Estimating the Value Added Product Life Cycle

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Abstract

This research analyzes factors affecting product and profit life-cycles for new value added products. The methodology used shows how sales and profits evolve and how exogenous factors affecting sales and profits. Results indicate that producers can increase the level of sales and profits over time through initial marketing efforts.

Keywords: Brand Value, Marketing, Product Life-Cycle, Profit Life-Cycle, Value Added

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Estimating the Value Added Product Life Cycle

Value added agriculture has roots with using a commodity to produce a product that is either a complement or substitute for an existing end-use or industrial product. Sometimes the product relays little differentiation (e.g., ethanol) and sometimes the product relays significant differentiation (e.g., Blue Diamond Almonds). Most economic questions surround significantly differentiated value added products. The most important of which are, how profitable will the new product be and how long will the profit stream last. General wisdom is that; the profit stream will be short as others can duplicate a product derived from a commodity. The objective of this study is to show through comparative statics how agricultural producers can extend the product and profit life-cycle for value added products. Then, data from a quality cattle program is used to show how this procedure can be applied to arrive at the product and profit life-cycle for a value added good.

Product life-cycle theory is widely used in the marketing strategy literature to evaluate the expected sales and profit of new products. The theory predicts that profits will increase as sales increase, profits reach a maximum and then profits go to zero because of competitive factors. However, some factors can change the length of product life-cycle, hence the period for which of profits are positive. These factors are; initial marketing efforts (delay factor), time at which profits obtain the maximum (inflection point) and the projected sales maximum.

The study provides a framework to analyze how evaluation of sales and profits over time. The current research differs from other previous in this field by providing analytical framework to analyze the changes in product and profit life-cycles. Analytical framework helps to drive the profit curve from product life-cycle and to forecast the profit curve for new products. In the next section a theoretical justification for modeling the change in product and profit life-cycles is
presented. Next, an empirical model is specified and data from quality (branded) bred heifer program is used to estimate product and profit life-cycles and the brand value. In the last section conclusions are made.

**Theoretical Model**

The product life-cycle approach has been used to analyze and forecast level of sales and profits. Cox (1965) showed that the life-cycle of a new product is characterized by four stages; introduction, growth, maturity and decline. The introduction stage is the stage when the product is first marketed and sales are less then 5% of the market share. During this period profits are moving from negative to positive. In the growth phase, total sales increases rapidly and profits continue to increase. The growth phase ends with profits reaching a maximum level. The next phase is the maturity phase, in which the rate of increase in total sales and profits begin to decrease. The last stage is the decline stage. During this phase, both total sales and profits decline rapidly. Figure 1 shows the stages of product life-cycle as mentioned by Cox and figure (2) shows the corresponding profit Life-Cycle. Because profits are negative in the decline stage, the model does not take into account the decline state. Therefore, the production ends when sales reach the maximum, $S$, before the decline stage begins.

The formulation of the product life-cycle model is (Cox):

$$F_t = \frac{S}{1 + e^{A(t-I)}} \quad \text{for } t=1,2...T,$$

where $t$ is the index for time, $F_t$ is the cumulative sales level at time $t$, $S$ is the saturation level of the sales of the product, $T$ is the time where the maximum of sales are reached $S$ is reached, $I$ is the inflection point and the time at which profits reach the maximum and the maturity phase begins. $A$ is the delay factor, which shows how long the sales of a product will stay in the
introductory phase. The value of $A$ ranges between zero and one. If buyers consume rapidly, then the value of $A$ will be close to zero, the introductory phase to be short. Otherwise, $A$ may be close to 1 and the introductory phase will be long. In general, marketing efforts in the introduction phase will make $A$ approach zero. Figure 1 shows the product life-cycle (1) and figure (2) is the profit life-cycle for the data for $S=21000$, $A=1$, $I = 7$ and $T=12$.

The impact of a change in the levels of $S$, $I$ and $A$ on the product life-cycle was presented graphically by Morrison (1995). The present study provides the comparative static analysis for how changes in levels of $S$, $I$ and $A$ impact the shape of the product life-cycle curve. In particular, the present study analyzes how the changes in the level of $A$ impact a products life-cycle, as this can be impacted by marketing efforts. The change in $F_t$ due to a change in $A$ can be represented as

$$ \frac{dF_t(S(A), I(A), t, A)}{dA} = \frac{\partial F_t(.)}{\partial S} \frac{dS}{dA} + \frac{\partial F_t(.)}{\partial I} \frac{dI}{dA} + \frac{\partial F_t(.)}{\partial A} $$

for $t=1,2,...,T$.

Assuming that $S$ and $I$ are implicitly function of $A$\(^1\) and applying the implicit function theorem to equation (2) yields

$$ \frac{dS(A)}{dA} = -\frac{\frac{\partial F_t(S(A), I(A), A)}{\partial A}}{\frac{\partial F_t(S(A), I(A), A)}{\partial S}} = -\frac{Se^{A(I-t)}(I-t)}{(1+e^{A(I-t)})^2} = \frac{S(I-t)e^{A(I-t)}}{(1+e^{A(I-t)})} $$

As sales approach $S$ during the last period of the product life-cycle, $t = T$, the sign of the derivative in equation (2) is

$$ \left. \frac{dS(A)}{dA} \right|_{t=T} = \frac{S(I-T)e^{A(I-t)}}{(1+e^{A(I-t)})} < 0 \quad \text{as } T>I. $$

\(^1\) Previous studies did not make this assumption. The data for this analysis provides evidence that when $A$ changes both $S$ and $I$ also change.
This result indicates that as $A$ decreases, the saturation level of sales will increase. Hence, the firm benefits from marketing efforts by realizing an increase in the saturation level of sales. The impact of an increase in sales in each year can be calculated as

$$\frac{\partial F_t(S(A), I(A), A)}{\partial S} = \frac{1}{(1 + e^{A(I-t)})} > 0 \quad \text{for } t=1,2\ldots T.$$  

Hence, by decreasing $A$, $S$ will increase, which will cause $F_t$ to increase for each year.

The sign of $\frac{dl}{dA}$ is calculated by applying the implicit function theorem to equation (1) to obtain

$$\frac{dl(A)}{dA} = -\frac{\frac{\partial F_t(S(A), I(A), A)}{\partial A}}{\frac{\partial F_t(S(A), I(A), A)}{\partial l}} = -\frac{Se^{A(I-t)}(I-t)}{(1 + e^{A(I-t)})^2} = -\frac{(I-t)}{A} \bigg|_{t<I} < 0$$

This shows that an increase in $A$ will increase $I$. The impact of an increase in $I$ on cumulative sales point in time is given by

$$\frac{\partial F_t(S(A), I(A), A)}{\partial I} = -\frac{Se^{A(I-t)}A}{(1 + e^{A(I-t)})^2} < 0 \quad \text{for all } t=1,2\ldots T,$$

As $I$ can also be interpreted as the year at which half of the total sales are reached, Morrison(1995), the intuition behind this inverse relationship is that an increase in $I$ will cause producers to delay in reaching a given level of $S$. Hence, without an increase in $S$, this delay will lead a decrease in $F_t$ each year. The sign of partial impact of $A$ on cumulative sales, $\frac{\partial F_t(.)}{\partial A}$, is given by,

$$\frac{\partial F_t(.)}{\partial A} = -\frac{Se^{A(I-t)}(I-t)}{(1 + e^{A(I-t)})^2} = \begin{cases} < 0 & \text{for } t < I \\ > 0 & \text{for } t > I \end{cases}$$

This result shows that when $A$ decreases the partial impact causes cumulative sales to increase in years prior to $I$ and to decrease in years following $I$. 

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The overall impact of a change in $A$ on $F_i(.)$ is analyzed and reported in Table 1, which shows that for $t<I$, $F_i(.)$ will increase by decreasing $A$, $\frac{dF_i(.)}{dA} < 0$. This is due to the magnitude of the cross-partial derivatives for $S$ being bigger than of $I$ for $t<I$. Hence, making the total impact negative. However, for $t>I$, the sign of $\frac{dF_i(.)}{dA}$ is ambiguous and depends on the magnitude of partial effects.

**Profit Life-Cycle**

To observe the change in the level of profits at each period the profit function is specified as

$$\pi_t = \frac{\partial F_i(S(A), I(A), t, A)}{\partial t} = P_t \frac{\partial F_i(.)}{\partial t} - c(A) - b \frac{\partial F_i(.)}{\partial t} \quad \text{for } t=1,2...,T.$$  

$\frac{\partial F_i(.)}{\partial t}$ is the derivative of cumulative life-cycle function with respect to time, which gives the instantaneous sales amount at time $t$, $P_t$ is the price of the product at time $t$. Linear cost function is assumed with respect to instantaneous sales amount, which makes $b$, slope of cost curve and $c(A)$ be constants with respect to instantaneous sales amount. $c(A)$ includes the initial marketing cost and fixed production costs. Hence, $\frac{dc(A)}{dA} < 0$, as marketing efforts indicate $A$ is decreasing. The impact of $A$ on the level of profits attained in each period is calculated as

$$\frac{d\pi(.)}{dA} = (P_t - b) \left( \frac{\partial^2 F_i(.)}{\partial t \partial I} \frac{dI}{dA} + \frac{\partial^2 F_i(.)}{\partial t \partial S} \frac{dS}{dA} + \frac{\partial^2 F_i(.)}{\partial t \partial A} \right) - \frac{dc(A)}{dA} \quad \text{for } t=1,2...,T.$$  

The corresponding derivatives are calculated to be

$$\frac{\partial^2 F_i(.)}{\partial t \partial I} \frac{dI}{dA} = \left[ - \frac{2SA^2 e^{2A(I-t)}}{(1 + e^{A(I-t)})^3} + \frac{SA^2 e^{A(I-t)}}{(1 + e^{A(I-t)})^2} \right] \left( - \frac{I-t}{A} \right) = \frac{2SAe^{2A(I-t)}(I-t)}{(1 + e^{A(I-t)})^3} - \frac{SA(I-t)e^{A(I-t)}}{(1 + e^{A(I-t)})^2}.$$  

$^2$ Figure (2) shows the profit curve for $S=21000$, $A=1$, $I = 7$ and $T=12$.
\[ \frac{\partial^2 F_t}{\partial t \partial A} = -\frac{2SA(I-t)e^{2A(I-t)}}{(1+e^{A(I-t)})^3} + \frac{Se^{A(I-t)}}{(1+e^{A(I-t)})^2} + \frac{SA(I-t)e^{A(I-t)}}{(1+e^{A(I-t)})^2} \]

\[ \frac{\partial^2 F_t}{\partial t \partial S} \bigg|_{t=T} = \frac{Ae^{A(I-T)}}{(1+e^{A(I-T)})^2} - \frac{SAe^{2A(I-T)}(I-T)}{(1+e^{A(I-T)})^3} \]

Adding up (11), (12), (13) and marketing cost leads to

\[ \frac{d\pi_t(.)}{dA} = (P_t - b) \left[ \frac{Se^{A(I-t)}}{(1+e^{A(I-t)})^2} + \frac{SAe^{2A(I-T)}(I-T)}{(1+e^{A(I-T)})^3} \right] \frac{dc(A)}{dA} \quad \text{for } t=1,2,\ldots,T \]

\[ \frac{d\pi_t(.)}{dA} = \begin{cases} +/ - & \text{for } t \in \{\text{Introductory Phase}\} \\ - & \text{for } t \in \{\text{Growth Phase}\} \\ -/ + & \text{for } t \in \{\text{Maturity Phase}\} \end{cases} \]

The negative sign in (15) indicates initial marketing efforts increase profits get higher. As the term in the bracket [ ] in (14) is always negative and \( \frac{dc(A)}{dA} \) exist only for initial years, profits are expected to be lower in the introductory phase (initial years), but to be higher in the years following the introductory phase. Depending on the magnitude of the marketing costs, profits may be realized in the introductory phase.

**Data and Empirical Model**

The Show-Me-Select (SMS) Heifer Program® was initiated in 1997 to develop quality branded bred heifers, hence to create brand value for the heifers. The long-term profit of the program brand value created is a crucial criterion for farmers to participate in the program.

The data for cumulative sales and profits are obtained from SMS Heifer Program®, which is available for 1998 through 2004 (Patterson and Randle, 2005). The data is obtained by aggregating the individual data from producers. Therefore, the results reflect changes in industry level. Bayus (1998) shows that the length of product life-cycle can differ for individual
producers and recommends that the inferences about the product life-cycle should be made based on industry level data.

The use of the product life-cycle method requires the sales data of the products. For the new products, as there is no pre-sales data available, sales forecasts are sufficient. Morrison (1995) provides approximation methods for parameters in the product life-cycle of new products. More specifically; the expectations augmented version of equation (1) can be used to arrive at

\[
E_t[F_i(t(A), A)] = \frac{\hat{S}}{1 + e^{A(t-t)}} \quad \text{for } t=1,2,.T.
\]

Where \( E_t[F_i(t(A), A)] \) is the expected value of accumulated sales value at time \( t \), \( \hat{S} \) is the approximated value of the saturation level of the sales value, \( \hat{A} \) is the estimated value of the delay factor and \( \hat{I} \) is the estimated value of the inflection point. According to Morrison (1995), the value of \( \hat{S} \) can be found as the maximum amount of total sales that the product can obtain.

Morrison (1995) uses a simple example to show how to arrive at \( \hat{S} \); there is a producer who wants to sell a new type of tomato in a small town. It is known by this producer, from market research, that there are 1000 people in the town that buy tomatoes, two every year. Also, from through consumer surveys it was learned that only 25% of the potential consumers are willing to purchase this new tomato. From this knowledge, the producer can approximate the saturation level of sales as; \( \hat{S} = 1000 \times 2 \times 0.25 = 500 \) tomatoes a year at some period in the future.

For the current analysis a similar method is used to approximate \( \hat{S} \) for the SMS Heifer Program®. The total number of registered buyers is 595 for 2003, the average herd size for the average registered buyer is 90 animals, 61% of buyers bought SMS Heifers and 73% of those
who bought SMS Heifers indicated that they want to continue buying SMS Heifers (Parcell et al., 2005). The number of actual buyers is found by multiplying the total number of registered buyers (595) with the percentage of buyers who actually bought Heifers (61%). To approximate the number of future buyers the number of actual buyers (595*0.61) is multiplied by the number of buyers who want to continue buying SMS Heifers® (73%). Finally, to find the future sales of SMS Heifers multiply the average herd size with the approximate future SMS Heifer buyers, which gives \( S \) as

\[ S = 595 \times 0.61 \times 0.73 \times 90 = 23,845 \text{ heifers.} \]

The value of \( A \) and \( I \) are calculated by a non-linear optimization procedure as used in Kros (2005). Using the calculated value of \( S \), Excel Solver is used to select the values of \( A \) and \( I \) that minimize the sum of squared errors between the actual cumulative sales data and the expected value of cumulative sales date for 1997 through 2004. Specifically,

\[
\min_{A,I} \sum_{t=1}^{8} (F_t - \frac{23845}{1 + e^{A(t-t^*)}})^2 \\
\text{s.t.} \\
0 \leq A \leq 1, \\
0 \leq I.
\]

Where \( F_t \) is the actual cumulative sales data for year \( t \) and the value of \( A \) is expected to be between zero and one. The value of \( A \) was calculated to be 0.37. This implies that there were
significant marketing efforts in the beginning of SMS Heifer Program®, which is consistent with actual observation.³ The value of \( I \) was estimated to be 7.5.

**Results and Implications**

The shape of cumulative life-cycle (CLC) curve for SMS Heifer Program® is shown for two different values of \( A \), 0.37 and 1 (figure 3). As can be seen in figure 3 when \( A = 1 \), the shape of CLC curve is similar to the shape generally represented in text books. Total sales are low initially, grow rapidly and reach maturity. The assumption that \( A = 1 \) implies no initial marketing efforts. When \( A = 0.37 \) a higher amount of sales can be attained earlier in the introduction phase. This scenario can be seen better when, for example, the amount of animal sales are compared for year two. In year two total sales are projected to be 2,646 heifers for \( A = 0.37 \) and 200 heifers for \( A = 1 \). The actual data indicates that year two sales are 1,844 heifers.

As can be seen in the figure 3, a decrease in \( A \) causes both \( S \) and \( I \) to increase. The increase in \( S \) causes the sales level in each year to be higher than that of the same level \( A \) with a lower level of \( S \). This point was mentioned by Morrison (1995). However, he does not incorporate the increase in \( S \) due to a decrease in \( A \). However, the increase in \( S \) causes the sales level to increase for any given level of \( A \). For SMS Heifer Program®, the decrease in \( A \) causes the sales level to increase for the period of years one through six and thirteen through nineteen.

³ Producer-owner representatives at sale locations advertise widely, with the University of Missouri Extension system serving as catalyst for marketing efforts. The consignment cost for each heifer marketed through registered sales ranges from $15 to $20 per animal. Most of this consignment fee goes toward marketing. In addition, the University of Missouri Extension service offered free news releases for this program because of it being initiated through University of Missouri Extension monies.
For the period years seven through twelve sale quantities are projected to decrease due to the decrease in $A$, which led to increased quantity sold in the initial years. If $A$ decrease sufficiently, then there may be no period of time for which sales levels are decreasing (figure 4).

For the case of the SMS Heifer Program® the level of $S$ is maximized at 28,213 animals when $A=0$, which does not prevent a decrease in quantity sold in some periods. If 30,666 animals is a minimum for $S$, then SMS Heifer Program® heifers sales would not decrease for any period because of initial marketing efforts, i.e., decrease in $A$. This scenario can also been seen in figure 4.

It is possible to compare the net gain in sales from a decrease in $A$, i.e, increase marketing efforts. SMS animal sales were computed to be 15,489 for period one to six and 12,887 for period thirteen to nineteen. The decrease in sales for the period seven to twelve is computed to be 14,929 Heifers. Hence, adding up these three values yields a net increase in SMS Heifer Program® sales of 13,447 animals due to early marketing efforts. Figure 5 shows the change in cumulative sales for corresponding years, when $A$ is 1 and 0.37.

The decrease in $A$ causes $I$ to increase. For the SMS Heifer Program® the inflection point $I$ increased from $t = 6.5$ to $t = 7.5$ by a decrease in $A$ from 1 to 0.37.

**Profit Curve**

The profit function for the SMS Heifer-Program® is estimated following the cumulative life-cycle as

$$
\min_{b,c} \sum_{r=1}^{7} \left( \pi_t - p_t \frac{\partial F_t(\cdot)}{\partial t} - c(A) - b \frac{\partial F_t(\cdot)}{\partial t} \right)^2
$$

s.t.

$p_t = 981$
For $\frac{\partial \hat{F}_t(.)}{\partial t}$, estimated instantaneous rate of sales, the values calculated from estimated SMS Heifer Program® heifer sales for a value of $A = 0.37$. For estimation of the profit levels it is assumed that the price of the SMS Heifer Program® is constant over time and the average of the price level in seven years $\$981$ is used as the constant price level. The Excel Solver calculated $\hat{b}=980$ and $\hat{c}=183$. The estimated profit curves, for $A=0.37$ and $A=1$ are shown in the figure 6.

Comparing the profit curve for $A=0.37$ and $A=1$ shows the impact of marketing effects on profit ability. Initially profits are lower for $A=0.37$ than for $A=1$. This is due to the expenditures for marketing. The profit levels for $A=0.37$ becomes significantly higher after year six. Profits diminish faster for $A=1$. This shows that producers receive more periods of positive profits when initial marketing efforts are made. The area under the profit curve for each level of $A$ are calculated to be $\$617$ for $A=1$ and $\$955$ for $A=0.37$, which reveals a net gain of $\$337$/head by decreasing $A$ from 1 to 0.37.

**Brand Value**

The General wisdom in agriculture is that the brand value created with product differentiation will last for a very short time period, as the replication of the product is easy in agriculture. The brand value ($B_t$) here is defined as the difference between the profit levels for the SMS Heifer Program and non-program bred heifers;

(16) \[ B_t = \pi_t^{SMS} - \pi_t^{non-SMS} \quad \text{for } t=1,2 \ldots T. \]

To calculate the brand value for each year the fitted profit values for SMS Heifer Program® and for non-program heifer sales are used. The values for SMS Heifer Program® are presented in the previous section. For non-program heifers sales, the sale quantities do not exist. Therefore, the
profit function is fitted with the same procedure that is used for SMS Heifer Program® with one
difference that profits and sales are fitted simultaneously

\[
\min \sum_{t=1}^{T} \left[ \pi_t - p_t \frac{\hat{S} A e^{\hat{A}(l-t)}}{(1 + e^{\hat{A}(l-t)})^2} - c(A) - b \frac{\hat{S} A e^{\hat{A}(l-t)}}{(1 + e^{\hat{A}(l-t)})^2} \right]^2
\]

s.t.
\[
p_t = 740,
0 \leq A \leq 1,
0 \leq \hat{t},
0 \leq \hat{S}
\]

Figure 7 shows the fitted brand value for SMS Heifer Program®. For SMS Heifer Program sales
the positive brand value is expected to last for 8.5 years for \(A=0.37\). As the profit life-cycle is
extended for 2 years by initial marketing efforts, it is expected that the length of brand value is
also extended by 2 years.\(^4\) The increase in the total value of product life-cycle also represents the
total increase in the brand value

\[
\frac{\partial B_t}{\partial A^{SM}} = \frac{\partial \pi_t^{SM}}{\partial A^{SM}} - \frac{\partial \pi_t^{non-SM}}{\partial A^{SM}} = \frac{\partial \pi_t^{SM}}{\partial A^{SM}} - 0 = \frac{\partial \pi_t^{SM}}{\partial A^{SM}} \quad \text{for } t=1,2,...,T
\]

This leads to total change in brand value as

\[
\sum_{t=1}^{T} \frac{\partial B_t}{\partial A^{SM}} = \sum_{t=1}^{T} \frac{\partial \pi_t^{SM}}{\partial A^{SM}}
\]

The value of (18), total increase in the value of brand value, is calculated to be $337/head for
SMS Heifer Program\(^\circ\).

\(^4\) As seen in figure (6), \(T\) is 12 years for \(A=0.37\), which is two years more than the case for \(A=1\)
Conclusions

This analysis shows that the product life-cycle theorem without incorporating the impact of a change in $A$ is not capable of explaining profit behavior in the long run. When producers initially allocate resources for marketing efforts, producers realize higher profits throughout the product life-cycle. The increase in profits also gives rise to an increase in the brand value. This result suggests that value-added agricultural products can create brand premium that is sustained in the long-run.
References


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Figure 1. Product Life-Cycle

Figure 2: Profit Life-Cycle
Figure 3: Cumulative Life-Cycle

Figure 4: Cumulative Life-Cycle with No-Decrease in Sales
Figure 5: Change in Cumulative Sales

![Cumulative Sales Chart]

Figure 6: Profit Curve

![Profit Curve Chart]
Figure 7: Brand Value

![Graph showing the Brand Value over the years from 1 to 9. The graph displays the Actual Brand Value and the Estimated Brand Value for A=0.37. The values range from -$100 to $300 per head. The years are labeled on the x-axis, and the brand value is labeled on the y-axis. The graph shows a peak at year 5 followed by a decline.]