OPTIMAL COLLECTIVE INVESTMENT IN GENERIC ADVERTISING, EXPORT MARKET PROMOTION AND COST-OF-PRODUCTION REDUCING RESEARCH

by

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ABSTRACT

Optimal investment rules are developed for a producer agency investing in domestic market generic advertising, export market promotion, and cost of production reducing research. Analytical results show fundamental difference in optimal investment rules when the producer group is assumed to maximise either producers' surplus or social surplus. Incorporating a constraint limiting total expenditure on the three activities substantially alters the structure of the optimal investment rules. Results highlight the importance of accounting for the financing mechanism when modelling optimal producer investment. Simulation of the optimal intensities suggests the proposed budget of the Canadian Beef Cattle Research Market Development and Promotion Agency underestimates the optimal level of investment.

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INTRODUCTION

Agricultural producer organisations have a strong history of investing in activities designed to either increase the size of the domestic market, the size of the export market, or to help producers achieve cost savings through production research. Historic examples of agencies engaging in such activities include the National Cattlemen's Beef Association and National Pork Producer's Council in the U.S., the Beef Information Centre in Canada, the Canadian Beef Export Federation, Canada Pork International, and the U.S. Beef Export Federation, among others. Given that agricultural producers typically contribute to such activities through check-offs, many have asked whether such investment has paid for itself (e.g., Cranfield and Goddard; Piggott et al.; Ward and Lambert). The general conclusion is that net returns are positive and large. However, much of the progress in this area considers investment in a single activity, a closed economy, or both. Although, Alston et al. (1994, 1995) and Kinnucan consider advertising a traded good in both large and small country scenarios, Ding and Kinnucan considered optimal investment in domestic and export market promotion, while Sellen and Goddard, and Chyc and Goddard, considered optimal investment in domestic advertising and production research. The mode by which investment funds are generated has also received attention, typically with the question "what lump sum investment is optimal?" or "what is the optimal per unit check-off?" (e.g., Alston et al., 1994; Kinnucan and Myrland).

These studies notwithstanding, recognise that the manner by which multiple activities are financed has a bearing on the optimal investment policy. If investment in generic advertising, export promotion and production research is financed via a
centralised check-off, then investment in each alternative will compete for a share of total available funds. In such instances, one would expect the investment in a specific option to be commensurate with the impact that alternative has on producer benefits, but relative to the impact of other alternatives. In contrast, if each investment option is financed in isolation of the other alternatives, one would not expect to see a direct relationship between investment alternatives.

The objective of this paper is to develop optimal investment rules when a producer agency invests in domestic market generic advertising, export market promotion, and cost-of-production reducing research. Specific consideration is given to a country holding an export position, with two distinct financing scenarios. In the first scenario, total investment across the three activities is not constrained \textit{a priori}; that is, optimal investment in each activity dictates the optimal lump sum check-off needed to finance that activity. In the second scenario, a producer agency is assumed to have a fixed amount of money to optimally allocate across the three investment activities. The distinction lies in the structure of the optimisation problem. The first case is an unconstrained problem, while the second scenario is a maximisation problem subject to an expenditure constraint where financing is centralised. To add further richness, the objective function being maximised is varied. Consideration is first given to optimal producer investment by maximising producers' surplus. As some producer groups have been given a legislative mandate to use check-off levies, and given that consumers may bear a burden of any check-off, it is equally important to consider optimal investment
from a social perspective. In this regard, optimal investment rules are derived when the
objective is to maximise social surplus.

The underlying economic framework is presented next. Optimal investment rules
for generic advertising, export promotion and research are then derived with and without
an expenditure constraint. In the former case, the optimal rules are examined for
sensitivity to changes in advertising, export promotion and research elasticities.
Numerical analysis is then conducted to provide estimates of optimal investment
strategies for the Canadian beef industry. Finally, the paper is summarised and policy
implications drawn.

**ECONOMIC FRAMEWORK**

The market environment is assumed to be static, certain, and competitive, where the law
of one price holds, trade occurs without barriers (i.e., no transport or marketing costs, no
trade barriers) and the exporter is assumed to be large enough to affect the market-
clearing price. Furthermore, a single market level is assumed. Figure 1 shows a
graphical representation of the framework. The left-hand side of the diagram represents
the domestic market, while the right hand side represents the export market. Domestic
demand is represented by \( D(P,A) \) where \( P \) is price, \( A \) is advertising, \( \partial D/\partial P < 0 \) and
\( \partial D/\partial A > 0 \), while domestic supply is represented by \( S(P,R) \), where \( R \) is cost-of-
production reducing research, \( \partial S/\partial P > 0 \) and \( \partial S/\partial R > 0 \). An export position is assumed,
so the exporter's excess supply curve is the difference in domestic supply and demand,
and denoted by \( ES \). Demand for the country's exports is represented by the export
function \( T(P,M) \), where \( M \) is export market promotion, \( \partial T/\partial P < 0 \) and \( \partial T/\partial M > 0 \).
The equilibrium price, denoted by $P^*$, occurs where export demand equals excess supply. Markets clear with $Q_t^*$ units exported and domestic demand and supply equalling $Q_{D}^*$ and $Q_{S}^*$ respectively.

Assuming simultaneous increases in advertising, research and export promotion, the equilibrium price and quantities will change. Suppose domestic demand shifts to $D(P,A')$, domestic supply to $S(P,R')$, and export demand to $T(P,M')$. As drawn, these changes raise the equilibrium price to $P^{**}$, equilibrium trade to $Q_{T}^{**}$, domestic demand to $Q_{D}^{**}$ and domestic supply to $Q_{S}^{**}$. Given these parallel shifts the increase in advertising, research and export promotion raises domestic producers' surplus.\textsuperscript{3,4} Recognise, however, that the direction and magnitude of the change in producers' surplus will differ according to the size of shifts. This then raises the question of what level of investment in advertising, export promotion and research would maximise producers' surplus.

As such, one would like to have analytical formulae relating optimal investment in generic advertising, export promotion and production research to measurable data and elasticities. To derive such formulae it is assumed that domestic producers collectively engage in research, advertising and export promotion to maximise producers' surplus. Furthermore, investment decisions are implemented through a producer agency where a manager (or group of managers) within the agency acts on behalf of producers in choosing optimal investment levels. However, some governments have passed legislation enabling the creation of agencies that undertake generic advertising, export promotion and research activities for a group of producers. Given the legislative mandate
underlying formation of such agencies, consideration ought to be given to the socially optimal level of investment. In this case, the objective is to maximise social surplus defined as domestic consumers' and producers' surplus.

A factor that cannot be overlooked is that producer agency budgets are often fixed a priori. In such instances investment alternatives must compete for a share of a fixed amount of money. Consequently, optimal investment is limited by an expenditure constraint that says investment in $A, M$ and $R$ must add-up to a fixed, known level, $F$. The manager(s) within the agency is then faced with a constrained optimisation problem that will affect the investment level in each activity compared to a situation without an expenditure constraint.

Since a multitude of optimisation problems come about from the described options, it is convenient to write the optimisation as:

$$\max_{A,R,M,\lambda} L = \delta_C \left[ \int_0^{\theta_p} D^{-1}(\tau, A) d\tau - P Q_D \right] + P Q_S - C(Q_S, R)$$

$$- (\delta_C + \Omega (1 - \delta_C))(A + R + M) + \delta_{PEC} \lambda (F - A - R - M)$$

where $\delta_C$ and $\delta_{PEC}$ are binary indicator parameters to be defined latter,

$$\int_0^{\theta_p} D^{-1}(\tau, A) d\tau - P Q_D$$ represents consumers' surplus, $D^{-1}(\tau, A)$ is the inverse demand function, $P Q_S - C(Q_S, R)$ represents producers' surplus, where $C(Q_S, R)$ is the aggregate variable cost function, $\Omega$ shows the burden of the check-off borne by producers, and $\lambda$ is the Lagrange multiplier associated with the expenditure constraint. The use of indicator parameters allows one to express a number of different optimisation problems with one
formulation. For example, including consumers' surplus in the producer group's optimisation means $\delta_C = 1$ (otherwise $\delta_C = 0$), if expenditure is constrained \textit{a priori}, then $\delta_{PEC} = 1$ (otherwise $\delta_{PEC} = 0$).

The term $\delta_C + \Omega(1 - \delta_C)$ merits explanation. It reflects the fact that if the objective is to maximise producers' surplus (i.e., $\delta_C = 0$), then the optimisation problem takes account of the producers' portion of the tax-burden arising from any check-off. In this case, $\delta_C + \Omega(1 - \delta_C)$ reduces to $\Omega$. If, however, a social surplus perspective is taken, so $\delta_C = 1$, then the optimisation problem takes account of the total (i.e., producers' and consumers' burden) portion of the tax-burden arising from any check-off, and $\delta_C + \Omega(1 - \delta_C)$ reduces to a value of 1.

In deriving the first order conditions it is important to recognise that equilibrium quantities vary with $A$, $R$, and $M$. As such, the upper limit of integration in the producer group's optimisation varies with the investment level. Given this, the first order conditions can be written as:

$$\frac{\partial L}{\partial A} = \frac{\partial P}{\partial A}(Q_S - \delta_C Q_D) + \delta_C \int_0^{\Omega} \frac{\partial D^{-1}(\tau, A)}{\partial A} d\tau - (\delta_C + \Omega(1 - \delta_C)) - \delta_{PEC} \lambda = 0 \quad (1)$$

$$\frac{\partial L}{\partial M} = \frac{\partial P}{\partial M}(Q_S - \delta_C Q_D) - (\delta_C + \Omega(1 - \delta_C)) - \delta_{PEC} \lambda = 0 \quad (2)$$

$$\frac{\partial L}{\partial R} = \frac{\partial P}{\partial R}(Q_S - \delta_C Q_D) + \frac{\partial C(Q_S, R)}{\partial R} - (\delta_C + \Omega(1 - \delta_C)) - \delta_{PEC} \lambda = 0 \quad (3)$$

$$\frac{\partial L}{\partial \lambda} = \delta_{PEC} (F - A - R - M) = 0 \quad (4)$$
Assuming research lowers the variable cost of production, the partial of $C(Q_s, R)$ with respect to research is negative. To make the analysis transparent, this sign condition has been explicitly included in equation (3). Depending on the values taken by $\delta_C$, and $\delta_{PEC}$ different goals can be achieved. In what follows, the case of unconstrained expenditure ($\delta_{PEC} = 0$) is first explored, followed by constrained expenditure ($\delta_{PEC} = 1$).

**Optimal Unconstrained Investment**

When expenditure on generic advertising, export promotion and production research is unconstrained, equation (4) falls out of the first order conditions, as does the Lagrange multiplier. Assuming a parallel shift in domestic demand, the integrand in (1) is independent of quantity and equal to $\partial P/\partial A$. After cancelling terms related to $\partial P/\partial A$ in (1) and turning the relevant derivatives into elasticities, the first order conditions can be solved to arrive at formulae defining the optimal investment intensity for each activity:

$$\gamma_A = \frac{\pi_A}{(\delta_C + \Omega(1 - \delta_C))}$$  \hspace{1cm} (1')

$$\gamma_M = \frac{\pi_M \left(1 - \frac{\delta_C}{1+k}\right)}{(\delta_C + \Omega(1 - \delta_C))}$$  \hspace{1cm} (2')

$$\gamma_R = \frac{\pi_R \left(1 - \frac{\delta_C}{1+k}\right) + \xi_S}{(\delta_C + \Omega(1 - \delta_C))}$$  \hspace{1cm} (3')

where $\gamma_i$ is the ratio of optimal investment in the $i$th activity to producers' market revenue and is referred to as the investment intensity, $i \in Z = \{A, R, M\}$, $\pi_i$ is the elasticity of price with respect to the $i$th activity, $k = Q_T/Q_D$ ($> 0$), $\xi$ is the elasticity of
variable production costs with respect to research, and \( s \) is the ratio of variable production cost to producers’ market revenue (i.e., \( C/(PQ_s) \)).

Notice that if \( \pi_i = 0 \), then \( \gamma_A = \gamma_M = 0 \) but \( \gamma_R \neq 0 \). This means that if the exporting country is characterised as small and open, advertising and export promotion activities should not be used, as investment in these activities will not change price. This reflects earlier work by Alston et al. (1994) and Kinnucan. However, investment in research, which shifts the cost function, and therefore the domestic supply curve, should be undertaken. To see this, set \( \pi_r = 0 \) in (3’), so the optimal research intensity becomes

\[
\gamma_R = \xi s (\delta_c + \Omega(1 - \delta_c))^{-1}.
\]

Since \( \xi \) is assumed to be positive so too is \( \gamma_R \). Thus, even when the market is small and open, the producer group should invest in production research that shifts the domestic supply function out provided \( \xi > 0 \).

To operationalise (1’), (2’) and (3’), values for \( \pi_i \), \( \xi \), \( s \) and \( \Omega \) are needed. The value for \( \xi \) can be determined empirically, \( s \) is datum, while \( \Omega \) can be derived from the structure of the market model. Values for \( \pi_i \) are derived using an equilibrium displacement model based on a market model represented with the following equations:

\[
Q_D = D(P(A,R,M),A) 
\]

\[
Q_s = S(P(A,R,M),R) 
\]

\[
Q_T = T(P(A,R,M),M) 
\]

\[
Q_T = Q_s - Q_D. 
\]
Equation (5) is the domestic demand function, (6) is the supply function, (7) is an export demand function, while (8) provides a market closure rule. That equilibrium price and quantities are functions of $A$, $M$ and $R$ has been explicitly included.

Taking the logarithmic partial derivative of equations (5) through (8) with respect to advertising results in the following system of equations:

\[
\frac{\partial \ln Q_D}{\partial \ln A} = \frac{\partial \ln D}{\partial \ln P} \frac{\partial \ln P}{\partial \ln A} + \frac{\partial \ln D}{\partial \ln A} = -\eta \pi_A + \beta 
\]

(5')

\[
\frac{\partial \ln Q_S}{\partial \ln A} = \frac{\partial \ln S}{\partial \ln P} \frac{\partial \ln P}{\partial \ln A} = \varepsilon \pi_A 
\]

(6')

\[
\frac{\partial \ln Q_T}{\partial \ln A} = \frac{\partial \ln T}{\partial \ln P} \frac{\partial \ln P}{\partial \ln A} = -e \pi_A 
\]

(7')

\[
\frac{\partial \ln Q_T}{\partial \ln A} = \frac{Q_S}{Q_T} \frac{\partial \ln Q_S}{\partial \ln A} + \frac{Q_D}{Q_T} \frac{\partial \ln Q_D}{\partial \ln A} 
\]

(8')

where $\eta(>0)$ and $\beta(>0)$ are the price and advertising elasticities of demand, respectively, $\varepsilon(>0)$ is the supply elasticity and $e(>0)$ is the elasticity of demand for the country's exports. After substituting (5'), (6') and (7') into (8') and manipulating, one can derive the following:

\[
\pi_A = \frac{\beta}{\zeta} 
\]

(9)

where $\zeta = (1 + k)\varepsilon + \eta + ke$, which is positive for normally considered values of the elasticities and corresponding trade-to-demand ratio. A similar procedure can be performed for the export market promotion elasticity:
\[ \pi_M = \frac{k\varphi}{\zeta} \]  

(10)

and for the cost-of-production reducing research elasticity:

\[ \pi_R = -\frac{(1+k)\theta}{\zeta} \]  

(11)

where \( \varphi > 0 \) is the elasticity of export demand with respect to export market promotion and \( \theta > 0 \) is the elasticity of supply with respect to research.

Substituting (9) into \( (1') \) and (10) into \( (2') \) gives optimal investment intensity rules for domestic generic advertising and export market promotion:

\[ \gamma_A = \frac{\beta}{\zeta(\delta_C + \Omega(1-\delta_C))} \]  

(12)

\[ \gamma_M = \frac{\varphi k \left(1 - \frac{\delta_C}{1+k}\right)}{\zeta(\delta_C + \Omega(1-\delta_C))} \]  

(13)

both of which are positive in normal cases \( (i.e., \text{when elasticities have expected signs}). \)

For the optimal research intensity rule substitute (11) into \( (3') \):

\[ \gamma_R = -\frac{\theta(1+k)\left(1 - \frac{\delta_C}{1+k}\right) - \xi s \zeta}{\zeta(\delta_C + \Omega(1-\delta_C))} \]  

(14)

which is positive when \( \theta(1+k)\left(1 - \frac{\delta_C}{1+k}\right) < \xi s \zeta \). To understand the intuition underlying this last equation, note that all things being equal, an increase in research is assumed to lower the cost function, \( C(Q_s, R) \). However, equilibrium quantities are assumed to increase in \( R \). If the quantity increase is sufficiently large, the variable cost of production
at the new equilibrium quantity may actually rise in response to research. This leads to
the result that the commodity agency should invest in cost-of-production reducing
research when \( C(Q_s, R) > C(Q'_s, R') \), where \( Q_s < Q'_s \) correspond to research levels
\( R < R' \), respectively. This will be true when the cost reducing effect of research is large
and the quantity increasing effect of research is small. In this setting, a small elasticity of
the quantity supplied with respect to research is actually preferred.

To re-iterate an earlier point, in a small open economy, which is the case when
\( e = \infty \), the agency should not engage in domestic advertising, as shifts in the domestic
demand curve have no effect on price, nor should the agency use export market
promotion expenditure for the same reason. However, even in the case of a small open
economy, the agency should engage in production related research. To see this last point,
apply l'Hôpital's rule (with respect to \( e \)) to equation (14) and note that the limiting
behaviour of \( \gamma_r \) as export demand becomes perfectly elastic is \( \xi_s \).

Equations (12), (13) and (14) show that optimal advertising, export market
promotion and research intensities depend on price elasticities, the elasticity of quantity
demanded (or supplied) with respect to the respective investment alternative, \( k, \xi, s \)
and the incidence parameter. For comparison, the top half of Table 1 shows the optimal
investment intensities when the producer group maximises producers' surplus and social
surplus (denoted below as \( \gamma_i(\delta_c = 0) \) and \( \gamma_i(\delta_c = 1) \) respectively). It can be shown that
\( \gamma_i(\delta_c = 0) > \gamma_i(\delta_c = 1) \) provided \( \Omega < 1 \), but are equal when \( \Omega = 1 \). Thus, when producers
bear a fraction of the check-off, the optimal advertising intensity falls when domestic
consumer surplus is added to the optimisation problem. When optimal export market
promotion intensities are compared, the following ranking results:

\[ \gamma_M(\delta_c = 0) > \gamma_M(\delta_c = 1) \text{ provided } \Omega < 1 \text{ and } k > 0. \]

Although trivial, if trade does not occur export market promotion investment will not occur. Finally, to compare optimal production research intensities note that the denominator in \( \gamma_R(\delta_c = 0) \) is less than the denominator in \( \gamma_R(\delta_c = 1) \) provided \( \Omega < 1 \), but the numerator in \( \gamma_R(\delta_c = 0) \) is less than that in \( \gamma_R(\delta_c = 1) \) provided \( \theta > 0 \). Consequently, there is an ambiguous ranking of \( \gamma_R(\delta_c = 0) \) and \( \gamma_R(\delta_c = 1) \).

**OPTIMAL CONSTRAINED INVESTMENT**

When producer agency budgets are fixed, the producer group must decide on the optimal allocation of scarce funds across the three investment options. Here, the aim is to maximise producers' surplus (or social surplus) through investment in generic advertising, research and export market promotion subject to an expenditure constraint.

This means \( \delta_{pec} = 1 \) in the first order conditions (1) to (4), which can now be written as:

\[ \frac{\partial L}{\partial A} = \frac{\partial P}{\partial A}(Q_S - \delta_c Q_D) + \delta_c \int_0^{Q_0} \frac{\partial D^{-1}(\tau, A)}{\partial A} d\tau - (\delta_c + \Omega(1 - \delta_c)) - \lambda = 0 \tag{1'} \]

\[ \frac{\partial L}{\partial M} = \frac{\partial P}{\partial M}(Q_S - \delta_c Q_D) - (\delta_c + \Omega(1 - \delta_c)) - \lambda = 0 \tag{2'} \]

\[ \frac{\partial L}{\partial R} = \frac{\partial P}{\partial R}(Q_S - \delta_c Q_D) + \frac{\partial C(Q_s, R)}{\partial R} - (\delta_c + \Omega(1 - \delta_c)) - \lambda = 0 \tag{3'} \]

\[ \frac{\partial L}{\partial \lambda} = F - A - R - M = 0. \tag{4'} \]
As before, the integrand in (1") is assumed to be independent of quantity. However, the Lagrange multiplier can be eliminated. To do so, elasticitise (1") to (4"), substitute in equations (9), (10) and (11) and manipulate to derive the following:

\[
\begin{align*}
\hat{\gamma}_A &= \frac{\beta}{\zeta(\delta - \Omega(1 - \delta_c) + \lambda)} \\
\hat{\gamma}_M &= \frac{\phi k}{\zeta(\delta_c + \Omega(1 - \delta_c) + \lambda)} \\
\hat{\gamma}_R &= -\frac{\theta(1 + k) - \zeta \xi}{\zeta(\delta_c + \Omega(1 - \delta_c) + \lambda)} \\
\end{align*}
\]

\[\Gamma - \hat{\gamma}_A - \hat{\gamma}_R - \hat{\gamma}_M = 0.\]  

where \( \Gamma = F/(PQ_s) \) and \( \hat{\gamma}_i \) represent the optimal investment intensity when the producer group faces an expenditure constraint. Since (15) to (18) describe the first order conditions for a maximisation problem subject to an expenditure constraint, use the Hotelling-Wold identity to eliminate \( \delta_c + \Omega(1 - \delta_c) + \lambda \) and arrive at the following:

\[
\begin{align*}
\hat{\gamma}_A &= \beta \Xi^{-1} \Gamma \\
\hat{\gamma}_M &= \phi k \left(1 - \frac{\delta_c}{1 + k}\right) \Xi^{-1} \Gamma \\
\hat{\gamma}_R &= -\left\{\theta(1 + k) \left(1 - \frac{\delta_c}{1 + k}\right) - \xi \xi \zeta\right\} \Xi^{-1} \Gamma \\
\end{align*}
\]

where \( \Xi = \beta + (\phi k - \theta(1 + k)) \left(1 - \frac{\delta_c}{1 + k}\right) + \xi \xi \zeta.\)
As with the unconstrained case, the distinction between a small and large country affects the optimal investment intensity. If the market is characterised as small, the producer agency should not invest in domestic market generic advertising and export market promotion, as shifts in domestic and export demand do not effect price. However, the producer group should invest all of their funds in production research. (As with the unconstrained case, apply l'Hopital's rule to equation (21) and note that $\hat{R}_\gamma$ approaches $\xi_s$ as export demand becomes perfectly elastic). Even though production research does not affect the equilibrium price, the cost savings increases producers' surplus thereby providing an incentive to invest in production research.

The bottom half of Table 1 shows $\hat{\gamma}_i$ when the producer group maximises domestic producers' surplus ($\delta_c = 0$) and social surplus ($\delta_c = 1$). Contrary to the unconstrained case, it is not possible to unambiguously rank the optimal investment intensities when expenditure is constrained. However, one can investigate how changes in advertising, research and export promotion elasticities affect the optimal constrained intensities by taking the partial derivative of $\hat{\gamma}_i$ with respect to the $j$th activity’s elasticity.

These qualitative results are shown in Table 2. At first blush, it appears that when the objective is to maximise producers' surplus, $\hat{\gamma}_A$ may increase or decrease in $\beta$, while $\hat{\gamma}_M$ and $\hat{\gamma}_R$ unambiguously fall in $\beta$. However, note that since $\Gamma$ is fixed, the total effect of a change in $\beta$ on $\Gamma$ must be zero. Since $\partial\hat{\gamma}_M/\partial \beta < 0$, $\partial\hat{\gamma}_R/\partial \beta < 0$ and $\partial\hat{\gamma}_A/\partial \beta < 0$, $\partial\hat{\gamma}_M/\partial \beta + \partial\hat{\gamma}_R/\partial \beta + \partial\hat{\gamma}_A/\partial \beta = 0$, $\partial\hat{\gamma}_A/\partial \beta$ must actually be positive. By the same argument, $\partial\hat{\gamma}_M/\partial \phi$ must also be positive. These revised qualitative results are shown in
bracketed terms in Table 2. Note that identical qualitative results are derived when social surplus is considered.

Note that regardless of whether domestic or social surplus is maximised, the price elasticities of demand and supply still play a role through the $\zeta$ term appearing in the denominator of each optimal investment intensity equation. In contrast, Ding and Kinnucan, who considered optimal promotion investment in domestic and export markets, derived optimal investment rules that were not directly dependent upon quantity-price elasticities of demand or supply. Differences in this qualitative result stem from the inclusion of cost-of-production reducing research in this paper.

AN APPLICATION TO THE CANADIAN BEEF INDUSTRY

To cast the optimal investment intensity rules in better light, they are applied to the Canadian beef industry. This industry offers an interesting application as they lobbied for and were granted permission to establish an umbrella organisation to co-ordinate producer investment in domestic and export market promotion and production research. Because the "Canadian Beef Cattle Research Market Development and Promotion Agency" (Agency) was established under the auspices of the Farm Products Agencies Act, the legislative powers of government were used in its formation. To some, this would suggest that consideration be given to socially optimal investment when establishing investment strategies. Furthermore, the application made by Canadian Cattleman's Association (the national body representing beef-cattle producers in Canada) to the National Farm Products Council (NFPC) indicated a benchmark level of funding to
be made available for distribution across advertising, promotion and research. As such, consideration ought to also be given to the constrained optimal investment intensities.

Numerical simulation is conducted to show the range of values for $\gamma_i$ and $\hat{\gamma}_i$ based on "best-guess" estimates of elasticities for the Canadian beef cattle complex in a post-WTO environment. (For convenience, the optimal advertising intensity will be reported as a percent.) Table 3 shows the assumed elasticity and data values used in calculating the optimal investment intensities and sources. Goddard and Griffith reported own-price and advertising elasticities for beef in Canada equal to -0.23 and 0.004, respectively, which are used here. Since reliable estimates of $e$ are not available and Canada holds a net export position live cattle and beef, $e$ assumes the value -5, which is thought to be reflective of the demand response for Canadian beef-cattle exports. Estimates of $\varphi$, the elasticity of export demand with respect to export market promotion, are not prevalent in the literature. As such, $\varphi$ is assumed to equal $\beta$.

Cranfield and Goddard reported own-price supply elasticities for live-cattle in western Canada equal to 0.431. This value is consistent with supply elasticities reported for the literature and is used here. Widmer et al. estimated supply response elasticities for production research in the Canadian beef industry. Short run elasticities range from 0.01 to 0.04, with the long run elasticity reported at 0.36. For this paper, a middle ground approach is taken, with $\theta$ set equal to 0.1. Chan-Kang et al. estimated the elasticity of cost with respect to research for the Canadian food-manufacturing sector to equal 0.092. While this value is for food-processing, other references to the cost-research elasticity are not readily apparent in the literature. As such, $\xi$ is set equal to 0.092.
The value for $k$ is set at 0.31, which is the average value of $k$ using 1995-1998 data obtained from Agriculture Canada’s Livestock Market Review. Variable costs' share of market revenue ($s$) is pegged at 0.64, which is the 1995-1998 average value of the ratio of operating costs (net interest) to the value of farm product in the Ontario feedlot sector (OMAFRA). Following Kinnucan, the tax-incidence parameter is calculated using:

$$\Omega = \frac{\eta + k|\epsilon|}{\eta + k(1 + k)\nu}$$

where $\nu = p/(p - t)$, $t$ is the per unit levy used to finance generic advertising activity and $|\epsilon|$ is the absolute value operator. The per unit levy is set equal to equal $1$ per head, stated in live-weight equivalent (a slaughter weight animal is assumed to be 1,200 pounds, so the equivalent check-off is $1/1200$ per pound). Price is the average price (over 1995-1998) of slaughter-weight cattle as reported in Agriculture Canada's Livestock Market Review.

Lastly, to calculate the constrained optimal investment intensities, the ratio of the fixed amount of funds to market revenue needs to be determined. In the CCA's submission to the NFPC, the budget for the proposed agency was estimated to be between $8$ and $8.5$ million. Agriculture Canada’s Livestock Market Review places farm-level market revenue for fed cattle at about C$3.39$ billion. This suggests $\Gamma$ be set at approximately 0.24 (percent of market revenue).

Table 4 shows the simulation results of the constrained and unconstrained investment strategies under the assumption of producers' surplus and social surplus.
maximisation. Regardless of whether the objective being maximised is producers' or social surplus, the expenditure constrained intensities are all less than the unconstrained intensities. This suggests the budget proposed by the CCA under-estimates the optimal level of total investment. When producers' surplus is maximised the sum of the unconstrained intensities is 0.71, which is about three times as large as $\Gamma$. When social surplus is maximised, the sum of the unconstrained intensities is 0.49, which is about twice as big as $\Gamma$. Table 4 also shows the proportion of the "total" intensity allocated to advertising, export market promotion and research. When producers' surplus is maximised, the proportion of the total investment intensity allocated to the three alternatives is identical regardless of whether investment is constrained. However, when social surplus is maximised, the imposition of the expenditure constraint changes the relative magnitude of the optimal allocation scheme. Research receives considerably more investment when expenditure is constrained, at the expense of both advertising and export market promotion.

One issue of concern is how changes in the various parameters of equations (12)-(14) and (19)-(21) affect the optimal investment intensity in the various activities. Trivially, one can see that in the unconstrained case a given increase in the domestic advertising elasticity or export promotion elasticity will increase investment in those alternatives by amount equal to the given change. For example, if $\beta$ increased by 5%, then so too would the optimal investment intensity in domestic advertising. Furthermore, the optimal investment intensity in the other activities would not change. However, if the elasticity of cost with respect to research or supply with respect to research changes, then
the effect on the optimal research intensity will not be proportional. To illustrate this last point Table 5 shows the optimal investment intensities when $\xi$ and $\theta$ increase and decrease by 5% as well as the percent difference between the base case investment intensities and the shocked intensities. When $\xi$ ($\theta$) is shocked, the optimal research intensities move in the same (opposite) direction as the change in $\xi$, as expected, but by a different magnitude than the respective change in either $\xi$ or $\theta$. This reflects the additive nature of the numerator in equation (14).

One further point to note regarding the unconstrained optimal investment intensities is the role of the trade-to-demand ratio, $k$. Except for the optimal advertising intensity, $k$ appears in the numerator and denominator of the optimal intensities for export promotion and research. As such, changes in $k$ will be expected to affect the optimal investment intensities in something other than a proportional relationship. To illustrate, Table 5 shows that a 5% increase (decrease) in $k$ lowers (raises) $\gamma_A$ by less than 5%, as expected given $k$ appears in the denominator of $\gamma_A$. The effect on export promotion depends on the nature of the optimisation problem one assumes. A 5% increase in $k$ increases optimal investment in export promotion by less than 5% when the producer agency is assumed to maximise producers' surplus. When the producer agency is assumed to maximise social surplus $\gamma_M$ actually increases (decreases) by more than 5%. Lastly, changes in $k$ have a dramatic effect on $\gamma_R$. Specifically, $\gamma_R$ moves in the same direction as $k$, but by a magnitude of about 40-45%. These results suggest that if Canadian beef exports grow relative to the domestic market, that less money ought to be
invested in domestic advertising (relative to market revenue), and more in export market promotion and production research. Furthermore, results underscore the importance of research and trade when assessing producers' optimal investment patterns through a commodity agency.

When the producer agency has a fixed amount of money to allocate across the three investment alternatives, a change in the elasticity for one of these activities will have an impact on investment for all activities. This is evident by examining equations (19)-(21), which show that the denominator for all optimal investment intensities are a function of the elasticities for domestic advertising, export market promotion and research. Hence an change in any one of these elasticities will have bearing on the optimal allocation of the fixed amount of money, $F$. Table 6 shows the optimal investment intensities when the base values of $\xi$, $\theta$ and $k$ increase and decrease by 5%. Given the structure of the expenditure constrained optimal investment intensity rules, the directional affects of the assumed shocks are expected. The magnitude of these effects is an empirical issue that depends on the assumed elasticities and data values.

Of particular note, however, is that changes arising from shocks to $\xi$, $\theta$ and $k$ are of a far greater magnitude (in absolute value) when producers' surplus is maximised than when social surplus is maximised. This arises because of differences in the term multiplied by $\theta$ (which is $k$ in the case of producers' surplus maximisation and $1+k$ in the case if social surplus maximisation). Furthermore, the optimal investment intensity equations differ according to the objective function being maximised. Lastly, a change in the trade-to-demand ratio has a different effect on the optimal investment intensity for
export market promotion depending on the objective function. When producers' surplus is maximised, $\hat{\gamma}_M$ falls when $k$ increases, while $\hat{\gamma}_M$ increases in $k$ when social surplus is maximised. Again, this reflects differences in the nature of both the numerator and denominator of the respective optimality rules.

**SUMMARY AND DISCUSSION**

Optimal investment rules are developed for the case of a producer agency investing in generic advertising in the domestic market, export market promotion, and cost-of-production reducing research. These rules were developed assuming the objective is to maximise either producers' surplus or social surplus, with and without an expenditure constraint limiting the amount of money available for investment. The primary implication of this work relates to the mechanism by which the producer group raises the funds needed to finance their investment activities. If each activity's funds are raise independent of the other activities, then the optimal investment intensities derived without an expenditure constrained should be used to determine the optimal investment level. When the available funds are fixed *a priori*, for example when a producer organisation has a budget laid out via a business plan subject to producer approval, the optimal investment intensity rules developed with the expenditure constraint should be used in determining the optimal allocation. Note too that if, in the short run, the producer group uses a per unit check-off applied to the sale of the agricultural commodity to finance their activities, and if supply is fixed in the short run, then the available funds are also fixed and should be allocated using the expenditure constrained investment intensity rules. Such is the case in the Canadian dairy sector where milk production is rationed via
production quota. Finally, if financing is raised through a centralised check-off that is allocated across investment options, the optimal investment rules derived with the expenditure constraint should be used to calculating the optimal pattern of investment. Doing so allows the manager(s) within the producer agency to account for the role of competing investments when determining the optimal allocation across the three activities.

Simulation of the optimal intensities using "best-guess" estimates of the underlying elasticities and actual market data for the Canadian beef-cattle industry indicates the proposed budget for the recently established "Canadian Beef Cattle Research Market Development and Promotion Agency" under-estimates the optimal level of investment. Regardless of whether producers' surplus or social surplus was being maximised, the sum of unconstrained investment intensities for advertising, export market promotion and cost-of-production reducing research was greater than the investment intensity implied by the proposed budget for the aforementioned agency. Sensitivity of the optimal investment intensities was also investigated. Analysis suggests that the trade-to-demand ratio and cost-of-production related research elasticities are key factors in calculating optimal investment strategies for the Canadian beef-cattle industry.

The investment rules developed here are predicated on parallel shifts in the respective curves. Clearly, this is a limiting assumption. Without appeal to specific functional forms the analysis of pivotal changes is difficult. Nevertheless, the role of a pivotal change is worthy of discussion, especially in the context of the producer group's
objective function. For some pivotal changes, the welfare changes will differ in sign and order of magnitude compared to welfare changes arising from parallel shifts.

Further work could focus on a number of areas. First, a static model has been used to simplify the analysis. Extending the current model to account for the dynamic nature of returns stemming not only from production research but also from advertising and export promotion would allow for the incorporation of a discount rate reflecting producers' time value of money. Relaxing the certainty assumption would prove useful in developing a portfolio approach to allocating a fixed investment budget across activities with potentially uncertain returns. In this sense, one could view the producer groups' optimisation problem to be one of maximising expected utility of wealth of a representative producer by constructing an optimal portfolio of advertising, export promotion, and research investments. The dynamic and uncertain extensions of the model could also be cast in a real options framework where the managerial decision is to make irreversible investments in the three activities with an uncertain return. Lastly, the analysis conducted here assumed an interior solution in the sense that the producer group invests in all three activities. However, scope exists for the producer group to use a subset of the given activities. Consequently, extending this framework to allow for corner solutions (even in the case of a large open economy) would prove useful in developing guidelines that assist producer groups in make investment decisions.
REFERENCES


Table 1. Optimal investment intensity rules

<table>
<thead>
<tr>
<th>Maximise producers' surplus</th>
<th>Maximise social surplus</th>
</tr>
</thead>
<tbody>
<tr>
<td>Unconstrained Investment ($\gamma_i$)</td>
<td></td>
</tr>
<tr>
<td>Advertising</td>
<td>$\beta$</td>
</tr>
<tr>
<td></td>
<td>$\zeta\Omega$</td>
</tr>
<tr>
<td>Export market promotion</td>
<td>$\phi k$</td>
</tr>
<tr>
<td></td>
<td>$\zeta\Omega$</td>
</tr>
<tr>
<td>Research</td>
<td>$-\frac{\theta(1 + k) - \xi s\zeta}{\zeta\Omega}$</td>
</tr>
<tr>
<td>Expenditure Constrained ($\hat{\gamma}_i$)</td>
<td></td>
</tr>
<tr>
<td>Advertising</td>
<td>$\frac{\beta}{\Theta}$</td>
</tr>
<tr>
<td>Export market promotion</td>
<td>$\frac{\phi k}{\Theta}$</td>
</tr>
<tr>
<td>Research</td>
<td>$-\frac{\theta(1 + k) - \xi s\zeta}{\Theta}$</td>
</tr>
</tbody>
</table>

Where $\zeta = (1 + k)\epsilon + \eta + ke$

$$\Theta = \beta + \phi k - \theta(1 + k) + \xi s\zeta$$

$$\Phi = \beta + \frac{\phi k^2}{1 + k} - \theta k + \xi s\zeta$$
Table 2. Directional relationship between optimal investment intensity rules and advertising, export market promotion and research elasticities.

<table>
<thead>
<tr>
<th></th>
<th>$\frac{\partial \gamma_i}{\partial \beta}$</th>
<th>$\frac{\partial \gamma_i}{\partial \phi}$</th>
<th>$\frac{\partial \gamma_i}{\partial \theta}$</th>
<th>$\frac{\partial \gamma_i}{\partial \xi}$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Maximise Producers’ Surplus</td>
<td>Maximise Social Surplus</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\hat{\gamma}_A (\delta_C = 0)$</td>
<td>+/- (+)</td>
<td>-</td>
<td>+</td>
<td>-</td>
</tr>
<tr>
<td>$\hat{\gamma}_M (\delta_C = 0)$</td>
<td>-</td>
<td>+/- (+)</td>
<td>+</td>
<td>-</td>
</tr>
<tr>
<td>$\hat{\gamma}_R (\delta_C = 0)$</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>+</td>
</tr>
<tr>
<td>$\hat{\gamma}_A (\delta_C = 1)$</td>
<td>+</td>
<td>-</td>
<td>+</td>
<td>-</td>
</tr>
<tr>
<td>$\hat{\gamma}_M (\delta_C = 1)$</td>
<td>-</td>
<td>+</td>
<td>+</td>
<td>-</td>
</tr>
<tr>
<td>$\hat{\gamma}_R (\delta_C = 1)$</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>+</td>
</tr>
</tbody>
</table>
Table 3. Elasticity and Parameter Values

<table>
<thead>
<tr>
<th>Elasticity/Data (in absolute value)</th>
<th>Symbol</th>
<th>Assumed Value/ Mean</th>
</tr>
</thead>
<tbody>
<tr>
<td>Own-price demand elasticity(^a)</td>
<td>(\eta)</td>
<td>0.23</td>
</tr>
<tr>
<td>Domestic advertising demand elasticity(^a)</td>
<td>(\beta)</td>
<td>0.004</td>
</tr>
<tr>
<td>Own-price export demand elasticity</td>
<td>(e)</td>
<td>5</td>
</tr>
<tr>
<td>Export promotion demand elasticity</td>
<td>(\varphi)</td>
<td>0.004</td>
</tr>
<tr>
<td>Own-price elasticity of supply(^b)</td>
<td>(\varepsilon)</td>
<td>0.431</td>
</tr>
<tr>
<td>Research supply elasticity(^c)</td>
<td>(\theta)</td>
<td>0.1</td>
</tr>
<tr>
<td>Elasticity of cost with respect to research(^d)</td>
<td>(\zeta)</td>
<td>0.092</td>
</tr>
<tr>
<td>Trade's share of farm demand(^e)</td>
<td>(k)</td>
<td>0.31</td>
</tr>
<tr>
<td>Variable cost as a share of market revenue(^f)</td>
<td>(s)</td>
<td>0.64</td>
</tr>
<tr>
<td>Price ($ CWT)(^e)</td>
<td>(P)</td>
<td>82.22</td>
</tr>
</tbody>
</table>

- \(^a\) Source: Goddard and Griffith
- \(^b\) Source: Cranfield and Goddard
- \(^c\) Source: Widmer \textit{et al.}
- \(^d\) Source: Chan-Kang \textit{et al.}
- \(^e\) Based on average of annual values over 1995-1998
- \(^f\) Source: OMAFRA
Table 4. Base case optimal investment intensities under alternative optimisation assumptions

<table>
<thead>
<tr>
<th></th>
<th>Unconstrained Investment</th>
<th>Constrained Investment</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Maximise Producers' Surplus</td>
<td>Maximise Social Surplus</td>
</tr>
<tr>
<td>Intensity</td>
<td>Share$^a$</td>
<td>Intensity</td>
</tr>
<tr>
<td>Domestic Generic</td>
<td>0.231</td>
<td>0.325</td>
</tr>
<tr>
<td>Export Market</td>
<td>0.072</td>
<td>0.101</td>
</tr>
<tr>
<td>Production</td>
<td>0.407</td>
<td>0.574</td>
</tr>
<tr>
<td>Total</td>
<td>0.71</td>
<td>0.485</td>
</tr>
</tbody>
</table>

a. Share shows the proportion of total investment accounted for by the respective investment alternative
Table 5: Sensitivity of optimal unconstrained investment intensities under alternative optimisation assumptions

<table>
<thead>
<tr>
<th>Value of the shocked parameter</th>
<th>Producers' Surplus</th>
<th>Social Surplus</th>
<th>Producers' Surplus</th>
<th>Social Surplus</th>
<th>Producers' Surplus</th>
<th>Social Surplus</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\xi = 0.097$</td>
<td>0.231</td>
<td>0.171</td>
<td>0.231</td>
<td>0.171</td>
<td>0.221</td>
<td>0.165</td>
</tr>
<tr>
<td>$\gamma_A$</td>
<td>(0.0)</td>
<td>(0.0)</td>
<td>(0.0)</td>
<td>(0.0)</td>
<td>(-4.2)</td>
<td>(-3.5)</td>
</tr>
<tr>
<td>$\gamma_M$</td>
<td>0.072</td>
<td>0.013</td>
<td>0.072</td>
<td>0.013</td>
<td>0.072</td>
<td>0.013</td>
</tr>
<tr>
<td>$\gamma_R$</td>
<td>(97.9)</td>
<td>(97.9)</td>
<td>(-92.9)</td>
<td>(-92.9)</td>
<td>(42.0)</td>
<td>(43.2)</td>
</tr>
<tr>
<td>$\xi = 0.087$</td>
<td>0.231</td>
<td>0.171</td>
<td>0.231</td>
<td>0.171</td>
<td>0.241</td>
<td>0.177</td>
</tr>
<tr>
<td>$\gamma_A$</td>
<td>(0.0)</td>
<td>(0.0)</td>
<td>(0.0)</td>
<td>(0.0)</td>
<td>(4.6)</td>
<td>(3.7)</td>
</tr>
<tr>
<td>$\gamma_M$</td>
<td>0.072</td>
<td>0.013</td>
<td>0.072</td>
<td>0.013</td>
<td>0.071</td>
<td>0.012</td>
</tr>
<tr>
<td>$\gamma_R$</td>
<td>(97.9)</td>
<td>(97.9)</td>
<td>(92.9)</td>
<td>(92.9)</td>
<td>(45.9)</td>
<td>(46.4)</td>
</tr>
</tbody>
</table>

a. Values in cells show shocked optimal investment intensities, values in parentheses show percent change from base case values.
Table 6: Sensitivity of optimal constrained investment intensities under alternative optimisation assumptions\(^a\)

<table>
<thead>
<tr>
<th>Value of the shocked parameter</th>
<th>Producers' Surplus</th>
<th>Social Surplus</th>
<th>Producers' Surplus</th>
<th>Social Surplus</th>
<th>Producers' Surplus</th>
<th>Social Surplus</th>
</tr>
</thead>
<tbody>
<tr>
<td>ξ = 0.097</td>
<td>0.050</td>
<td>0.008</td>
<td>0.167</td>
<td>0.009</td>
<td>0.061</td>
<td>0.008</td>
</tr>
<tr>
<td></td>
<td>(-36.0)</td>
<td>(-5.8)</td>
<td>(114.1)</td>
<td>(1.4)</td>
<td>(-22.0)</td>
<td>(-3.0)</td>
</tr>
<tr>
<td>θ = 0.105</td>
<td>0.016</td>
<td>0.001</td>
<td>0.052</td>
<td>0.001</td>
<td>0.020</td>
<td>0.001</td>
</tr>
<tr>
<td></td>
<td>(-36.0)</td>
<td>(-5.8)</td>
<td>(114.1)</td>
<td>(1.4)</td>
<td>(-18.1)</td>
<td>(5.7)</td>
</tr>
<tr>
<td>k = 0.3255</td>
<td>0.174</td>
<td>0.231</td>
<td>0.021</td>
<td>0.231</td>
<td>0.159</td>
<td>0.231</td>
</tr>
<tr>
<td></td>
<td>(26.7)</td>
<td>(0.2)</td>
<td>(-84.8)</td>
<td>(-0.1)</td>
<td>(15.7)</td>
<td>(0.1)</td>
</tr>
</tbody>
</table>

| ξ = 0.087                      | 0.178              | 0.009          | 0.051              | 0.009          | 0.109              | 0.009          |
|                                | (128.1)            | (6.6)          | (-34.8)            | (-1.4)         | (39.3)             | (3.2)          |
| θ = 0.095                      | 0.055              | 0.001          | 0.016              | 0.001          | 0.032              | 0.001          |
|                                | (128.1)            | (6.6)          | (-34.8)            | (-1.4)         | (32.4)             | (-5.8)         |
| k = 0.2945                     | 0.007              | 0.230          | 0.173              | 0.231          | 0.099              | 0.231          |
|                                | (-95.2)            | (-0.3)         | (25.8)             | (0.1)          | (-28.0)            | (-0.1)         |

\(^a\) Values in cells show shocked optimal investment intensities, values in parentheses show percent change from base case values.
Figure 1. Schematic of the economic framework
Footnotes

1 A variety of related topics have been examined, including optimality of observed investment levels (Holloway et al.), advertising and promotion effectiveness (Brester and Schroeder; Comeau et al.; Cranfield and Goddard; Ding and Kinnucan; Kinnucan et al.; Richards et al.; Richards and Patterson; Piggott et al.; Ward and Lambert), the relationship between market power and benefits of production research (Alston et al., 1999; Hamilton and Sunding), and the distribution of advertising and research benefits along the marketing channel (Wohlgenant 1993, 1999; Chung and Kaiser).

2 Assuming a fixed proportion processing technology allows for consideration of one market level rather than a vertically related market structure.

3 Recognise that the shifts do not have to be parallel, all of the curves could undergo pivotal shifts, see, for example, Chung and Kaiser. However, the welfare outcome may differ depending on the nature and magnitude of different types of shifts.

4 Returns to primary production related research tend to be dynamic in nature. By assuming a static model, it is assumed the change in producers' surplus due to research equals the discounted stream of research benefits.

5 See Kinnucan (1999) for further discussion related to this export demand function.

6 This is due to the complex nature of and differences in the denominators under different optimisation scenarios.