The optimal single forest rotation under climatic fluctuation effect

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Abstract: The stochastic impact of the “climatic fluctuation effect” is introduced in the traditional determinant timber growth model and optimal harvesting strategies are derived under different stochastic “climatic fluctuation effect” patterns. Results show that properties of different patterns significantly affect the forms of the optimal harvesting rules.

Keywords: Forest management, optimal harvesting, dynamic programming, climatic fluctuation effect
Section one: Introduction

Most of the current literature on the optimal forest rotation problem considers tree growth as a determinant process, which usually is described as a logistic function of the age of the timber. However, timber’s growth is not a determinant process. It is not only affected by its genetic factors, but also affected by the weather condition under which the plant grows.

The study in Rice university showed that “the climate variation does produce detectable effects on tree and sapling growth in natural forest.” The climate fluctuation can be defined as the short-term fluctuation in temperature, precipitation, wind and all other aspects of the earth’s climate. These factors affect the timber growth rate through controlling photosynthesis, respiration, transpiration and the uptake of nitrogen and water of tree stands. Research results also show that it affects the tree growth rate in an uncertain way and magnitude. Thus, omitting the climatic fluctuation effect and, consequently, its uncertain effect on tree growth will easily cause biased or even wrong harvest decisions.

In this paper, we consider loblolly pine in Piedmont region of North Carolina and generalize its growth factors into two main categories, genetic and “climatic fluctuation effect”. And we assume that genetic effect has a determinant effect on tree growth and climatic fluctuation effect has stochastic effect on tree growth. This paper addresses
optimal tree harvesting under both genetic effect and climate fluctuation effect. Dynamic programming is used to determine optimal harvest policies.

The aim of this study is to introduce climatic fluctuation effect in tree growth function and explore how it affects the optimal harvest policy. The specific objectives of this research include: (1) To identify and justify the fact that the climate fluctuation affects the growth rate of the timber; (2) To develop a valid framework to incorporate the climate fluctuation effect and investigate how it affects the optimal forest rotation decision. Loblolly pine data from the Piedmont region of North Carolina, USA, will be used. (3) To compare results under different parameters by doing a sensitivity analysis.

This paper is organized as follows. The next section briefly reviews the existing studies of optimal timber cutting problem. In section 3, we describe the modified model that is based on the previous research and discuss the methodology that will be used. In section 4 primary results of dynamic programming will be presented and discussed. In the last section, we summary the main results.

**Section two: previous research review**

In forest economics, Fausmanian framework is the starting model to analyze optimal rotation problem. The model assumes the perfect capital markets and perfect foresight. Under the assumption of constant timber price, harvesting and replanting cost and
replanting, people maximizes the present value of forest stand over an infinite time horizon. The model leads to a constant rotation period that depends on the timber price, cost of harvesting and replanting, nature of forest growth as well as the interest rate.

The constant timber price assumption has been relaxed and was assumed to follow GBM (geometric Brownian motion) in many papers, eg. Morck et al(1989), Yin and Newman(1997), and Thomson(1992). The assumption of GBM in much of the these literature embodies some unrealistic implication for the behavior of real commodity prices, for example, the expected value and variance of price rise with bound. Insley(2001) consider the price follow MR (mean reverting) process. Haight and Holmes(1991) and Plantinga (1998) consider the policy implication of assuming price follow a driftless random work. Researches have demonstrated the critical dependence of the optimal harvest rule on the specification of the price process. In discrete-time and stationary price assumption, optimal harvesting follows a reservation price policy in which cutting takes place when price is above the historical average (Norstrtim [1975], Lohmander [1988], Brazee and Mendelsohn [1988], Haight and Smith [1991]). With a non-stationary random walk model, optimal harvesting depends on fixed costs: with none, the policy is a fixed rotation age; otherwise, the policy is price dependent (Thomson [1992]). In continuous time, Clarke and Reed [1989] show that a fixed rotation age is optimal when price follows geometric Brownian motion (i.e., the logarithm of price grows linearly with additive error), there are no fixed costs, and there is a single rotation.
Saphores (2000) examines the impact of jumps on the harvesting decision. Reed (1984) considers the effect of the forest fire risk on the optimal rotation period. Clarke and Reed (1989, 1990) and Willassen (1998) and Alvarez (2001b) consider the stochastic forest growth. Alvarez and Koskela (2003) relax the constant interest rate assumption and model the interest rate as mean reverting process. These papers in general have similar conclusion that timber price risk, interest rate risk, fire risk and forest growth risk will tend to lengthen the rotation period.

Haight and Holmes (1991) estimated the price process and tree growth function for loblolly pine using data from the Piedmont region of North Carolina. Their model develops optimal strategy for a mid-rotation stand that maximizes the expected present value of the stand. The estimate results of tree growth will be used in our model.

Section 3: Model description and the dynamic programming formulation

By generalizing tree growth factors into two main categories, genetic and “climatic fluctuation effect”, we assume that genetic effect has a determinant effect on tree growth and “climatic fluctuation effect” has stochastic effect on tree growth. A modified timber growth model will be developed to include both effects and the dynamic programming approach will be used in solving our model. The framework of our model is as follow. The growth function of timber, considering only genetic effect, is assumed to be a logistic
function of age of timber $n$ at year $t$, which is formulated $Y(nt) = a^* n_t + b^* n_t^2 - d^* n_t^3$.

Here we directly use the estimation results from Haight and Holmes’ (1991) research which applies an ordinary least squares methods to data from North Carolina State University Plantation Management Simulator. The equation is:

$$Y(nt) = -16.54 + 1.029 n_t - 0.005220 n_t^2 \quad (R^2=0.999) \quad (1)$$

where $Y(n_t)$ is saw timber yield (mbf/ac) obtained from a $n_t$ year-old forest-stand.

The natural new growth without considering weather effect within year $t$ ($\Delta Y(n_t)$) thus is equal to $\Delta Y(n_t) = Y(n_t+1) - Y(n_t)$, which is a function of age of timber $n$.

The “climatic fluctuation effect” is introduced by using a multiple $L$, which is a stochastic variable assumed following first-order autoregressive processes

$$L(n_t) = \alpha + \rho L(n_{t-1}) + \epsilon_t \quad (2)$$

This process means the weather effect in the long term is stationary and will revert back to its expected level.

Another possible stochastic processes for “climatic fluctuation effect” is random work model which is formulated as

$$L(n_{t+1}) = a + \rho L(n_t) + \epsilon \quad (3)$$

We will exam how harvesting policy works under both situation respectively later.

The growth function, after taking into account both effects, would be:
\[ Y^R(n_{t+1}) = Y^R(n_t) + L(n_t) \Delta Y(n_t) \]  \quad (4)

where \( Y^R(n_{t+1}) \) means Realized yield in year \( n_{t+1} \), \( Y^R(n_t) \) means Realized yield in previous year \( n_t \), \( L(n_t) \Delta Y(n_t) \) is the realized new growth in year \( n_t \), which is the product of nature growth and climatic fluctuation effect.

Using dynamic programming approach, we have three state variables: (i) Age of timber \( N_t \); (ii) realized yield at the beginning of each year \( Y_t \); (iii) climate fluctuation effect on tree growth \( L_t \).

The decision maker makes the decision early every year about whether to keep or cut certain existing trees. The decision variable is \( X_t \) (cut, keep).

Following the Haight and Holmes’ (1991) model, the revenue function is

\[ R[Y(n_t,L_t)] = \exp(P) Y(L_t,n_t) + K \]  \quad (4)

Where \( Y(L_t, n_t) \) is saw timber yield (mbf/ac) obtained from a \( t \) year-old forest-stand; \( K \) is an exogenously given value of bare-land, \( P \) is the timber price which is are exogenously given and constant.

The Belleman’s equation for a planning horizon of \( T \) years is given as

\[ V_t(Y_t, L_t, n_t, X_t) = \max \{ R(Y_t, L_t), \delta E R_{t+1}(Y_{t+1}, L_{t+1}) \} \]  \quad (5)

Where \( \delta = 1/(1+r) \) is the discount rate, \( V \) is a reward function, and \( E \) is the expected value of the timber. The decision will be made early every year to maximize the present value
of forest over a certain time horizon. The maximization problem is to choose between clear-cutting \((x = 1)\) and waiting \((x = 0)\) every year to maximize the present value of forest over a certain time horizon. Waiting will be optimal when the revenue \(R[Y(L, n_t)]\) is less than the expected present value of the stand in period \(n_{t+1}\).

The boundary condition in period \(T\) assumes that all tree are cut. The problem will then be solved back-wards using the decision in period \(T\) as a boundary condition.

**Section 4. Optimization results and discussion**

The value for the parameter of the “climatic fluctuation effect” \(\alpha, \rho\) will be chosen randomly and the parameter analysis will be conducted. And we chose 125$/mbf as the timber price and assume loblolly pine have 30 year growing life and harvesting horizon is 50 years. Bare land value is assumed to be $550/ac and discount rate is 4%.

4.1 Climatic fluctuation effect \(L\) follows first order autoregressive model

\[
L(n_t) = \alpha + \rho L(n_{t-1}) + \epsilon_t.
\]  

(2)

The probability transition matrix will be formed as follows.
Let $L(n_t)$ for $n = 1, \ldots, n$ represent $n$ discrete climatic fluctuation effect in period $n_t$. We will show how to find the probability of observing any price at period $n_{t+1}$, given we have drawn $L(n_t)$ from the solid line normal distribution curve.

Assume for example the mean of $L(n_{t+1})$, calculated from equation (2), is equal to $L(n_{t+1})$ as indicated on figure above. The probability of observing any price in year $n_{t+1}$ is then given by area to the left of $L(n_{t+1})$ less the area to the left of $L_{-1}(n_{t+1})$ using the dotted normal distribution curve. Thus for any $L(n_t)$ realizations, we have corresponding normal distributions with specific mean in year $n_{t+1}$. We can form a probability matrix $(P_{ij})$ accordingly.

The dynamic programming results show that threshold climatic effect level decreases with age $n_t$ as showed in figure 2. The area below the curve contains the climatic-age combination when harvesting should be postponed. The intuition is that for the first order regressive processes, it has the tendency to revert back to its mean level. When the observed climatic effect is favorable, it’s better to cut because the climatic effect will be more likely to revert to normal or less favorable situation in the future. Conversely, if the climatic effect is not favorable, it’s better to postpone because the climatic effect will be more likely to go better in the future.

If we increase the variance level of the stochastic processes for the climatic fluctuation
effect from 0.3 to 0.5, the threshold climatic effect level will shift upwards as showed in figure 2.

The expected present value of the 30-year-old loblolly pine is about $2509/ac. The expected rotation age is 33.6 years.

Figure 2: Threshold level for the climate fluctuation effect on a first-order regressive model

4.2. Climatic fluctuation effect \( L \) follows random walk model

\[
L(n_{t+1}) = a + \rho L(n_t) + \epsilon
\]  

(3)

This process means that one-period forecasts only depend on the observed current information. All past information cannot be used to produce a better estimate of the future price than the current information.

Opposite to that of first-order autoregressive processes, the dynamic programming results under random walk processes shows that threshold climatic effect level increase with age
as shown in figure 2. The area above and to the left of the curve contains the climatic-age combination when harvesting should be postponed. The intuition is that for any level of current climatic fluctuation, there is equal chance of fluctuation level to go up or go down in next period. When the observed climatic fluctuation effect is favorable (high), harvesting level is postponed because the climatic fluctuation effect is expected to remain high and also because the value of the growing stock is high relative to the fixed value of the bare land. Conversely, when the observed climatic fluctuation effect is low, harvesting will be pre-exercised because the climatic fluctuation effect is expected to stay unfavorable (low) and also because the value of the bare land is greater than the expected return from timber growing.

Same with that of first-order regressive processes, if we increase the variance level of the random work processes for the climatic fluctuation effect from 0.3 to 0.5, the threshold climatic effect level will shift upwards as showed in figure 3.

The expected present value of the 30-year-old loblolly pine is about $1865/ac. The expected rotation age is 42.3 years.
Figure 3. Threshold level for the climate fluctuation effect on a random walk model

**Section 5: Summary and conclusions**

This paper takes both “genetic” and “climatic fluctuation effect” into account in timber growth. And we assume that genetic effect has a determinant effect on tree growth and climatic fluctuation effect has stochastic effect on tree growth. Two potential stochastic processes, first order autoregressive processes and random walk processes, are assigned to the “climatic fluctuation effects” and optimal harvesting strategies are derived under each processes. Programming results show that threshold “climatic fluctuation effect” level decreases with timber age $n_t$ under first order autoregressive processes. Conversely, threshold “climatic fluctuation effect” level increases with timber age $n_t$ under random walk processes. Threshold “climatic fluctuation effect” level increases with the variance rate of the processes in both processes.
Reference

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