips made to the park. The more peripheral locations make the park less accessible, but also provide a higher quality recreation experience. The location that maximizes utility is, in fact, far beyond the city's edge. This location is associated with dramatically fewer total trips (12.6 vs. 23 million) and more vehicle miles traveled (2,062 vs. 1,839 million) than without the congestion effect.

The effect of congestion on utility is mediated both through definition of the congestion variable [equation (3)], and through the value of the parameter ρ. Increasing the value of the parameter ρ makes utility less responsive to greenbelt congestion; when ρ = 0.99, the importance of access increases relative to the importance of congestion in determining utility levels. Figure 4 shows that a lower weight on the quality variable brings the utility maximizing greenbelt park location closer to the center of town, with correspondingly more total trips and fewer vehicle miles traveled than when the weight on utility is higher.

Case 3: With a Substitute Recreational Amenity, No Congestion Effect

Given our contention that green space confers benefits on residents that are capitalized into the rent function and therefore affects city structure, it is reasonable to expect that the green space at the urban fringe would have a similar effect. In this section, we introduce the city's peripheral green space as a substitute recreational amenity available to residents. If sites are perfectly substitutable (ρ = 0), residents on the landscape would simply make all of their trips to whichever recreation area has a lower price/quality ratio (or, if both sites are of equal quality, to the closer site). If, on the other hand, the elasticity of substitution is unitary (ρ=1), residents spend a quality-weighted proportion of their recreation budget on travel to each area.[9] Because the recreation value of a drive through the countryside may be similar in some respects but different in other respects from the value derived from the trails and bike paths of an urban greenbelt park, we use an intermediate substitutability parameter of ρ = 0.50 as the base case in our simulations.

In the base case (Case 3a), the optimal park placement is slightly shifted in toward the CBD relative to Case 1, where no substitutes exist (18.7 vs. 19.6). Because the amenities are only partially substitutable, access to the urban park remains the critical element in determining a utility-maximizing park location. In contrast, when the substitutability parameter is assumed to be 0.99 (Case 3b), the optimal park location shifts much farther inward, to x_1 = 15.8. When the amenities are highly substitutable, it is no longer important that *all* residents have access to the urban park; the park is placed to maximize its usability by those more central residents who have diminished access to the amenity value of the urban fringe. By providing residents with greater incentives for decentralization of residential development, inclusion of the periphery as a source of recreation benefits has a significant

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[9] The budget share for trips to the interior park when ρ = 0 is

$$\frac{m + td_{x_1} w_1^*}{Y_d} = \frac{\gamma}{\alpha + \beta + \gamma} \frac{\mu_1}{\mu_1 + \mu_2}.$$

For trips to the periphery, the budget share is

$$\frac{m + td_{x_2} w_2^*}{Y_d} = \frac{\gamma}{\alpha + \beta + \gamma} \frac{\mu_2}{\mu_1 + \mu_2}.$$

The budget share for total recreation trips is the sum of these two, and equals the standard Cobb-Douglas ratio of preference parameters: $\gamma/(\alpha + \beta + \gamma)$. Budget shares for trips to each recreation site are modified by quality parameters, μ_1 and μ_2. In this section, quality levels are fixed and equal to one, so the budget share going to each site is half of the overall recreation budget. In the subsequent section, μ_1—and these budget shares—are endogenous functions of trips.

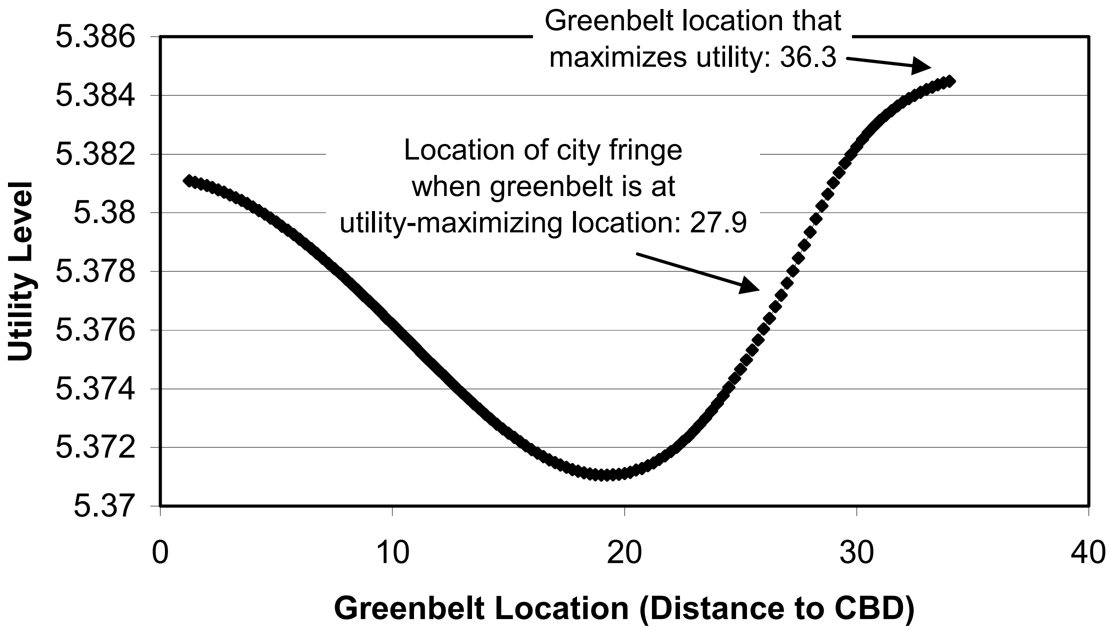

Figure 3: Relationship Between Park Location and Landscape-Wide Utility Level for Case 2a: With Greenbelt Congestion, ρ = 0.50, No Substitute Recreation Amenity

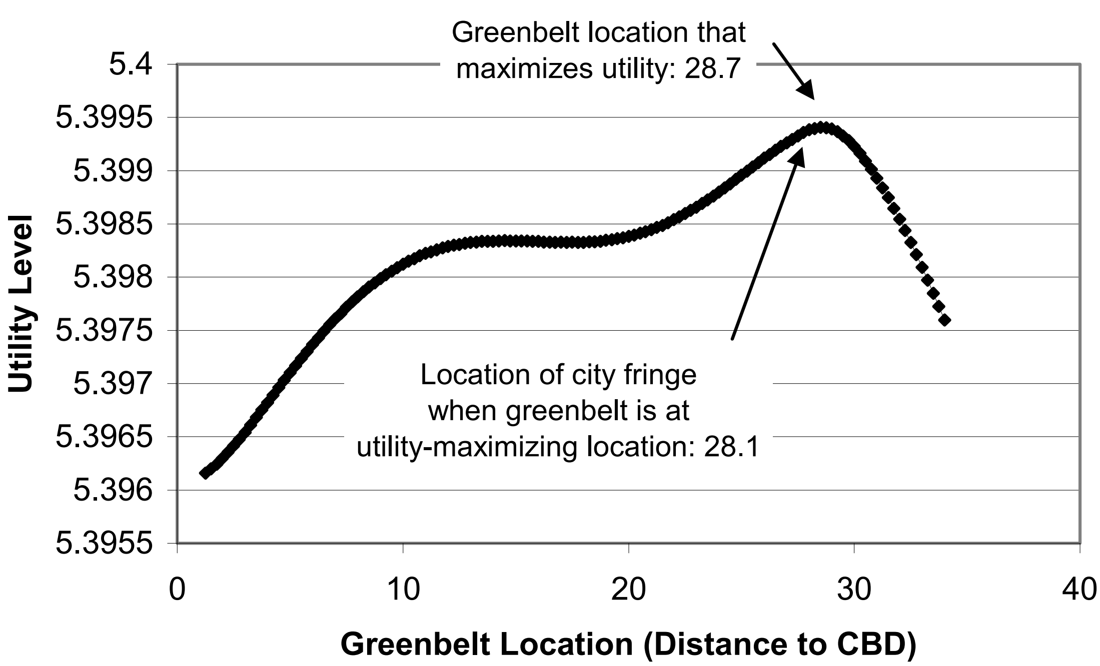

Figure 4. Relationship Between Park Location and Landscape-Wide Utility Level for Case 2b: With Greenbelt Congestion, ρ = 0.99, No Substitute Recreation Amenity

impact on urban structure. In general, a recognition of the amenity benefits of the urban fringe results in a larger equilibrium city size. City size in Case 3 is larger than the city size in Case 1, even when the optimal greenbelt in Case 3 is closer to the urban center. Because the benefits of the urban edge cause residents to settle more densely near the fringe, and higher density given a fixed population size implies a smaller city, one might have expected the opposite result. However, the larger the circumference of the city, the more residents can be accommodated in close proximity to the rural edge; in aggregate, this effect offsets the density effect and results in a larger city, with the more densely settled residents on the fringe living on relatively larger housing lots.

City size is also less sensitive to park placement in the current scenario; it is possible to manipulate the park location to contract and expand the city, but such an effect occurs over a much smaller range of city sizes. The smallest possible city size is larger in Case 3 than in Cases 1 and 2 (Table 4). This range contracts even further as the substitutability of the recreation destinations increases, but city size never becomes invariant to park location.

The effect of park location on total vehicle miles traveled is similar to that found earlier, except that again total vehicle miles traveled fluctuates over a smaller range in response to changes in park location. This makes sense both because city size varies over a much smaller range and because many residents have the option of responding to changes in the accessibility of the park by substituting lower cost (i.e., shorter) trips to the periphery.

Case 4: With Congestion, with a Substitute Recreational Amenity

In the final scenario, we included the possibility that the urban greenbelt park could suffer congestion-induced quality decline, while the quality of the peripheral recreation area remains constant at one. We can think of the difference as due to the characteristics of the two types of park, where the greenbelt has paths that can become congested and the substitute site has a more open layout, with no limit on the area available for recreation.

The congestion function used to represent quality of the urban park is again equation (3). When a substitute recreational area is available, the strong congestion effect seen earlier is no longer as important. Because landscape residents are now splitting their recreation activities between two areas, fewer trips are made to the urban park. Therefore, it is no longer necessary to place the park beyond the urban boundary in order to reduce the number of visits. In Case 4, we see the smallest number of visits to the park at its preferred location, even though it is centrally located. Regardless of location, the park is generally less congested and of higher quality than it was in the absence of a substitute. Also, residents have the option of substituting trips to the periphery as the quality of trips to the park declines; this flexibility in sources of recreation benefits means that overall landscape utility is not as sensitive to urban park quality changes as when no substitute is available.

The peripheral green space therefore serves dual roles in mediating the effect of congestion: it siphons off park visitors, thereby reducing the urban park congestion, and it provides an alternative source of benefits should the quality of the urban park decline too greatly. Because the two areas are only partially substitutable, however, the peripheral green space is unable to completely compensate for the congestion effect.

Concluding Remarks

A recent spate of papers has looked at the relationship between urban green space amenities and urban structure, a particularly relevant topic in an age when substantial public and private funds are being used to establish land preserves in urban areas. Unplanned purchases may have unintended and undesirable spatial consequences. Possibilities include an increase in automobile use along with its associated ills of traffic congestion and air pollution, and provision of incentives for increased development at a city’s periphery or even leapfrog development. Wise placement of urban green space may, on the other hand, contribute to urban planning goals such as higher-density housing development and compact city size.

We model the urban green space as a recreational amenity, introducing the idea that residents must travel to the green space in order to enjoy it.

An immediate consequence of this modeling approach is that we are able to assess the implications of the existence and placement of an urban park on the number of trips taken and vehicle miles traveled by urban residents. Additionally, we model features of recreational amenities that are common in models of recreation demand, but not in models that examine the effects of urban natural area preserves on urban structure. In particular, consumers choose among a variety of substitute sites with varying quality levels, and the quality of recreation sites can be affected by congestion at those sites.

Our results suggest that site substitutability and site congestibility can have dramatic influences on both the optimal location of an urban greenbelt and on the efficacy of using greenbelt placement to achieve urban planning goals. With a single site and no congestion, a centrally located greenbelt provides the highest utility for residents due to its easy accessibility. When congestion becomes a factor, accessibility becomes a liability that reduces the quality of the recreational experience. In this case, the best location is less accessible to inner city residents who, as a consequence, take fewer trips. Taking fewer trips implies reduced congestion at the site and higher site quality, and inner city residents are compensated for their loss in accessibility with lower rental rates.

The effect of site substitutability depends, naturally, on the degree of substitutability among sites. If residents do not consider the two sites to be substitutable, then all residents will continue to have a strong interest in access to the greenbelt, as they do when the greenbelt is the only recreation site. If, however, the two sites are considered to be perfect substitutes, the preferred location of the greenbelt will significantly change. Outer-city residents are no longer as concerned with accessibility to the greenbelt, as the trip to the periphery is cheap and provides a comparable level of satisfaction. The best placement at that point depends largely on the accessibility of inner-city residents to the greenbelt and the congestion generated by their trips; the greater the congestion of the greenbelt, the larger the ring of peripheral residents who will choose to travel to the periphery instead. The availability of a substitute peripheral amenity also dampens the effectiveness of urban park placement as an urban planning tool, as utility, city size, and vehicle trips become much less sensitive to location of the urban greenbelt.

Our results suggest that, although recreation access to open space is often a primary reason for the establishment of urban and suburban parks in our communities, wise placement of those parks must concern itself with more than a simple evaluation of access. Establishment of urban parks has impacts that reverberate landscape-wide, and those impacts are highly sensitive to congestion, quality of recreational experience, and availability of substitutes. Increased access to parks may ensure greater use of parks, but at the expense of recreational quality; the effects of degraded park quality proliferate landscape-wide through changes in land rental rates, development patterns, and commuting patterns. If park quality and the value of the recreational experience are sufficiently sensitive to congestion, landscape-level utility may in fact be maximized by providing parks in *less* accessible locations and allowing those residents with diminished park access to be compensated with lower housing rates. It is therefore important to understand the nature of recreational benefits that residents derive from open space, how those benefits are affected by congestion, and the degree to which residents consider different types of open space substitutable in determining appropriate criteria for locating urban green spaces.

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Appendix

The demand functions are derived by maximizing utility subject to the budget constraint to find the optimal quantities of goods consumed as a function of prices and income:

(A1)
$$h^*(x,j,t,Y)=\frac{\alpha(Y+SD-jtx)}{(\alpha+\beta+\gamma)r(x)}$$

(A2)
$$c^*(x,j,t,Y)=\frac{\beta(Y+SD-jtx)}{(\alpha+\beta+\gamma)}$$

(A3)
$$w_1^*(x,mj,t,Y)=\frac{\gamma(Y+SD-jtx)}{(\alpha+\beta+\gamma)}\frac{\mu_1(m+td_{x_1})^{\frac{1}{1+\rho}}}{\mu_1^{\frac{1}{1-\rho}}(m+td_{x_1})(m+td_{x_2})^{\frac{1}{1+\rho}}+\mu_2^{\frac{1}{1-\rho}}(m+td_{x_1})^{\frac{1}{1+\rho}}(m+td_{x_2})}$$

(A4)
$$w_2^*(x,mj,t,Y)=\frac{\gamma(Y+SD-jtx)}{(\alpha+\beta+\gamma)}\frac{\mu_2(m+td_{x_2})^{\frac{1}{1+\rho}}}{\mu_1^{\frac{1}{1-\rho}}(m+td_{x_1})(m+td_{x_2})^{\frac{1}{1+\rho}}+\mu_2^{\frac{1}{1-\rho}}(m+td_{x_1})^{\frac{1}{1+\rho}}(m+td_{x_2})}.$$

Inserting the optimal quantities of c, h, w_1, and w_2 into the utility function yields an indirect utility function expressing the maximum achievable utility for a set of commodity prices and income:

(A5)
$$\bar{v}(x,m,j,t,Y)=\alpha\ln\left[\frac{\alpha(Y+SD-jtx)}{r(x)(\alpha+\beta+\gamma)}\right]+\beta\ln\left[\frac{\beta(Y+SD-jtx)}{(\alpha+\beta+\gamma)}\right]+$$
$$\frac{\gamma}{\rho}\ln\left[\frac{\left(\gamma(Y+SD-jtx)\right)^{\rho}\left(\mu_1\left(\mu_1(m+tdx_1)\right)^{\frac{\rho}{1+\rho}}+\mu_2\left(\mu_2(m+tdx_2)\right)^{\frac{\rho}{1+\rho}}\right)}{\left((\alpha+\beta+\gamma)\left(\mu_1^{\frac{1}{1+\rho}}(m+td_{x_1})(m+td_{x_2})^{\frac{1}{1+\rho}}+\mu_2^{\frac{1}{1+\rho}}(m+td_{x_1})^{\frac{1}{1+\rho}}(m+td_{x_2})\right)\right)^{\rho}}\right].$$

In this model, rent is endogenous: for a given landscape (fixed x_1), it varies with x, and for a given x, it varies with x_1.

In equilibrium, individuals are distributed on the landscape so that their utility is equalized. The rent function is found by setting the indirect utility equal to this constant utility level ($\overline{v}$) and solving for $r(x)$:

$$\text{(A6)}\quad r(x) = \frac{1}{(\alpha+\beta+\gamma)}\left(e^{-\frac{\overline{v}}{\alpha}} \alpha(Y+SD-jtx)\left(\frac{\beta(Y+SD-jtx)}{(\alpha+\beta+\gamma)}\right)^{\frac{\beta}{\alpha}} (\mu_1\mu_2)^{\frac{\gamma}{\alpha\rho}} (\mu_1\mu_2(m+td_{x_1})(m+td_{x_2}))^{\frac{A\gamma}{\alpha(1-\rho)}} \right)$$

$$\left[\frac{\gamma(Y+SD-jtx)}{(\alpha+\beta+\gamma)\left(\mu_1^{\frac{1}{1+\rho}}(m+td_{x_1})(m+td_{x_2})^{\frac{1}{1+\rho}} + \mu_2^{\frac{1}{1+\rho}}(m+td_{x_1})^{\frac{1}{1+\rho}}(m+td_{x_2})\right)}\right]^{\frac{\gamma}{\alpha}} .$$

Two additional equations close the system. The first sets the rent level at the extensive margin, $r(x_2)$, where x_2 is the boundary of the city, equal to the agricultural rent generated by the land at the city's periphery (r_A):

$$\text{(A7)}\qquad r(x_2) = r_A .$$

A second equation equates supply and demand for housing for a given population size, N:

$$\text{(A8)}\qquad N = \int_0^{x_1} n(x)dx + \int_{x_1+b(x_1,a)}^{x_2} n(x)dx = \int_0^{x_1} \frac{2\pi x}{h(x)}dx + \int_{x_1+b(x_1,a)}^{x_2} \frac{2\pi x}{h(x)}dx ,$$

where $n(x)$ is the number of households living at distance x, $2\pi x$ is the supply of housing at that distance, and $b(x_1, a)$ represents the width of the area removed from residential development.