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ABSTRACT---A catastrophe (CAT) bond is designed for peanut production as a means of transferring natural disaster risks from insurance purveyors to the global capital market. The CAT bond so designed is priced using state-level historical yields for peanut production in the southern part of the United States in the State of Georgia. The index triggering the CAT bond contract was based on percent deviation from state average yield. The principal finding of the study is that it appears feasible for crop insurance purveyors to issue insurance-linked securities. CAT bonds can reduce the variance of the loss ratio when issued optimally with regard to the number of bonds and contract specifications. CAT bonds could therefore be used in hedging catastrophic risk effectively in peanut production given that crop insurance purveyors normally seek to minimize the variance of the loss ratio. CAT bonds were found to be feasible as hedging instruments even in the range of normal losses commonly covered by crop insurance and reinsurance.

Key words: Insurance, Reinsurance, Pricing, and Hedging

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Abstract

A catastrophe (CAT) bond is designed for peanut production as a means of transferring natural disaster risks from insurance purveyors to the global capital market. The CAT bond so designed is priced using state-level historical yields for peanut production in the southern part of the United States in the State of Georgia. The index triggering the CAT bond contract was based on percent deviation from state average yield. The principal finding of the study is that it appears feasible for crop insurance purveyors to issue insurance-linked securities. CAT bonds can reduce the variance of the loss ratio when issued optimally with regard to the number of bonds and contract specifications. CAT bonds could therefore be used in hedging catastrophic risk effectively in peanut production given that crop insurance purveyors normally seek to minimize the variance of the loss ratio. CAT bonds were found to be feasible as hedging instruments even in the range of normal losses commonly covered by crop insurance and reinsurance.

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1. Introduction

The agriculture industry is susceptible to the influence of various hazards. Farmers’ unique and substantial exposure to natural disasters such as adverse weather and epidemic diseases affecting crops and livestock has led to the development of various insurance programs. For agricultural crop production in the U.S., such insurance is mainly available in the form of multiple-peril crop insurance (MPCI), which covers most causes of crop loss. The program is subsidized by the federal government, while individual policies are sold by private insurers. Some private companies also offer crop hail insurance that covers damage caused by hail, lightning, and transit between farm and market.

The aforesaid crop insurance products provide farmers with an important means to manage production risks (Gardner, 1994). However, the capacity of private insurance markets is insufficient to adequately cover broad catastrophic risk exposures across large geographic areas due to a high systemic component with an attendant potentially high magnitude of disaster losses. The inability of private insurance companies to protect against catastrophic risks in agriculture has forced the government to intervene in crop insurance markets and fostered legislative solutions to support the agriculture industry (Glauber and Collins, 2001). Since the 1980s, the government has provided support both through subsidies and reinsurance of crop insurance products and through direct disaster payments authorized on an ad hoc basis. No private market can adequately compete with a program of direct disaster payments provided by the government; yet, political incentives make it very difficult to obviate government intervention (Skees and Barnett, 1999).
Indeed, many problems exist with the dual disaster aid system. Since the federal crop insurance program is run by a government agency, private companies are wary of assuming the risk of policies that might be misclassified and often require more than generous compensation in the form of subsidies and reinsurance provisions. The review procedures administered by the Risk Management Agency (RMA) are costly, thus increasing transaction costs (Skees and Barnett, 1999).

Significant changes have occurred in recent years in an attempt to address the limitations of crop insurance programs such as high program costs, low participation, poor actuarial performance of federal agricultural risk management programs, and the continued existence of ad hoc disaster payments (Skees, Black, and Barnett, 1997). Several Acts of Congress have been passed to encourage farmers to buy crop insurance including catastrophe insurance and thus reduce the potential need for disaster aid. These Acts include the 1994 Federal Crop Insurance Reform Act, the 1996 Federal Agricultural Improvement and Reform Act, the 2000 Agricultural Risk Protection Act (ARPA), and the 2002 Farm Security and Rural Investment Act (Coble and Knight, 2001; CATO Institute, 2002).

Innovative financial instruments such as catastrophe options and catastrophe bonds have been utilized to hedge against catastrophe risks, for example, from hurricanes and earthquakes in populated areas, by insurers and reinsurers in recent years (Bantwal and Kunreuther, 2000; Croson and Kunreuther, 1999; George, 1999; Froot, 1999; Hommel, 2000). CAT bonds provide a mechanism that insurance or reinsurance purveyors can use to gain access to the capital market to cover uninsurable losses from natural disasters (Lewis and Davis, 1998; Jaffee and Russell, 1997). CAT bonds are sold on the open market to private and public investors who are interested
in adding to their portfolios instruments with returns that are uncorrelated with traditional financial instruments such as stocks and bonds. CAT bonds are similar in design to normal bonds in that they are in essence loans given to firms by investors who expect payment of interest in return and repayment of principal at the end of some agreed period (Cummins, Lalonde, and Phillips, 2000). Unlike with traditional bonds, however, investors agree to forfeit the interest and/or principal under certain well-defined conditions, such as the occurrence of a catastrophic event. The issuer may then utilize the proceeds from selling the bond in order to offset losses caused by the event.

CAT bonds belong to a family of index-based instruments, because the occurrence of the catastrophic event is usually determined based on an objectively measured parameter called an index. An application of index instruments to risk management in crop production is area-yield crop insurance, also known as the Group Risk Plan (GRP), introduced in 1994 (Skees, Black, and Barnett, 1997; Baquet and Skees, 1994). The feasibility of rainfall index insurance has been examined (Martin, Barnett, and Coble, 2001; Vedenov and Barnett, 2004) and the World Bank is considering the possibility of underwriting such insurance in developing countries (Skees et al., 2001). The common feature of these instruments is that the payoff also depends on an index related to the risk being hedged.

CAT bonds have yet to be used to hedge disaster risks in agriculture. Based on their use in other sectors of the economy, it seems logical that CAT bonds could play a similar role in agriculture especially for crops particularly subject to the elements such as peanuts. Through the CAT bond instrument, it may be possible for crop insurance purveyors to shift catastrophic
agricultural risk to the capital market and away from taxpayers while keeping crop insurance affordable.

Although the existing federal crop insurance program may be considered adequate for ordinary crop production risks; additional losses incurred through targeted disaster payments and subsidies on crop insurance could be enormous in the event of major disasters. Issuing CAT bonds is essentially an alternative to federal crop reinsurance with the private sector tapped to provide reinsurance through the vehicle of CAT bonds.

Under the current system, every federal crop insurance product developed by a private company must undergo scrutiny since such products may put the government at excessive risk (Skees and Barnett, 1999). If CAT bonds with a transparent underlying index are used for the transfer of risk, there is no need for insurance product scrutiny by the government which may reduce transaction costs substantially. Further, there would be no need for a complicated reinsurance agreement with associated significant barriers to entry and unique rent-seeking opportunities for crop insurance companies (Skees and Barnett, 1999). The private crop insurance market may be willing and able to provide a wider variety of risk management mechanisms for farmers, thus reducing the need for government emergency programs and helping the system to work more efficiently and with lower social costs (Skees and Barnett, 1999).

The Property Claims Services (PCS), an insurance industry statistical agency defines catastrophes as losses from catastrophic perils that cause insured property damage of $5 million or more (Cummins, Lewis, and Phillips, 1998). In agricultural production, farmers deal with perils of nature on an annual basis and crop insurance is provided based on cropping season
cycles. Therefore, it is hard to define agricultural catastrophes in terms of insured losses based on the PCS’ definition.

A different way of defining catastrophes in agriculture is presented in this study. It is assumed that the area average yield loss (i.e. deviation of realized yield from long-term average) for a given crop every year adequately represents insured losses suffered by insurance companies writing crop insurance policies in a region. Therefore, the yield loss information can be used to define catastrophes in agriculture. Indeed, realized yield incorporates a cumulative representation of losses due to catastrophes, no matter the kinds of catastrophic events and how many occur during the growing season. In this regard, different yield loss percentages could be used to define different levels of catastrophes and concomitant alternative triggers for CAT bonds.

The hypothetical CAT bond for this study is crop and location-specific with peanuts as representative crop and the state of Georgia in the U.S. as the location. Peanuts were chosen for the analysis because of economic importance in Georgia and high ranking in liability among all crops by the Federal Crop Insurance Corporation (FCIC). A specific location is necessary because different areas have different exposures to the same types of risk. It is assumed that a hypothetical crop insurance company SERVO is the issuer of the CAT bond.

The purpose of the CAT bond is to provide protection from catastrophic losses for a given year based on the insurer’s ability to absorb losses. The suggested CAT instruments are zero-coupon bonds where the premium rate is a certain percentage over the LIBOR. Design aspects of the agricultural CAT bond contract in this study include the following:

1. The CAT bond contract is sold annually with a maturity of one year, and provides disaster coverage for insured losses for peanuts suffered by the insurance company SERVO over
the course of a year. The CAT bond contracts can cover insured crop losses from drought, floods, excess moisture, extreme temperature, hail, freezes, insect infestations, plant diseases, and the like.

2. Trigger levels of the CAT bond contract are set as specified percent losses of state average yield. By design, once the percent loss of state average yield exceeds the specified trigger, the CAT bonds default paying either nothing or only a part of the principal to the investor depending on the design.

3. The payout function on CAT bond contracts is a function of the percent loss of state average yield and is fixed for the contract when issued. The CAT bond contract stipulates the amount of the face value of a CAT bond and interest premium. The face value is one dollar in this case.

4. Expected yield losses are derived from the probability distribution of state yields, which in turn, is obtained from detrended data on state average yields. Since the actual distribution of yield losses for peanuts for all years is unknown, it must be estimated from limited historical data.

The financial structure underlying the hypothetical agricultural CAT bond is explained via the following simple illustration. The insurance company SERVO issues a CAT bond to hedge its exposure to disaster risk defined as an event when the percent loss of realized state yield for the insured crop exceeds a certain trigger, e.g. 50% of the long-term average. The crop insurance company sells the CAT bond at a discount on the open market via a special purpose vehicle (SPV) or an intermediary. The bond seller invests the funds in risk-free instruments such as government securities, e.g. treasury bills. If no triggering event occurs during the contract
lifetime, the investors receive the face value of the bond which includes both the principal and accumulated interest, e.g. the LIBOR plus a risk premium. The difference between the interest earned on the principal and that paid to the bondholder represents the cost of reinsurance for SERVO.

In the event of a catastrophe, the bond is triggered and investors lose the interest and part or all of the principal (depending on the contract). The company SERVO then uses the funds received from selling the bonds and accumulated interest (if any) in order to offset their losses due to the catastrophic event. Thus, from SERVO’s standpoint, CAT bonds are equivalent to a reinsurance agreement.

2. Methodology

Since the payoff of the proposed CAT bond depends on the occurrence of peanut yield losses, the distribution of yield losses is necessary for correct bond pricing. This can be derived from historical yield data. In order to evaluate CAT bond performance in hedging catastrophic risks, data are also required on premiums and indemnities associated with underwriting crop insurance for peanut production in the state of Georgia.

Official yield data were obtained from the National Agricultural Statistics Services, USDA (NASS data, 2003). The annual peanut yield data cover the period between 1909 and 2002. However, because of major changes in government policies (Becker, 1999), only data for 1963 to 2002 were used. Insured loss data including dollar values of premiums and liabilities were obtained for the same period from the Risk Management Agency, USDA (RMA Data, 2003).
Note that raw crop yield data cannot be used when modeling yield loss in agriculture because changes in technology foster higher yields over time. Yield data can be divided into two components: the central tendency and the deviation from central tendency. Central tendency captures the effects of technology change and the deviation from central tendency indicates natural risks. CAT bonds can be fashioned to provide risk protection against insured loss from low yields caused by natural disasters.

In order to separate yield risks from the deterministic trend, yield data were detrended by subtracting central tendency. There are a number of ways to estimate central tendency in yields, including ARIMA models, robust double exponential smoothing, and spline regression (Skees, Black, and Barnett, 1997). The yield data in this analysis are detrended using spline regression. Spline regression is widely used and provides stability in estimates. The spline procedure involves fitting a series of linear regressions representing different time segments and piecing the estimated relationships together into a spline function. The relationship is specified as follows:

(1) \[ \ln Y_T = \{A_1 + B_1(T - T_0)\}D_1 + \{A_2 + B_2(T - T_1)\}D_2 + \{A_3 + B_3(T - T_2)\}D_3 + u , \]

where \( \ln Y_T \) is the natural logarithm of the yield at time \( T \), \( D_i \) is a dummy variable whose value is 1 for all observations such that \( T_{i-1} \leq T < T_i \), and 0 otherwise, and \( T_i \) are the knots of the spline regression (where the slope of the spline function changes). The exact spline equation can be determined using maximum likelihood estimation.

Based on equation (1), the percent yield loss can be computed as follows:

(2) \[ M = \frac{Y_T - \exp(\ln Y_T)}{\exp(\ln Y_T)} = \frac{Y_T}{\exp(\ln Y_T)} - 1 = \exp(\hat{u}) - 1 , \]
where $M$ is percent deviation of yield from average, and $\hat{u}$ are the residuals from the detrending function (1).

Historical premium and loss data also have to be adjusted to reflect nonstochastic changes over time. In agriculture, for example, climatological changes may increase the chances of lower yields and thus dollar losses for the insurance company; changes in the amount of liabilities can affect premiums. In addition, premiums and losses are expressed in nominal terms and thus are affected by inflation. However, the ratios of premiums to liabilities (premium rates) and indemnities to premiums (loss ratios) are free of such problems. Therefore, it is assumed that historical premium rates and loss ratios correctly represent corresponding distributions. The historical premiums are then adjusted to their 2002 equivalents based on liabilities of the Federal Crop Insurance Corporation (FCIC) in Georgia in that year. The year 2002 was chosen as a benchmark since it is the most recent year of the study period.

The historical premiums for each year are adjusted to their 2002 equivalents as follows:

$$P_{t02} = P_t \frac{A_{02}}{A_t}$$

where $P_{t02}$ are the 2002 equivalents of FCIC total premiums for peanuts in Georgia in year $t$, $P_t$ are the actual (historical) FCIC total premiums for peanuts in Georgia in year $t$, $A_{02}$ is the total FCIC liability for peanuts in Georgia in 2002, and $A_t$ are total liabilities for peanuts in Georgia in year $t$.

2.1. Pricing of CAT bonds

A general approach to pricing a CAT bond involves two basic steps: (1) Estimating the frequency of catastrophe and the catastrophic loss distribution, and (2) incorporating these
estimates and the interest rate into the bond contract price (Cummins, Lewis, and Phillips, 1998; Baryshnikov, Mayo, and Taylor, 1998; Burnecki and Kukla, 2002; Cox and Pedersen, 2000; Lee and Yu, 2002; Litzenberger, Beaglehole, and Reynolds, 1996). Since the definition of catastrophes in agriculture is unique, a more appropriate way of modeling catastrophe in agriculture is chosen where the CAT bond pricing model involves the discounted expectation of various payoffs incorporating a constant interest rate.

2.1.1. Payout structure of cat bonds

The CAT bonds in the analysis are zero-coupon bonds, issued at time 0 with face value $F$ and time to maturity $T$. $D$ is the trigger value in terms of percent loss below the average state yield, and $L_T$ is the percent deviation from the state average yield at maturity. $V_T$ is the payoff of the CAT bond at time $T$, and $A$ is the portion of the face value repaid to bondholders if the CAT bond is triggered.

The payoff function of the bond depends on the relationship between the percent deviation $L_T$ and the trigger value $D$:

$$V_T = \begin{cases} 
F & \text{if } L_T \leq D \\
A \cdot F & \text{if } L_T > D 
\end{cases}$$

In other words, part or all of the bond proceeds are retained by the insurance company when the index $L_T$ exceeds the specified trigger $D$, otherwise, the bond pays its face value which includes the principal and the interest (e.g. LIBOR plus a risk premium). Recall that the CAT bond is a zero-coupon bond sold at discount, i.e. the investors’ profit is the difference between the face value and the initial price of the bond. The value of $A$ in equation (4) can range from 0 to 1. For example, when $A=0.5$, the bond pays the bondholder half of the face value of
the bond if the index exceeds the trigger level. Similarly, when \( A=0 \), the bond pays nothing if the index exceeds the trigger value.

The bond contract described above can offer flexible bond designs with differing triggers, interest rates, and proportions of face value repaid in case of a catastrophic event. However, all of these parameters need to be fixed in order to specify a particular bond contract. Different values of the parameters are used in the analysis to ascertain a range of results.

2.1.2. Pricing formulas for CAT bonds

A CAT bond is valued by taking the discounted expectation of its possible payoffs under the assumptions of the yield-loss trigger-index distribution and interest rate. The formula for pricing a CAT bond issued at time 0 with maturity time \( T \) is given as:

\[
P(0) = \mathbb{E}_{\theta,\eta} \left[ V_T e^{-\int_0^T r_s ds} \right],
\]

where \( \mathbb{E}_{\theta,\eta} \) means taking expectations with respect to two states. It is reasonable to assume that the state variable \( \theta \), which for the purpose of valuing catastrophe risk bonds essentially encompasses the term structure of interest rates, is independent of state variable \( \eta \) which pertains to catastrophe risk.

The CAT bond price then becomes

\[
P(0) = \mathbb{E}_{\eta} \left[ V_T \right] \mathbb{E}_{\theta} \left[ e^{-\int_0^T r_s ds} \right],
\]

where \( \mathbb{E}_{\eta}[V_T] \) is the expected payoff of the CAT bond, and \( \mathbb{E}_{\theta}[e^{-\int_0^T r_s ds}] \) is the expected value of a risk-free zero-coupon bond.
It is relatively straightforward to obtain the pricing formula for the risk-free bond. For a constant interest rate, the solution denoted as \( B(0,T) \) is found by discounting the face value at the one-year Treasury bill or LIBOR rate for time period \( T \). The expected payoff of the CAT bond can then be written as:

\[
\]

where \( P[L_T \leq D] \) denotes the probability of the percent yield deviation to be less than or equal to the trigger level \( D \), and \( P[L_T > D] \) is the probability of the opposite event.

Therefore the general pricing formula for CAT bonds can be written as follows:

\[
P(0) = B(0,T)[F^*P[L_T \leq D] + A^*F^*P[L_T > D]].
\]

In other words, the price of the CAT bond can be interpreted as the product of the price of a risk-free bond and the expected payoff from the CAT bond.

The pricing model assumes that the financial market is liquid and there are no arbitrage opportunities. The exact pricing formula for a specific CAT bond depends on the assumptions about the interest rate and probability distribution of the trigger index. The trigger index density function is critical to pricing agricultural CAT bonds. The model yields different market prices of CAT bonds for different values of parameters \( D, F, T \), and interest rate \( r \).

2.1.3. The distribution of the trigger index

In the absence of a traded underlying asset, insurance-linked securities have been structured to pay-off conditional on three types of variables – insurance industry catastrophe loss indices, insurer-specific catastrophe losses, and parametric indices based on the physical characteristics of catastrophic events. The choice of a triggering variable involves a trade-off between moral
hazard and basis risk (Doherty, 1997). In order to eliminate or minimize the prospects for moral hazard, a trigger related to state-level yield loss is appropriate because it depends on objectively measured state average yield. Thus there is no incentive for insurers to over-report losses in an attempt to increase recoveries if such an index trigger is used to determine settlements.

In determining the payout of the CAT bond, it is important to estimate the probability distribution of the underlying triggering index which is measured in this study by percent deviation of state average yields from long-term average. A nonparametric technique — kernel density estimation — was used in order to derive the distribution of the index from historical yield data. Kernel density estimation was preferred to parametric estimation because it better preserves the information contained in data and does not impose any distributional assumptions.

Generally, kernel density estimation involves constructing the probability distribution of a random variable $x$ as a sum of specially selected functions or kernels of the form

$$f(x) = \sum_{i=1}^{n} I\left(\frac{x - x_i}{H}\right),$$

where $f(x)$ is the kernel density function, $x_1, \ldots, x_n$ are observations (realizations) of the random variable $x$, $H$ is a smoothing parameter called bandwidth, and $I(u)$ is the kernel function.

A variety of functions can be used as kernels (Wand and Jones, 1995). Since yields are always positive and cannot exceed a certain upper limit determined by agronomical and climatic conditions, the finite support kernels are the best suited to model yield distributions. For the present study, two finite-support kernel functions were selected to model the distribution of percent deviation of state average yield:
(9) \textbf{Epanechnikov Kernel:} \quad I(u) = \begin{cases} \frac{3}{4}(1-u^2), & |u| \leq 1 \\ 0, & \text{otherwise} \end{cases}

(10) \textbf{Quartic Kernel:} \quad I(u) = \begin{cases} \frac{15}{16}(1-u^2)^2, & |u| \leq 1 \\ 0, & \text{otherwise} \end{cases}

In order to obtain reasonable kernel density estimates, the choice of bandwidths is far more important than the choice of kernel functions. The commonly used “optimal” bandwidth is determined as

(11) \quad H = \sqrt{5} \times 0.9 \times N^{-0.2} \times \min[SR / 1.34],

where $N$ is the number of observations used to construct the kernel density estimator, and $SR$ is the sample interquartile range (Wand and Jones, 1995).

2.1.4. Interest rate

Interest rate is another important factor affecting the pricing of CAT bonds. A constant interest rate is assumed for the sake of simplicity in the analysis. Three different constant rates are used for sensitivity analysis.

2.2. Hedging catastrophic risk in agriculture with CAT bonds

It is assumed that the objective of a hypothetical insurance company SERVO is to decrease its aggregate risk exposure by reducing the variance of its loss ratio. To model the insurer’s loss ratio and hedging strategy, let $X$ be total losses for insurance company SERVO from crop insurance before issuing CAT bonds, let $P$ be total premiums for crop insurance company SERVO from crop insurance, let $L$ be total losses for crop insurance company SERVO from crop insurance after issuing CAT bonds, and let $Y$ be the insurer’s total gain or loss from...
sells a CAT bond which depends on whether the CAT bond is triggered or not and is determined as

\[
Y = \begin{cases} 
V - A F & \text{if the bond is triggered} \\
V - F & \text{otherwise}, 
\end{cases}
\]

where \( V \) is the price of one CAT bond, \( F \) is the face value of the bond (set to one dollar in our analysis), and \( A \) is the proportion of the face value paid to investors when the bond is triggered. Then

\[
(12) \quad L = X - N Y + C, 
\]

where \( N \) is the number of the CAT bond contracts sold and \( C \) is the fixed cost of issuing CAT bonds.

Company SERVO’s loss ratio before issuing the CAT bond can be defined as:

\[
(13) \quad LR = \frac{X}{P}. 
\]

Assuming that crop insurance company SERVO issues a certain number \( N \) of CAT bond contracts, its loss ratio after issuing CAT bonds can be expressed as:

\[
(14) \quad LR = \frac{X}{P} - \frac{N Y}{P} + \frac{C}{P}. 
\]

In order to derive an optimal hedging strategy, it is necessary to minimize the variance of the loss ratio given by:

\[
(15) \quad Var(LR) = Var\left(\frac{X}{P}\right) + N^2 Var\left(\frac{Y}{P}\right) - 2NCov\left(\frac{X}{P}, \frac{Y}{P}\right), 
\]
where $Var\left(\frac{X}{P}\right)$ is the variance of the insurer’s loss ratio before issuing CAT bonds, $Var\left(\frac{Y}{P}\right)$ is the variance of the ratio of net gain/loss to the insurance premium by selling a CAT bond, and $Cov\left(\frac{X}{P}, \frac{Y}{P}\right)$ is the covariance between the loss ratio before issuing CAT bonds and the ratio of net gain/loss to the insurance premium by selling a CAT bond.

To find the number of contracts that minimizes the variance of the loss ratio, it is necessary to differentiate equation (15) with respect to $N$ and solve the first-order condition for optimal $N$, which results in

\[
N = \frac{Cov\left(\frac{X}{P}, \frac{Y}{P}\right)}{Var\left(\frac{X}{P}\right)}.
\]

Thus, in order to calculate $N$, it is necessary to compute $Cov\left(\frac{X}{P}, \frac{Y}{P}\right)$ and $Var\left(\frac{X}{P}\right)$. Based on loss ratio data, the variance of the loss ratios can be estimated, and the distribution of $Y$ based on the payout structure of CAT bonds can be derived. For a specified CAT bond with given parameter values, it is possible to compute the variance and expectation of the gain from selling the bond. However, it is not possible to calculate $Cov\left(\frac{X}{P}, \frac{Y}{P}\right)$ exactly because the joint distribution of the two variables cannot be reconstructed based on two marginal distributions alone. However, the correlation can be calculated empirically based on historical data of losses and derived distribution of the triggering index (and thus the CAT bond payoffs).
3. Empirical results

3.1. Detrending and index density estimation

The yield detrending model was estimated using maximum likelihood. The model has an R-square of 0.67 and adjusted R-square of 0.64. The model tracks the data well, provides a reasonably good fit, and reflects the general tendency of peanut yields in Georgia. Based on the estimated yield-detrending model, the peanut yield percent loss for each year is computed according to equation (2).

The next step after detrending the yield data is to construct the probability density function for the trigger index. In order to fit the yield loss data in the best way possible, four different bandwidths, $0.5H$, $H$, $1.5H$, and $2H$, where $H$ is the “optimal” bandwidth defined in (11), were used to construct kernel densities with two different kernel functions. From the array of bandwidths, $2H$ appears to be a good choice and provides relatively smooth density curves without losing the basic structure of the peanut yield data. The estimated value of $H$ is 0.0894 based on 40 data points.

Since there are no explicit functional forms for the kernel density estimators, numerical integration using Simpson’s rule with 1,000 nodes was used to compute probabilities of triggering CAT bonds. Given a certain trigger, one can compute the probability of the bond being triggered based on the appropriate density curve. For example, for CAT bonds with triggers equal to 40%, 35%, and 30%, of state average yield, probabilities of the bond being triggered based on the Quartic kernel density are 1.51%, 3.86%, and 12.00%, respectively. The general tendency is that the higher the trigger level, the lower the probability the bond is triggered, and vice versa.
3.2. Empirical Pricing Results

For a one-year zero-coupon bond with a face value of one dollar, it is assumed that the security has three notes – A1, A2, and A3 which have premiums of 7.5%, 10%, and 12.5%, respectively. These can be interpreted as certain premiums over LIBOR at the time the bond is sold. The repayment of principal is indexed to the percent deviation of the state average yield. If the bond triggers, the bondholder receives a repayment of a certain percentage of the face value depending on the specified parameters of the CAT bond contract. For example, with $A = 0.5$ and trigger threshold of 50%, the bondholders would receive repayment of half of the face value if the state yield falls below 50% of average, and the face value stipulated in the CAT bond (i.e. one dollar), otherwise.

In order to illustrate the basic properties of CAT bond contracts, the parameters of the contract are selected specifying eight trigger values from 15% to 50% in increments of 5% and proportions of repayment $A$ equal to 0.0, 0.3, and 0.5. Shown in Table 1 are prices of CAT bond contracts for different triggers, values of $A$, and interest rates. All else equal, CAT bond prices increase with trigger values and values of parameter $A$. This is reasonable since a bond that is easily triggered or repays a lower proportion of the face value in case of default is more risky from an investor standpoint and thus has a lower price. Recall that for a zero-coupon bond, a lower price means a higher return. Further, it can be observed, all else equal, that bond prices decrease as the premium rate increases and vice versa which is a standard result of financial theory.

Note that the bond prices were computed based on two kernel density estimators – Quartic and Epanechnikov kernels. While prices based on the Epanechnikov estimator tend to
be slightly lower than those based on the Quartic estimator, the differences are negligible. The Quartic kernel density estimator resulted in slightly higher probabilities of triggering the bond than those of the Epanechnikov kernel density estimator. Therefore, results based only on the Quartic estimator are reported in the rest of the paper.

3.3. Hedging catastrophic risk with CAT bonds

In order to find the optimal number of $1 bond contracts for a specific crop, equation (16) is employed. It is assumed that SERVO underwrites all crop insurance policies in Georgia. Therefore, the loss ratio for SERVO each year is exactly the same as that for the FCIC so that the FCIC loss data can be used in the analysis.

In the subsequent analysis, it is assumed that SERVO issued one-year zero-coupon CAT bonds each year from 1974 to 2002. Payout information for each issue can then be generated based on the specified payout structure of the CAT bonds and historical yield loss data.

Since the loss ratio is highly correlated with the percent yield loss, and the payout of the CAT bond depends on the yield loss for a specific year, there should be a high correlation between the loss ratio and the gain from the CAT bond. As indicated previously, the joint density of the two variables — loss ratio and gain from issuing CAT bonds – cannot be derived analytically. However, the correlation coefficient for the two variables can be computed empirically using historical data. Different scenarios are analyzed for parameter $A$ equal to 0, 0.3, and 0.5 with triggers from 15% to 50% in order to derive the optimal number of contracts the insurance company SERVO should issue to minimize the variance of the loss ratio.

Table 2 reports the optimal number of peanut CAT bonds with premium rates 7.5%, 10%, and 12.5%, respectively. Table 2 shows that for a given trigger level, the optimal number of CAT
bonds increases as the portion $A$ of face value repaid to investors increases. The bond issuer in this case needs more capital to hedge the same risk when $A$ is higher. Similarly, for a given $A$, the optimal number of CAT bonds increases monotonically when the trigger increases from 15% to 35%. This is true because the loss and thus the number of bonds needed to compensate are cumulative moving from lower to higher trigger levels. In other words, if the percent yield loss is high enough to trigger CAT bonds with a 35% trigger parameter, it is certainly high enough to trigger all bonds with lower trigger parameters as well. Thus, the optimal number of bonds at the 35% trigger level includes all of the bonds issued at trigger levels up to and including the 35% level. The results also show that beyond the 35% level, the optimal number of bonds falls to zero. This is because the risk of yield loss at higher trigger levels is insufficient to warrant CAT bonds given the historical data series for peanut yield.

Table 2 also shows, all else equal, that the optimal number of bonds decreases as premium rates rise from 7.5% to 12.5%. Higher premium rates mean more expensive bonds (see Table 1), and thus it is economically optimal to issue fewer bonds.

Since the objective of insurer SERVO is to minimize the variance of the loss ratio, it is necessary to measure the variance reduction of the loss ratio given that the company issues the optimal number of CAT bonds. Different scenarios are considered with the repayment parameter $A$ set to 0 and 0.5 with trigger levels 25% and 35%. Table 3 illustrates the potential effectiveness of issuing CAT bonds for reducing overall risk exposure. Table 3 shows the in-sample reduction in the variance of the loss ratio with respect to trigger level, parameter $A$, and level of risk premium.
There seems to be little or no difference in variance reduction at different premium rates or at various values of parameter $A$ for the same trigger level. From insurer’s prospective this means that it can issue more attractive bonds for investors while still achieving its goal of reducing the variance of loss ratio. On the other hand, reduction in the variance of loss ratio is shown to vary substantially across trigger levels, all else equal. The general trend is a monotonic rise in variance reduction level as trigger level increases with maximum variance reduction achieved at the 35% trigger level.

Thus far, the optimal number of CAT bonds for different contract parameters has been determined in sample with full information. In reality, the performance of CAT bonds for risk reduction is measured in an out-of-sample environment. For example, the determination of the number of bonds to sell this year with selected contract parameters is based on the data from previous years, while the efficiency of the bond for risk reduction is determined by events that are yet to happen.

As a result, an out-of-sample analysis is important to verify the relevance and plausibility of the in-sample results. In other words, results from in-sample and out-of-sample analyses must be consistent. In order to perform the out-of-sample analysis, the sample is usually divided into two subsamples: one subsample is then used to price the bond, while the other is used to perform the efficiency analysis. Two pairs of subsamples denoted as (19,21) and (21,19) were created randomly from the original sample. The notation (19,21), for example, indicates 19 data points were randomly chosen and used for bond pricing, while the remaining 21 data points were used to calculate the optimal number of bonds to be issued. The (21,19) sample was created and used
in the same way. This procedure was then repeated five times and averaged across runs. The same analysis used for pricing and hedging in-sample was used for out-of-sample analysis.

For the out-of-sample analysis, two $A$ values (0 or 0.5) and two triggers (25% and 40%) were selected for presentation of results for peanuts. The high-end in-sample trigger was 35% as there were no in-sample observations to trigger 40%. The same premium rates (7.5%, 10%, and 12.5%) used in sample were used here as well. The results for the variance reduction of the loss ratio are shown in Table 4. The results indicate the average reduction of variance of the loss ratio from five random samples for peanuts.

The out-of-sample results in Table 4 mirror the in-sample results in Table 3. Neither the parameter $A$ of the CAT bond contract nor risk premium level seems to have an impact on variance reduction of the loss ratio. However, variance reduction changes substantially with trigger level.

4. Discussion

CAT bonds have been used widely during the past decade to hedge catastrophic risk facing insurance companies. The introduction of CAT bonds has been driven both by the dramatic increase in catastrophe losses over the last decade and by insufficient mechanisms for financing losses from catastrophes provided by conventional insurance and reinsurance markets. However, CAT bonds have yet to be used in the field of agriculture.

This paper is an attempt to design a series of CAT bonds for peanut production based on state-level average yield data. Different CAT bond contracts with different parameters were designed and priced. Risk reduction analysis was conducted to ascertain the feasibility of the CAT bonds.
The CAT instruments designed in this study are zero-coupon bonds with the yield set at a certain percentage level (e.g. at some risk premium above LIBOR). The triggers specified in the CAT bond contracts are in the form of a certain magnitude of percent loss of average state yield. Once the percent average yield loss exceeds the specified trigger, the CAT bonds default paying nothing or part of the face value to the investor depending on the design. The payout function on CAT bond contracts is a function of the percent loss of state average yield and is fixed when the contract is issued.

The exact calculation of CAT bond prices mainly depends on the interest rate and estimated probability distribution of the trigger. Trigger density estimation determines the accuracy of the probability of the CAT bond being triggered and thus affects CAT bond prices. Quartic kernel density estimation with appropriate bandwidth was used to provide empirical density estimators. The procedure captures the basic structure of the yield data and generates accurate probabilities of CAT bond triggering. The different index triggers in the agricultural CAT bond contracts are based on percent deviation from state average yield, computed based on the spline detrending models. Yield losses generated from the detrending model were consistent with actual yield losses based on historical data.

The study reveals that the proportion $A$ of face value paid to investors if the bond is triggered has a strong influence on CAT bond prices, that is, the higher the value of $A$, the higher the CAT bond price, all else equal. Similarly, higher risk premiums result in lower CAT bond prices, all else equal. Lower triggers lead to lower CAT bond prices since the bonds are much more easily triggered and thus more risky for buyers. Thus, designing a CAT bond for
agricultural risk management requires consideration of contract specifications as they impact the optimal number of bonds required to reduce the loss ratio of bond issuer.

CAT bond parameters play an important role in determining the exact amount of money the company needs. Higher values of parameter $A$, lower premium rates, and higher trigger levels require higher levels of secured funds. Choosing an appropriate trigger level is of primary importance in issuing CAT bonds for reinsurance.

5. Conclusion

The principal finding of the study is that it is feasible for crop insurance companies to issue insurance-linked securities such as CAT bonds that can be used effectively in hedging catastrophic risk in agricultural production. CAT bonds, when issued optimally in terms of the number of bonds and contract specifications, can reduce the variance of the loss ratio as demonstrated in this study.

While the initial hypothesis was that only CAT bonds with high trigger levels may be feasible for crop production (if at all), the results suggest that CAT bonds are feasible even at more moderate trigger levels, e.g., in the range of normal losses commonly covered by crop insurance and reinsurance. Thus, the usefulness of CAT bonds in agriculture may be much greater than anticipated. The findings of this study are consistent with those of Vedenov, Epperson, and Barnett (2006).
References


George, J.B., 1999. Alternative reinsurance: using catastrophe bonds and insurance derivatives as a mechanism for increasing capacity in the insurance markets. CPCU Journal 52(spring), 50-54.


Table 1. CAT bond prices for peanuts

<table>
<thead>
<tr>
<th>Premium</th>
<th>A</th>
<th>15%</th>
<th>20%</th>
<th>25%</th>
<th>30%</th>
<th>35%</th>
<th>40%</th>
<th>45%</th>
<th>50%</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.5121</td>
<td>0.5627</td>
<td>0.6787</td>
<td>0.8186</td>
<td>0.8943</td>
<td>0.9162</td>
<td>0.9276</td>
<td>0.9302</td>
<td></td>
</tr>
<tr>
<td>7.5%</td>
<td>0.6375</td>
<td>0.6730</td>
<td>0.7542</td>
<td>0.8521</td>
<td>0.9051</td>
<td>0.9204</td>
<td>0.9284</td>
<td>0.9302</td>
<td></td>
</tr>
<tr>
<td></td>
<td>0.5</td>
<td>0.7212</td>
<td>0.7465</td>
<td>0.8045</td>
<td>0.8744</td>
<td>0.9123</td>
<td>0.9232</td>
<td>0.9289</td>
<td>0.9302</td>
</tr>
<tr>
<td></td>
<td>0</td>
<td>0.5005</td>
<td>0.5499</td>
<td>0.6633</td>
<td>0.8000</td>
<td>0.8740</td>
<td>0.8954</td>
<td>0.9066</td>
<td>0.9091</td>
</tr>
<tr>
<td>10%</td>
<td>0.6204</td>
<td>0.6577</td>
<td>0.7370</td>
<td>0.8327</td>
<td>0.8845</td>
<td>0.8995</td>
<td>0.9073</td>
<td>0.9091</td>
<td></td>
</tr>
<tr>
<td></td>
<td>0.7048</td>
<td>0.7295</td>
<td>0.7862</td>
<td>0.8546</td>
<td>0.8916</td>
<td>0.9022</td>
<td>0.9078</td>
<td>0.9091</td>
<td></td>
</tr>
<tr>
<td></td>
<td>0</td>
<td>0.4893</td>
<td>0.5377</td>
<td>0.6485</td>
<td>0.7822</td>
<td>0.8546</td>
<td>0.8755</td>
<td>0.8864</td>
<td>0.8889</td>
</tr>
<tr>
<td>12.5%</td>
<td>0.6092</td>
<td>0.6431</td>
<td>0.7206</td>
<td>0.8142</td>
<td>0.8649</td>
<td>0.8795</td>
<td>0.8872</td>
<td>0.8889</td>
<td></td>
</tr>
<tr>
<td></td>
<td>0.5</td>
<td>0.6891</td>
<td>0.7133</td>
<td>0.7687</td>
<td>0.8356</td>
<td>0.8717</td>
<td>0.8822</td>
<td>0.8876</td>
<td>0.8889</td>
</tr>
</tbody>
</table>

Note: Trigger is the percent loss of state average yield for peanuts; A is the portion of the face value paid to the investor when the bond is triggered, and prices are computed based on Quartic kernel density estimation.
Table 2. Optimal number of CAT bonds issued for peanuts

<table>
<thead>
<tr>
<th>Premium</th>
<th>A</th>
<th>15%</th>
<th>20%</th>
<th>25%</th>
<th>30%</th>
<th>35%</th>
<th>40%</th>
<th>45%</th>
<th>50%</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>30,602,723</td>
<td>43,188,932</td>
<td>45,582,036</td>
<td>46,997,182</td>
<td>47,074,174</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>7.5%</td>
<td>0.3</td>
<td>42,873,037</td>
<td>60,590,703</td>
<td>64,379,284</td>
<td>66,907,539</td>
<td>67,285,195</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>0.5</td>
<td>58,376,587</td>
<td>82,585,216</td>
<td>88,531,947</td>
<td>93,001,151</td>
<td>94,040,807</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>0</td>
<td>30,375,250</td>
<td>42,864,469</td>
<td>45,324,584</td>
<td>46,905,403</td>
<td>47,099,265</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>10%</td>
<td>0.3</td>
<td>42,269,621</td>
<td>59,729,339</td>
<td>63,698,578</td>
<td>66,596,465</td>
<td>67,220,017</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>0.5</td>
<td>56,972,516</td>
<td>80,535,793</td>
<td>86,853,148</td>
<td>92,062,665</td>
<td>93,592,962</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>0</td>
<td>30,154,576</td>
<td>42,544,313</td>
<td>45,060,504</td>
<td>46,790,140</td>
<td>47,092,256</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>12.5%</td>
<td>0.3</td>
<td>41,679,310</td>
<td>58,869,044</td>
<td>62,986,032</td>
<td>66,211,354</td>
<td>67,061,668</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>0.5</td>
<td>55,594,640</td>
<td>78,473,199</td>
<td>85,074,260</td>
<td>90,916,018</td>
<td>92,889,909</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

Note: Trigger is the percent loss of state average yield for peanuts; A is the portion of the principal paid to the investor when the bond is triggered.
Table 3. In-sample variance reduction of the loss ratio for peanuts

<table>
<thead>
<tr>
<th>Premium</th>
<th>(A=0, T=35%)</th>
<th>(A=0, T=25%)</th>
<th>(A=0.5, T=35%)</th>
<th>(A=0.5, T=25%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>7.5%</td>
<td>75.79%</td>
<td>66.37%</td>
<td>72.83%</td>
<td>63.00%</td>
</tr>
<tr>
<td>10%</td>
<td>74.30%</td>
<td>65.94%</td>
<td>71.35%</td>
<td>60.89%</td>
</tr>
<tr>
<td>12.5%</td>
<td>73.76%</td>
<td>65.17%</td>
<td>69.74%</td>
<td>58.77%</td>
</tr>
</tbody>
</table>

Note: $T$ is the trigger level determined as percent loss of the state average yield for peanuts, and $A$ is the portion of the principal paid to the investor when the bond is triggered.
<table>
<thead>
<tr>
<th>Sample</th>
<th>Contract</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$(A = 0, T = 40%)$</td>
</tr>
<tr>
<td>(19,21)$^a$</td>
<td>12.13%$^b$</td>
</tr>
<tr>
<td></td>
<td>12.13%$^c$</td>
</tr>
<tr>
<td></td>
<td>12.13%$^d$</td>
</tr>
<tr>
<td>(21,19)</td>
<td>16.00%$^b$</td>
</tr>
<tr>
<td></td>
<td>16.00%$^c$</td>
</tr>
<tr>
<td></td>
<td>16.00%$^d$</td>
</tr>
</tbody>
</table>

Note: $T$ is the trigger level determined as percent loss of the state average yield for peanuts, and $A$ is the portion of the principal paid to the investor when the bond is triggered. $^a$ For example, the first 19 data points were used to price the bond and the last 21 data points were used to calculate the optimal number of bonds for issue. $^b$ 7.5% premium, $^c$ 10% premium, and $^d$ 12.5% premium.