Combining Annual Econometric Forecasts with Quarterly ARIMA Forecasts: A Heuristic Approach

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Data limitations often limit the time framework in which agricultural commodities are modeled and prices forecasted. Our research provides a technique to alleviate this constraint. By combining an annual econometric model with a quarterly ARIMA model, quarterly forecasts can be made which utilize the theoretical and structural foundations in econometric modeling.

The topic of price forecasting has long been an area of interest to economists. Though much has been written on alternative forecasting methods (e.g., Brandt and Bessler; Granger and Newbold; Leuthold et al.; Helmers and Held; Pierce and Porter), any one technique is not satisfactory for all situations. Often the researcher is faced with a situation requiring forecasts with data available only at a higher level of temporal aggregation. This paper addresses this problem by suggesting a technique of forecasting a quarterly price variable with data available for most explanatory variables only on an annual basis.

The objective of this study is to evaluate an ad hoc procedure of combining annual forecasts of alfalfa hay prices from an econometric model with quarterly alfalfa hay price forecasts from an ARIMA model. In this manner, the benefits of both modeling methodologies are incorporated, forecasting frequency is increased, and forecasting accuracy is improved.

For the eleven western states, alfalfa hay is an important input in the beef and dairy industries. Econometric analysis on the alfalfa hay market is generally limited to an annual framework due to data limitations for various explanatory variables. For example, lower level temporal aggregation (e.g., quarterly data) is not available for alfalfa hay consumption and production. Generally, there is no information available on alfalfa hay movements. While an annual forecast provides useful information, decision makers would prefer more frequent forecasts for planning marketing strategies.

Recently, Brandt and Bessler argued that combining econometric, ARIMA and other methods is superior to any one method. They showed that any particular forecasting method can result in large errors and combining more than one method may reduce the risk of creating a large error.

The ARIMA model is attractive from the standpoint of data requirements in that only one series of data is required. Also, this technique can be used to identify sea-

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1 The eleven western states considered in this study are Arizona, California, Colorado, Idaho, Montana, Nevada, New Mexico, Oregon, Utah, Washington, and Wyoming.
sonal variability in the series. Furthermore, the ARIMA model allows for adaptive forecasting in which weights can be calculated that will allow forecast adjustments as new information becomes available.

**Econometric Model**

Alfalfa hay demand is a derived demand from livestock products (primarily beef and dairy). The major inputs for cattle production are feed grains, alfalfa and other hay, and labor. To derive the input demand in the case of multiple products, the profit function is formed in terms of output prices and production cost (Henderson and Quandt, pp. 80–81 and 97–98). It is assumed that individual producers are expected profit maximizers. Optimal allocation of product by each producer, assuming perfect competition in input and output markets, leads to optimal allocation for the region (eleven western states).

The input demand equation derived from profit maximization is transformed into price dependent form. This inverse demand form is appropriate for models where total output is assumed to be predetermined in the short run as in the case of perennial crops like alfalfa hay. The demand equation estimated is specified as:

\[
\frac{\text{ALFPR}}{\text{WPI}} = a_0 + a_1 \frac{\text{ALFPROD}}{\text{WPI}} + a_2 \frac{\text{OTHHYPR}}{\text{WPI}} + a_3 \frac{\text{CORNPR}}{\text{WPI}} + a_4 \frac{\text{WAGE}}{\text{WPI}} + a_5 \text{CATTLE} + a_6 \text{DUM73} + a_7 \text{FPI} + u \quad (1)
\]

where ALFPR is alfalfa hay price #2 leafy Petaluma market ($/ton), WPI is the wholesale price index (1967 = 1.0), ALFPROD is alfalfa hay production in the 11 western states (1,000 tons), OTHHYPR is the price of U.S. other hay ($/ton), CORNPR is the average annual corn price received by U.S. farmers ($/bu), WAGE is the wage index of all hired farm workers (1967 = 1.0), CATTLE is cattle and calves January 1 inventory, 11 western states (1,000), DUM73 is an intercept shifter (0/1) 1953–72 = 0 and 1973–78 = 1, FPI is the farm productivity index (1967 = 1.0), and u is the error term.

The demand equation is homogeneous of degree zero in all prices. The quantity of alfalfa hay produced is assumed to equal the amount of alfalfa hay used by ranchers and farmers for a given year. The real price of alfalfa hay is expected to vary negatively with the quantity of hay consumed. The real price of other hay is expected to be positively related to the deflated alfalfa hay price since other hay is a substitute for alfalfa hay. The coefficient on the deflated price of corn should be negative, indicating that corn is a complement with alfalfa hay in livestock feed rations. However, for high corn prices, alfalfa hay may be a substitute for corn as a source of protein. The real wage index, as a measure of labor input, is expected to vary negatively with real alfalfa hay prices since labor is a complementary input in livestock production. The estimated annual derived demand model uses cattle numbers instead of product prices for beef and dairy to reduce collinearity of prices.

The coefficient on CATTLE should be positive indicating that changes in the

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2 See Heien for a thorough discussion of price and quantity dependent models.

3 A single equation model was chosen to demonstrate this proposed methodology and estimate ex ante price forecasts (see Pindyck and Rubinfeld, p. 204). The authors recognize that a system of equations can be used in lieu of a single equation model for the case of stochastic regressors.

4 Originally, product prices for beef and dairy were used as explanatory variables. However, coefficient signs different from a priori expectations and statistical evidence of high correlation between these two variables led to the aggregation of beef and dairy effects in cattle numbers.

<table>
<thead>
<tr>
<th>Equation</th>
<th>Coefficient</th>
<th>t-statistic</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>RALFPR = 0.64143 – 0.000555<em>ALFPROD + 0.31756</em>ROTHHYPR</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.001)</td>
<td>(0.39)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>+ 1.146375<em>RCORNPR – 23.05721</em>RWAGE</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(4.28)</td>
<td>(10.46)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>+ 0.00209<em>CATTLE + 6.87138</em>DUM73 + 19.12261*FPI</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.001)</td>
<td>(1.89)</td>
<td>(18.21)</td>
</tr>
<tr>
<td>rho = 0.35398</td>
<td></td>
<td>t-statistic</td>
<td>1.93</td>
</tr>
<tr>
<td>R-square = 0.85</td>
<td></td>
<td>R-square</td>
<td>0.85</td>
</tr>
</tbody>
</table>

Transformations: \[ \begin{align*} 
RALFPR &= \frac{ALFPR}{WPI} \quad \text{ROTHHYPR} = \frac{OTHHYPR}{WPI} \\
RCORNPR &= \frac{CORNPR}{WPR} \quad \text{RWAGE} = \frac{WAGE}{WPI} 
\end{align*} \]

Mean values of variables: RALFPR = 37.169; ALFPROD = 20299.462; ROTHHYPR = 23.965; RCORNPR = 1.256; RWAGE = 0.943; CATTLE = 19371.654; DUM73 = 0.231; and FPI = 0.963.

Notes: Figures in parentheses for the GLS regression are estimated standard errors. The R-square statistic is viewed only as a measure of goodness of fit (Kmenta, p. 234) and is calculated as \[ 1 - \frac{\text{SSR}}{\text{SST}} \], where SSR = sum of squares of residuals and SST = total sum of squares.

Data Sources: Alfalfa Hay, California Market Summary; Agricultural Statistics; Survey of Current Business; Changes in Farm Population and Efficiency, 1978; Economic Indicators of the Farm Sector: Products and Efficiency Statistics, 1979.

The quantity of final products will shift the demand for alfalfa hay in the same direction. The dummy variable is an intercept shifter to account for exogenous effects on the alfalfa hay market not captured by explanatory variables. Factors such as the 1973 oil embargo, that consequently led to increases in energy costs in later years, and increased grain trade with Russia, manifest structural changes in the agricultural sector. A positive intercept shift is expected. Since time series data are used, the effects of shifts in the production function due to technological change should be considered. The farm productivity index is a proxy for technological change with a positive expected sign indicating that technological change in the cattle industry (e.g., artificial insemination, cross breeding, refrigeration and packaging, among others) should increase the demand for alfalfa hay.

Time Series Model

The ARIMA model in its general form can be written as:

\[ (1 - B^1 - B^2 \ldots - B^p)(1 - B)^dZ_t = (1 - B^1 - B^2 \ldots - B^q)a_t \]

where B is the backward shift operator, \((1 - B)\) is the differencing operator such that \((1 - B)Z_t = Z_t - Z_{t-1}\) and \(d\) is the number of differencing. The term \(Z_t\) is the value of the series at time \(t\), \(i\) is the autoregressive parameter for \(i = 1, 2, \ldots, p\), \(j\) is the moving average parameter for \(j = 1, 2, \ldots, q\) and \(a_t\) is a white noise term. Box and Jenkins describe a model building process that involves the steps of identification, estimation, and forecasting. Forecasts from the ARIMA model are based on these estimated parameters.
on past observations of the data in question. Thus, the forecasts are not based on a theoretical structure as defined in the econometric model.

The ARIMA model used in this study is:

$$(1 + .153B^4)(1 - B)Z_t = (1 + .130B)(1 - .850B^4)a_t.$$  

Figures in parentheses represent standard errors. The Box and Pierce Q statistic is 21.28 with 20 degrees of freedom. This ARIMA model with seasonal autoregressive and moving average parameters was estimated from quarterly data for the time period 1953-78.

Results

The annual derived demand equation for alfalfa hay was initially estimated using ordinary least squares regression for the time period 1953-78. These statistical results showed explanatory variables to be consistent with previously hypothesized coefficient signs. However, the Durbin-Watson statistic (D.W.) and residual plot of the regression equation revealed potential serial correlation. A Cochrane-Orcutt generalized least squares (GLS) corrective procedure was used which showed a statistically significant rho value (Table 1).  

The calculated R-square statistic implies that the equation explains 85 percent of the variation in the dependent variable. The sign for deflated corn price is positive but statistically insignificant. This perhaps reflects the reasoning presented earlier that whether or not corn is a substitute or complement for alfalfa hay depends on their absolute price levels.

Quarterly price forecasts from the ARIMA model and the combined GLS econometric-ARIMA model are shown in

Table 2. Price forecasts for the combined model were derived by first calculating an ARIMA annual price for a specific year, as a simple average of the four quarterly prices. Quarterly weights were then calculated by dividing the quarterly ARIMA prices, which were used to derive the average annual price, by the calculated average annual price. The GLS annual forecasts were then multiplied by these weights to derive the quarterly price forecasts of the combined model. In 1979, the weights were 1.0109, 1.0164, 0.9675 and 1.0051 for the four quarters. The quarterly weights for 1980 were 1.0088, 1.0090, 0.9786 and 1.0035.

This technique of combining price forecasts would not be reasonable if the ARIMA forecasts more closely predicted the actual prices than the proposed method.

The ARIMA model forecasts are consistently lower than actual prices because of the large alfalfa price increases occurring during the mid-1970s. Adding more current data will tend to alleviate this underforecasting.
TABLE 3. Revised Quarterly Price Forecasts for Combined Econometric and ARIMA Model.

<table>
<thead>
<tr>
<th>Quarterly Price Forecasts</th>
<th>Combined Econometric (GLS) and ARIMA Model</th>
<th>Actual</th>
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</thead>
<tbody>
<tr>
<td>1979</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1st quarter</td>
<td>93.80</td>
<td>82.95</td>
</tr>
<tr>
<td>2nd quarter</td>
<td>98.19</td>
<td>100.16</td>
</tr>
<tr>
<td>3rd quarter</td>
<td>94.80</td>
<td>99.43</td>
</tr>
<tr>
<td>4th quarter</td>
<td>102.73</td>
<td>105.33</td>
</tr>
<tr>
<td>1980</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1st quarter</td>
<td>98.50</td>
<td>122.66</td>
</tr>
<tr>
<td>2nd quarter</td>
<td>118.34</td>
<td>128.33</td>
</tr>
<tr>
<td>3rd quarter</td>
<td>119.71</td>
<td>121.50</td>
</tr>
<tr>
<td>4th quarter</td>
<td>125.19</td>
<td>123.75</td>
</tr>
<tr>
<td>Root Mean Square Error</td>
<td>10.24</td>
<td></td>
</tr>
<tr>
<td>Theil Inequality Coefficient</td>
<td>0.09</td>
<td></td>
</tr>
</tbody>
</table>

To test forecasting accuracy, root mean square errors and Theil inequality coefficients ($U_2$) were used and are presented in Table 2. These results indicate that the combined model produces more accurate quarterly price forecasts for 1979 and 1980.

It should be noted that this weighting scheme is only one of several alternatives. Another method would be to use the error information of the quarterly ARIMA forecasts to adjust the quarterly weights. For example, the 1978 ARIMA forecast errors can be used to adjust the 1979 ARIMA quarterly forecasts and the corresponding derived 1979 quarterly weights. In this case, the revised quarterly weights are 0.9632, 1.0083, 0.9735 and 1.0549, respectively. Similarly, the 1979 ARIMA forecast errors are used to adjust the 1980 ARIMA quarterly forecasts. The revised weights for 1980 are 0.8533, 1.0255, 1.0370 and 1.0845. Multiplying the GLS annual price forecasts for 1979 and 1980 by these weights yields an alternative set of quarterly price forecasts shown in Table 3. Comparison of root mean square error and Theil inequality coefficient evaluation statistics (with forecasts shown in Table 2) indicates that this alternative weighting scheme does not improve overall forecasting accuracy.

Forecasting and Prediction

In order to forecast alfalfa hay prices, our technique requires estimates of explanatory variables for the annual econometric model. This is not a stringent assumption because of available forecasts from private forecasting firms (e.g., Wharton, Data Resources Incorporated, and Chase Econometrics) and the U.S. Department of Agriculture. However, it is recognized that these forecasts of contemporaneous, exogenous variables involve error which should be incorporated in the total estimated error of forecast.

A technique incorporating errors of forecast for explanatory variables into the model is outlined by Pindyck and Rubinfeld. For the case where values of explanatory variables are not known with certainty, conditional forecasts are derived by estimating confidence intervals for forecasts of each explanatory variable. Next, this information is utilized to estimate the variance of the error of forecast for the dependent variable.

Alternative econometric model specification also could be used, e.g., models with lagged explanatory variables or simultaneous equations models. Considering the work done by Brandt and Bessler, which shows that forecasts taken solely from individual models are not likely to provide users with the most accurate forecasts, and given that very little work has been done on alfalfa hay modeling, there was no a priori reason to suggest that one forecasting model would perform better than another.

Summary

Often in commodity modeling, data availability limits the time framework of
Combining Forecasts

analysis (e.g., monthly, quarterly, annually, etc.). The suggestions outlined in this paper utilize an annual econometric model and a quarterly ARIMA model to obtain a quarterly forecast. This combination technique is relatively easy to apply and enables the use of the annual econometric model with its statistical and economic appeal. The results for alfalfa hay price forecasts indicate that this technique has potential use in modeling other agricultural commodities.

Additionally, the ARIMA procedure can be used to update forecasts as new information becomes available. Nelson refers to this method as adaptive forecasting. In the case of quarterly price forecasts, when the first quarter price is known, the other three quarter forecasts can be updated using the error information of the first quarter.

Depending on the structure of the commodity being analyzed and the availability of forecasts for exogenous variables, the single equation econometric model can be replaced by a system of simultaneous equations. For instance, the assumption of predetermined output levels in the short run or a "small" international trade component may not be applicable for some commodities, suggesting a system of equations as more appropriate. Also, this ad hoc technique can be revised if market structure of the commodity changes.

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